

TRANSIENT DYNAMICS OF COMPOSITE SANDWICH PLATES USING 4-, 8-, 9-NODED ISOPARAMETRIC QUADRILATERAL ELEMENTS

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Abstract. A finite element method based on Mindlin's theory is employed in the prediction of the dynamic transient response of multilayered composite sandwich plates. Numerical convergence and stability of 4-noded linear, 8-noded serendipity, and 9-noded Lagrangian elements are established using an explicit time integration technique. A special mass matrix diagonalization scheme is adopted which conserves the total mass of the element and includes the effects due to rotary inertia terms. The parametric effects of the time step, finite element mesh, lamination scheme, and orthotropy on the transient response are investigated. Numerical results for deflections and stresses are presented under various boundary conditions and loadings. The results presented herein should be of interest to composite-structure designers, experimentalists, and numerical analysts in verifying their results.

Introduction

The high strength and stiffness versus weight obtainable in sandwich panels has led to a wide variety of applications. Most structures, whether they are used in land, sea, or air, are subjected to dynamic loads during their operation. There exists a need for assessing the transient response of laminated plates. However, transient analyses of composite sandwich plates of finite dimensions have not received adequate consideration.

A large amount of literature has been devoted to the development of theories for conventional sandwich structures and to the study of their static and dynamic behaviours by analytical and numerical methods. A detailed historical review is given in the books by Plantema [19] and Allen [2] and in two papers by Habip [4,5]. The pioneer workers are Reissner [22–24], who also studied the finite deflection problem, and Yu [27–30], who published a series of papers on vibrations of sandwich plates including viscous damping and large deflections. Exact elasticity solutions for some particular sandwich plate bending problems were obtained by Pagano [17]. Many researchers have adopted the versatile finite element method in analysing conventional sandwich and laminated plates (e.g. see [1,15] for sandwich plates and [6,10,16,18] for laminated plates). Khatua and Cheung [11,12] successfully solved a variety of linear problems for multilayer plates and shells using non-conforming rectangular and triangular plate and shell finite elements. All of these studies were confined to either static analyses or free-vibration analyses.

In the present paper, transient response of layered, composite sandwich plates is investigated using a shear deformable finite element. The present study is the first to consider the transient response of sandwich plates of arbitrary construction and finite dimensions.

Theory and formulation

A multilayered sandwich plate, as shown in Fig. 1, having n stiff faces and $n - 1$ soft cores is considered. The displacement field of such a plate can be assumed to be

$$\begin{aligned}
 u(x, y, z, t) &= u_0(x, y, t) + z\theta_x(x, y, t), \\
 v(x, y, z, t) &= v_0(x, y, t) + z\theta_y(x, y, t), \\
 w(x, y, z, t) &= w_0(x, y, t),
 \end{aligned}
 \tag{1}$$

where t is the time and $u, v,$ and w are the displacements in the $x, y,$ and z directions, respectively, of any generic point in the plate space. The variables $u_0, v_0,$ and w_0 are the associated midplane displacements; and the parameters θ_x and θ_y are the rotations of the normals to the midplane in the $x-z$ and $y-z$ planes due to membrane-flexural deformations.

The strain expressions derived from the displacement model of equation (1) are:

$$\begin{aligned}
 \epsilon_x &= \epsilon_{x0} + z\chi_x; & \epsilon_y &= \epsilon_{y0} + z\chi_y; \\
 \gamma_{xy} &= \epsilon_{xy0} + z\chi_{xy}; & \gamma_{yz} &= \phi_y; & \gamma_{xz} &= \phi_x;
 \end{aligned}
 \tag{2}$$

in which

$$\begin{aligned}
 [\epsilon_{x0}, \epsilon_{y0}, \epsilon_{xy0}] &= \left[\frac{\partial u_0}{\partial x}, \frac{\partial v_0}{\partial y}, \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right], \\
 [\chi_x, \chi_y, \chi_{xy}] &= \left[\frac{\partial \theta_x}{\partial x}, \frac{\partial \theta_y}{\partial y}, \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \right], \\
 [\phi_x, \phi_y] &= \left[\theta_x + \frac{\partial w_0}{\partial x}, \theta_y + \frac{\partial w_0}{\partial y} \right].
 \end{aligned}
 \tag{3}$$

The stress-strain relations for a typical layer L with reference to the material axes (1-2-3) are given by

$$\begin{aligned}
 \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}^L &= \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix}^L \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix}^L, \\
 \begin{bmatrix} \tau_{23} \\ \tau_{13} \end{bmatrix}^L &= \begin{bmatrix} \alpha C_{44} & 0 \\ 0 & \alpha C_{55} \end{bmatrix}^L \begin{bmatrix} \gamma_{23} \\ \gamma_{13} \end{bmatrix}^L,
 \end{aligned}
 \tag{4}$$

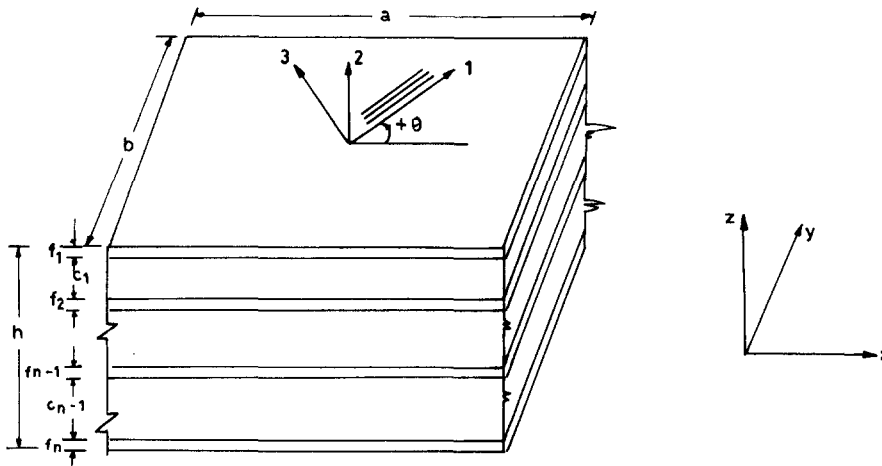


Fig. 1. Multilayer arrangement.

in which $(\sigma_1, \sigma_2, \tau_{12}, \tau_{23}, \tau_{13})$ are the stress and $(\epsilon_1, \epsilon_2, \gamma_{12}, \gamma_{23}, \gamma_{13})$ are the linear strain components referred to the material axes (1–2–3) as shown in Fig. 1, and C_{ij} 's are the plane stress reduced elastic constants of the L th lamina and these are related to the engineering elastic constants [9]. The coefficient α has been given various values by different authors; e.g., $\alpha = 2/3$ according to Timoshenko [25], Reissner [21] assumes a value of $\alpha = 5/6$, and Mindlin [14] uses $\alpha = \pi^2/12$, which is very close to the value used by Reissner. Unless otherwise stated, the value used throughout this work is Reissner's value of $\alpha = 5/6$ for stiff face layers and unity for core material.

Following the usual transformation rule of stresses/strains between the layer (material) and the plate coordinate systems [9], the stress–strain relations for the L th layer in the plate coordinate, (x, y, z) are written in a compacted form as

$$\sigma = Q\epsilon, \tag{5}$$

in which $\sigma = (\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz})^T$ and $\epsilon = (\epsilon_x, \epsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})^T$ are the stress and strain vectors with respect to the plate axes, and

$$Q = \begin{bmatrix} Q_{ij} & \mathbf{0} \\ \mathbf{0} & Q_{lm} \end{bmatrix} \quad \begin{matrix} i, j = 1, 2, 3 \\ l, m = 4, 5 \end{matrix} \tag{6}$$

are the transformed reduced elastic constants in the plate axes of the L th layer.

The constitutive relations involving membrane forces, bending moments and shear forces are defined as:

$$\begin{bmatrix} N_x, N_y, N_{xy} \\ M_x, M_y, M_{xy} \\ Q_x, Q_y \end{bmatrix} = \sum_{L=1}^n \int_{h_L}^{L+1} \begin{bmatrix} \sigma_x, \sigma_y, \tau_{xy} \\ z\sigma_x, z\sigma_y, z\tau_{xy} \\ \tau_{xz}, \tau_{yz} \end{bmatrix} dz. \tag{7}$$

Upon integration, these expressions are rewritten in a matrix form which defines the stress-resultant/strain relations of the sandwich plate and is given by, $\bar{\sigma} = D\bar{\epsilon}$,

$$\begin{bmatrix} N \\ M \\ Q \end{bmatrix} = \sum_{L=1}^n \begin{bmatrix} Q_{ij}H_1 & Q_{ij}H_2 & \mathbf{0} \\ Q_{ij}H_2 & Q_{ij}H_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & Q_{ml}H_1 \end{bmatrix} \begin{bmatrix} \epsilon_0 \\ \chi \\ \phi \end{bmatrix}, \tag{8a}$$

where

$$\begin{aligned} N &= [N_x, N_y, N_{xy}]^T; & \epsilon_0 &= [\epsilon_{x_0}, \epsilon_{y_0}, \epsilon_{xy_0}]^T; \\ M &= [M_x, M_y, M_{xy}]^T; & \chi &= [\chi_x, \chi_y, \chi_{xy}]^T; \\ Q &= [Q_x, Q_y]^T; & \phi &= [\phi_x, \phi_y]^T. \end{aligned} \tag{8b}$$

In the above relations, n is the number of layers and

$$H_i = (h_{L+1}^i - h_L^i)/i, \quad i = 1, 2, 3. \tag{8c}$$

Finite element formulation

In the C^0 finite element theory, the continuum displacement vector within the element is discretized such that

$$a(x, y, t) = \sum_{i=1}^{NN} N_i(x, y) a_i(t), \tag{9a}$$

in which the term $N_i(x, y)$ is the shape function associated with node i , NN is the number of nodes in the element, and $\mathbf{a}_i(t)$ is the value of the displacement vector $\mathbf{a}(x, y, t)$ corresponding to node i .

With the generalized displacement vector $\mathbf{a}(x, y, t)$ at all points within the element, the generalized strain vector $\bar{\boldsymbol{\epsilon}}(x, y, t)$ at any point is expressed as follows:

$$\bar{\boldsymbol{\epsilon}}(x, y, t) = \sum_{i=1}^{\text{NN}} \mathbf{B}_i(x, y) \mathbf{a}_i(t), \quad (9b)$$

where

$$\bar{\boldsymbol{\epsilon}} = (\epsilon_{x0}, \epsilon_{y0}, \epsilon_{xy0}, \chi_x, \chi_y, \chi_{xy}, \phi_x, \phi_y)^T, \quad (9c)$$

$$\mathbf{a}_i = (u_{0i}, v_{0i}, w_{0i}, \theta_{xi}, \theta_{yi})^T. \quad (9d)$$

\mathbf{B}_i is the strain–displacement matrix and is given as follows:

$$\begin{aligned} B_{11} = B_{32} = B_{44} = B_{65} = B_{73} &= \partial N_i / \partial x, \\ B_{22} = B_{31} = B_{55} = B_{64} = B_{83} &= \partial N_i / \partial y, \\ B_{74} = B_{85} &= N_i. \end{aligned}$$

With finite elements for the space discretization, the dynamic problem (in the absence of damping) gives rise to a set of ordinary differential equations of the form

$$\mathbf{M}\ddot{\mathbf{a}}(t) + \mathbf{K}\mathbf{a}(t) = \mathbf{P}(t), \quad (10)$$

in which the dots denote differentiation with respect to time, t , $\mathbf{a}(t)$ is the nodal displacement vector, \mathbf{M} is the mass matrix, and $\mathbf{P}(t)$ is the vector of nodal forces which varies with time, t . The elements of the stiffness matrix \mathbf{K} can readily be computed using the standard relation,

$$\mathbf{K}_{ij} = \sum_{e=1}^{\text{NE}} \mathbf{K}_{ij}^e = \sum_{e=1}^{\text{NE}} \int_{-1}^{+1} \int_{-1}^{+1} \mathbf{B}_i^T \mathbf{D} \mathbf{B}_j |J| d\xi d\eta. \quad (11)$$

The mass matrix \mathbf{M} in equation (10) is given by

$$\mathbf{M} = \sum_{e=1}^{\text{NE}} \mathbf{M}^e = \sum_{e=1}^{\text{NE}} \int_{\text{Area}} \mathbf{N}^T \mathbf{m} \mathbf{N} dA, \quad (12a)$$

where

$$\mathbf{m} = \begin{bmatrix} I_1 & & & \mathbf{0} \\ & I_1 & & \\ & & I_1 & \\ \mathbf{0} & & & I_2 \\ & & & & I_2 \end{bmatrix}$$

and

$$I_1, I_2 = \sum_{L=1}^n \int_{h_L}^{h_{L+1}} (1, z^2) \rho^L dz. \quad (12b)$$

in which I_1, I_2 are normal inertia and rotary inertias respectively; ρ^L is the material density of the L th layer.

The time integration scheme

As previously mentioned, the integration in time of equation (10) is performed using the explicit central difference time integration scheme. This scheme can be written as

$$\mathbf{a}^{n+1} = \mathbf{M}^{-1} [(\Delta t)^2 (-\mathbf{K}\mathbf{a}^n + \mathbf{P}^n) - \mathbf{M}\mathbf{a}^{n-1} + 2\mathbf{M}\mathbf{a}^n], \quad (13a)$$

where superscripts $n-1$, n , $n+1$ stand for three successive time stages and Δt is the time step length. The main advantage of this approach is that if \mathbf{M} is diagonal, the computation at each time step is trivial. Unfortunately, for the quadrilateral isoparametric plate elements used in the spatial discretization, \mathbf{M} is not diagonal and a special mass lumping scheme is used to bring about the diagonalization of \mathbf{M} .

A diagonal mass matrix that is more sophisticated than a lumped mass matrix can be derived from a consistent mass matrix and is discussed elsewhere [7].

After the mass matrix is diagonalised, equation (13a) can be rewritten as

$$a_i^{n+1} = [(\Delta t)^2 / M_{ii}] \left[- \sum_{j=1}^m K_{ij} a_j^n + P_i^n \right] - a_i^{n-1} + 2a_i^n, \quad (13b)$$

where m denotes the number of nodal variables. If values of a^0 and \dot{a}^0 are prescribed as initial conditions, a special starting algorithm can be written by noting that

$$\dot{a}^0 = (a^1 - a^{-1}) / 2\Delta t \quad (13c)$$

and eliminating a^{-1} from equation (13b) leads to the expression

$$a_i^1 = (\Delta t^2 / 2M_{ii}) \left[- \sum_{j=1}^{NN} K_{ij} a_j^0 + P_i^0 \right] + a_i^0 + a_i^0 \Delta t. \quad (13d)$$

Numerical results and discussion

In the present study the four-node linear, the eight-node serpendipity, and the nine-node Lagrangian quadrilateral isoparametric elements were employed. Since the element accounts for the transverse shear strains, reduced integration was employed to evaluate the shear terms numerically. That is, the 3×3 Gauss rule was used to integrate the bending and the inertia terms and the 2×2 Gauss rule was used to integrate the shear energy terms. All the computations were carried on a CYBER 180/840 Computer in single precision. Due to the biaxial symmetry of the problems discussed, only one quadrant of the plate was analysed except for angle-ply laminates, which are analysed as full plates. In all of the numerical examples presented herein, zero initial conditions were assumed.

The following three sets of data were used in obtaining the numerical results:

$$a = b = 25 \text{ cm}, \quad q = 10 \text{ N/cm}^2$$

Data 1.

Face sheets [3]

$$E_1 = 1.308 \times 10^7 \text{ N/cm}^2, \quad E_2 = 1.06 \times 10^6 \text{ N/cm}^2,$$

$$G_{12} = G_{13} = 6 \times 10^5 \text{ N/cm}^2, \quad G_{23} = 3.9 \times 10^5 \text{ N/cm}^2,$$

$$\rho = 1.58 \times 10^{-5} \text{ N s}^2/\text{cm}^4, \quad \nu = 0.28.$$

Core [2]

$$G_{23} = 1.722 \times 10^4 \text{ N/cm}^2, \quad G_{13} = 5.206 \times 10^4 \text{ N/cm}^2,$$

$$\rho = 1.009 \times 10^{-6} \text{ N s}^2/\text{cm}^4.$$

Data 2 [20]

$$E_2 = 2.1 \times 10^6 \text{ N/cm}^2, \quad E_1/E_2 = 25, \quad G_{12} = G_{23} = G_{13} = 0.5E_2,$$

$$\rho = 8 \times 10^{-6} \text{ N s}^2/\text{cm}^4, \quad \nu = 0.25.$$

Data 3

Face sheets [8]

$$E_x = E_y = 6.895 \times 10^6 \text{ N/cm}^2, \quad \nu = 0.33,$$

$$G_{xy} = G_{yz} = G_{xz} = 2.592 \times 10^6 \text{ N/cm}^2,$$

$$\rho = 2.821 \times 10^{-5} \text{ N s}^2/\text{cm}^4.$$

Core [8]

$$\rho = 1.242 \times 10^{-6} \text{ N s}^2/\text{cm}^4,$$

Case I: $G_{yz} = 5.17 \times 10^4 \text{ N/cm}^2, \quad G_{xz} = 1.344 \times 10^5 \text{ N/cm}^2,$

Case II: $G_{yz} = 5.17 \times 10^3 \text{ N/cm}^2, \quad G_{xz} = 1.344 \times 10^4 \text{ N/cm}^2,$

Case III: $G_{yz} = 5.17 \times 10^2 \text{ N/cm}^2, \quad G_{xz} = 1.344 \times 10^3 \text{ N/cm}^2.$

$\nu_{12} = (E_1/E_2) \nu_{21}$; directions 1 and x are coincident.

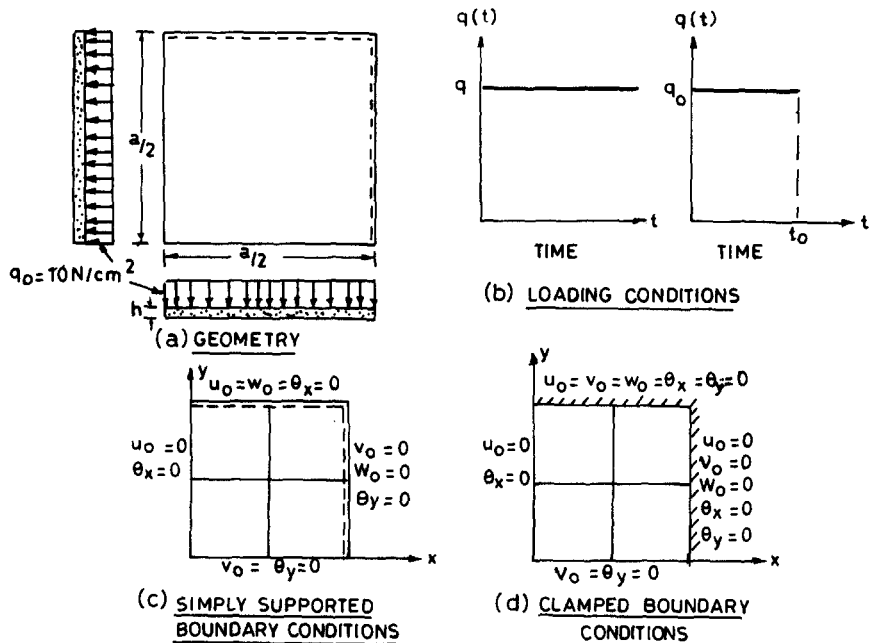


Fig. 2. Geometry, loading and boundary conditions.

Table 1
Convergence of centre deflection and stress for different time steps and meshes with 4-noded elements

Time (μs)	$w_0 \times 10^{-3}$ σ_x^{top}	$\Delta t = 4 \mu s$			$\Delta t = 5 \mu s$			$\Delta t = 10 \mu s$		
		2×2	3×3	4×4	2×2	3×3	4×4 ^a	2×2	3×3 ^a	4×4 ^a
40	w_0	0.2340	0.2350	0.2348	0.2339	0.2349		0.2337		
	σ_x	51.656	50.548	48.656	51.847	50.621		53.458		
80	w_0	0.9538	1.0019	1.0259	0.9541	1.0024		0.9569		
	σ_x	181.22	204.32	215.45	181.15	204.90		181.46		
120	w_0	1.8251	1.8784	1.8843	1.8256	1.8788		1.8302		
	σ_x	345.14	387.96	404.82	344.72	387.03		340.67		
160	w_0	2.1375	1.9669	1.9163	2.1372	1.9661		2.1345		
	σ_x	403.84	420.81	418.09	404.25	420.59		404.54		
200	w_0	1.6977	1.5661	1.5438	1.6970	1.5666		1.6905		
	σ_x	329.63	340.02	346.97	330.25	340.51		337.67		

Data 1, 5 layers, 0°/90°/core/0°/90°, $h_0 = h_{90} = 0.5$, $h_{core} = 3.0$, $a/h = 5$

^a Unstable

In order to investigate the numerical convergence and accuracy of the transient behaviour of the composite sandwich plate, a simply supported plate with a suddenly applied uniform pulse loading was analysed using Data 1. The plate geometry, a typical finite element mesh, boundary conditions of the quarter model, and applied loading are shown in Fig. 2. The estimate of the critical time step is crucial in transient analysis of composite sandwich plates. Tables 1, 2, and 3 present center deflection and normal stress of 4-, 8-, and 9-noded elements, respectively, for different meshes and time steps. From these tables it is found that the safe estimate of the critical time step length given by Leech [13], and Tsui and Tong [26] are valid only for isotropic plates. The estimate of the critical time step length given by Tsui and Tong [26] is used with minor modification in this study. It is thus given as

$$\Delta t \leq L_e \left[\frac{\rho(1 - \nu^2)/E_2 R}{2 + 0.83(1 - \nu)(1 + 1.5(L_e/h))^2} \right]^{1/2},$$

Table 2
Convergence of centre deflection and stress for different time steps and meshes with 8-noded elements

Time (μs)	$w_0 \times 10^{-3}$ σ_x^{top}	$\Delta t = 0.5 \mu s$			$\Delta t = 1 \mu s$			$\Delta t = 2 \mu s$		
		2×2	3×3	4×4	2×2	3×3	4×4	2×2	3×3 ^a	4×4 ^a
40	w_0	0.1573	0.2353	0.2487	0.1569	0.2346	0.2489	0.1553		
	σ_x	41.851	42.349	45.686	41.805	42.382	45.661	41.535		
80	w_0	1.0532	1.0679	1.0638	1.0536	1.0669	1.0661	1.0556		
	σ_x	228.62	227.34	227.22	228.71	227.23	227.47	229.07		
120	w_0	1.9578	1.8598	1.8680	1.9578	1.8587	1.8685	1.9574		
	σ_x	440.25	430.63	424.65	440.20	430.32	424.53	439.85		
160	w_0	1.9208	1.8650	1.8917	1.9199	1.8644	1.8943	1.9164		
	σ_x	425.16	425.41	424.73	425.01	425.08	425.33	424.37		
200	w_0	1.4901	1.5270	1.5189	1.4922	1.5289	1.5171	1.5009		
	σ_x	345.64	357.19	359.18	345.86	357.27	359.48	346.97		

Data 2, 5 layers, 0°/90°/core/0°/90°, $h_0 = h_{90} = 0.5$, $h_{core} = 3.0$, $a/h = 5$

^a Unstable

Table 3
Convergence of centre deflection and stress for different time steps and meshes with 9-noded elements

Time (μs)	$w_0 \times 10^{-3}$ σ_x^{top}	$\Delta t = 0.5 \mu\text{s}$			$\Delta t = 1 \mu\text{s}$			$\Delta t = 2 \mu\text{s}$		
		2x2	3x3	4x4	2x2	3x3	4x4	2x2	3x3	4x4 ^a
40	w_0	0.2344	0.2343	0.2346	0.2344	0.2343	0.2345	0.2343	0.2343	
	σ_x	45.419	45.093	45.510	45.418	45.099	45.487	45.412	45.142	
80	w_0	1.0779	1.0784	1.0798	1.0780	1.0785	1.0800	1.0784	1.0791	
	σ_x	236.14	234.72	234.54	236.24	234.77	234.59	236.65	235.09	
120	w_0	1.8508	1.8531	1.8473	1.8507	1.8531	1.8471	1.8505	1.8530	
	σ_x	425.06	422.86	418.83	424.98	422.83	418.78	425.05	422.81	
160	w_0	1.8947	1.8925	1.8923	1.8946	1.8926	1.8922	1.8944	1.8931	
	σ_x	424.74	422.96	422.24	424.53	422.91	421.99	423.61	422.50	
200	w_0	1.5279	1.5192	1.5247	1.5281	1.5192	1.5248	1.5286	1.5197	
	σ_x	368.00	364.94	366.39	368.02	364.93	366.25	367.57	364.96	

Data 2, 5 layers, $0^\circ/90^\circ/\text{core}/0^\circ/90^\circ$, $h_0 = h_{90} = 0.5$, $h_{\text{core}} = 3.0$, $a/h = 5$

^a Unstable.

where $R = E_1/E_2$, L_e is the smallest distance between adjacent nodes in any element used. The solution obtained by using a four-node element with $\Delta t = 4 \mu\text{s}$, and eight- and nine-node elements with $\Delta t = 1 \mu\text{s}$ is found suitable for all cases.

Table 4
Comparison of transverse deflections $w_0 \times 10^{-3}$ (cm) obtained in the present study with those obtained by the classical plate theory (CPT) and closed-form solution (CFS) [20] for a composite square plate (Data 2, $a/h = 5$) under suddenly applied sinusoidal pulse load

Lamination scheme ^a		Layers							
		1	2	3	4	5	6	7	8
CP, $R = 25$	Present	0.3582 (85)	0.4606 (98)	0.3399 (83)	0.2951 (77)	0.2929 (77)	0.2815 (75)	0.2825 (75)	0.2776 (75)
	CFS	0.3566 (90)	0.4604 (100)	0.3386 (85)	0.2947 (80)	0.2924 (80)	0.2809 (80)	0.2817 (80)	0.2765 (80)
	CPT	0.1272 (55)	0.3153 (85)	0.1272 (55)	0.1493 (60)	0.1272 (55)	0.1366 (55)	0.1272 (55)	0.1325 (55)
CP, $R = 40$	Present	0.3243 (82)	0.3833 (89)	0.2993 (78)	0.2448 (70)	0.2473 (70)	0.2352 (68)	0.2376 (69)	0.2323 (68)
	CFS	0.3233 (85)	0.3824 (90)	0.2985 (80)	0.2438 (75)	0.2473 (75)	0.2352 (70)	0.2376 (70)	0.2323 (70)
AP, $R = 25$	Present		0.3394 (84)		0.2283 (67)		0.2205 (66)		0.2180 (66)
	CFS		0.3387 (85)		0.2277 (70)		0.2196 (70)		0.2170 (70)
AP, $R = 40$	Present		0.2830 (76)		0.1986 (63)		0.1932 (62)		0.1915 (62)
	CFS		0.2826 (80)		0.1977 (65)		0.1922 (65)		0.1885 (65)

Values between parentheses denote the time (in μs) at which the maximum centre deflection occurred

^a CP = cross-ply ($0^\circ/90^\circ/0^\circ/\dots$); AP = angle-ply ($45^\circ/-45^\circ/45^\circ/\dots$); $R = E_1/E_2$; $q_0 = 10 \text{ N/cm}^2$. Boundary conditions used for angle-ply laminates are the same as given in [20]

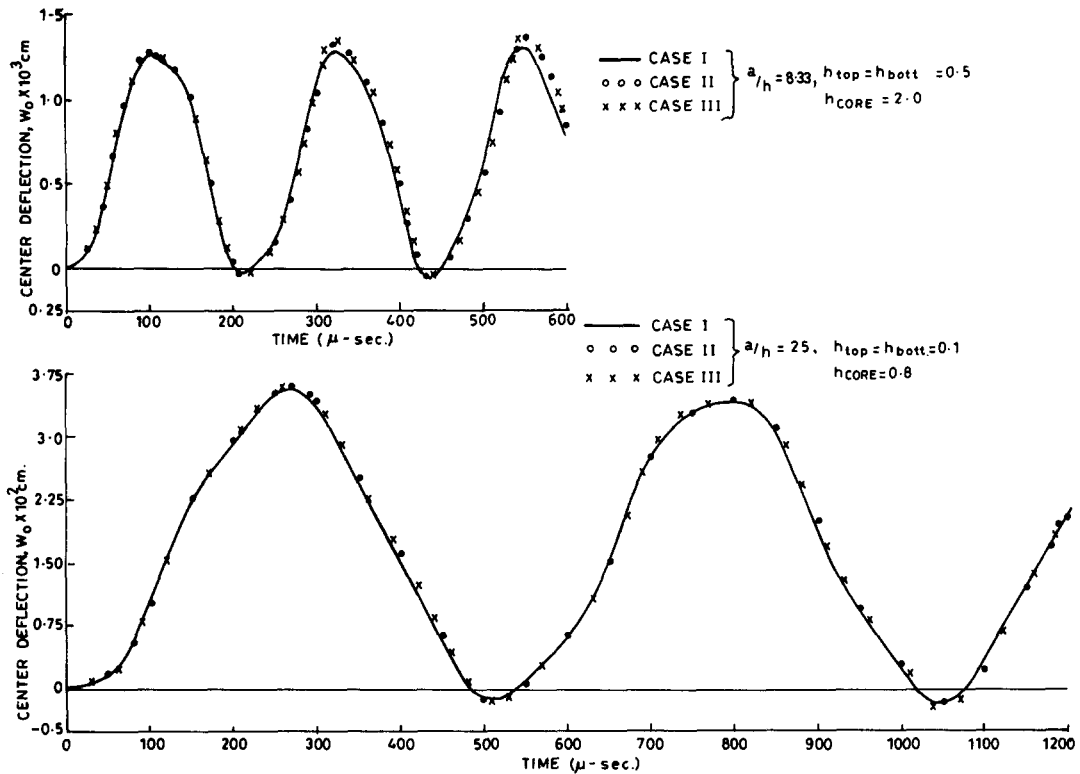


Fig. 3. Effect of shear and side-thickness ratio on the transient response of a three-layer clamped, square plate subjected to suddenly applied pulse loading (Data 3, $\Delta t = 2 \mu s$).

In order to validate the present element, a problem for which the analytical solution exists is solved. The problem consists of a simply supported square plate (with Data 2) subjected to a suddenly applied sinusoidally distributed pulse loading. A comparison of the effect of layers, shear deformation, lamination scheme, and orthotropy on the center transverse deflection obtained by the present element and by the closed-form solution [20] is shown in Table 4. It is seen that classical plate theory predicts significantly lower values of the deflection and the period. Since the present finite element solution is in excellent agreement with the exact solution, the 2×2 mesh with a nine-noded Lagrangian element was used in all of the problems to be discussed.

The effect of shear moduli of core materials and side-thickness ratio on the transient response of a three-layer clamped, square plate subjected to a suddenly applied uniform pulse loading (Data 3) is shown in Fig. 3. Although the effect of transverse shear moduli (G_{yz} and G_{xz}) of core material is not prominent, the deflection and period decrease with increasing thickness of the plate. The effect of plate thickness on the amplitude and period of the deflection is clear.

Figures 4 and 5 show plots of center deflection and normal stress for a five-layer simply supported plate (Data 1). From these figures, it is found that the effect of transverse shear moduli of stiff layers is very significant on the transient behaviour of sandwich plates. The composite sandwich plate, neglecting transverse shear moduli of stiff layers predicts higher values of deflection, period, and stresses. The effect of transverse shear moduli of stiff layers on the transient response of a sandwich plate is clear.

The material properties of Data 2 for facings and core material properties of Data 1 with a full plate model were used in the following example. To further investigate the effect of the

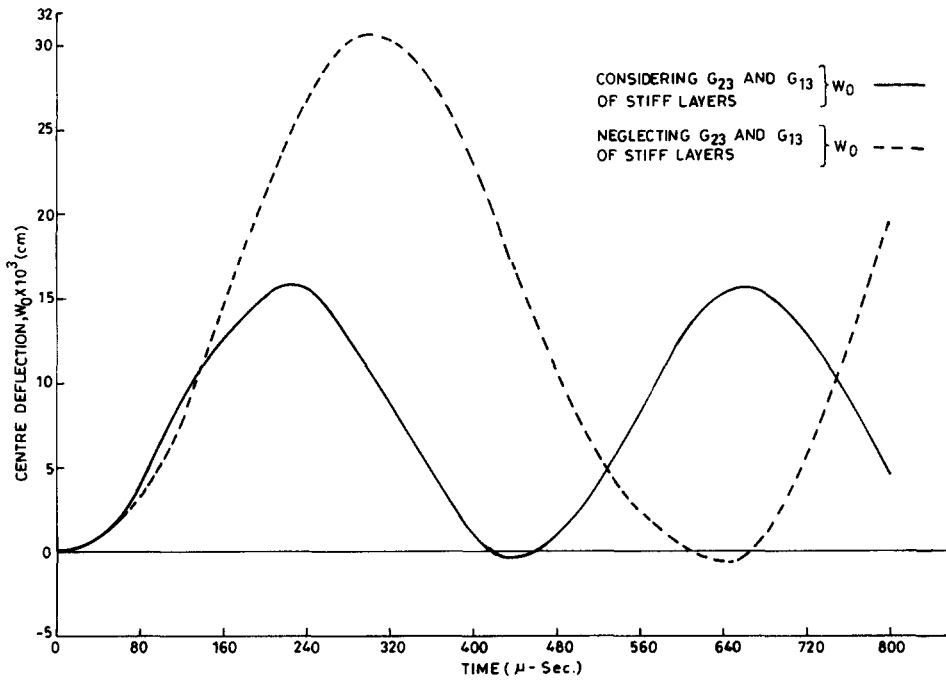


Fig. 4. Effect of transverse shear moduli of stiff layers on the transient response of five-layer ($0^\circ/90^\circ/\text{core}/90^\circ/0^\circ$) simply supported plates under suddenly applied pulse loading (Data 1, $\Delta t = 2 \mu\text{s}$, $a/h = 10$, $h_0 = h_{90} = 0.125$, $h_{\text{core}} = 2.0$).

angle and material orthotropy on the center transverse deflection and normal stress, a simply supported plate subjected to a suddenly applied sinusoidal loading was analysed by considering the full plate model (Fig. 6). The effects on the amplitude and period of the deflections and

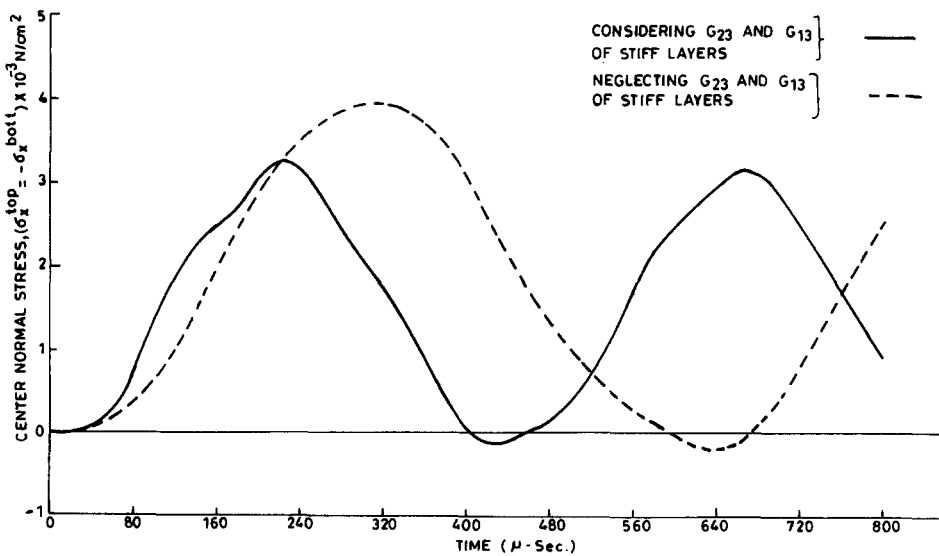


Fig. 5. Center normal stress versus time for five-layer ($0^\circ/90^\circ/\text{core}/90^\circ/0^\circ$) simply supported plates under pulse loading (Data 1, $\Delta t = 2 \mu\text{s}$, $a/h = 10$).

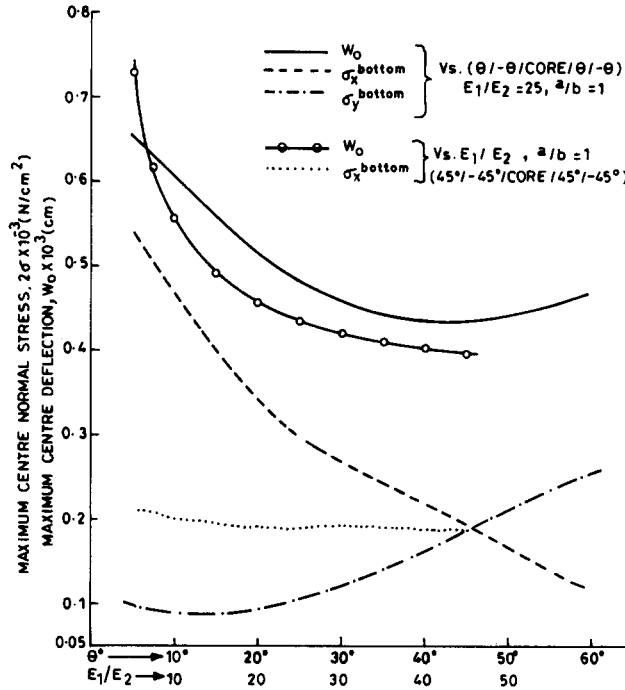


Fig. 6. Effect of lamination angle and material orthotropy on the solution of five-layer simply supported plates under suddenly applied sinusoidal loading (Data 1, $\Delta t = 1 \mu s$, 4×4 full plate, $h_\theta = 0.5$, $h_{core} = 3.0$, $a/h = 5$).

stresses are clearly nonlinear. Since no damping or internal friction is included in the present model, the solutions do not decay with time.

Conclusions

Numerical results of the transient dynamic analysis of composite sandwich plates based on Mindlin’s theory are presented. The shear flexible element employed herein is stable and accurate in predicting the transient response of sandwich plates. The transverse shear rigidity moduli of stiff layers on the transient response of sandwich plates are significant. All of the results presented herein for sandwich plates must be validated by an independent investigation. However, on the basis of the excellent agreement of the present results with analytical results for generally orthotropic plates, it is fair to say that the element is accurate in predicting the transient response of composite sandwich plates.

The extension of these analyses to nonlinear (large rotations) transient response of composite sandwich plates is awaiting attention. An accurate prediction of the wave propagation in composites can be made only if a three-dimensional theory of inhomogeneous plates is used. The analysis presented here can also be extended to certain shell problems.

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