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Shape Control of Intelligent Composite Stiffened Structures Using Piezoelectric Materials—A Finite Element Approach

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ABSTRACT: A finite element procedure is adopted for the analysis of composite stiffened structures with piezoelectric materials attached to it either at the top or bottom surfaces or inside the substrate as a composite layer. In the large space structures made of strong and light weight materials, the necessity of using self controlling strategies becomes important day by day for deformation control and suppression of undesired oscillation, specially for the unmanned space vehicles in the outer space. As the importance of stiffened composite configuration in aerospace structures has already been established for their optimum use of materials with high strength to weight ratio, a study of electro-mechanical interaction problem with these structures become necessary. Analysis of composite stiffened plates with piezoelectric sensors/actuators attached to it has been carried out for the first time. Static analysis of intelligent structures, i.e., structures integrated with distributed actuators and sensors, is presented in this work. The distributed sensors and actuator layers, made of piezoelectric materials, are perfectly bonded to the structures. The substrate is a laminated composite stiffened plate made of graphite/epoxy materials. To validate the present procedure, several experimental and analytical examples already solved by the previous researchers, have been compared. Finally, some new results for the stiffened composite plates, hitherto not published, have been included for future research.

INTRODUCTION

A NEW CLASS of structures, called "intelligent structures" has been developed recently in space research. The active research on intelligent structures started in late eighties after researchers realized the immense potential of these types of struc-

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tures having self controlling capabilities. The control of shape under static loading and vibration and noise control under dynamic situation becomes very important for unmanned vehicles in the outer space where sudden unaccounted load may come at any time. The piezoelectric materials, which have the capability of sensing when subjected to deformation and actuating when subjected to charge/voltage, are used as a distributed sensor/actuator to control the deformed shape of the structures. The entire process of shape control by actuation is done by piezoelectric materials with the help of induced strain. A special purpose FORTRAN code has been written for this purpose in Civil Engineering Department, IIT Bombay, for the theoretical simulation.

Induced strain actuation is the process by which actuation strain in some elements of the structures induces deformation of the overall structures. Actuation strain is the general term applied to strain components other than those caused by stress. Natural mechanisms that cause actuation strains include thermal expansion, piezoelectricity, electrostriction, magnetostriction or moisture absorption.

When the actuation strain mechanism is regulated, as in the case of piezoelectricity, induced strain actuation can be used to control the deformations of intelligent structures, i.e., the structures with distributed sensor and actuator systems. It can be used to control bending, extension and twisting of plate like structures without producing rigid body forces and torques or inertial loads. It has advantages over other types of actuation because the actuators can be easily integrated with load bearing structures bysurface bonding or embedding and do not significantly alter the passive static or dynamic stiffness characteristics of the structures. Used in conjunction with tailored anisotropic composite materials, control of specific static deformations or modes can be greatly enhanced. Some of the interesting applications of intelligent piezoelectric mounted structures are shape control of optical system mirror or reflector, antenna, etc., in static situation and flutter suppression, control of movement in flexible robotic arm, acoustic noise control, etc., in dynamic situation.

As the concept of intelligent structures emerges and the immense potential of distributed controlling parameters (sensors/actuators) gradually becomes well established, a good number of research papers [1–10] have been published on this subject. Analytical solutions of electroelastic problems using piezoelectric materials on the surface of plated structures have been noted in some of the recent papers [1–3]. Analytical methods give exact solutions to very specific cases of structural configurations with simple loading and boundary conditions having regular geometry. But the practical aerospace structures are too complicated to be solved by any analytical tools. So, the research has been directed to solve the aerospace structural problems (composite plates/shells) using some popular numerical technics, such as the finite element method. Some research workers concentrated their work on improving the mechanics part of piezoelectric effect on the structures [4,5]. Crawley and Luis [4] developed the necessary analytical tools for the structural analysis whereas, Lee [5] developed the equations of composite plates and piezoelectric materials considering both direct and reciprocal relationships using classical lamination theory. Crawley and Lazarus [6] developed the theoretical and numerical model to analyse the composite plates embedded with piezoelectric layers. Ritz's method has been used for this study. In this model, piezoelectric layers are considered as anisotropic laminae in the total laminate, this makes the modelling very easy and effective for composite structures. References [7–10] deal with the finite element application on structures having distributed sensors/actuators attached to it on the surface.Tzou and Tseng [7] developed a moderately thick solid finite element for the analysis of plates on which piezoelectric materials are attached on its surface. This element has internal degrees of freedom in the formulation which has to be condensed out for the final calculation which makes the entire process complicated. A higher order theory of Lo et al. [11] has been used by Ray et al. [8] to formulate a two-dimensional finite element for composite plates with piezoelectric materials. Otherwise acceptable, this formulation has the disadvantage of having large number of nodal degrees of freedomwhich makes it computationally expensive. Ha et al. [9] attempted another finite element formulation with eight noded brick element.To overcome the poor bending response of this element, some incompatible modes have been added to formulate the element. Robbins and Reddy [10] proposed a layer wise laminate theory considering actuation strain. The theory is applied to a composite beam problem where the actuation strain is simply assumed as an input to the problem.

Here, a simple formulation considering piezoelectric sheets as discrete anisotropic layers integrated to the structural system has been adopted using two-dimensional Lagrangian family of finite elements. The Mindlin-Reissner first order shear deformation theory has been considered in the formulation which makes this formulation applicable to both thin to moderately thick plates. In this formulation, the piezoelectric sheets can be placed anywhere in the structure either as a bonded layer at the plate surface or as an embedded layer inside the substrate. This formulation is computationally economic and applicable to most of the practical aerospace structures containing composite/isotropic plates/shells where engineering accuracy is required.

FORMULATION

Consistent Plate Model

The formulation of the laminated plate model follows the model of Crawley and Lazarus [6]. In the consistent plate actuation model, both the actuators and the structural plate substrates have been treated as plies of an integrated laminated plate. Hence, no assumptions regarding the actuator placement, actuator symmetry or the component isotropy have been made in the analysis. Any number of arbitrarily positioned and oriented surface bonded or embedded actuators, combined with substrates of arbitrary stiffness properties, can be modeled (Figure 1).

The term "consistent plate model" comes from the assumption of consistent deformations in the actuators and substrates. The in-plane extensional strain is assumed as constant thickness wise whereas the bending strain varies linearly through the thickness. This assumption performs extremely well in almost all the situations. The strain in the system will therefore depend on the midplane strain ε^0 and κ.

$$\varepsilon = \varepsilon^0 + z,\kappa$$

$$\varepsilon^0 = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} \tag{1}$$

$$\kappa = \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \theta_x}{\partial x} \\ \dfrac{\partial \theta_y}{\partial y} \\ \dfrac{\partial \theta_x}{\partial y} + \dfrac{\partial \theta_y}{\partial x} \end{Bmatrix}$$

The constitutive relation for a homogeneous plate, or for any ply of a laminated plate is

$$\sigma = Q(\varepsilon - \Omega) = Q\varepsilon - Q\Omega \tag{2}$$

where the stress vector and the actuation strain vector are

$$\sigma = [\sigma_x \sigma_y \sigma_{xy}]^T \qquad \Omega = [\Omega_x \Omega_y \Omega_{xy}]^T \tag{3}$$

The matrix Q is the transformed reduced stiffness of the plate or one of its plies in the plate axis system [12]. The second term $Q\Omega$ in Equation (2) represents the equivalent

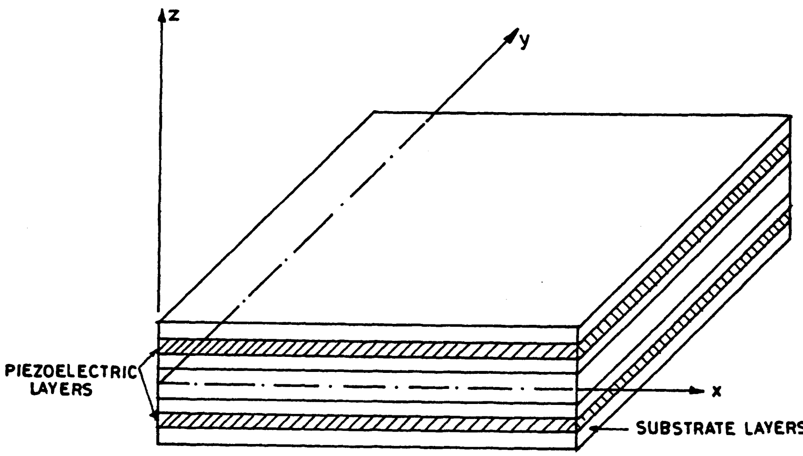

Figure 1. Laminated plate with piezoelectric and substrate layers bonded together.

stress created as a result of the actuation strains. The actuation strain vector Ω contains in-plane normal and shear strain components, and enters into the elasticity equations in the same manner as does thermal strain. Actuation strain is the strain that physically causes induced strains to be produced, and can be due to thermal expansion, piezoelectricity, electrostriction, etc. The consistent plate load deformation relations can be found by substituting the assumed deformation [Equation (1)] into the stress-strain [Equation (2)] and integrating through the thickness h of the plate

$$\begin{bmatrix} \overline{N} \\ \overline{M} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} - \begin{bmatrix} N_\Omega \\ M_\Omega \end{bmatrix} \tag{4}$$

where the conventional mechanical force and moment resultants are

$$\overline{N} = \int_h \sigma dz \qquad \overline{M} = \int_h \sigma z dz \tag{5}$$

The matrices A, D and B are the usual extensional, bending and extensional-bending coupling stiffnesses of the plate [12]. The equivalent actuator forces N_Ω and moments M_Ω per unit length are

$$N_\Omega = \int_h Q\Omega dz \qquad M_\Omega = \int_h Q\Omega z dz \tag{6}$$

Care must be taken in performing the necessary integrations to obtain the correct stiffness and actuator forcing terms. Both actuator and substrate plies contribute to the stiffness matrices A, B and D whereas only actuator plies contribute to the actuator forcing vectors N_Ω and M_Ω, which are dependent on the mode of actuation (prescribed actuation strains).

Extensional actuation is effected by prescribing actuation strains that are symmetric about the neutral axis, while bending actuation results when the actuation strains in the actuator above the neutral axis are prescribed to be in opposition (180° out of phase) to those below the neutral axis.

Equation (4) relates the resultant total strains and curvatures found in the actuator/substrate system to the actuation strains, external loads, and stiffness properties of the system in a general and compact form. The presence of numerous coupling terms shows that it is possible to create a variety of deformations (e.g., bending or twisting) using several different actuation modes (e.g., extension or bending). Thus, by careful selection of the laminate ply orientations, an actuator/substrate system can be designed to effect control for a variety of applications. In addition, various couplings may be introduced by the boundary conditions, as would be the case for a swept cantilever wing.

As the exact analytical solutions for induced strain plate equations are difficult and applicable to very limited number of cases, the application of some numerical tool is essential to solve complex practical problems. Here, finite element method is used to solve the equations in hand. The consistent plate model can be used to formulate the strain energy relations for a laminated plate.The total potential energy stored in a plate undergoing induced strain actuation is

$$U = \frac{1}{2}\iint_A \{\varepsilon^{0T}\kappa^T\}\begin{bmatrix} A & B \\ B & D \end{bmatrix}\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} d(A) - \iint_A [N_\Omega M_\Omega]\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} d(A) \quad (7)$$

This strain energy equation, along with finite element method has been used to solve for the approximate strains and curvatures induced in the plate actuator systems.

Finite Element Solution—Matrix Form

As closed form solutions are intractable for most of the practical problems with complicated geometry, boundary conditions and loading, approximate solution procedures like finite element have been resorted to to solve the problem at hand. Finite element procedure is simple but accurate enough for the engineering problems to give insight into the physics of induced actuation.

Solutions are sought for extensional and bending actuation of systems with extensive stiffness couplings. Therefore, it is essential that the in-plane as well as out-of-plane deformations are adequately represented. The plate element has got both in-plane and out-of-plane displacements, and the displacement vector is defined as

$$\delta = \{uvw\theta_x\theta_y\}^T \tag{8}$$

where u and v are the longitudinal (along x-axis) and transverse (along y-axis) displacements respectively, w is the out-of-plane displacement (normal to x-y plane) and θ_x and θ_y are the total rotations about y and x axes respectively. From isoparametric relationships, the coordinates and displacements inside the element can be defined as

$$x_i = \sum_{i=1}^{NN} N_i x_i \qquad y_i = \sum_{i=1}^{NN} N_i y_i \qquad \delta_i = \sum_{i=1}^{NN} N_i \delta_i \tag{9}$$

Using the expressions for the displacements, Equation (8), the strains and the curvatures can be expressed as

$$\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} = [H]\{\delta\} \tag{10}$$

where $[H]$ is the standard strain-displacement matrix.

An expression for the strain energy in terms of the assumed shape functions can be found by substituting Equation (10) into strain energy Equation (7)

$$U = \frac{1}{2}\delta^T K\delta - \Psi_\Omega \delta$$

$$K = \iint_A [H]^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} [H]\, d(A) \tag{11}$$

$$\Psi_\Omega = \iint_A [H]^T \begin{Bmatrix} N_\Omega \\ M_\Omega \end{Bmatrix} d(A)$$

In the energy expression above, K is the element stiffness matrix and Ψ_Ω can be described as the actuation strain forcing vector, since it represents the forces acting on the assumed shape functions that develop from the actuation strains.

The static and dynamic response can be calculated by combining the strain energy expression with appropriate energy principles, such as Lagrange's equations.

RESULTS AND DISCUSSIONS

The present finite element formulation and the computer code developed has been verified by comparing the analysis results from other published work. The simulation results from the present work have been found to be quite close to the other available datas.

Cantilever Composite Plate $[0/\pm 45]_s$

This problem has been solved previously by Crawley et al. [6] and Ha et al. [9]. Crawley et al. [6] analysed the problem with a Ritz's model and also through experiment and Ha et al. [9] used 3-D brick finite element model. The cantilever laminated composite plate has been surface bonded by distributed G-1195 piezoelectric ceramics. Figure 2 shows the dimensions of the plate and the positions and thickness of piezoelectric ceramics. The material properties have been taken from Ha et al. [9].

Figure 3 shows the comparison of the deflections of a $[0°/\pm 45°]_s$ composite plate between the prediction and the test result. W_L, W_T and W_R in the figure are non-dimensional deflections (longitudinal bending, transverse bending and the lateral twisting respectively) and are defined as follows:

$$W_L = \frac{M_2}{C}$$

$$W_T = \frac{M_2 - (M_1 - M_3)/2}{C}$$

$$W_R = \frac{M_1 - M_3}{C}$$

where C is the width of the plate and M_1, M_2 and M_3 are the lateral deflections from the sensors as shown in the Figure 2. The simulation results from the present analysis (shown as firm line in the figure) matches quite well with the other published results.

Cantilever Composite Plate $[+30^\circ_2/0^\circ]$

This cantilever plate has been solved by Crawley and Lazarus [6] by experimental method and Ritz's solution technique. Figure 4 shows the analysis results of the problem from the present simulation and that of Crawley et al. [6]. The results show close agreement. All the parameters of the plate and distributed piezoelectric layers

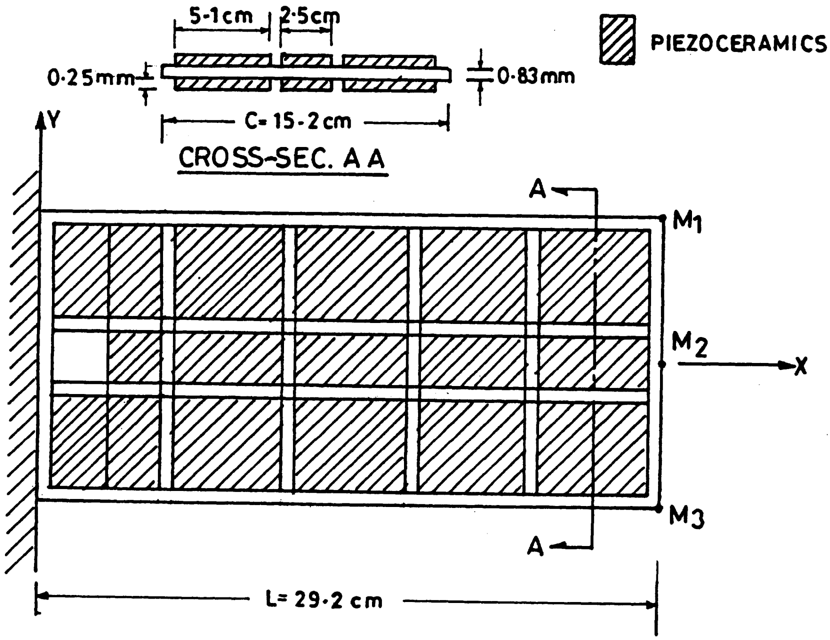

Figure 2. *Cantilevered composite plate containing surface-bonded distributed piezoelectric actuators.*

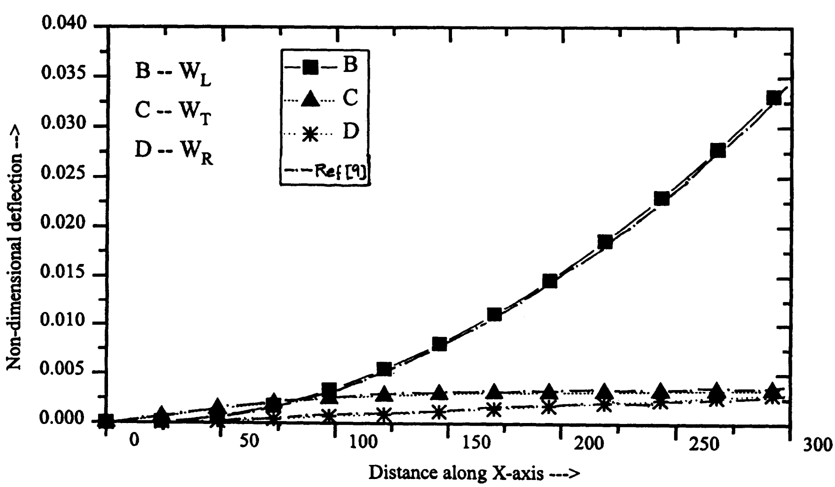

Figure 3. *Cantilever plate with distributed piezoelectric layers.*

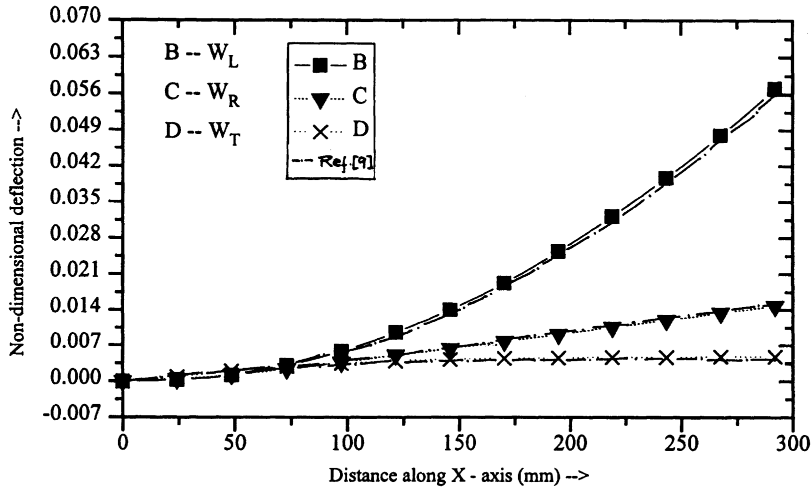

Figure 4. *Cantilever composite plate with piezoelectric layers.*

and also material properties are same as that of the previous problem (Figure 2). The results are shown in the form of non-dimensional deflections and the corresponding deflection parameters are also kept unchanged (W_L, W_T and W_R).

Composite Plate under Thermal Loading

A composite plate of laminae orientation $[0°/\pm45°]_s$ and of dimension 22.8 × 37.2 cm^2 has been considered next. This problem has previously been solved by Ha et al. [9]. All the material properties used in this study are same as that used by Ha et al. [9]. The plate was originally flat and was simply supported along two parallel edges and free on the other two edges. The plate has been exposed to an elevated environment with temperature increase of 50°F on one side and with temperature drop by –50°F on the other side. It is assumed that the temperature distribution throughout the plate has reached thermal equillibrium (stationary). Due to the thermal gradient, the structure has deformed into a curved shape, which can be determined from the code. The calculated deformed configuration of the structure resulting from the temperature changes has been ploted along the central line of the plate and shown in Figure 5.

The objective of the numerical simulation was to determine the amount of the electric potential to be applied to the piezoceramics such that the out-of-plane deflection of the plate could be minimized at all times. In this calculation, all the piezoceramics were considered as actuators to control the deformations of the structure. In the present study, a slightly higher potential was required to minimize the vertical deflection than that of Ha et al. [9] which can be attributed to the difference of approaches of the two works.

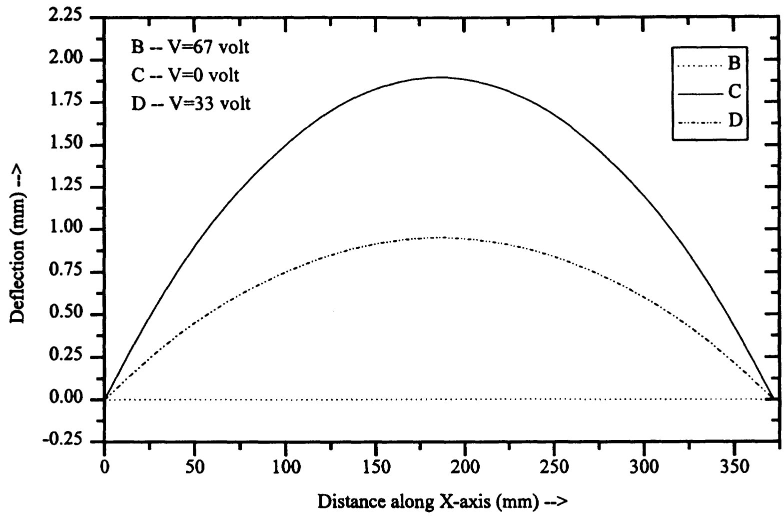

Figure 5. *Shape control of composite plate under thermal loading using piezoelectric layers.*

Shape Control of Stiffened Composite Plates

Stiffened structures are considered as the most effective aerospace structures in terms of higher strength-to-weight ratio and better stiffness-to-weight ratio. Moreover, when these stiffened structures are made of composite materials, the advantage of light weight and high strength material properties can be best exploited. The stiffened composite configurations are very common in aircraft, missiles, under water vehicles and space vehicles. The control of undesired deformations are of prime importance in those structures. Here, piezoelectric materials have been bonded at the top and bottom surface of the stiffened composite plates to actuate the induced strain to effectively control the unwanted deflections. As far as the authors' knowledge goes, the shape control of stiffened composite plates using piezoelectric materials attached have been carried out for the first time.

For all the examples considered here, plate dimensions considered are 200 × 200 mm^2, the total thickness of the plate is 1.0 mm. The stiffener dimensions are 4.0 mm (depth) × 1.0 mm (width). The plates considered here are cross-stiffened with two stiffeners and are placed centrally in global x and y directions. The laminae orientations for stiffeners are always $[0°/0°]_s$ for x-directional and $[90°/90°]_s$ for y-directional stiffener (i.e., the plies are running along the stiffener length). Uniformly distributed load considered over the entire plate is 0.0005 N/mm^2 for all the cases. For all the cases, the boundary conditions are simply supported with in-plane motion perpendicular to the edges restricted. For all the problems analysed, the material properties are taken from Ha et al. [9].

Symmetrically Stiffened Composite Plates $[0°/\pm 45°]_s$

For the first case, the $[0°/\pm 45°]_s$ ply orientation has been studied. The composite stiffened composite plates with piezoceramics attached top and bottom have been studied for the first time in the open literature. Figure 6 shows the transverse deflections along the centerline of the plate for three different conditions. The deflection profiles of the plate without any voltage across the piezoelectric layers and with two different voltage across have been shown in the figure. Externally applied 433V/mm is found to be good enough to reduce the plate deflection to almost horizontal level.

Symmetrically Stiffened Composite Plates $[+30^{\circ}_{2}/0°]_s$

For the second case of this study, $[+30^{\circ}_{2}/0°]_s$ laminae orientations for plate have been analysed. Configuration of plate and stiffeners, and also the positions of the piezoelectric materials are exactly same as the previous case. Figure 7 shows the deflected shape of the plate without and with two different applied voltages. Externally applied 432 V/mm across the piezoceramic layers at the top and bottom is found to be well enough to give engineering accuracy for vertical deformation control.

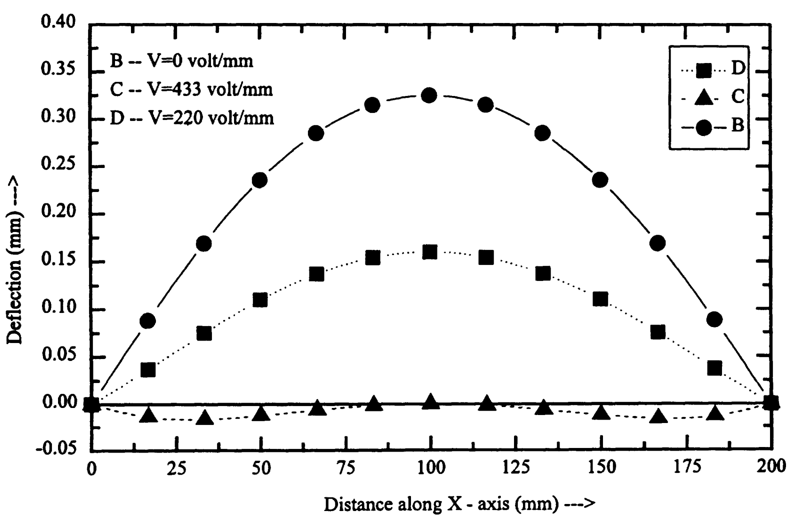

Figure 6. *Symmetrically stiffened composite plate (symmetric angle-ply, $[0/+45/-45]_S$).*

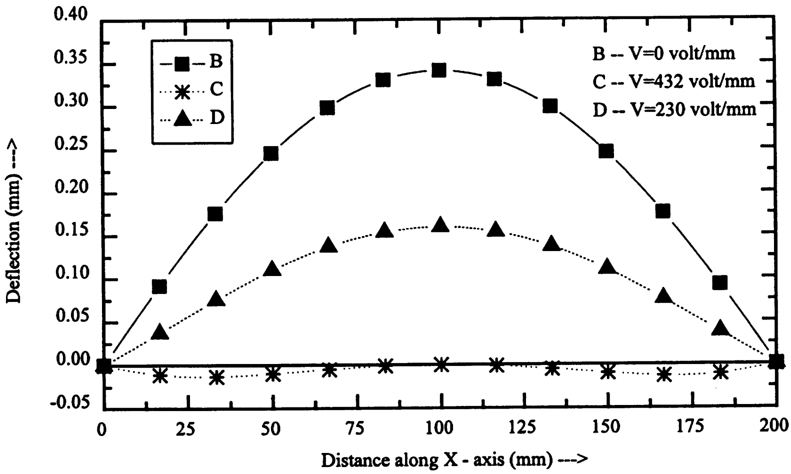

Figure 7. *Symmetrically stiffened composite plate ([+30/+30/0]$_s$ angle-ply).*

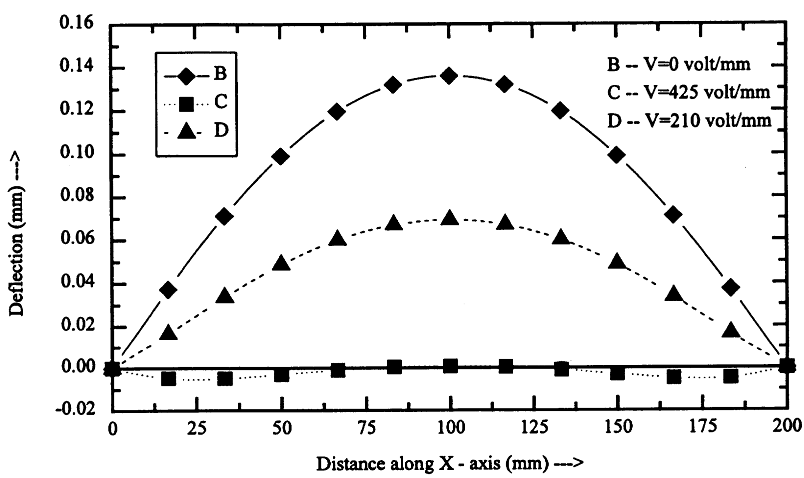

Figure 8. *Eccentrically stiffened composite plate (symmetric cross-ply, [0/90/0]$_s$).*

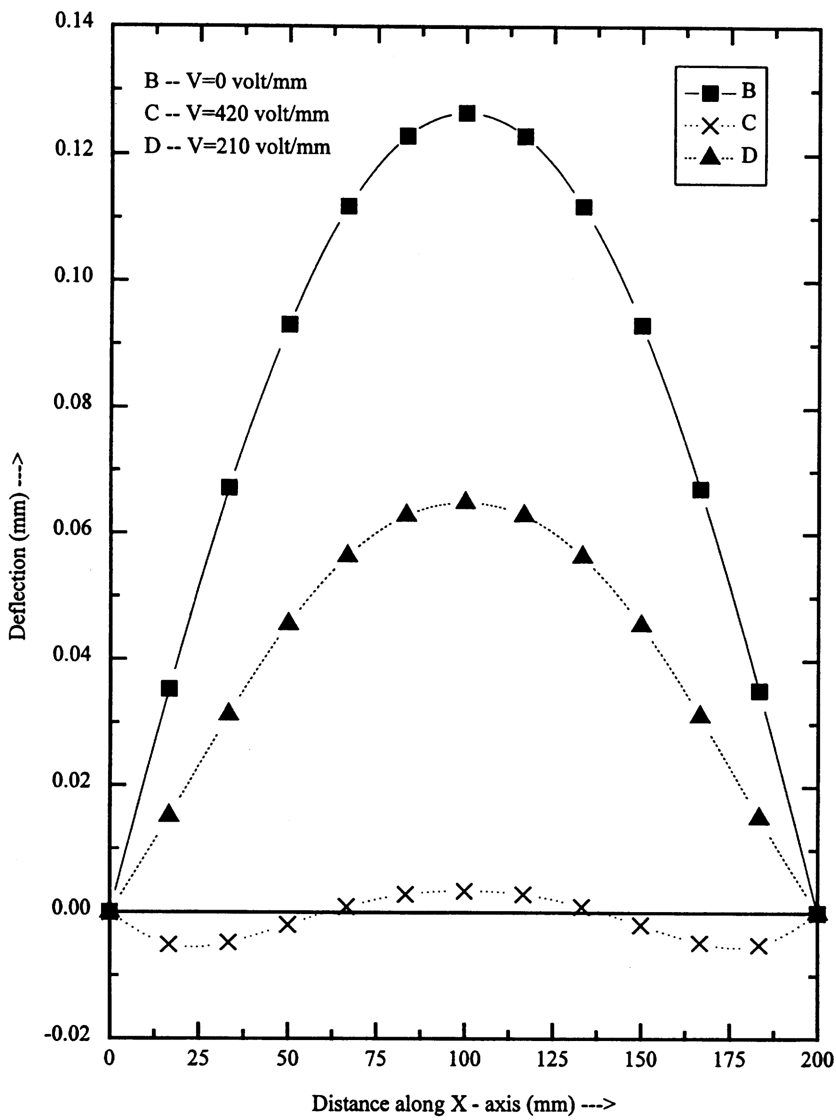

***Figure 9.** Eccentrically stiffened composite plate (symmetric angle-ply $[+45/-45/0]_s$).*

Eccentrically Stiffened Composite Plates

The eccentricity of the stiffener helps to increase the stiffness of the plate to a great extent. So, eccentricity makes the stiffened structure to use the material most effectively with highest possible achievement of strength and stiffness. Hence, eccentrically stiffened composite plates are also studied in this work.

Case 1: Laminae Orientation—$[0°/90°/0°]_s$

As the eccentrically stiffened configurations are more suitable for aerospace structures for its higher load bearing capacity with minimum materials used, some problems on this type of structures have also been solved here. For the first case, a cross-stiffened composite plate with $[0°/90°/0°]_s$ laminae orientation is studied. Figure 8 shows the deflection pattern of the plate with and without applied voltage. Externally applied 425 V/mm is found to be strong enough to achieve a shape control of the structure with very little vertical deformation.

Case 2: Laminae Orientation—$[\pm45°/0°]_s$

In the second case, the plate laminae orientations are $[\pm45°/0°]_s$. For the stiffeners, ply orientations are as mentioned above. Figure 9 shows the deflection pattern of the plate under mechanical and electromechanical condition. Externally applied 420 V/mm is found suitable for the effective shape control of this plate with very small transverse deflection.

CONCLUSIONS

A finite element formulation for the electromechanical problem for stiffened composite plates using piezoelectric materials has been presented. The piezoelectric layer can be placed anywhere in the plate system either at the surfaces (top and bottom) or embedded inside the substrate. The piezoceramic layer is treated as an anisotropic layer along all the other layers in the substrate. This flexibility of modelling makes the formulation very attractive to apply when the structure is made of composite materials. The formulation is verified by comparing present simulation results with other available published results. Finally, hitherto unpublished results of the shape control of composite stiffened plates have been carried out using the present formulation to give some new results for future reference.

ACKNOWLEDGEMENTS

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