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NUMERICAL ANALYSIS OF ELASTIC PLATES WITH TWO OPPOSITE SIMPLY SUPPORTED ENDS BY SEGMENTATION METHOD

TARUN KANT†
Department of Civil Engineering, Indian Institute of Technology, Bombay 400 076, India

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Abstract—A method for the numerical analysis of elastic plates with two opposite simply supported ends is presented. A variety of boundary conditions including the mixed and the nonhomogeneous types can be prescribed along either of the remaining two opposite edges. Numerical results are presented for the three examples. Based on the comparisons made with the results available elsewhere, it is concluded that the present method is efficient, economical, reliable and very accurate.

INTRODUCTION

The present paper is concerned with the numerical analysis of rectangular plates with different boundary conditions. A less known formulation which is originally due to Goldberg *et al.*[1] is used in the present study. The system of equations are then numerically integrated by using a so-called "segmentation method" which is comprehensively documented in a recent publication[2]. This method is found to be efficient, reliable, accurate and computationally economical for a certain class of plate problems. The plate is assumed to be simply supported along two opposite edges. Different boundary conditions are then prescribed along the other two opposite edges.

PROBLEM FORMULATION

The present study is based on the thin plate theory[3] due to Kirchhoff with the following assumptions:

(1) Material is homogeneous, isotropic and linear elastic.

(2) Deflections are small compared to the thickness of the plate.

(3) Normals to the reference surface before deformation remain straight and normal to the deformed reference surface and their length remain unchanged ($\gamma_{xz} = \gamma_{yz} = \epsilon_z = 0$).

(4) Transverse normal stress components acting on planes parallel to the reference surface is neglected compared to other stress components ($\sigma_z = 0$).

The well-known governing equations of such a theory which defines a boundary value problem are summarised in Appendix A. Numerical integration of such a boundary value problem by the segmentation method[2] which is originally due to Goldberg *et al.*[1] involves first algebraic manipulation of the basic equations so as to obtain a set of first order differential equations—called the "intrinsic equations" involving only some particular dependent variables—called the "intrinsic variables", the number of which equals the order of the partial differential equation system of such a theory (fourth order in the present case). Then out of the two independent coordinates which describe the problem, one is chosen to be the preferred one. In the present analysis, x-coordinate is selected as the preferred one. Intrinsic equations are then derived consisting of a system of first order partial differential equations each of which contains necessarily the first derivative with respect of x of one of the so-called intrinsic dependent variables which appear naturally in the boundary conditions on the edge x = a constant. In the present analysis, $\underset{\sim}{Y}$ defining the vector of intrinsic variables, consist of the dependent variables w, θ_x, V_x and M_x. After the required manipulations, the system of equations are obtained in the following form:

$$\frac{\partial w}{\partial x} = -\theta_x \tag{1a}$$

$$\frac{\partial \theta_x}{\partial x} = \nu \frac{\partial^2 w}{\partial y} + \frac{1}{D} M_x \tag{1b}$$

$$\frac{\partial V_x}{\partial x} = D(1-\nu^2)\frac{\partial^2 w}{\partial y^4} - \nu \frac{\partial^2 M_x}{\partial y^2} - (p_z^+ + p_z^- + \rho h) \tag{1c}$$

$$\frac{\partial M_x}{\partial x} = -2D(1-\nu)\frac{\partial^2 \theta_x}{\partial y^2} + V_x. \tag{1d}$$

The other dependent variables are expressed as functions of intrinsic variables by simple algebraic relations, called "the auxiliary relations" in the following form:

$$M_y = -D(1-\nu^2)\frac{\partial^2 w}{\partial y^2} + \nu M_x \tag{2a}$$

$$M_{xy} = D(1-\nu)\frac{\partial \theta_x}{\partial y} \tag{2b}$$

$$Q_x = D(1-\nu)\frac{\partial^2 \theta_x}{\partial y^2} + \frac{\partial M_x}{\partial x} \tag{2c}$$

$$Q_y = -D(1-\nu)\frac{\partial^3 w}{\partial y^3} + \frac{\partial M_x}{\partial y} \tag{2d}$$

$$V_y = -D(1-\nu)^2\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial M_x}{\partial y}. \tag{2e}$$

The generalised displacement conponents and the corresponding stress resultants which form the vector $\underset{\sim}{Y}$ of the intrinsic variables are functions of x and y, and for a plate with two opposite edges, $y = 0$ and $y = b$ as simply supported, these may be represented in the form of a Fourier series which automatically satisfies both the displacement and the force boundary conditions along these edges, to any desired degree of accuracy as follows:

$$w(x, y) = \sum_m w_m(x) \sin(2m-1)\frac{\pi y}{b} \tag{3a}$$

†Assistant Professor; Presently a Visitor at the Department of Civil Engineering, University of Wales, Swansea SA2 8PP (U.K.) under the Jawaharlal Nehru Memorial Trust (U.K.) Scholarship—1979 Award.

$$\theta_x(x, y) = \sum_m \theta_{xm}(x) \sin (2m-1)\frac{\pi y}{b} \quad (3b)$$

$$V_x(x, y) = \sum_m V_{xm}(x) \sin (2m-1)\frac{\pi y}{b} \quad (3c)$$

$$M_x(x, y) = \sum_m M_{xm}(x) \sin (2m-1)\frac{\pi y}{b}. \quad (3d)$$

Substitution of the expansions (3a–d) in the system of differential equations (1a–d) and analytic integration of these equations with respect to the indpendent variable (coordinate) y coupled with the use of the orthogonality conditions of the basic beam functions used in the y-direction in the aforesaid expansions reduces the set of partial differential equations (1a–d) into the following set of simultaneous first order ordinary differential equations (say for the mth harmonic) involving only the four intrinsic variables. It may be mentioned that it is not necessary to express the external loads in the form of a Fourier series in the y-direction unlike the analytical [3] and the semi-analytical [4] methods. Further it is noted that the series uncouple with respect to the harmonic m leading to a term-by-term analysis which enables storage of only the final discrete values of the intrinsic and the auxiliary dependent variables corresponding to a particular harmonic analysis to be added to the values of the subsequent harmonic analyses.

$$\frac{dw_m\dagger}{dx} = -\theta_{xm} \quad (4a)$$

$$\frac{d\theta_{xm}}{dx} = -\nu(2m-1)^2\left(\frac{\pi}{b}\right)^2 w_m + \frac{1}{D}M_{xm} \quad (4b)$$

$$\frac{dV_{xm}}{dx} = D(1-\nu^2)(2m-1)^4\left(\frac{\pi}{b}\right)^4 w_m + \nu(2m-1)^2 \times\left(\frac{\pi}{b}\right)^2 M_{xm} - \frac{4}{\pi(2m-1)}(p_z^+ + p_z^- + \rho h) \quad (4c)$$

$$\frac{dM_{xm}}{dx} = 2D(1-\nu)(2m-1)^2\left(\frac{\pi}{b}\right)^2 \theta_{xm} + V_{xm} \quad (4d)$$

and the auxiliary relations (2a–e) take the form:

$$M_y = \sum_m [D(1-\nu^2)(2m-1)^2\left(\frac{\pi}{b}\right)^2 w_m + \nu M_{xm}] \sin (2m-1)\frac{\pi y}{b} \quad (5a)$$

$$M_{xy} = \sum_m D(1-\nu)(2m-1)\left(\frac{\pi}{b}\right)\theta_{xm} \cos (2m-1)\frac{\pi y}{b} \quad (5b)$$

$$Q_x = \sum_m \left[D(1-\nu)(2m-1)^2\left(\frac{\pi}{b}\right)^2 \theta_{xm} + V_{xm}\right] \sin (2m-1)\frac{\pi y}{b} \quad (5c)$$

$$Q_y = \sum_m \left[D(1-\nu)(2m-1)^2\left(\frac{\pi}{b}\right)^2 w_m + M_{xm}\right](2m-1) \times\left(\frac{\pi}{b}\right)\cos (2m-1)\frac{\pi y}{b} \quad (5d)$$

$$V_y = \sum_m \left[D(1-\nu)^2(2m-1)^2\left(\frac{\pi}{b}\right)^2 w_m + (2-\nu)M_{xn}\right] \times (2m-1)\left(\frac{\pi}{b}\right)\cos (2m-1)\frac{\pi y}{b} \quad (5e)$$

†For the sake of brevity $w_m(x)$, $M_{xm}(x)$, etc. are hereafter written simply as w_m, M_{xm}, etc.

The equations (4a–d) are numerically integrated by the segmentation method [2] for th mth harmonic at a time and the discrete point values of all the dependent variables are obtained by summing the corresponding values got for the given number of harmonics as given by the relations (3a–d) and (5a–e).

NUMERICAL EXAMPLES

Numerical results are presented for a square plate of side "a" and thickness "h", simply supported along the two opposite edges, $y=0$ and $y=a$ with the boundary conditions $w_m = M_{ym} = 0$ and loaded with a uniformly distributed load p_z^+. Discrete numerical values of the dependent variables are presented in the non-dimensional form as follows:

$$w = \alpha\frac{p_z^+ a^4}{D} \quad (6a)$$

$$M_x = \beta p_z^+ a^2 \quad (6b)$$

$$M_y = \beta_1 p_z^+ a^2 \quad (6c)$$

$$M_{xy} = \frac{n}{2} p_z^+ a^2 \quad (6d)$$

$$Q_x = \gamma p_z^+ a \quad (6e)$$

$$Q_y = \gamma_1 p_z^+ a \quad (6f)$$

$$V_x = \delta p_z^+ a \quad (6g)$$

$$V_y = \delta_1 p_z^+ a. \quad (6h)$$

The same geometric and material properties, viz, $a/h = 50$ and $\nu = 0.3$ are used throughout. Five equal segments and five subdivisions within each segment for the Runge-Kutta-Gill algorithms [5] have been found suitable for the half plate analysis in the x-direction taking advantage of the symmetric conditions along the centre line in all the examples considered in this section. On an edge with outward unit normal vector $\hat{n}$ and unit tangent vector $\hat{t}$, the following nomenclature has been used for the designation of the prescribed boundary conditons.

$$S \text{ for } w = M_n = 0 \quad (7a)$$

$$C \text{ for } w = \theta_n = 0 \quad (7b)$$

$$F \text{ for } V_n = M_n = 0. \quad (7c)$$

Results obtained in the present study are compared with those available elsewhere [3]. Most of the discrete values of the dependent variables are tabulated for the first 20 harmonics. Convergence is seen to be excellent. It is observed that while the value of w converges at the second or the third harmonic, the values of M_x, M_y, M_{xy}, Q_x and V_x take six to seven harmonics to converge. Convergence of the values of Q_y and V_y is seen to be slow.

Example 1. Plate with boundary conditions "S" along all the four edges

The maximum values of w, M_x, M_y, M_{xy}, Q_x, Q_y, V_x and V_y are presented in Table 1 while the variation of M_x and M_y along the centre line $y/a = 0.5$ is tabulated in Table 2. Variations of Q_x and V_x along the centre line $y/a = 0.5$ and that of M_{xy}, Q_y and V_y along the supported edge $y/a = 0.0$ are plotted in Figs. 1–3.

Example 2. Plate with boundary conditions "C" along $x = 0$, a and "S" along $y = 0$, a

Table 1. Maximum values of transverse deflection and stress resultants in a square plate simply supported on all edges ($w = M_x = 0$ along $x = 0$, $w = M_y = 0$ along $y = 0, a$) under U.D.L. based on Kirchhoff plate theory formulation of segmentation method—a convergence study ($\nu = 0.3$, $a/h = 50$)

	w_{max}	$(Mx)_{max}$	$(My)_{max}$	$(Mxy)_{max}$	$(Qx)_{max}$	$(Qy)_{max}$	$(Vx)_{max}$	$(Vy)_{max}$
	$=\alpha \frac{pza^4}{D}$	$= \beta pza^2$	$= \beta_1 pza^2$	$= \frac{\eta}{2} pza^2$	$=\gamma pza$	$=\gamma_1 pza$	$=\delta pza$	$=\delta_1 pza$
m	α	β	β_1	$\eta/2$	γ	γ_1	δ	δ_1
1	.00410	.04915	.05164	- .03012	.37145	.24366	.46612	.32503
2	.00405	.04760	.04709	- .03179	.32646	.28785	.40541	.37055
3	.00406	.04791	.04812	- .03215	.34266	.30404	.42728	.38677
4	.00406	.04780	.04774	- .03228	.33440	.31230	.41612	.39504
5	.00406	.04785	.04792	- .03234	.33939	.31730	.42287	.40004
6	.00406	.04783	.04782	- .03238	.33605	.32065	.41835	.40339
7	.00406	.04784	.04788	- .03240	.33844	.32304	.42159	.40578
8	.00406	.04783	.04784	- .03241	.33664	.32484	.41916	.40758
9	.00406	.04784	.04787	- .03242	.33805	.32624	.42105	.40898
10	.00406	.04783	.04785	- .03243	.33692	.32737	.41954	.41010
15	.00406	.04784	.04786	- .03244	.33765	.33073	.42052	.41347
20	.00406	.04784	.04786	- .03245	.33730	.33242	.42004	.41516
	.00406[a]	.0479	.0479	.0325	.338	.338	.420	.420

[a]Values quoted in Ref. 3 based on Kirchhoff Plate Theory

Table 2. Bending moments in a square plate simply supported on all edges ($w = M_x = 0$ along $x = 0, a$; $w = M_y = 0$ along $y = 0, a$) under U.D.L. based on Kirchhoff plate theory formulation of segmentation method—a convergence study ($\nu = 0.3$, $a/h = 50$)

	$Mx = \beta pza^2, \frac{y}{a} = 0.5$					$My = \beta_1 pza^2, \frac{y}{a} = 0.5$				
m	$\frac{x}{a} = 0.1$	$\frac{x}{a} = 0.2$	$\frac{x}{a} = 0.3$	$\frac{x}{a} = 0.4$	$\frac{x}{a} = 0.5$	$\frac{x}{a} = 0.1$	$\frac{x}{a} = 0.2$	$\frac{x}{a} = 0.3$	$\frac{x}{a} = 0.4$	$\frac{x}{a} = 0.5$
1	.02211	.03573	.04372	.04787	.04915	.01863	.03314	.04345	.04960	.05164
2	.02063	.03403	.04208	.04630	.04760	.01633	.02958	.03926	.04513	.04709
3	.02099	.03438	.04240	.04661	.04791	.01703	.03052	.04027	.04615	.04812
4	.02086	.03426	.04229	.04650	.04780	.01673	.03015	.03989	.04578	.04774
5	.02092	.03431	.04234	.04655	.04785	.01689	.03033	.04007	.04595	.04792
6	.02088	.03428	.04231	.04652	.04783	.01680	.03023	.03997	.04586	.04782
7	.02090	.03430	.04233	.04654	.04784	.01685	.03029	.04003	.04592	.04788
8	.02089	.03429	.04232	.04653	.04783	.01682	.03025	.03999	.04588	.04784
9	.02090	.03430	.04233	.04654	.04784	.01684	.03028	.04002	.04590	.04787
10	.02089	.03429	.04232	.04653	.04783	.01682	.03026	.04000	.04588	.04785
15	.02090	.03429	.04232	.04653	.04784	.01683	.03027	.04001	.04590	.04786
20	.02090	.03429	.04232	.04653	.04784	.01683	.03026	.04001	.04589	.04786
	.0209[a]	.0343	.0424	.0466	.0479	.0168	.0303	.0400	.0459	.0479

[a]Values quoted in Ref. 3 based on Kirchhoff Plate Theory

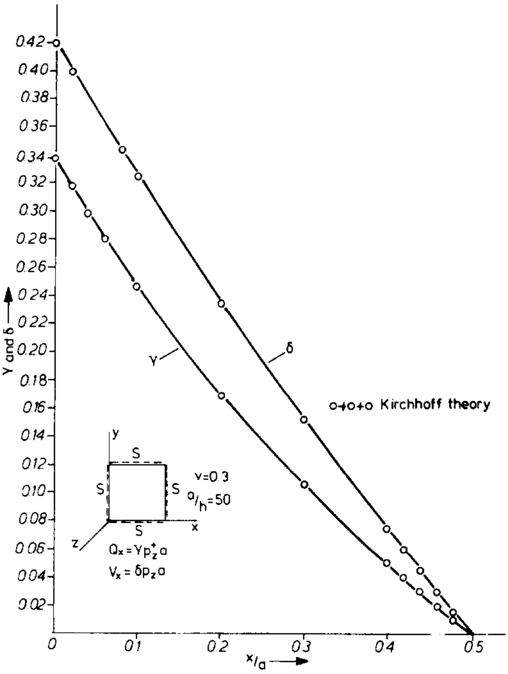

Fig. 1. Variation of Q_x and V_x along $y/a = 0.5$.

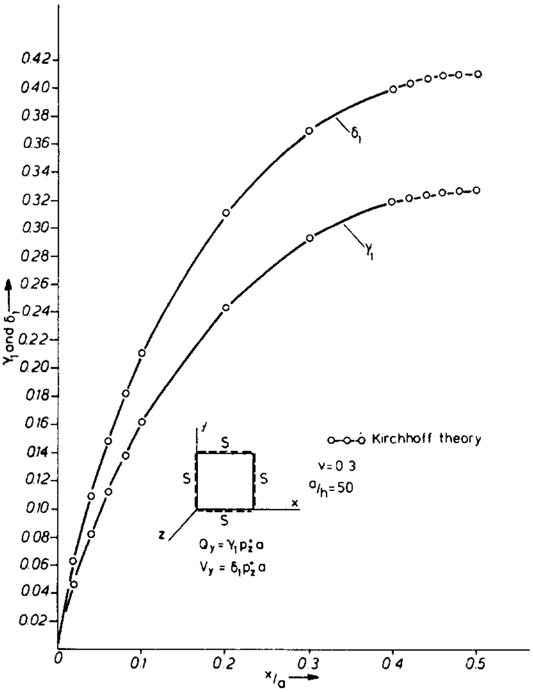

Fig. 3. Variation of Q_y and V_y along the supported edge (y/a = 0).

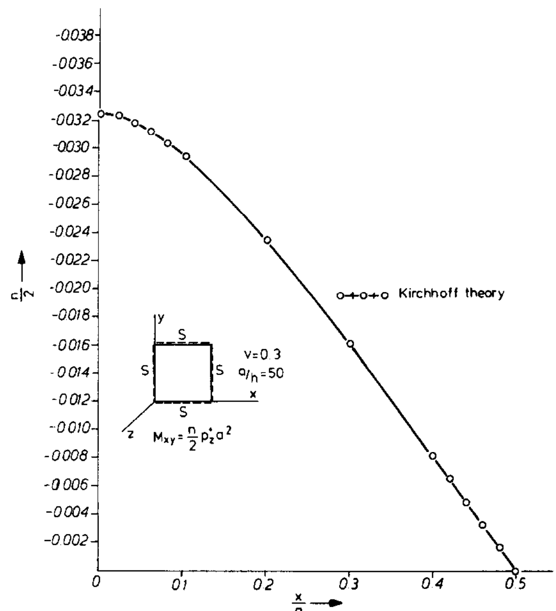

Fig. 2. Variation of M_{xy} along the supported edge ($y/a = 0$).

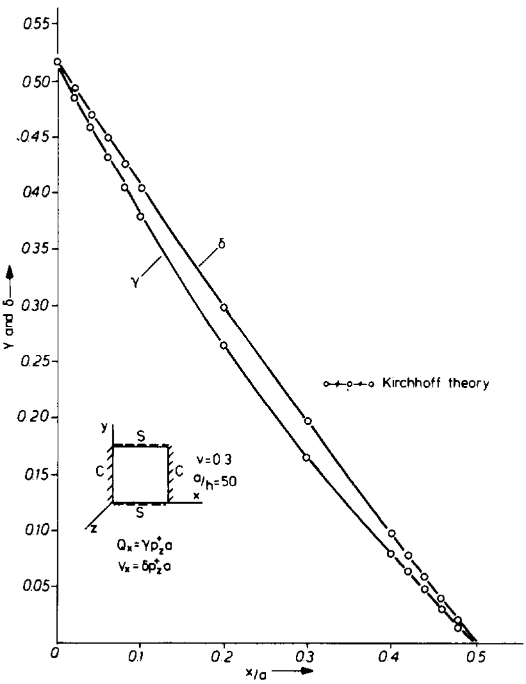

Fig. 4. Variation Q_x and V_x along $y/a = 0.5$.

Discrete numerical values of all the dependent variables are tabulated in Tables 3 and 4, while the variations of Q_x, V_x, M_{xy}, Q_y and V_y are plotted in Figs. 4–6. The occurrence of a maxima of Q_y at the corner of the plate is unrealistic. But this is not due to the method used in the present study. This is due to the inherent limitation of the Kirchhoff plate theory itself. This point has been clarified in a recent paper[6].

Example 3. *Plate with boundary conditions "F" along $x = 0$, a and "S" along $y = 0$, a*

Relevant discrete numerical values are presented in Tables 5 and 6. Plots of Q_x and V_x along the centre line $y/a = 0.5$, and M_{xy}, Q_y and V_y along the supported edge $y/a = 0.0$ are shown in Figs. 7–9. These plots are quite revealing. It gives a feel for the variation of the quantities. The value of Q_x is seen to be inconsistent at the free edge. This is due to the well-known problem of satisfaction of the boundary conditions at a free edge in a Kirchhoff plate, wherein a new quantity V_x is introduced in the theory. In a recent publication[6] it is shown that barring its value at the free edge the variation of Q_x is realistic in a physical situation. Thus it is

Table 3. Maximum values of transverse deflection and stress resultants in a square plate with two opposite edges simply supported and the other two edges clamped ($w = \theta_x = 0$ along $x = 0, a$; $w = M_y = 0$ along $y = 0, a$) under U.D.L. based on Kirchhoff plate theory formulation of segmentation method—a convergence study ($\nu = 0.3$, $a/h = 50$)

m	$w_{max} = \alpha \frac{pza^4}{D}$	$(Mx)_{max} = \beta pz\,a$ POS	NEG	$(My)_{max} = \beta_1 pz\,a^2$ POS	NEG	$(Mxy)^{\dagger}_{max} = \frac{n}{2}\,pz\,a^2$	$(Qx)_{max} = \gamma pz\,a$	$(Qy)^{\dagger\dagger}_{max} = \gamma_1 pz\,a$	$(Vx)_{max} = \delta p\,za$	$(Vy)^{\dagger\dagger}_{max} = \delta_1 pz'a$
	α	β	β	β_1	β_1	n/2	γ	γ_1	δ	δ_1
1	.00196	.03459	- .07378	.02800	- .02213	- .01332	.58413	- .23187	.58413	- .39418
2	.00191	.03299	- .06902	.02362	- .02071	- .01426	.49429	- .27673	.49429	- .47044
3	.00192	.03330	- .07005	.02464	- .02102	- .01436	.52669	- .29293	.52669	- .49798
4	.00192	.03319	- .06967	.02427	- .02090	- .01437	.51016	- .30119	.51016	- .51203
5	.00192	.03324	- .06985	.02444	- .02096	- .01438	.52016	- .30619	.52016	- .52053
6	.00192	.03321	- .06975	.02435	- .02093	- .01438	.51347	- .30954	.51347	- .52622
7	.00192	.03323	- .06981	.02441	- .02094	- .01438	.51826	- .31194	.51826	- .53029
8	.00192	.03322	- .06978	.02437	- .02093	- .01438	.51466	- .31374	.51466	- .53335
9	.00192	.03322	- .06980	.02439	- .02094	- .01438	.51746	- .31514	.51746	- .53573
10	.00192	.03322	- .06978	.02438	- .02093	- .01438	.51522	- .31626	.51522	- .53764
15	.00192	.03322	- .06979	.02439	- .02094	- .01438	.51667	- .31963	.51667	- .54337
20	.00192	.03322	- .06979	.02438	- .02094	- .01438	.51597	- .32131	.51597	- .54623
-	.00192[a]		- .0697	.0244						

[a]Values quoted in Ref. 3 based on Kirchhoff Plate Theory

†Occurs along the supported edge at $\frac{x}{a} = 0.2$ and $\frac{y}{a} = 0$; ††Occurs at corners of the supported edges.

Table 4. Bending moments in a square plate with two opposite edges simply supported and the other two edges clamped ($w = \theta_x = 0$ along $x = 0, a$; $w = M_y = 0$ along $y = 0, a$) under U.D.L. based on Kirchhoff plate theory formulation of segmentation method—a convergence study ($\nu = 0.3$, $a/h = 50$)

	$Mx = \beta pza^2$, y/a = 0.5						$My = \beta_1 pza^2$, y/a = 0.5					
m	$\frac{x}{a} = 0.0$	$\frac{x}{a} = 0.1$	$\frac{x}{a} = 0.2$	$\frac{x}{a} = 0.3$	$\frac{x}{a} = 0.4$	$\frac{x}{a} = 0.5$	$\frac{x}{a} = 0.0$	$\frac{x}{a} = 0.1$	$\frac{x}{a} = 0.2$	$\frac{x}{a} = 0.3$	$\frac{x}{a} = 0.4$	$\frac{x}{a} = 0.5$
1	- .07378	- .02644	.00364	.02185	.03155	.03459	- .02213	- .00540	.00875	.01932	.02581	.02800
2	- .06902	- .02669	.00219	.02020	.02993	.03299	- .02071	- .00652	.00589	.01551	.02156	.02362
3	- .07005	- .02642	.00253	.02053	.03024	.03330	- .02102	- .00601	.00676	.01650	.02258	.02464
4	- .06967	- .02654	.00241	.02042	.03013	.03319	- .02090	- .00626	.00641	.01612	.02220	.02427
5	- .06985	- .02648	.00247	.02047	.03018	.03324	- .02096	- .00612	.00658	.01630	.02238	.02444
6	- .06975	- .02652	.00244	.02044	.03015	.03321	- .02093	- .00621	.00648	.01620	.02228	.02435
7	- .06981	- .02650	.00246	.02046	.03017	.03323	- .02094	- .00615	.00654	.01626	.02234	.02441
8	- .06978	- .02651	.00245	.02045	.03016	.03322	- .02093	- .00619	.00650	.01622	.02230	.02437
9	- .06980	- .02650	.00245	.02046	.03017	.03322	- .02094	- .00616	.00653	.01625	.02233	.02439
10	- .06978	- .02651	.00245	.02045	.03016	.03322	- .02093	- .00618	.00651	.01623	.02231	.02438
15	- .06979	- .02650	.00245	.02045	.03017	.03322	- .02094	- .00617	.00652	.01624	.02232	.02439
20	- .06979	- .02650	.00245	.02045	.03016	.03322	- .02094	- .00618	.00652	.01624	.02232	.02438
	- .0697[a]	-	-	-	-	.0332	-	-	-	-	-	.0244

[a]Values quoted in Ref. 3 based on Kirchhoff Plate Theory

Table 5. Maximum values of transverse deflection and stress resultants in a square plate with two opposite edges simply supported and the other two edges free ($M_x = V_x = 0$ along $x = 0, a$; $w = M_y = 0$ along $y = 0, a$) under ($\nu = 0.3$, $a/h = 50$)

m	$w^{\dagger}_{max} = \alpha \frac{pza^4}{D}$	$(Mx)_{max} = \beta pza^2$	$(My)^{\dagger}_{max} = \beta_1 pza^2$	$(Mxy)_{max} = \frac{\eta}{2} pza^2$	$(Qx)_{max} = \gamma pza$	$(Qy)_{max} = \gamma_1 pza$	$(Vx)^{\dagger\dagger}_{max} = \delta pza$	$(Vy)_{max} = \delta_1 pza$
	α	β	β_1	$\frac{\eta}{2}$	γ	γ_1	δ	δ_1
1	.01504	.02825	.13519	.02280	.07167	.37384	.00514	.35048
2	.01498	.02682	.13011	.02367	.06349	.41869	.00433	.39538
3	.01498	.02713	.13121	.02386	.06643	.43488	.00447	.41158
4	.01498	.02702	.13081	.02393	.06493	.44315	.00444	.41984
5	.01498	.02707	.13100	.02396	.06584	.44815	.00445	.42484
6	.01498	.02704	.13089	.02398	.06523	.45149	.00444	.42819
7	.01498	.02706	.13096	.02399	.06566	.45389	.00444	.43058
8	.01498	.02705	.13091	.02399	.06534	.45569	.00444	.43238
9	.01498	.02706	.13094	.02400	.06559	.45709	.00444	.43378
10	.01498	.02705	.13092	.02400	.06539	.45821	.00444	.43491
15	.01498	.02705	.13093	.02401	.06552	.46158	.00444	.43828
20	.01498	.02705	.13093	.02401	.06546	.46327	.00445	.43996

† Occurs at $\frac{x}{a} = 0$ and $\frac{y}{a} = 0.5$

†† Occurs at $\frac{x}{a} = 0.2$ and $\frac{y}{a} = 0.5$

Table 6. Bending moments in a square plate with two opposite edges simply supported and the other two edges free $V_x = M_x = 0$ along $x = 0, a$; $w = M_y = 0$ along $y = 0, a$) under U.D.L. based on Kirchhoff plate theory formulation of segmentation method—a convergence study ($\nu = 0.3$, $a/h = 50$)

m	$Mx = \beta pza^2$, $y/a = 0.5$					$My = \beta_1 pza^2$, $y/a = 0.5$					
	$\frac{x}{a} = 0.1$	$\frac{x}{a} = 0.2$	$\frac{x}{a} = 0.3$	$\frac{x}{a} = 0.4$	$\frac{x}{a} = 0.5$	$\frac{x}{a} = 0.0$	$\frac{x}{a} = 0.1$	$\frac{x}{a} = 0.2$	$\frac{x}{a} = 0.3$	$\frac{x}{a} = 0.4$	$\frac{x}{a} = 0.5$
1	.01186	.01972	.02467	.02739	.02825	.13519	.13119	.12875	.12734	.12660	.12638
2	.01087	.01842	.02327	.02596	.02682	.13011	.12641	.12402	.12260	.12186	.12163
3	.01114	.01872	.02358	.02627	.02713	.13121	.12744	.12505	.12363	.12289	.12266
4	.01103	.01861	.02347	.02616	.02702	.13081	.12706	.12467	.12325	.12251	.12228
5	.01108	.01866	.02352	.02621	.02707	.13100	.12724	.12485	.12343	.12269	.12246
6	.01105	.01863	.02349	.02618	.02704	.13089	.12714	.12475	.12333	.12259	.12236
7	.01107	.01865	.02351	.02620	.02706	.13096	.12720	.12481	.12339	.12265	.12242
8	.01106	.01864	.02350	.02619	.02705	.13091	.12716	.12477	.12335	.12261	.12238
9	.01107	.01865	.02351	.02620	.02706	.13094	.12719	.12480	.12338	.12264	.12241
10	.01106	.01864	.02350	.02619	.02705	.13092	.12717	.12478	.12336	.12262	.12239
15	.01106	.01865	.02350	.02619	.02705	.13093	.12718	.12479	.12337	.12263	.12240
20	.01106	.01864	.02350	.02619	.02705	.13093	.12718	.12479	.12337	.12263	.12240

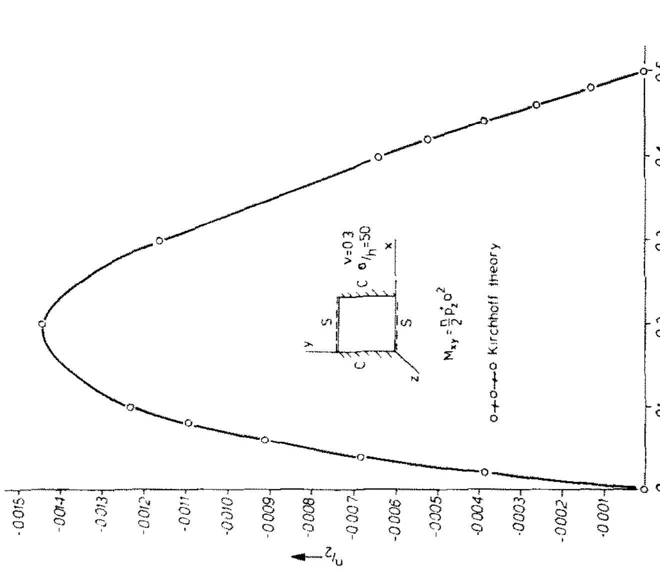

Fig. 5. Variation of M_{xy} along the supported edge ($y/a = 0$).

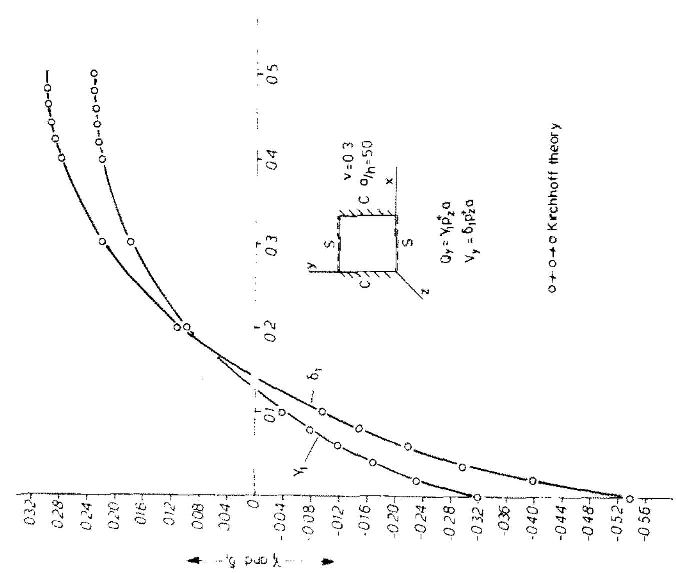

Fig. 6. Variation of Q_y and V_y along the supported edge ($y/a = 0$).

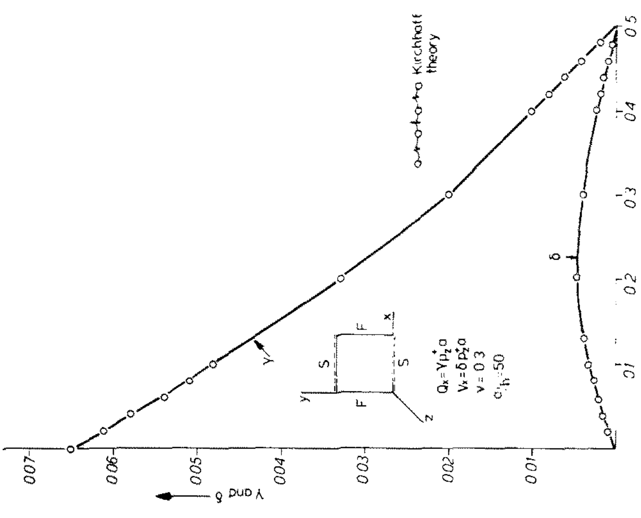

Fig. 7. Variation of Q_x and V_x along $y/a = 0.5$.

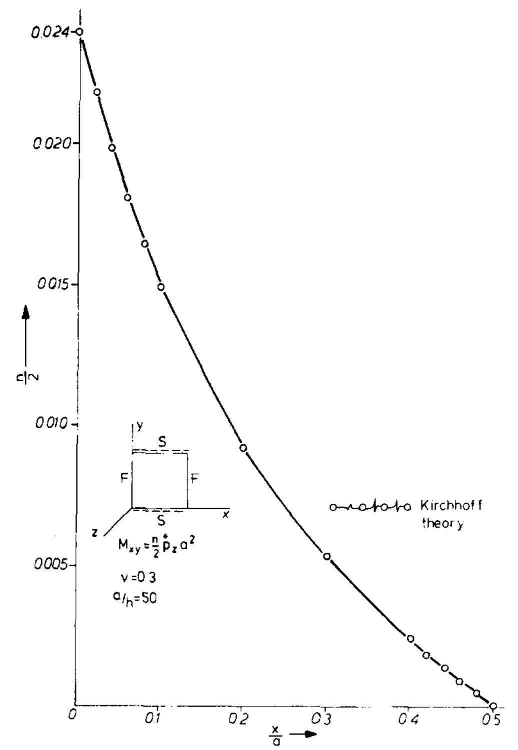

Fig. 8. Variation of M_{xy} along the supported edge ($y/a = 0$).

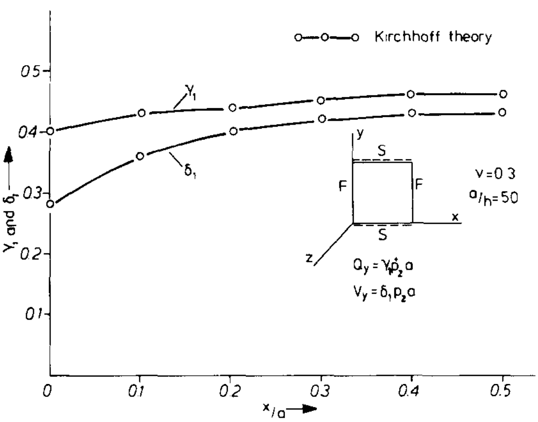

Fig. 9. Variation of Q_y and V_y along the supported edge ($y/a = 0$).

concluded that it is Q_x and not V_x which should be used in designs in such situations.

CONCLUSIONS

A less known formulation for economical numerical analysis of elastic plates is presented. The method is found to be very efficient and accurate. The numerical results obtained show excellent comparison with those of Timoshenko [3] which are presumably based on analytical solution. The main use of the method seems to lie in its adoption in design offices for preparation of reliable design charts at reasonable costs. Although the method has limitations in its applications to general problems when compared with the versatile finite element method [7], it appears to be superior to the finite strip method [4] because of its relatively accurate mathematical model.

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APPENDIX A

Basic equations of Kirchhoff plate theory

Displacement model (Ref. Fig. A1)

$$U(x, y, z) = z\theta_x(x, y) \tag{A1a}$$

$$V(x, y, z) = z\theta_y(x, y) \tag{A1b}$$

$$W(x, u, z) = w(x, y) \tag{A1c}$$

Strain-displacement relations

$$\epsilon_x = -z\frac{\partial^2 w}{\partial x^2} \tag{A2a}$$

$$\epsilon_y = -z\frac{\partial^2 w}{\partial y^2} \tag{A2b}$$

$$\gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y} \tag{A2c}$$

$$\theta_x = -\frac{\partial w}{\partial x} \tag{A2d}$$

$$\theta_y = -\frac{\partial w}{\partial y}. \tag{A2e}$$

Equilibrium equations (Ref. Fig. A2–4)

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + p_z^+ + p_z^- + \rho h = 0 \tag{A3a}$$

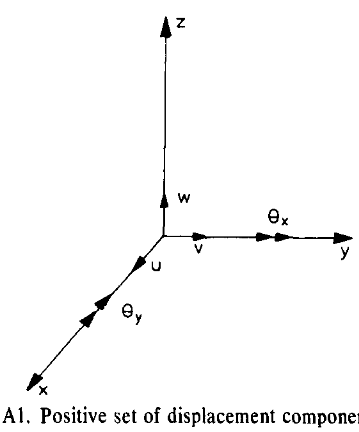

Fig. A1. Positive set of displacement components.

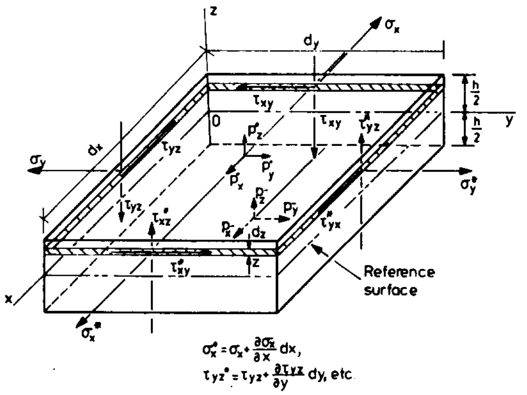

Fig. A2. Positive set of stress components.

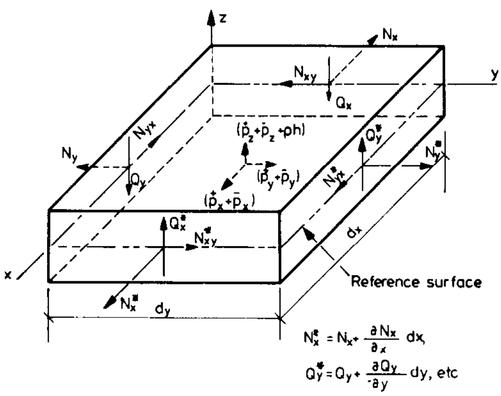

Fig. A3. Positive set of stress resultants—forces.

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0 \tag{A3b}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = 0. \tag{A3c}$$

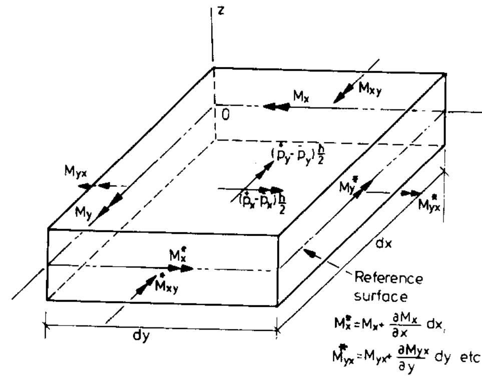

Fig. A4. Positive set of stress resultants—couples.

Force–displacement relations

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \tag{A4a}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \tag{A4b}$$

$$M_{xy} = -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}. \tag{A4c}$$

Kirchhoff shear and boundary conditions

$$V_x = Q_x + \frac{\partial M_{xy}}{\partial y} \tag{A5a}$$

$$V_y = Q_y + \frac{\partial M_{yx}}{\partial x} \tag{A5b}$$

$$w = \bar{w} \quad \text{or} \quad V_x = \bar{V}_x \tag{A6a}$$

$$\theta_x = \theta_x \quad \text{or} \quad M_x = \bar{M}_x$$

on edge $x = \text{const.}$ and (A6b)

$$\omega = \bar{\omega} \quad \text{or} \quad V_y = \bar{V}_y$$
$$\theta_y = \bar{\theta}_y \quad \text{or} \quad M_y = \bar{M}_y \tag{A7a}$$

on edge $y = \text{const.}$ (A7b)