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Review

A critical review of recent research on functionally graded plates

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ABSTRACT

In view of the significant increase in research activity and publications in *functionally graded materials* (FGMs) and structures in the last few years, the present article is an attempt to identify and highlight the topics that are most relevant to FGMs and structures and review representative journal publications that are related to those topics. A critical review of the reported studies in the area of thermo-elastic and vibration analyses of *functionally graded* (FG) plates with an emphasis on the recent works published since 1998. Because of the extensive growth in the body of knowledge in FGMs in the last two decades, it is prudent to reduce the review to a manageable level by concentrating on the FG plate problems only. The review carried out here, is concerned with deformation, stress, vibration and stability problems of FG plates. This review is intended to give the readers a feel for the variety of studies and applications related to graded composites. An effort has been made here, to include all the important contributions in the current area of interest. The critical areas regarding future research needs for the successful implementation of FGM in design are outlined in the conclusions.

Contents

1. Introduction

In the development of our society and culture, materials have played an essential role. The scientific use of available base materials into various inorganic and organic compounds has made the

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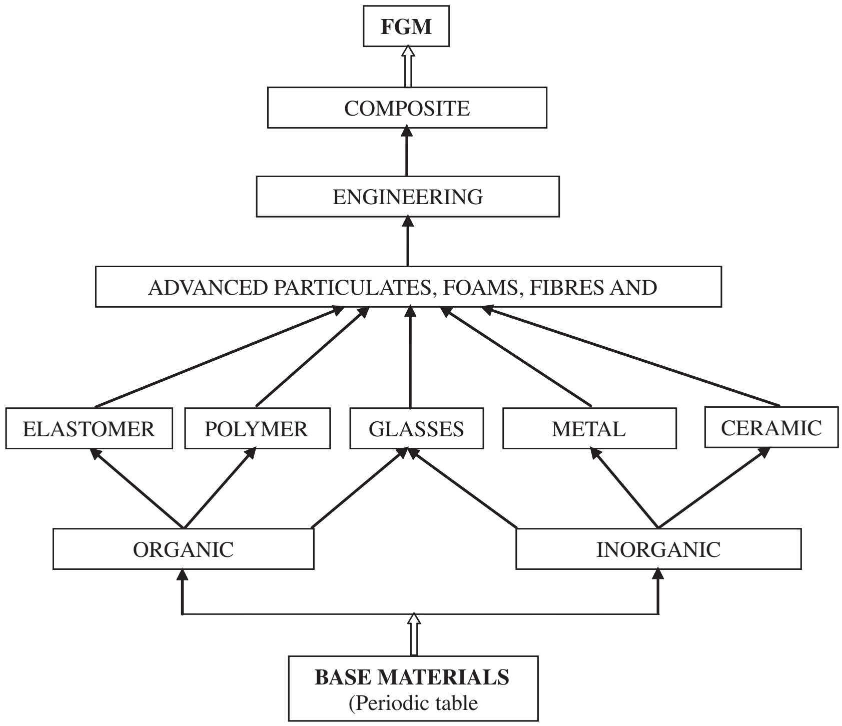

Fig. 1. Representation of modern material hierarchy.

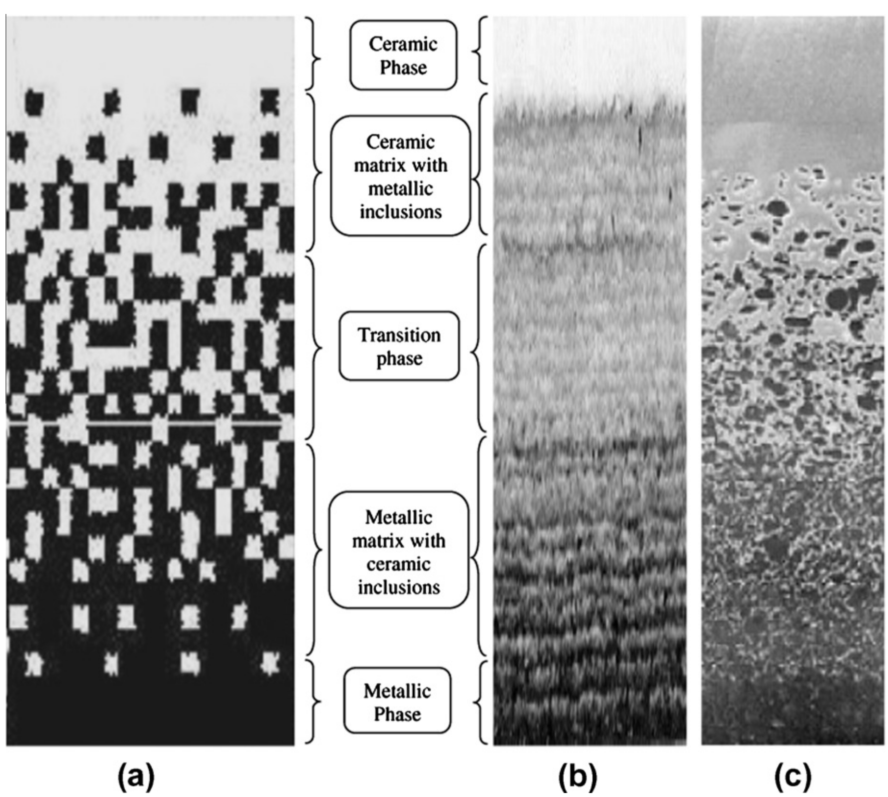

Fig. 2. Schematic of continuously graded microstructure with metal-ceramic constituents (a) Smoothly graded microstructure (b) Enlarged view and (c) Ceramic–Metal FGM.

path for developing the advanced polymers, engineering alloys, structural ceramics, etc. The structure of development of modern material is illustrated in Fig. 1. *Functionally graded materials* (FGMs) are the advanced materials in the family of engineering composites made of two or more constituent phases with continuous and smoothly varying composition [1]. These advanced materials with engineered gradients of composition, structure and/or specific properties in the preferred direction/orientation are superior to homogeneous material composed of similar constituents. The mechanical properties such as Young's modulus of elasticity, Poisson's ratio, shear modulus of elasticity, and material density, vary smoothly and continuously in preferred directions in FGMs. FGMs have been developed by combining the advanced engineering materials in the form of particulates, fibers, whiskers, or platelets. In the continuous drive to improve structural performance, FGMs are being developed to tailor the material architecture at microscopic scales to optimize certain functional properties of structures. These materials are gaining wide applications in various branches of engineering and technology with a view to make suitable use of potential properties of the available materials in the best possible way. This has been possible through research and development in the area of mechanics of FGMs for the present day modern technologies of special nuclear components, spacecraft structural members, and high temperature thermal barrier coatings, etc. These materials possess numerous advantages that make them appropriate in potential applications. It includes a potential reduction of in-plane and through-the thickness transverse stresses, improved thermal properties, high toughness, etc. FGMs consisting of metallic and ceramic components are well-known to enhance the properties of thermal-barrier systems, because cracking or de-lamination, which are often observed in conventional multi-layer systems are avoided due to the smooth transition between the properties of the components. By varying percentage contents of volume fractions of two or more materials spatially, FGMs can be formed which will have desired property gradation in spatial directions. De-lamination has been a problem of main concern in the reliable design of advanced fiber reinforced composite laminates. In laminated composites, the separation of layers caused by high local inter-laminar stresses result in destruction of load transfer mechanism, reduction of stiffness and loss of structural integrity, leading to final structural and functional failure. To eliminate these problems, FGMs have now gained importance, and are the latest advanced materials, discovered by material scientists for innovative engineering applications. The most common FGMs are metal/ceramic composites, where the ceramic part has good thermal resistance and metallic part has superior fracture toughness. A continuously graded microstructure with metal/ceramic constituents is represented in Fig. 2 schematically for illustration.

1.1. History of FGMs

Although the concept of FGMs, and our ability to fabricate them, appears to be an advanced engineering invention, the concept is not new. These sorts of materials have been occurring in nature. Some examples for natural FGMs have been included in Fig. 3 for

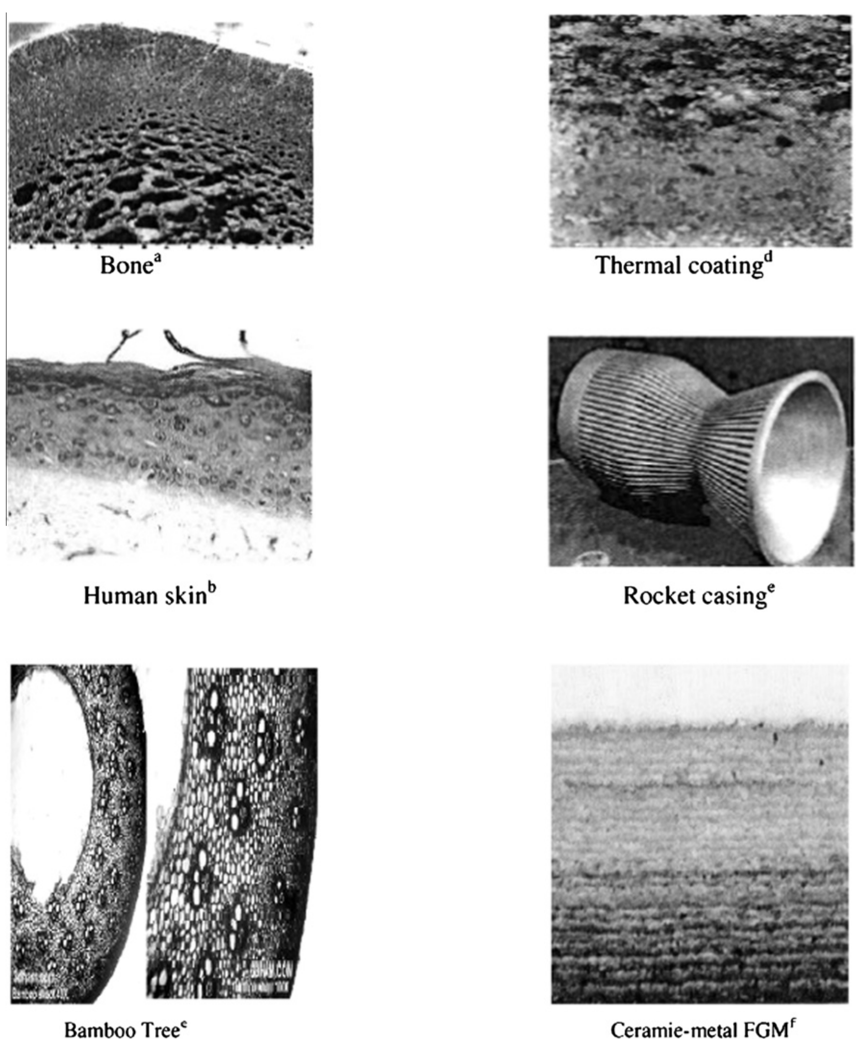

Fig. 3. Some examples of FGMs (naturally occurring and engineered by humans). (a) http://www.geo.ucaIgary.ca~-macrae/tt~rigins/~arbbones/dinobone.html, (date: 01.03.2012). (b) http://Nigb.fraunhofer.de/www/presse/jahn/9I99/dt/PIIHautmode11.dtthtml, (date: 01.03.2012). (c) http://www.3dham.com/microgallery/bamboo.html, (date: 01.03.2012). (d) [191]. (e) http://www.science.nasa.govheadlines/y2004/13apr_gradient.htm, (date: 01.03.2012). (f) http://www.docstoc.com/docs/48428855/Functionally-Graded-Materials-An-Introduction, (date: 01.03.2012).

illustration. Bones have functional grading. Even our skin is also graded to provide certain toughness, tactile and elastic qualities as a function of skin depth and location on the body. The FGM constituents engineered by humans commonly involve two isotropic material phases; although any numbers of chemically and spatially compatible configurations are possible. These components often include the engineering alloys of magnesium, aluminum, copper, titanium, tungsten, steel, etc. and the advanced structural ceramics such as zirconia, alumina, silicon-carbide, and tungsten-carbide. Some examples of human engineered FGM components currently under development are also included in Fig. 3.

1.2. Application of FGMs

FGMs have great potential in applications where the operating conditions are severe, including spacecraft heat shields, heat exchanger tubes, biomedical implants, flywheels, and plasma facings for fusion reactors, etc. Various combinations of the ordinarily incompatible functions can be implemented to create new materials for aerospace, chemical plants, nuclear energy reactors, etc. For example, a discrete layer of ceramic material is bonded to a metallic structure in a conventional thermal barrier coating for high temperature applications. However, the abrupt transition in material properties across the interface between distinct materials can cause large inter-laminar stresses and lead to plastic deformation or cracking [2]. These harmful effects can be eased by smooth spatial grading of the material constituents. In such cases, large concentrations of ceramic material are placed at corrosive, high temperature locations, while large concentrations of metal are placed at regions where mechanical properties need to be high. The application of these advanced materials was first visualized during a space plane project in 1984 in National Aerospace Laboratory of Japan to avoid the stress peaks at interfaces in coated panels for the space shuttle. Combination of materials used here served the purpose of a thermal barrier system capable of withstanding a surface temperature of 2000 K with a temperature gradient of 1000 K across a 10 mm thick section. Later on, its applications have been expanded to also the components of chemical plants, solar energy generators, heat exchangers, nuclear reactors and high efficiency combustion systems. The concept of FGMs has been successfully applied in thermal barrier coatings where requirements are aimed to improve thermal, oxidation and corrosion resistance. Two important research material systems in fabrication technology of FGMs are: Alumina 'Al_2O_3' [3] and Zirconia 'ZrO_2' [4] exterior protective ceramic layers on Ni-superalloy 'NiCrAlY' based substrates. Consequently, coatings were deposited by different metallurgical techniques. In thermoelectric field, the concept of graded material, such as doped BiTe/PBFe has been implemented for application in sensors and thermogenerators with metal–semiconductor transition with improved efficiency. FGMs can also find application in the communication and information techniques. Abrasive tools for metal and stone cutting are other important examples where gradation of surface layer has improved performance. As a final observation concerning FGMs, it can be noted that these graded materials concept has demonstrated that compositional micro/macrostructure gradient can not only dismiss undesirable effects such as stress concentration, but can also generate unique positive function [4]. The concept of FGMs is applicable to various fields as illustrated in Fig. 4.

1.3. Effective material properties (homogenization) of FGMs

The fabrication of the FGMs can be considered by mixing two discrete phases of materials, for example, a distinct mixture of a metal and a ceramic. Often, the accurate information of the shape and distribution of particles may not be available. Thus the effective material properties, viz. elastic moduli, shear moduli, density, etc. of the graded composites are being evaluated based only on the volume fraction distribution and the approximate shape of the dispersed phase. Several micromechanics models have been developed over the years to infer the effective properties of macroscopically homogeneous composite materials. The analytical approaches, both finite element methods and micromechanical models are frequently used for FGM modeling. The most important subjects of FGM modeling are: elastic strain, elastic stress, plastic yielding and deformation, creep at elevated temperature, crack propagation, etc. The various analytical approaches available in the literature for FGM modeling are presented in the following sections.

1.3.1. Self consistent estimates [5–7]

This method describes its estimates through the solution of an elastic problem in which an ellipsoidal inclusion is embedded in a matrix possessing the effective material properties of the composites. This method assumes that each reinforcement inclusion is embedded in a continuum material whose effective properties

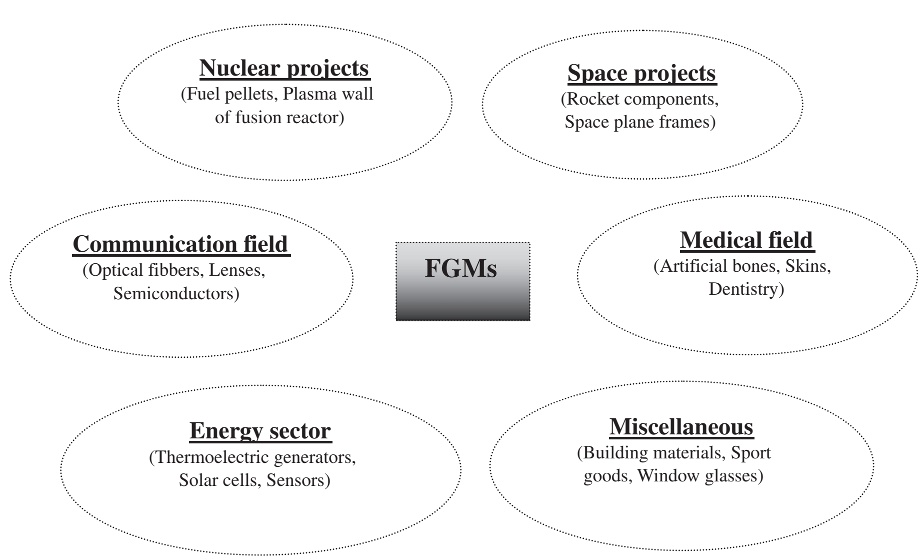

Fig. 4. Potential fields for the application of FGMs [192].

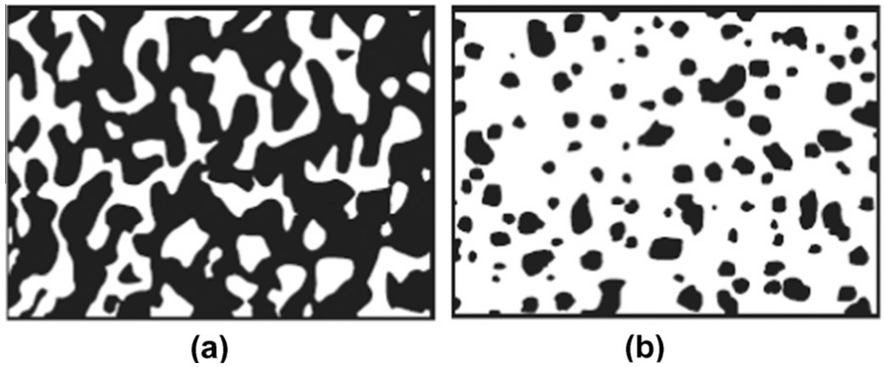

Fig. 5. Two-phase material with (a) skeletal microstructure, and (b) particulate microstructure [48].

are those of the composite. This method does not distinguish between matrix and reinforcement phases and the same overall moduli are predicted in another composite in which the roles of the phases are interchanged. This makes it particularly suitable for determining the effective moduli in those regions which have an interconnected skeletal microstructure as shown in Fig. 5a. This is a rigorous analytical method applicable to two-phase isotropic composite materials.

1.3.2. Mori–Tanaka scheme [8,9]

Such a method works well for composites with regions of the graded microstructure have a clearly defined continuous matrix and a discontinuous particulate phase as illustrated in Fig. 5b. This method assumes a small spherical particle embedded in a matrix. The matrix phase (denoted by the subscript 1), is assumed to be reinforced by spherical particles of a particulate phase (denoted by the subscript 2). K_1, G_1 and V_1 represents the bulk modulus, the shear modulus and the volume fraction of the matrix phase respectively; whereas K_2, G_2 and V_2 denote the corresponding material properties and the volume fraction of the particulate phase. It should be noticed that $V_1 + V_2 = 1$. The effective mass density at a point can be given by the rule of mixture ($\rho = \rho_1 V_1 + \rho_2 V_2$).

1.3.3. Composite sphere assemblage model [10,11]

In this model, the effective properties of isotropic composite materials have been determined analytically, which is based on the simplifying assumption that the composite material is filled with a fractal assemblage of spheres embedded in a concentric spherical matrix of different diameters such that the spheres completely fill the volume of the composite.

1.3.4. Composite cylindrical assemblage model [12,13]

This model is used for orthotropic composites and requires both the reinforcing fiber and matrix are isotropic, while the *representative volume elements* (RVEs) microstructure is transversely isotropic in material planes that are perpendicular to the fiber direction.

1.3.5. The simplified strength of materials method [14,15]

This is a popular modeling method due to its ease of implementation and computational efficiency. This method assumes that the matrix phase is reinforced with, and ideally bonded to, a periodic array of square fibers. This method can also be used to estimate the orthotropic strengths of fiber reinforced composite laminate from the strength properties of the fiber and matrix constituents and the fiber volume fraction.

1.3.6. The method of cells [16]

This is similar to Chamis's method [14] of simplified strength of materials, but more computationally rigorous since it assumes a representative volume element that involves a larger portion of matrix material.

1.3.7. Micromechanical models [17,18]

These models of representative volume elements may be constructed via FE simulations for either isotropic or orthotropic composite materials. Methods involving FE models attempt to accurately simulate the realistic microstructure of the RVE, and determine the thermo-mechanical response due to applied loads such that the effective material properties may be calculated for various volume fractions of constituent reinforcement. In this manner, various sets of curve fitted data may be collected for different material combinations. This is perhaps the most accurate method, since the microstructure under consideration is directly modeled via three-dimensional finite elements. Unfortunately, one drawback to this method is that multiple models must be constructed in order to determine material properties for various constituent material volume fractions; although this can be alleviated with proper computer software that can automate the process.

1.4. Mathematical idealization of FGMs

Although FGMs are highly heterogeneous, it will be very useful to idealize them as continua with their mechanical properties changing smoothly with respect to the spatial coordinates. The homogenization schemes are necessary to simplify their complicated heterogeneous microstructures in order to analyze FGMS in an efficient manner. Closed-form solutions of some fundamental solid mechanics problems can be obtained by this idealization and also it will help in evolving and developing numerical models of the structures made of FGMs. It is worth noting that, the distribution of material in FG structures may be designed to various spatial specifications. A typical FGM represents a particulate composite with a prescribed distribution of volume fractions of constituent phases. The material properties are generally assumed to follow gradation through the thickness in a continuous manner. Two types of variations/gradations are popular in the literature which covers most of the existing analytical models.

1.4.1. The exponential law

This particular idealization for FGM modeling is very common in the fracture mechanics studies [19]. For a structure made of FGM with uniform thickness 'h', the typical material properties '$P(z)$' at any point located at a distance 'z' from the reference surface is given by;

$$P(z) = P_t \exp\left(-\lambda\left(1 - \frac{2z}{h}\right)\right), \text{ where, } \quad \lambda = \frac{1}{2}\ln\left(\frac{P_t}{P_b}\right) \tag{1}$$

1.4.2. The power law

This is more common in the stress analysis of FGM [19] and given by;

$$P(z) = (P_t - P_b)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_b \quad (2)$$

Here '$P(z)$' denotes a typical material property, viz., Young's modulus of elasticity (E), shear modulus of elasticity (G), Poisson's ratio (υ), material density (ρ), etc. of the structures made of FGM. 'h' is the total thickness of structure. 'P_t' and 'P_b' are the material properties at the top-most ($z = +h/2$) and bottom-most ($z = -h/2$) surfaces. 'λ' in the exponential model, and 'k' in the power model are the material grading indexes respectively. Working range of these material grading indexes depend upon the design requirements.

2. Research studies reported on FG plates

Pagano [20,21], Srinivas and Rao [22] and Srinivas et al. [23] developed the exact solutions of simply supported laminated plates by using 3D elasticity theory. Their benchmark solutions have proved to be very useful in assessing the accuracy of various 2D approximate plate theories by various researchers [24–31]. Their methods are valid for laminated plates and shells, where the material properties are piecewise constant, but not applicable for finding solutions of plate problems with continuous in-homogeneity of material properties such as FGMs. The concept of FGMs was proposed in 1984 by Japanese material scientists [32]. Continuous changes in the composition, microstructure, porosity, etc. of these materials result in the gradients of the properties such as mechanical strength, thermal conductivity, and fracture toughness. Suresh and Mortensen [19] provided an excellent introduction to the fundamentals of FGMs. They published a very detailed literature review in FGM technology. Since then, numerous investigators have attempted variety of analytical, numerical methods for studying the mechanical, thermal and dynamic responses of structures made of FGMs. Birman and Byrd [33] have documented an exhaustive literature review of developments in FGM research addressing the recent progress in the characterization, modeling, and analysis of FGMs. Several topics relevant to theory and applications of FGMs are reflected in this review paper. It includes homogenization of particulate FGM, heat transfer issues, stress, stability and dynamic analyses, testing, manufacturing, design, applications, and fracture studies of FGMs. Here, the literature review is focussed on the research works in the area of thermo-elastic static, vibration and stability analyses of *functionally graded* (FG) plates published since 1998.

2.1. Thermo-elastic static analysis of FG plates

A review of the current state of the art in analytical and numerical thermo-elastic static studies of FG plates is presented. The integration of thermal protection system (ceramic) and load bearing mechanical component (metal) into a single construction is a desirable feature of FGMs. A candidate for FGM system for a particular application must exhibit its ability to resist thermal and mechanical loadings simultaneously.

The thermo-elastic behavior of FG rectangular ceramic–metal plates was presented using a four-noded rectangular isoparametric plate FE by Praveen and Reddy [34] based on *first order shear deformation theory* (FOST) including the von Karman nonlinear effects. This formulation accounts for the transverse shear strains, rotary inertia and moderately large rotations of FG plates. This is one of the earliest and widely used studies related to thermoelastostatic and thermoelastodynamic responses of FG plates subjected to pressure loads and through-thickness varying temperature fields. The FG plate was considered to be a single layer plate of uniform thickness, and its material properties were assumed to vary through the thickness in terms of a simple power law distribution. The general conclusion of this study was that, the response of the plates with material properties between those of the ceramic and metal is not intermediate to the responses of the ceramic and metal plates individually. Praveen and Reddy's formulation for rectangular FG plates was extended to an axi-symmetric formulation by Reddy et al. [35] for circular and annular FG plates in bending. They developed exact relationships between the bending solutions of the FOST and the *classical plate theory* (CPT). The solutions for the deflection, force resultants, and moment resultants were given in terms of corresponding equivalent quantities of isotropic plates based on CPT. The formulation adopted by Praveen and Reddy in 1998 was further extended by Reddy [36] for studying the static behavior of FG rectangular plates based on a *third order shear deformation theory* (TSDT). He derived the displacement based plate finite element models consistent with the TSDT (conforming element with eight degrees of freedom, and a nonconforming element with seven degrees of freedom per node) for the analyses of FG plates. This formulation accounts for the thermo-mechanical coupling, time dependency, and the von Karman type geometric nonlinearity. He has presented the Navier solutions based on the linear TSDT. Nonlinear static and dynamic finite element results based on the FOST were also presented in the study to show the effects of volume fractions and modulus ratio of the constituents on deflections and transverse shear stresses.

Mian and Spencer [37] presented a set of exact solutions of the 3D elasticity equations for traction-free rectangular and circular isotropic FG plates from the corresponding planar problems. They derived an exact solution of 3D elasticity equations for isotropic linearly elastic, inhomogeneous materials actually generalized from the solutions for stretching and bending of symmetrically inhomogeneous plates. The investigators showed that the exact 3D solutions are generated by 2D solutions of the thin-plate equations for a homogeneous plate. They actually developed a procedure for constructing the exact solutions of the linear elasticity equations of the plates in an inhomogeneous isotropic material assuming the elastic modulii depend in any specified manner on a specific direction.

Oatao and Tanigawa [38] have analysed the problems of transient thermal stresses in rectangular FG plates due to non-uniform heat flow. The same authors have also published the exact solutions for transient temperature and stress problems in a thick simply supported FGM strip [39]. The FG strip, assumed in the state of plane strain, was subjected to heat flow resulting from a sudden application of non-uniform surface temperatures. Further, Ootao et al. [40] applied a genetic algorithm to an optimization problem of material composition for step-formed FG plates, analyzing it as a laminated composite plate consisting of numerous layers with homogeneous and different isotropic material properties. The thermal stress components for infinitely long FG plate were formulated under the mechanical condition of being traction-free. They have carried out the numerical calculations using a genetic algorithm methodology accounting the effect of the temperature dependency of material properties without assuming a distribution function of material composition.

Cheng and Batra [41] have derived field equations for a FG plate and further these equations were simplified for a simply supported polygonal plate. They established an exact relationship between the deflection of a simply supported FG polygonal plate given by the FOST and TSDT to that of an equivalent homogeneous Kirchhoff plate. The effective material properties at a point of FG plate was assumed to be governed by the rule of mixture, and the volume fraction of the ceramic phase to follow a power law distribution through plate thickness. The same authors [42] studied the 3D thermo-mechanical deformations of an isotropic linear thermo-elastic FG elliptic plates, rigidly clamped at all the edges using the asymptotic expansion method considering the material properties having power-law dependence on the thickness coordinate. They obtained a closed form solution which shows that the distribution of the in-plane displacements and transverse shear stress along the thickness of a FG plate do not agree with those assumed in classical and shear deformation plate theories. Furthermore, a new set of field equations in terms of displacement and stress potential functions for inhomogeneous plates was presented and reformulated by Cheng [43] employing mixed Fourier series technique to solve the equations. Solution for nonlinear bending of transversely isotropic symmetric shear-deformable FG plates is attempted in this paper. A further advancement in FG plate analysis was provided by Reddy and Cheng [44] to address the 3D thermo-elastic behavior of simply supported FG rectangular plates under thermal and mechanical loads on its top and/or bottom surfaces. Using an asymptotic expansion approach for the heat conduction problem, the distributions of temperature, displacements and stresses in the plate were calculated for different volume fraction of ceramic constituent in this paper. The interesting conclusion of this study was that while the standard assumption of a constant through-the-thickness deflection is acceptable in the case of mechanical loads, it may become invalid in the case of thermal loading.

Han et al. [45] proposed analytical and numerical methods for analyzing the response of FG plate to an incident pressure wave using the Fourier transform techniques. They also characterized the material property of FGM excited by a pressure wave, whose elastic constants and mass density vary quadratically in the thickness direction. The propagation of stress waves in a FG plate is computed in this study. Further, Han and Liu [46] presented a computational method to investigate the simple harmonic waves in FG plates by varying the volume fraction of the constituents assuming the material properties as a quadratic function in the thickness direction. The displacements and stresses in the frequency domain and time domain were obtained using inverse Fourier integration.

Woo and Meguid [47] have studied the nonlinear deformations of thin FG plates and shallow shells based on the von Karman classical nonlinear plate theory under thermo-mechanical loads. The solution was obtained with double Fourier series for deflections and for the stress functions. A comparison between stresses and displacements in purely ceramic, purely metallic, and FG plates are presented in this study. The authors concluded that, the deflections in a FG plate even with a small volume fraction of ceramic are significantly smaller than those in the pure metallic plate. Furthermore, while the stress distributions in isotropic metallic or ceramic plates are linear functions of the thickness coordinate, they become nonlinear in a FG plate, reflecting a nonuniform property distribution through the thickness. This observation reflects the previously emphasized potential for a better "tailoring" of FGM structures compared to their homogeneous counterparts.

Vel and Batra [48] presented the exact 3D elasticity solutions of the static thermo-mechanical problems of a simply supported rectangular thin and thick FG plates. The thermal and mechanical loads were imposed to the top and bottom surfaces of the plate either individually or simultaneously. Suitable temperature and displacement fields identically satisfying the boundary conditions at the edges were used to reduce the governing equations (partial differential equations) to a set of coupled ordinary differential equations in the thickness coordinate, subsequently solved by employing the power series method. The key assumptions in this study were that the FG plate had smoothly varying material properties graded through the thickness (no discrete jumps) and infinitesimally small, homogeneous, isotropic and perfectly bonded adjoining layers. The homogenization schemes employed in the paper included the Mori–Tanaka method, the self-consistent scheme, and a combination of these two methods. They assumed the distribution of ceramic and metallic phases through the thickness to follow a power law for material volume fractions. The exact solutions of displacements and stresses were then used to assess the accuracy of the solutions obtained by CPT, FOST and TSDT for FG plates. Further, Vel and Batra [49] have also presented the analytical solutions of the 3D transient heat conduction problem for a rectangular simply supported FG plate, based on the uncoupled, quasi-static linear thermo-elasticity theory. The uniform temperatures are prescribed at the edges and either time-dependent temperature or heat flux is considered on the top and the bottom surfaces of the FG plate. The transient thermally induced stresses in FG plates were related to the mode of the application of thermal load in this study. The authors concluded that rapidly applied temperature boundary conditions could result in thermal stresses in an Al/SiC FG plate exceeding the steady-state counterparts by the factor of 8. The stresses produced by a transient heat flux were smaller than the steady-state stresses.

Pitakthapanaphong and Busso [50] studied the stress distribution and the effect of material grading in a three-layered plate consisting of a FGM layer sandwiched between ceramic and metal layers subjected to a uniform thermal load. They also accounted the plastic effects in the metal phase. The results were validated by making a comparison with the finite element results. The critical temperature corresponding to the onset of plasticity was determined as a part of the solution. The stress distribution was shown to be effectively controlled by an appropriate gradation in the FGM layer.

Both, the nonlinear bending and buckling behavior of FG plates were considered by Shen and his collaborators. In particular, nonlinear bending of thin FG rectangular plates clamped along a pair of opposite edges and with various conditions on the other pair of edges was studied by Yang and Shen [51]. They used von Karman nonlinear plate theory in this study. The solution was obtained by a semi-analytical perturbation technique combined with the one-dimensional differential quadrature approximation and the Galerkin procedure. An elastic foundation was included into consideration. The loads could include transverse pressure as well as in-plane compression and the authors emphasized that classical buckling can occur only in clamped plates, while plates with other boundary conditions receive transverse deflections, even if the applied load is small, as a result of the bending-stretching coupling. The analysis was further extended by the same authors [52] to the geometrically nonlinear shear-deformable plates subject to thermo-mechanical loads and under various boundary conditions using Reddy's TSDT. Boundary conditions applied on the FG plates are shown to have a profound effect on deflections with a pair of in-plane movable and a pair of in-plane immovable edges subjected to a simultaneous effect of an elevated temperature and transverse pressure. Other paper of this research group dealing with various aspects of thermo-mechanical responses of FG plates includes [53]. Shen [54] has also presented the post-buckling responses of simply supported FG plates subjected to the combined action of mechanical, electrical and thermal loads. Huang et al. [55] have presented the exact 3D elasticity solutions of FG thick plates resting on Winkler–Pasternak type elastic foundation considered as the boundary condition. The material properties of FG plate were assumed to be varying exponentially through the thickness. The governing set of PDEs are solved using the state space method by reducing it to ordinary differential equations in the thickness coordinate by expanding the state variables into infinite dual series of trigonometric functions. The effects of foundation stiffness, loads, and material grading index on mechanical responses of the plates are studied in this paper. The main conclusion of this study is that, the mechanical behavior of the plate with the softer surface supported by elastic foundation differ significantly from that of the plate with the harder surface subjected to the same foundation, especially for the thick plates. The same problem using a *higher order shear deformation theory* (HOST) and general von Karman-type relations is presented by Shen and Wang [56] for nonlinear bending analysis. The FG plate is exposed to elevated temperature and is subjected to a transverse uniform or sinusoidal load combined with initial compressive edge loads. The main conclusion of this study is that the effect of material grading becomes weaker for the plate supported on an elastic foundation. The characteristics of nonlinear bending are significantly influenced by foundation stiffness, temperature rise, transverse shear deformation, the character of in-plane boundary conditions and the amount of initial compressive load.

Zhong and Shang [57] presented the 3D analysis for a rectangular plate made of orthotropic FG piezoelectric material, simply supported along its four edges. The mechanical and electrical properties of the material were graded according to the exponential-law along the thickness direction. They considered the material properties used by Cheng and Batra [41] for presenting the 3D asymptotic approach to inhomogeneous and laminated piezoelectric plates.

Pan [58] derived an exact solution for simply supported rectangular FG anisotropic laminated plate using the pseudo-Stroh formalism extending the Pagano's solution to the FG plates. Further, Pan and Han [59] also derived an exact solution for simply supported FG and layered magneto-electro-elastic plates.

Ma and Wang [60] investigated the axi-symmetric large deflection nonlinear problems of bending and buckling behavior of simply supported and clamped FG circular plates based on the classical nonlinear von Karman plate theory, under mechanical, thermal and combined thermal–mechanical loadings. The axi-symmetric thermal post-buckling behavior of FG circular plate is also investigated in this study. The mechanical and thermal properties of FGM are assumed to vary continuously through the thickness of the plate, and obey a simple power law of the volume fraction of the constituents. Governing equations were numerically solved using a shooting method. The effects of material constants and boundary conditions on the temperature distribution, nonlinear bending, critical buckling temperature and thermal post-buckling behavior of the FG plate were studied in this paper. The same authors, in 2004 further published their study of both axi-symmetric bending behavior due to a uniform pressure and buckling behavior due to radial compression applied to circular FG plates by modeling it on the basis of TSDT and CPT. They showed that the CPT can adequately predict the response of FG plates of typical dimensions.

The thermal stresses in a ceramic–metal plate subjected to through-thickness heat flux using the Mori–Tanaka scheme and the classical laminated plate theory (modified to deal with inelastic deformations) was examined by Tsukamoto [61].

Bilgili et al. [62] studied the non-homogeneous rubber-like slab considering shear strains subjected to a thermal gradient in the thickness direction. They found in their study that it is possible to design a FG rubberlike material with a minimal effect of temperature on the magnitude of shear stresses.

A 3D elasticity bending solution for the stresses in a simply supported FG plate subjected to transverse loading was also presented by Kashtalyan [63]. He assumed the Young's modulus of the FG plate to vary exponentially through the thickness with constant Poisson's ratio. His approach makes use of general solution of the equilibrium equations for inhomogeneous isotropic media developed earlier by Plevako [64].

Qian et al. [65] studied the transient thermo-elastic deformations of a thick FG plate with boundary conditions either simply supported or clamped. The stresses and deformations due to the simultaneous application of the transient thermal and mechanical loads were computed keeping the plate edges at uniform temperature. They modeled FG the plate by a HOST and solutions obtained by *mesh-less local Petrov–Galerkin* (MLPG) method. They found that the centroidal deflection and the axial stress induced at the centroid of the top surface of the plate are significantly influenced by boundary conditions applied at the plate edges. Furthermore, it was found that inertia forces often have a negligible effect on deformations and stresses of thick FG plates generated by transient thermal loads. Qian et al. [66] further presented the plane strain static thermostatic deformations of a thick rectangular simply supported FG elastic plate using the same methodologies. They assumed material modulii to vary only in the thickness direction. The plate material is made of two isotropic randomly distributed constituents and the macroscopic response is also modeled as isotropic. Displacements and stresses computed in the study were found to agree very well with those obtained from the 3D exact solutions of the problem.

Croce and Venini [67] developed a hierarchic family of finite element for the analysis of rectangular FG plates based on variational formulation arising from the Reissner–Mindlin's plate theory by assuming material properties to vary with a simple power rule of mixture in terms of volume fractions of the constituent. Chinosi and Croce [68] have further approximated the problem with a simple locking-free discontinuous Galerkin finite element of nonconforming type, choosing a piecewise linear nonconforming approximation for both rotations and transversal displacement. The capability of the proposed element to capture the properties of plates of various grading, subjected to thermo-mechanical loads was discussed with the help of several numerical simulations in this study.

Lanhe [69] derived equilibrium and stability equations of a moderately thick rectangular simply supported FG plate under thermal loads based on the FOST. He assumed the power law variation of material properties along thickness of plate. The buckling temperatures are derived in this study considering two types of thermal loading, viz. uniform temperature rise and temperature-gradient through the thickness.

Elishakoff [70] performed a 3D flexural analysis of clamped FG plates under uniformly distributed load based on the linear theory of elasticity and applying the Ritz energy method. They emphasized on the significant effect of material grading in particular, by showing the deformations and axial stresses in a FG ceramic–metal plate do not necessarily exist between the values of pure ceramic and pure metal plate.

Ferreira et al. [71,72] have investigated the static deformations of simply supported FG plates using TSDT and a meshless method considering the collocation multi-quadric *radial basis functions* (RBFs). The effective material properties were calculated by using the rule of mixtures and the Mori–Tanaka scheme.

Ramirez et al. [73] presented an approximate solution for the static analysis of 3D, anisotropic, elastic FG plates by a discrete layer approach incorporating the transition functions reflecting the effect of material gradation into the governing equations. In the numerical examples for simply supported graphite/epoxy FG plates, significant decrease in deflections and in-plane normal stresses were observed by the proper gradation of the material. They have also examined the homogeneous, graded, and bi-layer plates in order to study the potential advantages of using FGM.

The bending problem of simply supported FG sandwich ceramic–metal panels has been considered by Zenkour [74]. The panels made of isotropic and homogeneous ceramic core and FGM facings were studied, assuming the power law variation of ceramic and metal constituents through the thickness. The formulations were done by the CPT, FOST, and a "sinusoidal" version of a shear deformation plate theory. Zenkour [75] further presented the static response of FG plates using a generalized shear deformation plate theory considering the effective material properties vary according to the power-law through thickness. Further the same author, Zenkour [76] has presented the 2D trigonometric solution for FG plate bending problems assuming their material properties to vary exponentially in thickness direction. The influences of aspect ratio, side-to-thickness ratio and the exponentially graded parameter on the plate bending response were investigated by him. Zenkour [77] has also studied the static response of FG plates subjected to hygro-thermo-mechanical loadings and resting on elastic foundation using the sinusoidal plate theory. The stress and displacement response of the plates have been analyzed under uniform loading. The author concluded in this study that the bending response of the FG plate deteriorates considerably with the increase in temperature and moisture concentration.

Yang et al. [78] studied the effect of uncertainties in the material properties and loading on the bending response of thick FG plates subjected to lateral pressure and uniform temperature. The analysis was conducted using the Reddy's TSDT combined with a first-order perturbation technique accounting for the random nature of the problem.

Bhangale and Ganesan [79] carried out the static analysis of simply supported FG and layered magneto-electro-elastic plates exponentially graded in the thickness direction. They assumed series solution in the plane of the plate and adopted finite element procedure across the thickness of the plate accounting for coupling between magnetic, elasticity, and electric three-dimensional effects. The magneto electric coupling is neglected in the study. This study may be very useful for characterizing the FG magneto-electro-elastic system to be used in sensors or actuators.

Prakash and Ganapathi [80] investigated numerically the supersonic flutter of flat FG plates operating in a thermal environment. The analysis was conducted using a FOST and accounting for the effect of temperature on material properties.

GhannadPour and Alinia [81] carried out the large deflection analysis of rectangular FG plates under pressure loads using the von-Karman nonlinear theory. The mechanical properties of the plate were assumed graded through the thickness by a simple power law distribution in terms of the volume fractions of constituents. The effects of material properties on the stress field through the thickness were studied in this paper.

A FG plate theory for an unusual application has been developed by Hsieh and Lee [82]. The investigators applied the von Karman plate theory to elliptical FG plates, rigidly fixed around a boundary that was allowed to be slightly disturbed (i.e., not perfectly elliptical in shape). A perturbation technique was used to obtain the approximate solutions of the governing equations for displacements.

Chi and Chung [83,84] obtained the closed-form solutions based on CPT and Fourier series expansion for a rectangular simply supported FG plate of medium thickness subjected to transverse loads. They assumed the elastic modulus reflecting the actual volume fraction of constituent phases varying through the thickness according to a power law, sigmoid, or exponential function of the plate thickness. Poisson's ratio is kept unaffected by material grading in the study. Subsequently, the analytical closed-form solutions of the FG plates were proved by comparing the numerical results with finite element method. Chung and Chen [85] further analysed the transversely loaded laminated FG plates with two simply supported opposite edges and two free edges. Two configurations of the plates were considered by them. The first involves a two-layer plate in which an FGM layer is coated on a homogeneous substrate, named an FGM-coated plate. The other involves a three-layer plate in which an FGM is employed for the inter-medium layer and different homogeneous materials are in the top and bottom layers, named an FGM-undercoated plate. The Young's modulus of FG plate is assumed to vary in the thickness direction as a sigmoid function, and the Poisson's ratio kept constant. The differences between the flexibility behaviors of FGM-coated and undercoated plates are investigated in this study by evaluating the deflections, strains, and stresses.

Navazi et al. [86] studied the nonlinear cylindrical bending of a FG plate using the von Karman strains to construct the nonlinear equilibrium equations of the plates subjected to in-plane and transverse loadings. The authors concluded that the FG plates exhibit different behavior from plates made of pure materials in cylindrical bending, and the linear plate theory which neglects the membrane action is inadequate for analysis of FG plates even in the small deflection range.

Pai and Palazotto [87] introduced a sub-lamination theory for analyzing the response of FG plates. Essentially the through-thickness gradation was divided into individual sub-layers to increase the degrees of freedoms and improve the numerical accuracy. The sub-lamination theory was formulated in generalized sense to satisfy assumptions of either CPT or more complicated higher order shear deformable plate theories. Further, the governing equations for a plate composed of several sub-laminates converges to the usual governing equations of plate theories when the number of sub-laminate layers equals one. The theory was shown to match the exact solutions of plates extremely well to include the prediction of mode shapes.

Sladek et al. [88] carried out the static and dynamic analyses of FG plates by the MLPG method. The Reissner–Mindlin plate bending theory was employed to describe the displacement field. Numerical solutions were presented for simply supported and clamped plates.

Chung and Che [89] analysed elastic, rectangular, and simply supported FG plates with medium thickness subjected to linear temperature change in thickness direction. They assumed the Young's modulus and Poisson's ratio of the FG plates to remain constant throughout the entire plate. However, the coefficient of thermal expansion of the FG plate varies continuously throughout the thickness direction in relation to the volume fraction of constituents defined by power-law, sigmoid, and exponential functions.

Abrate [90,91] studied the problems of static deflections, free vibrations, and buckling of FG plates considering the material properties vary through the thickness. He demonstrated that FG plates behave more like homogeneous plates than originally thought. He showed that if the reference surface is chosen judicially such that the bending-stretching coupling disappears, the in-plane and bending stiffness matrices derived from the new reference surface are the only material parameters required to solve the governing equations for plate deflections and vibrations. It is shown in the study that, all other parameters remaining the same, the static deflections, critical buckling load parameter and natural frequencies of FG plates are always proportional to those of homogeneous isotropic plates and that the proportionality constant can be easily be predicted. Therefore, one can predict the behavior of FG plates knowing that of similar homogeneous plates. This new approach was shown to the dramatically simplify the analyses while simultaneously matching identically the results published by other investigators for FG plates assuming both CPT and HOST.

Zhang and Zhou [92] analysed the FG thin plates based on the physical neutral surface concept using CPT (assuming that there is no stretching–bending coupling effect in the constitutive equations in both small and large deflection problems). Some typical analytical solutions which include bending, vibration, bucking and nonlinear bending problems are presented in this paper. The authors emphasized that the physical neutral surface thin plate theory has more merits in the engineering application, because it is easier and simpler than classical laminated plate theory based on geometric middle surface.

Fares et al. [93] presented a 2D theory of FG plates using a mixed variational approach. This theory accounts for a displacements field in which the in-plane displacements vary linearly through the plate thickness, while the out-of-plane displacement is a second-degree function of thickness coordinate. They illustrated the influence of the transverse normal strain on the bending and vibration of the FG plates.

Khabbaz et al. [94] predicted the large deflection and through the thickness stress of FG plates using the energy concept based on FOST and TSDT. They studied the responses as a function of plate thickness and index of power law model which was considered for the through-thickness variation of the FG plate properties. They showed that these models are capable of predicting the effects of plate thickness on the deformation and the through-thickness stresses.

Recently, Aghdam et al. [95] presented a static analysis for bending of moderately thick FG clamped sector plates, based on FOST, by adopting an iterative procedure using the extended Kantorovich method. The authors have compared their solutions with the solutions of finite element code ANSYS, and obtained the close agreement. To model FG sector in ANSYS, the thickness of the sector was divided into 100 layers with isotropic material properties. The material properties of these isotropic layers gradually changes based on the FG power law model of distribution.

A 3D analysis of simply supported FG rectangular plates subjected to thermo-mechanical loads have also been presented by Alibeigloo [96]. The thermal and thermo-elastic constants of the plate were assumed to vary exponentially through the thickness, and the Poisson's ratio was held constant. Analytical solutions for the temperature, stress and displacement fields were derived by using the Fourier series expansion and state-space method. He concluded that the Influence of in-homogeneity is more in the case of thermal loading than that of due to mechanical loads. Further, Alibeigloo and Simintan [97] also investigated the axi-symmetric static analysis of FG circular and annular plates imbedded in piezoelectric layers with various boundary conditions based on 3D theory of elasticity using *differential quadrature method* (DQM).

Vaghefi et al. [98] presented 3D solutions for static analysis of thick FG plates by adopting MLPG method assuming the exponential function for the variation of Young's modulus through the thickness of the plate.

Sepahi et al. [99] examined the effects of three-parameter elastic foundation on axi-symmetric large deflection responses of a simply supported annular FG plate. The plate was loaded transversely with uniform load, non-uniform temperature, and also with the combined thermo-mechanical loads. The mechanical and thermal properties of the FG plate are assumed to be graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. They modeled the FG plate based on FOST in conjunction with nonlinear von Karman assumptions. The effects of nonlinear foundations stiffness, material grading index, and temperature, on the axi-symmetric large deflection response of the FG plate were studied in this paper.

Cheng and Cao [100] have carried out a 3D analysis of FG plates with medium components and different micro net structures using a new microelement method for the analyses of FG structures. Accuracy of this method is established by comparing the stress contour charts in the plane of FG plates with different net microstructures with the 3D elasticity analytical solutions for FG rectangular plates. The distributions of the macro mechanical responses along the thickness and plane direction are calculated by the microelement method in this paper.

Abdelaziz et al. [101] have studied the bending response of FG sandwich plate using two variables *refined plate theory* (RPT) originally developed by Shimpi [102] for isotropic plates, and was extended by Shimpi and Patel [103] for orthotropic plates.

Carrera et al. [104] studied the effects of stretching of thickness in FG plates and shells deriving the advanced theories for bending analysis adopting Reissner mixed variational approach. This was done by removing or retaining the transverse normal strain term in the kinematics assumptions of various refined plate/shell theories. They have compared the plate/shell theories keeping constant transverse displacement component with the corresponding models containing linear to fourth order expansion terms in the thickness direction. Single-layered and multilayered FG structures have been analysed in this study implementing various plate/shell models. The importance of the transverse normal strain effects in prediction of mechanical stresses for FG plates was pointed out in their work. In fact, this work is an extension of several other papers published using *Carrera's unified formulations* (CUFs), as described in Carrera et al. [105], Brischetto [106] and Brischetto and Carrera [107]. The similar works following the Reissner mixed variational approach and using Reddy's TSDT is carried out by Wu and Li [108,109].

Neves et al. [110] presented the static deformations of square FG plates using the radial basis function collocation method, on the basis of a sinusoidal shear deformation formulation for plates, and accounting for through-the-thickness deformations. The governing equations and the boundary conditions are obtained by CUF, and further interpolated by collocation with radial basis functions. The authors noticed the effect of considering the non-zero transverse normal deformations are significant. Further, Neves et al. [111,112] have published two more papers on the quasi-3D sinusoidal and hyperbolic shear deformation theories for the bending and free vibration analysis of FG plate accounting through thickness deformations following the similar approach.

Singha et al. [113] have studied the nonlinear behaviors of FG plates under transverse load using a plate bending finite element based on FOST considering the physical/exact neutral surface position assuming the power-law gradation of material properties in the thickness direction.

Golmakani and Kadkhodayan [114] studied the large deflection behavior of circular and annular FG plates under thermo-mechanical loading based on FOST. Material properties were assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The von Karman nonlinear plate theory was used to obtain the nonlinear equilibrium equations. The solutions of these nonlinear equations were obtained by *dynamic relaxation* (DR) numerical method combined with the finite difference technique. The effects of material grading, thermal loads, boundary conditions and different thickness-to radius ratios were studied in this paper. The same authors [115] have also published a separate paper carrying out the similar studies, this time using Reddy's TSDT. They have compared the results of both the plate theories, and concluded that the difference between TSDT and FOST results becomes greater with increasing thickness-to-external radius ratios.

Wen et al. [116] has presented the 3D analysis of isotropic and orthotropic FG plates with simply supported edges under static and dynamic loads. The governing equations of the 3D elastic problem for the FG plates were formulated based on the state-space approach in the Laplace transform domain, transforming it to a one-dimensional problem, and solved using the RBF method. Two types of FG plate (exponent-law and volume fraction law) were investigated and numerical solutions were presented in the time domain.

Mantari et al. [117] have studied the bending responses of FG plates using a HOST considering the material properties to follow power-law distribution across thickness. Navier-type analytical solutions were obtained for FG plates subjected to transverse bi-sinusoidal and distributed loads.

Akbarzadeh et al. [118] performed an analysis of coupled thermo-elasticity of simply supported FG plates based on the Reddy's TSDT. The plate was subjected to lateral thermal shock of step function type on the lower side and upper side of the plate having convection with the ambient. The material properties of the FG plate, except Poisson's ratio, were assumed to be graded in the thickness direction according to a power-law distribution in terms of the volume fractions of the constituents.

2.2. Vibration and stability analyses of FG plates

The previous sections summarized some of the important works in thermo-elastic static analysis of FG plates; this section will present an overview of the body of the literature available on FG plate dynamics specifically vibration and modal analyses of FG plates.

Cheng and Batra [119] have studied the buckling and steady-state vibrations of a simply supported FG polygonal plate resting on an elastic foundation and subjected to uniform in-plane hydrostatic loads based on Reddy's TSDT.

The dynamic response of initially stressed rectangular FG plates was studied by Yang and Shen [120]. The material properties of plate were assumed to be graded through the thickness with a power law distribution and governed by the classical rule of mixtures. Various boundary conditions of plate were considered in the analysis, viz. clamped on all sides or clamped on two sides and simply supported on two sides. The plate was allowed to rest on an elastic foundation and be subjected to initial in-plane uni-axial or bi-axial stresses, although both or neither of these features needs to be included. The plate was subjected to a variety of pulse loads, viz., constant, linear, sinusoidal, or exponential with respect to time over a rectangular patch of the plate surface and then, it was removed abruptly at a given time to ensure that the plate entered free vibration. The CPT was used to model the dynamic and transient responses of FG plates under these conditions. The results, as expected, were found to vary greatly depending on the power law exponent, plate aspect ratio, foundation stiffness, shape and duration of pulse load, and initial stress magnitude. Yang and Shen [121] extended their previous analyses to the thick, shear-deformable FG plates in thermal environments. They developed a general plate theory (minus the case of an elastic foundation) where the response of the plate could be semi-analytically determined under general load and boundary conditions, including the case of an initially stressed plate. Yang et al. [122] further presented a large vibration analysis of pre-stressed FG laminated plates based on Reddy's TSDT. Some more publications dealing with the same subject are Yang et al. [123], and Yang and Huang [124].

Kim [125] has also developed a theoretical method, based on Reddy's TSDT to investigate the vibration characteristics of initially stressed FG rectangular plates in thermal environment. He assumed temperature to be constant in the plane of the plate and, to vary in the thickness direction. Temperature dependent material properties of plate were assumed to vary smoothly through the thickness according to a power law distribution in terms of the volume fraction of the constituents. The equations of motion were then solved by the Rayleigh–Ritz procedure. The effect of material compositions, plate geometry, and temperature fields on the vibration characteristics is examined in this study.

He et al. [126] presented a FE formulation based on the CPT for the shape and vibration control of the FG plates with integrated piezoelectric sensors and actuators. The properties of the FG plates were assumed to be graded in the thickness direction according to a volume fraction power law distribution. The effects of the constituent volume fraction on the FG plate responses were studied in this paper. The authors concluded that the vibration amplitude of the FG plate attenuates at very high rates for appropriate gain values.

Javaheri and Eslami [127–129] studied the thermal and mechanical buckling of FG rectangular plates based on the classical and higher-order plate theories. The governing equilibrium and stability equations for FG plates are derived using variational approach, identical with the equations for homogeneous plates. They have studied the buckling behavior of simply supported FG plates subjected to in-plane loading conditions with linear composition of constituent materials and homogeneous plates.

The thermal buckling analysis of FG plate was performed by Na and Kim [130] using 18-noded 3D solid finite elements, though the sinusoidal and linear through-the-thickness temperature distributions considered in this paper do not actually reflect the actual temperature distribution in a FG plate. Temperature-dependent material properties were assumed to be varying continuously in the thickness direction according to a simple power law distribution in terms of the volume fraction of a ceramic and metal. They adopted strain mixed formulation to prevent locking as well as maintaining kinematic stability of the finite element model for thin plates. Subsequently, they published two more papers [131,132] on the thermal post-buckling responses, and nonlinear bending responses of FG composite plates adopting the similar procedure. The Green–Lagrange nonlinear strain–displacement relation was adopted to account for large deflection due to thermal load and the incremental formulation is applied for nonlinear analysis. Furthermore, the thermal buckling and post-buckling behaviors of FG plates due to temperature field, volume fraction distributions, and system geometric parameters were studied, in detail.

Najafizadeh and Eslami [133,134] presented the axi-symmetric buckling analysis of simply supported and clamped radially loaded solid circular plate made of FGM. The equations were based on Love–Kirchhoff hypothesis and the Sander's nonlinear strain–displacement relation. Their studies conclude that, while grading can improve thermal properties and reduce stress concentration; the buckling resistance of FG plates is inferior compared to the counterpart constructed of the stiffer phase.

Chen and Liew [135] studied the buckling of FG plates subjected to various non-uniform in-plane loads, including pin, partially uniform, and parabolic loads based on FOST. They obtained the shape control of the FG plates under a temperature gradient by optimization of the voltage distribution for the open loop control, and also the displacement control gain values for the closed loop feedback control. They also examined the effect of the constituent volume fractions on the optimal voltages and gain values. Further, Chen et al. [136] employed the element free Galerkin method to analyse buckling of piezoelectric rectangular FG plates subjected to non-uniformly distributed loads, heat and voltage. A two-step solution procedure is adopted in this study. In the first step, pre-buckling stresses of plates subjected to non-uniformly distributed loads are calculated based on a plane stress condition. In the second step, the buckling load and temperature parameters of the plates are obtained based on the FOST. The authors concluded that the buckling parameters for isotropic plates are bigger than those for FG plates.

Vel and Batra [137] extended their previous exact 3D thermo-static analysis to the problem of free and forced vibrations of simply supported rectangular FG plates with an arbitrary variation of properties in the thickness direction. The assumed displacement fields that identically satisfy the simply supported boundary conditions are used to reduce the governing steady state equations to a set of coupled ordinary differential equations. The effective material properties and the displacements were expanded as Taylor series in the thickness co-ordinate. The obtained set of ODEs with variable coefficients is then solved by the power series method. The exact solutions using the 3D elasticity solutions are then used to assess the accuracy of the results obtained by 2D plate theories, viz., CPT, FOST and Reddy's TSDT for FG plates. They observed that there are substantial differences between the exact solutions and results obtained from the CPT even when the transverse shear and the transverse normal stresses of the plates are computed by integrating the 3D elasticity equations. Further, the results from the FOST and TSDT are well comparable with the exact solution. The FOST performs better than the TSDT for the FG plates studied by them. These exact solutions presented by them may be considered as the benchmark results which can be used to assess the adequacy of different plate theories and other approximate methods such as the finite element method. These results have been used by Qian and Batra [138] to validate their numerical solutions obtained by them based on a higher order shear and normal deformable plate theory (originally derived by Batra and Vidoli [139] from a 3D variational principle for a piezoelectric plate, named as HOSNDPT) and MLPG method to analyse the static and dynamic deformations of simply supported FG plates. They too assumed the plate material to be macroscopically isotropic with material properties varying in the thickness direction only. The effective material modulii were computed using the Mori–Tanaka homogenization technique. This paper indicates that the application of normal deformation theory may be justified if the side to thickness ratio of the plate is equal to or smaller than 5. The authors also indicated that material property variations seem to have a small effect on the fundamental frequency of the plate.

The buckling analysis of a FG plate resting on a Pasternak-type elastic foundation was solved by Yang et al. [140] considering the material properties of the constituent phases and the foundation parameters as random independent variables. The plate was modeled by FOST in this study and the solution of the problem utilized the first-order perturbation procedure to account for the randomness of the problem.

Buckling and free vibrations of simply supported FG sandwich ceramic–metal panels were presented by Zenkour [141] extending his previous work of static analysis on such panels.

Sundararajan et al. [142] studied the nonlinear free vibration characteristics of FG plates under thermal environment. The nonlinear governing equations of motion were solved using finite element procedure coupled with the direct iteration technique. The material properties of plate was assumed to be temperature dependent and graded in the thickness direction according to the power-law distribution in terms of volume fractions of the constituents. The authors concluded that the temperature field and material grading have significant effect on the nonlinear vibration of the FG plate.

Bhangale and Ganesan [143] following their previous approach, carried out the free vibration analysis of magneto-electro-elastic FG plates exponentially graded in the thickness direction accounting for coupling between magnetic, elastic, and electric effects.

Ferreira et al. [144] used the global collocation method and approximated the trial solution with multi-quadric RBF to analyze the free vibrations of FG plates. The plate is modeled using the FOST and TSDT. The effective material properties of FG plate are derived based on the Mori Tanaka homogenization technique. They have compared their numerical solutions with the exact 3D elasticity solutions [137], and the numerical solutions based on the MLPG formulation [138].

Woo et al. [145] presented the analytical solution for the nonlinear free vibration of FG plates using their previous approach. The effects of material properties, boundary conditions and thermal loading on the dynamic behavior of the plates were studied in this paper. The authors concluded that the nonlinear coupling effects play a major role in dictating the fundamental frequency of FG plates.

Ganapathi and Prakash [146] performed a study of simply supported skew FG plates subjected to a temperature distribution obtained from the heat conduction equation, varying in thickness direction using a FOST. Both structural stability and modal frequencies were simultaneously considered using a customized finite element in this study. Further, the same authors Prakash and Ganapathi [147] have investigated the asymmetric free vibration characteristics and thermo-elastic stability of circular FG plates using a three-noded shear flexible plate finite element based on the field-consistency principle. Temperature field was assumed to be a uniformly distributed over the plate surface and varied in thickness direction only. The material properties of FG plate were assumed to be graded in the thickness direction according to simple power law distribution. They highlighted the variation in critical buckling load considering gradient index, temperature, radius-to-thickness ratios, circumferential wave number and boundary condition of the plate. Prakash et al. [148,149] have further presented the post-buckling behavior and thermal snapping behavior of FG plates under thermal load based on their previous shear deformable finite element approach. The nonlinear governing equations in this case were derived based on von Karman's assumptions and are solved employing the direct iterative technique. The same authors [150] have further published a paper studying the large amplitude flexural vibration characteristics of FG plates under aerodynamic load using the same methodologies.

Wu et al. [151] obtained analytically the post-buckling response of the FG plate, subjected to thermo-mechanical loads using fast converging finite double Chebyshev polynomials. The mathematical formulation was based on the FOST and von-Karman nonlinear kinematics. They indicated with the help of numerical examples that the critical temperature and buckling loads decrease with increase in volume fraction exponent of the FG plate. They observed that the buckling and post-buckling responses of the FG plate is almost same for the aspect ratios more than or equal to 3 with several volume fraction exponents.

The 3D solutions of simply-supported, FG magneto-electro-elastic rectangular plates using a modified Pagano method were presented by Wu et al. [152]. The material properties of FG plates were assumed to obey a power-law distribution of the volume fractions of the constituents through the thickness. The Pagano method was modified in a sense that a displacement-based formulation was replaced by a mixed formulation, the complex-values solutions of the system equations were transferred to the real-values solutions, and a successive approximation method was used to make the modified Pagano method feasible for the coupled analysis of FG plates. The accuracy of the present solutions was evaluated by comparing them with the available asymptotic solutions of FG plates.

Lanhe et al. [153] extended their previous work to the dynamic stability analysis of thick FG plates subjected to aero-thermo-mechanical loads, using the moving least squares differential quadrature method. The influence of gradient index, temperature, mechanical and aerodynamic loads, thickness and aspect ratios, as well as the boundary conditions on the dynamic instability region are studied in this paper.

Uymaz and Aydogdu [154] presented 3D vibration solutions for rectangular FG plates with different boundary conditions using Ritz method with Chebyshev displacement functions, based on the small strain linear elasticity theory, and assuming the power law variation of the material properties through the plate thickness.

The 3D vibration analyses of thick annular isotropic and FG plates were performed by Efraim and Eisenberger [155]. They used FOST in deriving the system of equations of motion for the free vibration analyses (a set of coupled partial differential equations with variable coefficients), and obtained the exact solutions of this set of PDE using the exact element method and the dynamic stiffness method.

Matsunaga [156] analysed the free vibration and stability of simply supported FG plates using a HOST. He presented the natural frequencies and buckling stresses of plates made of FGMs by taking into account the effects of transverse shear and normal deformations and rotatory inertia.

Khorramabadi et al. [157] have studied the free vibration of simply supported FG plates using FOST and TSDT to understand the effect of applying these two different shear deformation theories on inhomogeneous plates.

Ebrahimi and Rastgoo [158] investigated the free vibration characteristics of thin circular clamped FG plates integrated with two uniformly distributed actuator layers made of piezoelectric material, based on CPT. The material properties were considered smoothly graded through thickness modeled by a power-law variation. The distribution of electric potential field of piezoelectric layers was simulated by a quadratic function along the thickness. Further, Ebrahimi et al. [159] presented the free vibration analysis of moderately thick shear deformable annular FG plate coupled with piezoelectric layers based on Mindlin's FOST.

Aydogdu [160] investigated the conditions for bifurcation buckling of FG plates using CPT. He found in his study that, a bending moment is required for simply supported FG plates to remain flat under in-plane loading.

Allahverdizadeh et al. [161,162] analysed the nonlinear free and forced axi-symmetric vibration of a thin circular FG plate using a semi-analytical approach. The material properties were assumed to vary continuously through the thickness according to a power-law distribution of the volume fraction of the constituents. The governing equations were solved using assumed-time-mode method and Kantorovich time averaging technique for harmonic vibrations. Steady-state free and forced vibration problems of FG plate were investigated in this study. The nonlinear frequencies and associated stresses were found at large amplitudes of vibration. They also examined the effects of material compositions and thermal loads on the vibration characteristics and stresses of FG plate. They concluded that the natural frequencies are dependent on vibration amplitudes, and the material index has a significant role to play on the nonlinear response characteristics of the FG plate.

Li et al. [163] studied the free vibration of rectangular FG plates with simply supported and clamped edges in the thermal environment based on the 3D linear theory of elasticity. The plate was subjected to uniform, linear, and nonlinear temperature rise along the thickness. The in-plane and transverse displacements of the plates were expanded by a series of Chebyshev polynomials multiplied by appropriate functions to satisfy the essential boundary conditions. The natural frequencies were obtained by Ritz method. They performed a detailed parametric study for FG plates.

Shariyat [164] investigated the vibration and dynamic buckling responses of rectangular FG plates with surface-bonded or embedded piezoelectric sensors and actuators subjected to thermo-electro-mechanical loading. A nine-noded second order finite element formulation based on a HOST is used for the analysis accounting for both initial geometric imperfections of the plate and temperature-dependency of the material properties. Dynamic buckling of plates already pre-stressed by other forms of loading conditions is assumed to occur under suddenly applied thermal or mechanical loads in this study.

Bouazza et al. [165] investigated the buckling of simply supported FG plate subjected to uniform and linear temperature rise through the thickness based on the FOST and applying the von Karman type stability and compatibility equations. They observed in their study that transverse shear deformation has considerable effects on the critical buckling temperature of FG plate, especially for a thick plate or a plate with large aspect ratio.

Lee et al. [166] also carried out the post-buckling analysis of FG plates subjected to edge compression and thermal conditions based on the FOST and the von Karman relationship. The effective material properties of the FG plates were assumed to vary according to the power law through thickness. A set of mesh-free kernel particle functions were used for approximating the displacement fields. To eliminate the membrane and shear locking effects for thin plates, these terms were evaluated using a direct nodal integration technique. The effects of the volume fraction exponent, boundary conditions and temperature distribution on post-buckling behavior are examined in this paper.

Liu et al. [167] analysed the free vibration behavior of a FG elastic rectangular plate of uniform thickness using CPT. A Levy-type solution is obtained for plates with a pair of simply supported edges that are parallel with the material gradient direction. The effect of in-plane material in-homogeneity on the fundamental frequencies is studied in this paper.

Talha and Singh [168] investigated the free vibration and static analysis of FG plates using an efficient C^0 finite element with 13° of freedom per node, formulated using a HOST. They further studied the large amplitude free vibration behavior of FG plates using the same formulation, modified to account for the large deflection responses. They used the Green–Lagrange nonlinear strain–displacement relation with all higher order nonlinear strain terms for incorporating the large deflection behavior of FG plates.

Gunes et al. [169] studied the 3D free vibration behavior of an adhesively-bonded single lap joint with wide and narrow FG plates. These plates were composed of ceramic (Al_2O_3) and metal (Ni) phases, properties varying through the thickness. This investigation was carried out using both the finite element method and the back-propagation *artificial neural network* (ANN) method. They studied the effects of geometrical parameters, viz. plate width, thickness and overlap length on the free vibration parameters of the adhesive joint. The effect of the similar and dissimilar material composition variations through-the-thicknesses of both upper and lower plates on the natural frequencies and corresponding mode shapes of the adhesive joint were also investigated in this study.

Hoang and Nguyen [170] presented an analytical approach to investigate the stability of FG plates under in-plane compressive, thermal and combined loads using CPT. Geometrical nonlinearity and initial geometrical imperfection, both are accounted suitably in this study. Temperature independent material properties are assumed to be graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of constituents. The explicit expressions for the post-buckling load–deflection curves were obtained by solving the governing equations by Galerkin procedure. The effects of the volume fraction index, plate geometry, in-plane boundary conditions, and imperfection on post-buckling behavior of the plate were studied in this paper. Further, the same authors, Nguyen and Hoang [171], following the similar methodology, presented an analytical study on the buckling and post-buckling behaviors of thick FG plates resting on elastic foundations and subjected to in-plane compressive, thermal and thermo-mechanical loads. This analysis was carried out to show the effects of material and geometrical properties, in-plane boundary restraint, foundation stiffness and imperfection on the buckling and post-buckling loading capacity of the FG plates.

Jalali et al. [172] studied the thermal stability of laminated FG circular plates subjected to uniform temperature rise based on a FOST. They demonstrated that, the thermal stability of FG plate is significantly influenced by the thickness variation profile, aspect ratio, the volume fraction index, and the core-to-face sheet thickness ratio.

Hashemi et al. [173] employed an analytical method to analyse the vibration problems of thick annular FG plates with integrated piezoelectric layers. The plate with different boundary conditions at the inner and outer edges is modeled on the basis of the Reddy's TSDT. The material properties variation of the FG plate follows a power-law distribution. The distribution of electric potential along the thickness direction in the piezoelectric layer is assumed as a sinusoidal function. In this study closed-form expressions for characteristic equations, displacement components of the plate and electric potential are derived. The natural frequencies of this piezoelectric coupled annular FG plate are evaluated for different thickness–radius ratios, inner–outer radius ratios, thickness of piezoelectric, material of piezoelectric, power index and boundary conditions in this study. Recently, the same authors [174] have developed an exact closed-form solution for the free vibration of piezoelectric coupled thick circular/annular FG plates subjected to different boundary conditions on the basis of Mindlin's FOST. The effects of coupling between in-plane and transverse displacements on the frequency parameters are proved to be significant in this study. It is concluded in the paper, that the developed model can describe vibration behavior of smart FG plates in a more realistic way.

Fakhari et al. [175] presented a finite element formulation based on a HOST to analyse the nonlinear natural frequencies and time response of FG plate with surface-bonded piezoelectric layers under thermal, electrical and mechanical loads. They used von Karman relation to account for the large deflection of the plate. In this study, the material properties of FGM were assumed temperature-dependent and are graded in the thickness direction following a simple power law in terms of volume fraction of the constituents.

Shahrjerdi et al. [176] have studied the free vibration of rectangular simply supported FG plates using *second order shear deformation theory* (SSDT). Kumar et al. [177] have also carried out the same using a HOST without enforcing zero transverse shear stress conditions on the top and bottom surfaces of the plate. In the similar line, Benachour et al. [178] have also evaluated the natural frequency of plates made of FGMs by using a four variable refined plate theory with an arbitrary gradient considering only the four numbers of unknown functions taking account of transverse shear effects and parabolic distribution of the transverse shear strains through the thickness of the plate. The Free vibration analysis of FG and composite sandwich plates are carried out by Xiang et al. [179] using a displacement model consisting n-order polynomial satisfying zero transverse shear stress boundary conditions at the top and bottom of the plate.

Hao et al. [180] carried out the nonlinear dynamic analysis of a cantilever FG rectangular plate subjected to the transversal excitation in thermal environment using Reddy's TSDT. This is an extension of the work carried out by Zhang et al. [181] on the chaotic vibrations of a simply supported orthotropic FG rectangular plate based on TSDT. Material properties of FG are assumed to be temperature dependent. The equations of motion were derived by using Hamilton's principle, and are converted into a two-degree-of-freedom nonlinear system by employing Galerkin's approach. The authors concluded that, the nonlinear dynamic response of the cantilever FG rectangular plate is much more sensitive to transverse excitation compared with that of a simply supported FG plate.

Nguyen-Xuan et al. [182] analysed the static, free vibration and mechanical/thermal buckling problems of FG plates using the finite element approach in which a node-based strain smoothing is merged into shear-locking-free triangular plate elements. This work is the extension of the earlier works carried out by the same authors [183,184] on an *edge-based smoothed finite element method* (ES-FEM) with stabilized discrete shear gap (DSG) technique using triangular meshes (ES-DSG) to enhance the accuracy of the existing finite element methods for analysis of isotropic Reissner/Mindlin plates.

Jha et al. [185] have recently published a study on the evaluation of natural frequency of simply (diaphragm) supported rectangular FG plates based on a *higher order shear and normal deformations theory* (HOSNT). The material properties such as material density, and Young modulus of elasticity. of the graded plates are assumed to follow the power law model through the plate thickness. The obtained closed form solutions utilizing Navier solution technique are in the excellent agreement of the 3D elasticity solutions available in the literature.

Shen and Wang [186] have presented the small and large amplitude vibrations of a FG rectangular plate resting on a Pasternak-type elastic foundation (boundary condition) in thermal environments considering two kinds of micromechanics models of FGM using the same methodologies. Shen has made several other publications [187–190] in the area of nonlinear bending response, thermal post-buckling analysis, comparison of buckling and post-buckling behavior of FG plates with or without surface bonded piezoelectric actuators due to heat conduction and under different sets of electric loading conditions using a HOST.

3. Conclusions

A review of various investigations carried out in the existing literature for the stress, free vibration and buckling analyses of FG plates have been presented in the present article. An effort has been made to include all the important contributions in the current area of interest highlighting the most pertinent literature available to research engineers studying FG plate structures. The general remarks from the current literature survey are as follows:

- 3D analytical solutions for FG plates are very useful since they provide benchmark results to assess the accuracy of various 2D plate theories and finite element formulations, but their solution methods involve mathematical complexities and are very difficult and tedious to solve.
- In most of the 2D theories developed to predict the global responses of FG plates, only the transverse shear deformation effect has been considered and very few theories consider the effect of both transverse shear and transverse normal deformations effect.
- In most of the 2D shear deformation theories developed till date, the validation and accuracy of the global responses of FG plates are done by comparing the results with 3D elasticity solutions. Very limited studies are reported on comparison of the accuracy with analytically predicted global responses of FG plates using various higher order theories.
- Having reviewed a large segment of the FGM research available it is apparent that nearly all the research conducted has been purely analytical or with numerical simulation.
- Use of improved 2D theoretical models which are now seem to provide accuracy as good as the 3D models should be pursued in the interest of computational cost and efficient analyses.

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