#### ORIGINAL PAPER

# The Effect of Spatial Correlation of Cone Tip Resistance on the Bearing Capacity of Shallow Foundations

G. L. Sivakumar Babu · Satyanarayana Murthy Dasaka

Received: 2 June 2005/Accepted: 20 July 2007/Published online: 28 August 2007 © Springer Science+Business Media B.V. 2007

Abstract The effect of directional behaviour of correlation structure of cone tip resistance on the bearing capacity of shallow strip footing resting on cohesionless soil deposit in 2-D random field is analysed using probabilistic approach. The results obtained from the analysis show that the assumption of perfect (or infinite) correlation of cone tip resistance data leads to lower values of probability of failure. In contrast, the isotropic assumption of correlation behaviour based on vertical scale of fluctuation leads to higher values of probability of failure. The results also show that the transformation model would play a major role in the evaluation of variability of design property. In conclusion, the need for a proper evaluation methodology for calculation of correlation lengths of soil properties and their influence in foundation design is highlighted.

**Keywords** Spatial variability · Cone tip resistance · Spatial correlation · Anisotropy · 2-D random field · Shallow foundations · Bearing capacity · Stationarity · Reliability

Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012, India e-mail: gls@civil.iisc.ernet.in

S. M. Dasaka

#### 1 Introduction

The soil properties exhibit large variations and their directional behaviour has been observed by many researchers (Vanmarcke 1983; Jaksa et al. 1999, 2004; Phoon and Kulhawy 1999; Griffiths and Fenton 2000; Nobahar and Popescu 2002; Fenton and Griffiths 2003: Sivakumar Babu and Mukesh 2004: Uzielli et al. 2005; and Wei et al. 2005). Owing to the nature of soil formation and depositional processes, the vertical and horizontal correlation structures of soil properties are generally anisotropic, with greater variability in the vertical direction (Uzielli et al. 2005). The effect of anisotropy of soil properties on the bearing capacity in a probabilistic framework has not been studied extensively in the literature. In general, due to economic feasibility, speed of exploration, unavailability of equipment, and time constraints, vertical cone penetration data alone is obtained and used in the evaluation of strength properties (Wei et al. 2005). In this study the effect of anisotropy on the allowable bearing pressure of shallow strip foundation resting on cohesionless soil deposit is evaluated using cone tip resistance data.

This paper addresses two important issues. Firstly, analysis has been done for the evaluation of simplified variance reduction functions in 2-D space using the corresponding functions developed for a 1-D field. Secondly, the effect of scale of fluctuation on the reliability of foundations has been studied.

G. L. Sivakumar Babu (🖂)

Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400 076, India e-mail: dasaka@iitb.ac.in

The contribution from all the three phases of uncertainty (inherent variability, measurement uncertainty, and transformation uncertainty) has been recognised and considered appropriately in the analysis.

The effect of assumptions of isotropy or perfect spatial correlation of cone tip resistance on the performance of shallow strip foundation in shear criterion is analysed through the following five cases.

- 1. Effect of assuming isotropic correlation structure based on vertical correlation length
- 2. Effect of assuming isotropic correlation structure based on horizontal correlation length
- 3. Effect of assuming perfect correlation in horizontal direction
- 4. Effect of assuming perfect correlation in vertical direction
- 5. Effect of assuming perfect correlation in both directions

Equation 1 shows the variance reduction function suggested by Vanmarcke (1977) for theoretical triangular function fitted to experimental or detrended autocorrelation data, where d and L are respectively the scale of fluctuation and the spatial averaging distance of the soil property under consideration.

Vanmarcke (1977) also proposed an approximate and simplified resultant variance reduction factor in 2-D space as the product of individual variance reduction factors in vertical and horizontal directions in terms of scale of fluctuation and spatial averaging distance in the respective directions as shown in Eq. 2.

$$\begin{split} \Gamma_{\rm L}^2 &= \left[ \frac{\rm d}{\rm L} \left( 1 - \frac{\rm d}{\rm 3L} \right) \right] \quad \text{for } {\rm L}/{\rm d} \geq 1 \\ \Gamma_{\rm L}^2 &= 1 - \frac{\rm L}{\rm 3d} \quad \text{for } {\rm L}/{\rm d} \leq 1 \end{split} \tag{1}$$

$$\Gamma_{\rm A}^2 = \Gamma_{\rm v}^2 \times \Gamma_{\rm h}^2 \tag{2}$$

From Eq. 1, we can see that the variance reduction factor is a function of L/d ratio. Averaging distance depends on the nature of the problem in hand and for design of shallow foundations for shear criterion, this distance is equal to the extent of shear failure zones within the soil mass (Cherubini 2000). This distance for shallow foundations in cohesionless soil subjected to vertical loading is approximately taken as 2B below the base of footing in the vertical direction and 2.5B from the centre of footing in the horizontal

direction, where B is the width of the footing. However, the scale of fluctuation of cone tip resistance varies from site to site. Moreover, it also varies with type of soil, as Jaksa et al. (2004) reports smaller scales of fluctuation in sands than clays due to their nature of formation. Further, Fenton and Vanmarcke (1998) argue that the scale of fluctuation depends largely on the geological processes of transport of raw materials, layer deposition, and common weathering rather than on the actual property studied. In this paper a parametric study has been conducted and results on the effect of spatial correlation of the bearing capacity are presented.

In this problem, the averaging distance is constant as the analysis is done for a unique value of width of foundation (B). With increased value of scale of fluctuation (d), L/d ratio decreases and the value of variance reduction factor would be increased. The increased variance reduction factor results in increased variance of the data and produces higher probability of failure.

The effect of the above five cases on the probability of failure is illustrated by taking an example of bearing capacity of shallow strip footing of width 1 m resting on the surface of a cohesionless soil deposit. As mentioned in the last section, the failure wedges (zone of influence) are approximately extended to 2B and 5B in vertical and horizontal direction. By equating the averaging distance to the zone of influence, averaging distance in vertical and horizontal directions  $(L_v \mbox{ and } L_h)$  are  $2\mbox{ m}$  and  $5\mbox{ m}$ respectively. The analysis is done using the scales of fluctuation of cone tip resistance in vertical and horizontal directions for cohesionless soil deposits as 0.1-2.2 m and 3-80 m respectively, as reported by Phoon and Kulhawy (1999). Since, the analysis is 2-D, the scale of fluctuation in the out-of-plane direction is taken as infinity.

## 2 Analysis of Variance Reduction Factor in 2-D Space

# 2.1 Case (I): Isotropic Correlation Structure Based on Vertical Scale of Fluctuation

In majority of the cases due to lack of rigorous closely spaced data of the site in horizontal space, an autocorrelation function obtained from the analysis of borehole data in the vertical direction, is assumed to prevail in the horizontal direction too, and hence, the same correlation distance as evaluated in the vertical direction is still used in the horizontal direction. This means that the soil correlation properties are assumed to be isotropic. It is common in conventional geotechnical practice to assume isotropic behaviour of soil properties. Alonso (1976) presented a risk analysis of Green Creek Slide assuming the isotropic correlation behaviour, i.e., same correlation distance in all directions. But in general soil properties do exhibit directional behaviour, i.e., they behave differently in vertical and horizontal directions. The soil properties in the horizontal direction are generally more correlated than in the vertical direction, in other words, the correlation distance for soil properties in horizontal direction is higher than that in the vertical direction.

The horizontal scale of fluctuation is in general higher than the vertical scale of fluctuation, and in the process of isotropic assumption, we normally extend the same vertical scale of fluctuation even in the horizontal direction. In doing so we are analysing for a lesser value of horizontal scale of fluctuation than the actual one, and the smaller scale of fluctuation would imply reduced variance reduction factor. Hence the resultant variance reduction factor will also be reduced and produces lower probability of failure. Hence, the assumption of isotropic behaviour based on vertical scale of fluctuation underestimates the probability of failure.

If isotropic conditions based on vertical scale fluctuation are considered, corresponding to scale of fluctuation of 0.1 m,  $L_v/d_v$  is equal to 20. Similarly L<sub>h</sub>/d<sub>h</sub> corresponding to same scale of fluctuation is 50. Using Eqs. 1 and 2, the resultant variance reduction factor is 0.004. Similarly, the scales of fluctuation of 2.2 m in both directions, give rise to  $L_{\rm h}/d_{\rm h}$  and  $L_{\rm v}/d_{\rm v}$  of 2.27 and 0.91 respectively. These values correspond to a variance reduction factor of 0.402. But, the actual value of scale of fluctuation in horizontal direction is usually higher than that in the vertical direction. If we replace the actual value of scale of fluctuation in the horizontal direction (3–80 m) in the above calculations, the variance reduction factors obtained for  $L_v/d_v$  equals to 20 (=2/0.1 m) are 0.059  $(=0.095 \times 0.616)$  and 0.093  $(=0.095 \times 0.9795)$  respectively for horizontal scales of fluctuation of 3 and 80 m. The corresponding values for  $L_v/d_v$  equals to 0.91 (=2/2.2 m) are 0.465 (=0.755 × 0.616) and 0.740 (=0.755 × 0.9795). These results are shown in Table 1 and Figs. 1 and 2. Fig. 1 and 2 shows decrease of the variance reduction factors with increase of  $L_v/d_v$  ratio for  $d_v$ equals to 0.1 m and 2.2 m respectively. They also show the results obtained for various combinations of averaging distance in horizontal direction ( $L_h$ ) and horizontal scale of fluctuation ( $d_h$ ). From these results, it is understood that isotropic assumption based on vertical scale of fluctuation results in lower variance reduction factor, and hence lower uncertainty in cone tip resistance, and subsequently underestimates the probability of failure.

# 2.2 Case (II): Isotropic Correlation Structure Based on Horizontal Scale of Fluctuation

If the horizontal scale of fluctuation is made available and the same is incorporated even in the vertical direction, it results in higher variance reduction factor, and hence overestimates the uncertainty as well as the subsequent probability of failure. Of course, it produces lower (conservative estimates of) probability of failure. The results are shown in Table 2 and Figs. 3 and 4. These figures correspond to vertical scales of fluctuation of 3 m and 80 m, which are observed extreme ranges for horizontal scale of fluctuation of cone tip resistance.

# 2.3 Case (III): Perfect Correlation of Soil Properties in Horizontal Direction

In some other cases, the correlation distance is assumed to be perfectly (infinitely) correlated in the horizontal direction and analysis is done considering only the vertical correlation distance. The assumption of infinite scale of fluctuation results in variance reduction factor of unity in that particular direction,

Table 1 Variance reduction factors in 2-D space for cone tip resistance for  $L_v = 2$  m and  $L_h = 5$  m

$d_v(m)$	Isotropic $(d_h = d_v)$	$d_h = 3 m$	$d_h = 80 m$	$d_h = \infty$
0.1	0.004	0.059	0.093	0.095
2.2	0.402	0.465	0.740	0.755
$\infty$	1	0.616	0.980	1

**Fig. 1** Variance reduction factors for cone tip resistance in 2-D space for isotropic, anisotropic and infinitely correlated in horizontal direction using dv = 0.1 m





and in this case, using Eq.2 the resultant variance reduction factor in 2-D space equals to that obtained in the vertical direction. It is obvious to note that this resultant variance reduction factor is more than that of actual resultant, which would have been obtained by multiplying the variance reduction factors obtained by using appropriate scales of fluctuation in vertical and horizontal directions. Hence, the assumption of infinite correlation length of soil strength properties in horizontal direction results in higher uncertainty and produces higher probabilities of failure than the actual estimates, and the designs based on this assumption are uneconomical. These results are shown in Figs. 1 and 2 corresponding to  $d_h = \infty$ . From Table 1, it can be seen that for  $d_v$ 

equals to 0.1 m, the variance reduction factor increases from 0.004 to 0.095 with increase in  $d_h$  from 0.1 to  $\infty$ , and in the case of  $d_v$  equals to 2.2 m, these values increase from 0.402 to 0.755 respectively.

# 2.4 Case (IV): Perfect Correlation of Soil Properties in Vertical Direction

In a similar way, the analysis of variability considering the estimated correlation distance in the horizontal direction and assuming infinite correlation of soil properties in vertical direction also produces conservative estimates of probabilities of failure and

Table 2 Variance reduction factors in 2-D space for cone tip resistance for  $L_v = 2$  m and  $L_h = 5$  m

d <sub>h</sub> (m)	Isotropic $(d_h = d_v)$	$d_v = 0.1 m$	$d_v = 2.2 m$	$d_v = \infty$
3	0.606	0.059	0.465	0.616
80	0.989	0.093	0.740	0.990
$\infty$	1	0.095	0.755	1

results in uneconomical designs. Table 2 shows the results obtained for various combinations of vertical and horizontal scales of fluctuation of cone tip resistance. The results presented in second and third rows of Table 2 show the effect of varying degrees of vertical scales of fluctuation on the variance reduction factor for horizontal scale of fluctuation of 3 m

**Fig. 3** Variance reduction factors for cone tip resistance in 2-D space for isotropic, anisotropic and infinitely correlated in vertical direction using dh = 3 m and 80 m respectively. For 3 m horizontal scale of fluctuation, the variance reduction factor increases from 0.059 to 0.616 with increase in vertical scale of fluctuation from 0.1 and  $\infty$ , and corresponding to 80 m horizontal scale of fluctuation these values range from 0.093 to 0.990.

# 2.5 Case (V): Perfect Correlation of Soil Properties in Horizontal and Vertical Directions

In conventional probabilistic analysis the correlation studies are totally ignored and soil properties are considered to be perfectly (infinitely) correlated in



**Fig. 4** Variance reduction factors for cone tip resistance in 2-D space for isotropic, anisotropic and infinitely correlated in vertical direction using dh = 80 m

space. But, in reality, the properties are correlated only to certain extent and generally they vary with direction of measurement. This assumption of infinite correlation of soil strength properties would results in higher variance reduction factors. It results in higher uncertainty than the actual and produces conservative values of probability of failure, leading to uneconomical designs. Tables 1 and 2 show variance reduction factors of unity corresponding to infinite correlation lengths of cone tip resistance in both directions.

#### 3 Reliability Analysis of Bearing Capacity

The cone tip resistance data in the vertical zone of influence (2B) is analysed for correlation structure. The data are checked for stationarity using Kendall's  $\tau$  test. The Kendall's  $\tau$  value for experimental data is obtained as 0.63. Since, the Kendall's  $\tau$  is significantly higher than zero, it states that the data follows a trend. Hence, it is decided to detrend the data to satisfy the stationarity condition. Jaksa (1999) reinstates that in random field theory, it is common practice to transform a non-stationary data to a stationary one by removing a low-order polynomial trend, usually no higher than a quadratic using the method of Ordinary Least Squares (OLS). Since complete removal of the trend is not possible, only linear as well as quadratic trends have been removed from the data and it is observed that the Kendall's  $\tau$ for the linear and quadratic detrended data are 0.1 and -0.05 respectively. Based on the above obtained  $\tau$ values, a quadratic trend has been selected from the experimental cone tip resistance data and the data are sufficiently trend-free. Autocorrelation function is then evaluated for the detrended data. A triangular function has been chosen to best fit the empirical autocorrelation data based on regression analysis, and autocorrelation distance for the vertical cone tip resistance data obtained from the parameter of the triangular fit is 0.32 m. The averaging distance, which is taken equal to zone of influence, is taken as 2B and 5B in vertical and horizontal directions respectively. The following four values of horizontal scales of fluctuation of cone tip resistance are assumed. They are 0.32 m, 3 m, 80 m, and infinity. The assumption of d<sub>h</sub> of 0.32 m corresponds to an isotropic assumption based on vertical scale of fluctuation obtained from statistical analysis of cone tip resistance. The extreme value of  $d_{\rm h}$  equals to  $\infty$ would imply a perfect (or infinite) correlation in the horizontal direction. The other two values (3 m and 80 m) considered in the analysis are the range of observed of fluctuation in horizontal direction (Phoon and Kulhawy 1999). The variance reduction functions in vertical and horizontal directions are evaluated separately using the respective scales of fluctuation and averaging distance based on Eq. 1. The point and spatial average statistical parameters (viz., mean and standard deviation) of the bearing pressure  $(Q_n)$  are obtained from that of cone tip resistance (q<sub>c</sub>) using the second-moment probabilistic techniques. The procedure is outlined by Phoon and Kulhawy (1999). The mean and standard deviation of point cone tip resistance data within zone of influence in 2-D field are evaluated as 53.45 kPa and 15.72 kPa. In the present analysis, the variability of bearing capacity is assumed to be characterized by a lognormal distribution, since soil properties never attain negative values and lognormal distribution has simple relationship with normal distribution (Fenton and Griffiths 2003).

$$Q_{\rm u} = 28 - 0.0052 \left(300 - q_{\rm c}\right)^{1.5} \tag{3}$$

$$Z = g(X_1, X_2, \dots, X_n) \tag{4}$$

The performance of the foundation against applied pressures can be expressed in terms of the reliability index ( $\beta$ ) evaluated for a limit state function of the form shown in Eq. 4. Where X<sub>1</sub>, X<sub>2</sub>, ...., and X<sub>n</sub> correspond to load and resistance parameters forming the transformation equation. Figure 5 shows the variation of reliability index with assumed horizontal scale of fluctuation.

Total three cases are analysed in this study. In the first case, the analysis is done taking into account only the effect of inherent variability of cone tip resistance. But, in the remaining two cases the effects of measurement uncertainty of cone tip resistance and transformation uncertainty of Eq. 3 have also been considered appropriately. The coefficient of measurement uncertainty of  $q_c$  (CoV<sub>eqc</sub>) is taken in the range of 5% and 15% (Phoon and Kulhawy 1999). Since, the data lead to the formulation of the Eq. 3 is not available to the authors, the transformation uncertainty of this equation is assumed at 15% and 25% of standard deviation of bearing capacity (SD<sub>equ</sub>). In the





43

analysis for reliability index, the variability in the load has been neglected. In the case when only the effect of inherent variability is considered, the reliability index corresponding to one third of the deterministic ultimate bearing pressure varies quite widely from 28 to 7 with increase of horizontal scale of fluctuation from 0.32 m to  $\infty$ . The reliability index of 28 corresponds to the assumption of isotropic correlation behaviour of cone tip resistance based on vertical cone tip resistance. Hence, it can be said that isotropic assumption of q<sub>c</sub> based on vertical scale of fluctuation (d<sub>v</sub>) produces higher reliability indices than that expected and produces unsafe designs. In contrast, the designs based on the assumption of infinite correlation lengths in horizontal direction produce lesser reliability indices, which are uneconomical. This necessitates proper evaluation of horizontal correlation structure of cone tip resistance by exploring the soil not only in the vertical direction but also in the horizontal direction. However, the remaining two cases do not show significant variations of reliability index with horizontal scales of fluctuation, even though the third case (CoVeqc = 15% and SDr = 25% SDqu) produces little lower reliability indices than that produced in the second case (CoVeqc = 5% and SDr = 15% SDqu). In case when the analysis is based on all the three uncertainties (inherent, measurement, transformation uncertainties) the reliability index is not sensitive to horizontal scale of fluctuation. Hence, it is understood that the transformation model has been identified as the primary factor influencing the degree of variability of design parameter.

Similarly, Fig. 6 shows the variation of reliability index with vertical scale of fluctuation. Due to lack of horizontal CPT data, the scale of fluctuation of tip resistance in the horizontal direction is assumed equal to 10 m. This value is well within an acceptable range of 3-80 m (Phoon and Kulhawy 1999). In case when only inherent variability is used, the reliability index reduces from 13.6 to 3 for increase in vertical scales of fluctuation from 0.1 to  $\infty$ . Isotropic assumption of qc based on horizontal scale of fluctuation ( $d_v = d_h = 10$  m) produces lower reliability indices than that expected, hence, leads to uneconomical designs. Similar to the isotropic assumption based on horizontal scale of fluctuation of  $q_c$ , an infinite correlation of  $q_c$  in the vertical direction produces lesser reliability indices and leads to uneconomical designs.

Figure 7 shows the variation of variance reduction factor in 2-D random field with the ratio of  $d_h/d_v$  for different d<sub>v</sub> values ranging from 0.1 m to 50 m. The variance reduction factor increases with increase in  $d_{\rm h}/d_{\rm v}$  ratio. Except for the case with  $d_{\rm v} = 0.1$  m, the variance reduction factor increases dramatically in all the other cases. It is seen from the previous studies that the changes in the soil properties in vertical direction are quite obvious even within a few meters of soil data. These small scale variations of soil properties in the vertical direction may be attributed to the soil depositional processes. Hence, the scales of fluctuation in the vertical direction are in general less than the corresponding values in the horizontal direction, which results in  $d_{\rm h}/d_{\rm v}$  values greater than unity. In a condition, where the chances of getting





information on horizontal scale of fluctuation are remote, either due to economic constraints or availability of limited time for in-situ tests, it is generally observed that scale of fluctuation in the horizontal direction be taken equal to either that obtained in the vertical direction, based on isotropic correlation structure, or infinite length, based on the assumption of perfect correlation of soil data in the horizontal direction. In the former case, the variance reduction factor obtained is lower than that obtained for latter case. Hence, in this case the analysis based on isotropic behaviour of correlation structure overestimates the bearing capacity, and in the case of perfect correlation the bearing capacity is underestimated. Moreover, based on the above results, it is suggested that in the absence of horizontal correlation structure for soil property, an upper range of horizontal scale of fluctuation evaluated for similar sites from the past records should be used rather than using the same scale of fluctuation evaluated for vertical data.

Figure 8 shows the variation of variance reduction factor in 2-D space with the ratio of  $d_h/d_v$  for  $d_h$ values ranging from 0.1 m to 100 m. In contrast to the above, it is inferred from the Fig. 8 that for any value of  $d_h$ , as the ratio of  $d_h/d_v$  increases the variance reduction factor decreases. This decrease is predominant, especially for  $d_h/d_v$  ratio greater than unity. As mentioned earlier, since, the scale of fluctuation of

**Fig. 7** Variation of variance reduction factor with dh/dv ratio for different values of vertical scales of fluctuation (dv)







soil property is greater in the horizontal direction than that in the vertical direction, the ratio of  $d_h/d_v$  is always greater than unity. Hence, if the information on horizontal scale of fluctuation is only available and the analysis is performed by assuming in the vertical direction either the same autocorrelation structure as obtained in the horizontal direction or a perfect correlation (having infinite correlation length), an increased variance reduction factor is obtained. This increased variance reduction factor produces higher probability of failure than that expected corresponds to  $d_h/d_v$  greater than unity, and results in conservative estimates of bearing pressure.

#### 4 Conclusions

- (1). Anisotropic behaviour is quite logical for natural soils because of its depositional processes. However, the assumption of isotropic correlation structure based on vertical cone tip resistance data underestimates the variability of design parameter and overestimates the bearing capacity. In contrast, isotropic correlation structure based on horizontal scale of fluctuation of cone tip resistance overestimates the variability and produces uneconomical designs.
- (2). Assumption of perfect correlation either in horizontal or vertical, or both directions, overestimates the variability of design parameters, and consequently produces conservative estimates of bearing capacity.

- (3). In general, horizontal scale of fluctuation is difficult to measure when compared to that in vertical direction, and hence in the absence of such data, it is recommended to assume perfect correlation in the horizontal direction, rather than isotropic behaviour based on vertical scale of fluctuation. This assumption guarantees the safety of the foundation as the analysis produces lower (or conservative) estimates of bearing capacity.
- (4). In the case of unavailability of scale of fluctuation in either direction for a particular site, it may be suggested to use an upper bound value from the range of observed values from the records of past experience within the similar sites, which obviously produces conservative estimates of bearing capacity.
- (5). The transformation model has been identified as the primary factor influencing the degree of uncertainty in the design. It plays a major role in the estimation of degree of variability of design parameter.

## References

- Alonso EE (1976) Risk analysis of soil slopes and its application to slopes in Canadian sensitive clays. Geotechnique 26:453–472
- Cherubini C (2000) Reliability evaluation of shallow foundation bearing capacity in c',  $\phi'$  soils. Can Geotech J 37:264–269
- Fenton GA, Griffiths DV (2003) Bearing capacity prediction of spatially random c-f soils. Can Geotech J 40:54–65

- Fenton GA, Vanmarcke EH (1998) Spatial variation in liquefaction risk. Geotechnique 48(6):819–831
- Griffiths DV, Fenton GA (2000) Influence of soil strength spatial variability on the stability of an undrained clay slope by finite elements. In: Slope stability 2000, ASCE Geotechnical Specialty Publication No. 101
- Jaksa MB, Kaggwa WS, Brooker PI (1999) Experimental evaluation of the scale of fluctuation of a stiff clay. In: Melchers RE, Stewart MG (eds) Proc. 8th Int. conf. on the application of statistics and probability, Sydney, 1, pp 415–422
- Jaksa MB, Yeong KS, Wong KT, Lee SL (2004) Horizontal spatial variability of elastic modulus in sand from the dilatometer. In: Proc. 9th Australia New Zealand conference on geomechanics, Auckland, pp 289–294
- Nobahar A, Popescu R (2002) Bearing capacity of shallow foundations on heterogeneous soils. In: Proc. 2nd

Canadian spec. conf. on computer applications in geotechnique, Winnipeg, M.A

- Phoon KK, Kulhawy FH (1999) Evaluation of geotechnical property variability. Can Geotech J 36(4):625–639
- Babu GLS, Mukesh MD (2004) Effect of soil variability on reliability of soil slopes. Geotechnique 54(5):335–337
- Uzielli M, Vannucchi G, Phoon KK (2005) Normal field characterization of stress-normalised cone penetration testing parameters. Geotechnique 55(1):3–20
- Vanmarcke EH (1977) Probabilistic modeling of soil profiles. J Geotech Eng Div ASCE 103(11):1227–1246
- Vanmarcke EH (1983) Random fields: analysis and synthesis. MIT Press, Cambridge
- Wei L, Tumay MT, Abu-Farsakh MY (2005) Field testing of inclined cone penetration. Geotech Test J ASTM 28(1):31–41