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Effect of spatial correlation structure and transformation model on the design of shallow strip footing

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Directional variability of spatial correlation is observed in natural soils due to their depositional characteristics and it influences the response of structures founded on these deposits. Nonetheless, the results presented in most of the available literature are based on the assumption of either isotropic spatial correlation or perfect spatial correlation of soil properties in horizontal and vertical directions. It is also observed from past studies that the effect of transformation model on the total uncertainty is quite significant. Hence, an effort has been made in this paper to study the effect of anisotropy of autocorrelation characteristics of cone tip resistance (q_c) and the transformation model on the bearing capacity of a shallow strip footing, founding on the surface of a spatially varying soil mass. The statistics in the vertical direction of the soil mass are taken from 8 Cone Penetration Test (CPT) records and statistics in the horizontal direction are assumed. For the case considered, it is observed that the transformation model significantly influences the degree of variability of design parameter. The results also show that isotropic correlation structure based on the vertical autocorrelation distance underestimates the variability of design parameter. On the other hand, perfect correlation in horizontal or vertical, or both directions, overestimates the variability of design parameters, and produces conservative estimates of allowable bearing capacity.

Keywords: soil variability; spatial correlation; transformation model; bearing capacity; reliability

Notations

$\overline{\phi}_{TC}$	Effective angle of internal friction
$\bar{\sigma}_{vo}$	Effective overburden pressure
B	Width of footing
C	Capacity
CoV_{eqc}	Coefficient of variation of measurement uncertainty of cone tip resistance
CoV_{wqc}	Coefficient of variation of inherent variability of cone tip resistance
CoV_{ϕ}	Coefficient of variation of angle of internal friction
D	Demand
e	Measurement uncertainty component
$E(\cdot)$	Error function
G	Performance function
L_h	Spatial averaging distance in horizontal direction
L_v	Spatial averaging distance in vertical direction
N_q	Bearing capacity factor for surcharge
N_{γ}	Bearing capacity factor for unit weight of soil
p_a	Atmospheric pressure
p_f	Probability of failure
q_c	Cone tip resistance
q_{ult}	Ultimate bearing pressure
T	Spatial averaging length
u	Soil property
u_T	Spatial average soil property
w	Inherent variability component
Γ_h	Standard deviation reduction factor for data in horizontal direction
$\Gamma_u(\cdot)$	Standard deviation reduction factor for soil property 'u'
Γ_v	Standard deviation reduction factor for data in vertical direction
Γ_A	Equivalent standard deviation reduction factor for data in 2D space
δ	Autocorrelation distance
δ_h	Autocorrelation distance in horizontal direction
δ_v	Autocorrelation distance in vertical direction
ε	Transformation uncertainty component
ϕ	Angle of internal friction
γ	Unit weight of soil
σ_i	Standard deviation of point property
σ_T	Standard deviation of spatial average property

Introduction

Directional variant autocorrelation structure is natural for soil properties due to the complex geological mechanisms involved

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in the formation and deposition of soil mass. This type of anisotropic structure of spatial variability is due to soil layering in sedimentary deposits (Vanmarcke 1983, Mostyn and Soo 1992, Fenton and Griffiths 2002, Nobahar and Popescu 2002). The above studies also indicated that spatial correlation of soil properties plays a prominent role in reduction of variance of data. For example, Fenton and Griffiths (2002) observed that higher autocorrelation distance of soil properties contributes to larger variability of settlement, and on the other hand, assumption of perfect correlation of soil property produces conservative estimates of probability of failure.

The *in-situ* and laboratory soil test data are often used to estimate the design parameters in the foundation design, through analytical, empirical or semi-empirical transformation models. The variability of a design parameter is estimated from the variability of transformation model adopted, and inherent and measurement variability of soil parameters involved in the transformation model. Phoon and Kulhawy (1999) evaluated the variability of various geotechnical design parameters and showed that the transformation model plays a major role in influencing the variability of design parameters.

Evaluation of variance reduction factor

The variability of soil property, u, from point to point is measured by standard deviation σi, and the standard deviation of the spatial average property u_T is measured by σ_T. The standard deviation of the spatially averaged property is inversely proportional to the size of the averaging length or volume, and the standard deviation reduction factor due to spatial averaging process is defined as:

$$\Gamma_u(T) = \frac{\sigma_T}{\sigma_i} \tag{1}$$

Vanmarcke (1983) presented simple relationships for variance reduction in terms of autocorrelation distance and averaging distance using theoretical triangular, exponential, and squared exponential functions. For example, the variance reduction function using squared exponential fit is:

$$\Gamma^2(T) = \left(\frac{\delta}{T}\right)^2 \left(\sqrt{\pi}\frac{T}{\delta}E\left(\frac{T}{\delta}\right) + \exp\left(-\left(\frac{T}{\delta}\right)^2\right) - 1\right) \tag{2}$$

where δ is the autocorrelation distance, or simply correlation distance and T is the averaging length over which the geotechnical properties are averaged over a failure surface. The correlation distance provides an indication of the distance within which the property values show relatively strong correlation. Phoon and Kulhawy (1999a) reported that typical values of vertical and horizontal correlation distances for cone penetration resistance (q_c) are in the range of 0.1–2.2 and 3–80 m, respectively. Cherubini (2000) suggested that the averaging length could be approximately taken as the length of zone of influence or the failure zone in the analysis. Cherubini (2000) and Sivakumar Babu *et al*. (2006) show that spatial averaging length (T) influences the results of reliability analysis of foundations.

In general, the following correlation structures have been used in literature to characterize the spatial variability of soil properties.

I. Perfect correlation in both horizontal and vertical directions (D'Andrea and Sangrey 1982, Chowdhury 1987 and many others).
II. Isotropic correlation structure in horizontal and vertical directions (Griffiths and Fenton 1997, Fenton and Griffiths 2002, 2003, 2005).
III. Anisotropic correlation structure in horizontal and vertical directions (Mostyn and Soo 1992, Nobahar and Popescu 2002, Popescu *et al*. 2002).

Vanmarcke (1983) also proposed an approximate and simplified variance reduction factor for the data in 2D space as the product of individual variance reduction factors in vertical and horizontal directions given by

$$\Gamma_A^2 = \Gamma_v^2 \times \Gamma_h^2 \tag{3}$$

Scope and methodology

The scope of the work reported in this paper is to analyse the influence of spatial variability of cone tip resistance on the stability of a shallow foundation. In this paper, parametric studies are conducted for the evaluation of variance reduction factor using simplified variance reduction function given by Equation 3. The effect of different correlation structures on the probability of failure of a hypothetical shallow strip footing resting on Texas A & M Riverside sand site of National Geotechnical Experimentation Sites (NGES) is studied. Cone tip resistance data of the site are used in the present analysis.

USACE (1992) suggested that for a vertically loaded strip foundation (of width B) placed on the horizontal surface of a cohesionless soils of loose to medium density, the shear failure envelope extends to 2B from base of the footing in the vertical direction, and 3.5B from centre of the footing in the horizontal direction. The present study uses a footing of width 1 m, and hence, spatial averaging distances in vertical and horizontal directions (L_v and L_h) are 2 and 7 m, respectively. For analysis purpose, eight cone tip resistance profiles are considered within the horizontal zone of influence. These cone tip resistance data were obtained at regular vertical intervals of 2 cm with an electric cone penetration test. Since the footing is placed on the surface, the data up to 2 m below the ground surface (zone of influence in the vertical direction) are used in the analysis. It is observed that a theoretical squared exponential function (Equation 2) best fits the vertical

Table 1. Statistical parameters of cone tip resistance (q_c)

Sounding	Mean (kPa)	Standard deviation (kPa)	Coefficient of variation of inherent variability (CoV_w)	Vertical autocorrelation Distance, δ_v (m)
CPT21	7677	3418	45%	0.20
CPT23	6891	6188	90%	0.18
CPT24	7789	2066	27%	0.14
CPT25	5535	2619	47%	0.21
CPT26	7826	1544	20%	0.13
CPT27	5390	1567	29%	0.20
CPT28	2011	778	39%	0.19
CPT29	4569	1672	37%	0.23
Average values	**5961**	**2482**	**42%**	**0.19**

autocorrelation functions for all eight cone tip resistance data sets. The details of probabilistic characterization of the above cone tip resistance profiles are presented in detail in Dasaka (2006). The averaged mean, standard deviation, and vertical autocorrelation distances for the above cone tip resistance profiles are evaluated and presented in Table 1. Due to insufficient data to estimate the horizontal correlation distance of cone tip resistance for this site, values of 3 to 80 m, the observed lower and upper bound values of horizontal autocorrelation distance of cone tip resistance reported by Phoon and Kulhawy (1999a) are used.

Evaluation of variance reduction factor

Assumption of perfect correlation of cone tip resistance

This case implicitly assumes that the cone tip resistance is perfectly (or infinitely) correlated in space, and the effect of spatial correlation on the reduction of point variance is ignored. The resulting variance reduction factor is obtained as unity:

$$\Gamma_A^2 = \Gamma_v^2 \times \Gamma_h^2 = 1 \times 1 = 1 \tag{3a}$$

Assumption of isotropic correlation structure of cone tip resistance

In most of the studies, variations of soil property with depth are obtained from which auto correlation distance is obtained. Due to lack of closely spaced data in horizontal direction, horizontal correlation distance has been taken equal to the estimated vertical autocorrelation distance, implicitly considering an isotropic correlation structure for cone tip resistance. In general, it is observed that the horizontal auto correlation distance is more than the vertical auto correlation distance. Hence, if the above consideration is used, the correlation distance assigned to the cone tip resistance in the horizontal direction is less than its likely value.

Hence, with reference to the data presented in Table 1, the vertical and horizontal autocorrelation distances of cone tip resistance (δ_v and δ_h) are taken as 0.19 m, L_v/δ_v is obtained as 10.53, and L_h/δ_h as 36.85. The variance reduction factors in vertical and horizontal directions estimated using Equation 2 are:

$$\Gamma_v^2 = \left(\frac{0.19}{2}\right)^2 \left(\sqrt{\pi}\frac{2}{0.19}E\left(\frac{2}{0.19}\right) + \exp\left(-\left(\frac{2}{0.19}\right)^2\right) - 1\right) = 0.159 \tag{3b}$$

$$\Gamma_h^2 = \left(\frac{0.19}{7}\right)^2 \left(\sqrt{\pi}\frac{7}{0.19}E\left(\frac{7}{0.19}\right) + \exp\left(-\left(\frac{7}{0.19}\right)^2\right) - 1\right) = 0.047 \tag{3c}$$

Using Equation 3, the equivalent variance reduction factor for the data in 2D space is computed as:

$$\Gamma_A^2 = \Gamma_v^2 \times \Gamma_h^2 = 0.159 \times 0.047 = 0.0075 \tag{3d}$$

Assumption of anisotropic correlation structure of cone tip resistance

The resulting variance reduction factors for different horizontal autocorrelation distances and estimated vertical autocorrelation distance for the q_c data are presented in Table 2. It can be noted that variance reduction factors for correlation lengths corresponding to $\delta_h/\delta_v > 1$ are higher than the values corresponding to isotropic correlation structure ($\delta_h = \delta_v$).

Table 2. Variance reduction factors in 2-D space for cone tip resistance with $L_v = 2$ m & $L_h = 7$ m

Autocorrelation distance of cone tip resistance in vertical direction, δ_v	$\delta_h = 0.19$ m	$\delta_h = 3$ m	$\delta_h = 80$ m	$\delta_h = \infty$
0.19 m	0.007	0.092	0.159	0.159

Assumption of perfect correlation of cone tip resistance in horizontal direction

When the information concerning horizontal correlation structure is not available, the cone tip resistance is sometimes assumed as perfectly (or infinitely) correlated in the horizontal direction. Using Equation 2 or 3, variance reduction factor is obtained as unity when infinite correlation length is used. For vertical correlation distance of 0.19 m, the variance reduction factor for the data in 2D space, using the assumption of perfect correlation in the horizontal direction is obtained as:

$$\Gamma_A^2 = \Gamma_v^2 \times \Gamma_h^2 = 0.159 \times 1 = 0.159 \tag{3e}$$

Hence, assumption of perfect correlation in horizontal direction results in lower variance reduction factor than those obtained using observed finite scale correlation lengths in horizontal direction. From the results shown in Table 2 it can be noted that when δ_v equals 0.19 m, the resulting variance reduction factor increases from 0.0075 to 0.159, as δ_h increases from 0.19 m to ∞.

Reliability analysis of bearing capacity

Reliability analysis is carried out to assess the influence of spatial correlation structure on the allowable bearing pressure of a strip footing using vertical cone tip resistance (q_c) data presented in Figure 1. The mean, standard deviation, coefficient of variation, and vertical autocorrelation distance of all the eight 'q_c' profiles are given in Table 1. The above parameters are averaged for all the profiles, and shown in the last row of the same table. These averaged values represent the mean properties over the zone of influence. To study effect of correlation distance on bearing capacity, horizontal correlation distances (0.19, 3, 80 m and infinity) of cone tip resistance data are used. The first value refers to vertical correlation distance of cone tip resistance, second and third values are the observed lower and upper bound correlation distances reported by Phoon and Kulhawy (1999a), and the last value corresponds to perfect correlation of cone tip resistance.

The bearing capacity of the strip footing is estimated from the *in-situ* cone tip resistance data, as follows. The *in-situ* cone tip resistance data within the significant zone of influence is used to evaluate the equivalent mean effective angle of internal friction of soil using a correlation model (Equation 4) developed by Kulhawy and Mayne (1990).

$$\overline{\phi}_{TC} = 17.6 + 11.0 \ \log_{10}\left(\frac{q_c/p_a}{\sqrt{\bar{\sigma}_{vo}/p_a}}\right) \tag{4}$$

The above equation was developed from 20 data sets, each obtained from laboratory calibration chamber tests on reconstituted sand, using triaxial compression effective stress angle of internal friction $\left(\overline{\phi}_{TC}\right)$ and the normalized cone tip resistance $\left(\frac{q_c/p_a}{\sqrt{\bar{\sigma}_{vo}/p_a}}\right)$. Parameters q_c, p_a, and $\bar{\sigma}_{vo}$ are cone tip resistance, atmospheric pressure, and effective overburden pressure, respectively. The regression coefficient and standard deviation of the above transformation model were reported as 0.64 and 2.8°, respectively (Kulhawy and Mayne 1990). The ultimate bearing pressure is evaluated using the following equations proposed by Meyerhof (1963).

$$q_{ult} = 0.5\gamma B N_\gamma \tag{5}$$

where γ, B, are the unit weight of soil and width of foundation, respectively. N_γ is bearing capacity factor for self weight of soil, which is approximated as:

$$N_\gamma = \left(N_q - 1\right)\tan(1.4\phi) \tag{6}$$

$$N_q = \exp(\pi \tan\phi)\tan^2\left(45 + \frac{\phi}{2}\right) \tag{7}$$

N_q and ϕ are bearing capacity factor for surcharge and the angle of internal friction, respectively. The equivalent mean effective angle of internal friction estimated using Equation 4 within the zone of influence is 41.2°. The unit weight of soil is taken as 18 kN/m^3. The ultimate bearing pressure computed using Equations 5, 6, and 7 is 1057 kPa and using a factor of safety of 3, the allowable bearing pressure is estimated as 352 kPa.

In the following section, probability based analysis is used to evaluate effect of spatial correlation on reliability of allowable bearing capacity. Four cases are analyzed to estimate the coefficient of variation of angle of internal friction using the individual sources of uncertainty, viz., inherent variability and measurement uncertainty of cone tip resistance, and uncertainty associated with transformation model given by Equation 4. The four cases are: (i) only inherent variability of cone tip resistance is considered in the analysis, (ii) inherent variability and measurement uncertainty of cone tip resistance are only considered, (iii) inherent variability and the uncertainty associated with transformation model are used, and (iv) when all the three sources of uncertainty are considered for the estimation of coefficient of variation of angle of internal friction.

The standard deviation of transformation model given by Equation 4 is reported as 2.8°. The average coefficient of variation of measurement uncertainty of q_c (CoV_{eqc}) for electric cone penetrometer, which was used at the Texas A&M Riverside sand site, is taken as 8% in accordance with Phoon

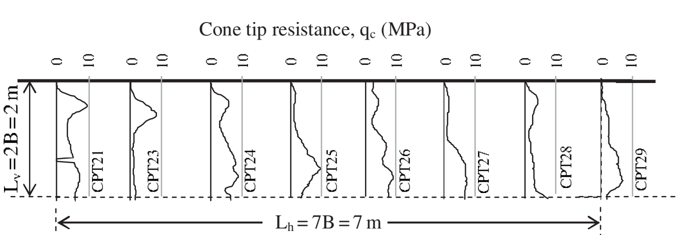

Figure 1. Cone tip resistance (q_c) profiles considered in the analysis.

Table 3. Coefficients of variation of angle of internal friction for the data within the zone of influence (autocorrelation distance in vertical direction, δ_v=0.19 m)

Combinations of different sources of uncertainty	Coefficient of variation of angle of internal friction (CoV$_\phi$ %): For autocorrelation distance of q_c in horizontal direction, δ_h: 0.19 m	3 m	80 m	∞	Perfect correlation in both directions
w	0.4	1.5	1.9	1.9	4.9
w+e	1	1.7	2.2	2.2	5
w+ε	6.8	7	7.1	7.1	8.4
w+e+ε	6.9	7	7.1	7.1	8.4

and Kulhawy (1999a). The mean and standard deviation of angle of internal friction are evaluated using Taylor series approach and followed by applying second-moment probabilistic technique (Phoon and Kulhawy 1999b). The variance of point friction angle within the zone of influence is reduced, due to spatial averaging, using approximate variance reduction function. These estimated moments of angle of internal friction are then used for the evaluation of probability of failure of allowable bearing pressure.

Table 3 presents the values of coefficients of variation of angle of internal friction (CoV$_\phi$) estimated using different values of horizontal and vertical autocorrelation distances of cone tip resistance. The terms w, e, and ε in Table 3 refer to inherent variability, measurement uncertainty, and transformation model uncertainty components. The following observations are made from the results.

- CoV$_\phi$ increases with increase of horizontal autocorrelation distance for all combinations of uncertainties. Higher CoV$_\phi$ is obtained for the case of perfect correlation of cone tip resistance.
- When only the effect of inherent variability of cone tip resistance is considered, the coefficient of variation of angle of internal friction varies from 0.4 to 1.9%.
- When the measurement uncertainty of cone tip resistance is also considered, the coefficient of variation of angle of internal friction varies from 1 to 2.2%. The above results show that the measurement uncertainty has no significant influence on the variability of angle of internal friction.
- However, when transformation model uncertainty is considered along with the inherent variability of cone tip resistance, moderate increase in the coefficient of variation of angle of internal friction is noted. This increase of CoV$_\phi$ is more than the corresponding increase when measurement uncertainty is considered along with inherent variability.

Though the inherent variability of q_c is 42%, the estimated CoV$_\phi$ does not exceed 8.4% even in case of perfectly correlated assumption for cone tip resistance. Thus the transformation model used can be identified as the crucial factor influencing the degree of estimated uncertainty in the angle of internal friction.

The first two moments of angle of internal friction obtained above are used in the evaluation of reliability of allowable bearing pressure of strip footing. The following limit state function is used in the reliability analysis.

$$G = C - D = q_{ult}(\gamma, B, \phi) - D \quad (8)$$

where C and D are capacity and demand. $q_{ult}(\gamma, B, \phi)$ is a random variable representing the ultimate bearing pressure of a strip footing, and demand, D, is the allowable bearing pressure obtained using a factor of safety of 3 on ultimate bearing pressure. The mean and variance of ultimate bearing pressure (capacity, C, in Equation 8) are estimated from the mean and variance of angle of internal friction using Monte Carlo Simulation procedure. A total of 10,000 random numbers are generated for friction angle and using the mean and variance of friction angle estimated in the above section. Lognormal distribution is chosen to represent its variation as it is commonly used to model soil properties (Przewłócki 2000). A set of 10,000 ultimate bearing pressures (q_{ult}) is computed corresponding to all the generated values of friction angle using Equations 5, 6, and 7. The probability of failure (p_f) of a shallow strip footing in ultimate limit state is estimated as:

$$p_f = \frac{\sum_{i=1}^{10,000} q_{ulti} < D}{10,000} = \frac{\sum_{i=1}^{10,000} q_{ulti} < 352 \text{ kPa}}{10,000} \quad (9)$$

Probability of failure of 0.0135, which corresponds to a reliability index of 3, which is generally accepted for foundation design (Cherubini 2000) is taken as reference for further discussion.

Table 4 indicates the variation of probability of failure estimated using Equation 9 with different values of horizontal autocorrelation distance. It may be observed that the assumption of isotropic correlation structure of q_c ($\delta_h = \delta_v = 0.19$ m) results in lower probabilities of failure than those obtained for cases with $\delta_v/\delta_h < 1$. When the analysis is based on the coefficients of variation obtained from all the three uncertainties (inherent, measurement, transformation uncertainties), the

Table 4. Probability of failure of allowable pressure in ultimate limit state (autocorrelation distance in vertical direction, δ_v=0.19 m)

Combinations of different sources of uncertainty	Autocorrelation distance of q_c in horizontal direction, δ_h				Perfect correlation in both directions
	0.19 m	3 m	80 m	∞	
w	≈0	≈0	≈0	≈0	0.0009
w+e	≈0	≈0	≈0	≈0	0.0012
w+ε	0.0128	0.0154	0.0156	0.0156	0.0351
w+e+ε	0.0131	0.0151	0.0152	0.0152	0.035

probability of failure shows increasing trend with horizontal autocorrelation distance. For the assumption of perfect correlation of cone tip resistance in both directions, the results show higher probability of failure.

When the effect of all the sources of uncertainty are considered in the analysis, probabilities of failure ranging from 0.0131 to 0.0152 are obtained for ultimate limit state with horizontal correlation distance varied from 0.19 m to infinity. There is no significant change in probabilities of failure corresponding to these two bounds of horizontal correlation distance (3 and 80 m) of q_c. On the other hand, when perfect correlation of cone tip resistance is assumed in both directions, the observed probability of failure is about 1.5 times the reference probability of failure of 0.0135. Hence, it can be noted that perfect spatial correlation of soil properties results in higher probabilities of failure.

Concluding remarks

This paper presents a few results illustrating the effect of spatial variability on the results of reliability analysis of a shallow foundation using a simple variance function for 2D random field model of Vanmarcke (1983). For the case considered, the results show that:

The assumption of isotropic correlation structure based on the vertical autocorrelation distance underestimates the variability of design parameter in a 2D space, than that obtained using appropriate autocorrelation distance in horizontal direction.

Assumption of perfect correlation both in horizontal or vertical, or both directions, overestimates the variability of design parameters, and produces conservative estimates of bearing capacity.

In general, horizontal autocorrelation is difficult to measure as more sampling points are necessary in that direction. Hence, in the absence of such data, it is recommended to assume acceptable values of correlation distance, rather than the values obtained from analysis of data in vertical direction.

In case of absence of data on autocorrelation distance in either direction for a particular site, it is suggested to use an upper bound value from the range of observed values for similar soils.

When inherent variability of cone tip resistance alone is considered, the variability of angle of internal friction is relatively small and the predicted probability of failure of allowable bearing pressure is underestimated compared to those obtained when all the uncertainties are considered appropriately. Hence, measurement and transformation model uncertainties should be evaluated and used in the analysis along with inherent variability.

Results of parametric studies using various combinations of uncertainties indicate that the transformation model has a significant effect on the design parameter uncertainty. The above remarks are valid for the specific case considered and the numerical values of correlation lengths used in the analysis. Both load distribution and value of friction angle influence the results as these factors influence the extent of failure zone and hence the values of reduced variance that need to be used in the analysis. Results based on stochastic finite elements address this issue to some extent. However, further studies are required in this direction to consider data from field tests, use appropriate transformation models and understand the role of other uncertainties to provide improved understanding of behaviour of geotechnical structures resting on variable deposits.

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