Discrete Traffic Flow Simulation Using Cellular Automata

Tom V Mathew

Indian Institute of Technology Bombay

Introduction

- **Traffic Simulation**
  - Mathematical relations to describe traffic flow
  - Scope
    - How one vehicle reacts to the other vehicles
    - Aggregate traffic stream characteristics
  - Microscopic simulation
    - Accurate Modeling
      - Time consuming and Data intensive
    - Practical application require less accuracy

Introduction

- **Discrete Simulation**
  - Examples
    - Cellular automata
    - Cell transmission model
  - Benefits
    - Simple and hence faster
    - Capture essential traffic phenomenon

Introduction

- **What is CA?**
  - Automata-Mechanism
    - It is an n-dimensional array of cells
    - Each cell can be in any one of k-states
  - Follow certain rules
    - Rules affect neighboring cells
    - Rules updates at each clock-tick
  - History
    - Stephen Wolfram -
    - Nagel-Schekenberg – traffic

Micro-Simulation

- **Governing Equations**
  - Velocity $$v(t+1) = v(t) + a(t) \times \Delta t$$
  - Position $$x(t+1) = x(t) + v(t) \times \Delta t + \frac{1}{2} a(t) \times \Delta t^2$$
  - Acceleration $$a^{t+\Delta t}{}_{n+t} = \frac{\alpha_{l,m} (v^{t+\Delta t}{}_{n+1})^m \left[v^{t-T}{}_n - v^{t-T}{}_{n+1} \right]}{(x_n{}^{t-T} - x^{t-T}{}_{n+1})^l}$$

General Motors: Car Following Model

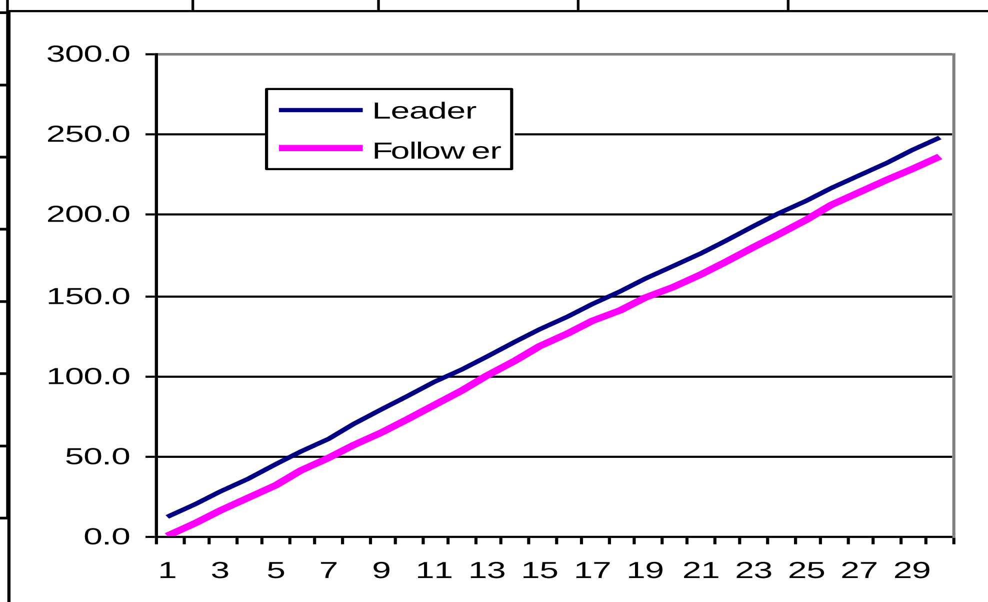
dt = 0.5 α = 13 l=0, m=0, T=1

t	L-a(t)	L-v(t)	L-x(t)	F-a(t)	F-v(t)	F-x(t)	dv	dx
0.0	**0.0**	**16.0**	**28.0**	**0.000**	**16.000**	**0.000**	0.000	28.000
0.5	**0.0**	16.0	36.0	0.000	16.000	8.000	0.000	28.000
1.0	**0.0**	16.0	44.0	0.000	16.000	16.000	0.000	28.000
1.5	**0.0**	16.0	52.0	0.000	16.000	24.000	0.000	28.000
2.0	**1.0**	16.0	60.0	0.000	16.000	32.000	0.000	28.000
2.5	**1.0**	16.5	68.1	0.000	16.000	40.000	0.500	28.125
3.0	**1.0**	17.0	76.5	0.000	16.000	48.000	1.000	28.500
3.5	**1.0**	17.5	85.1	0.231	16.000	56.000	1.500	29.125
4.0	**-1.0**	18.0	94.0	0.456	16.116	64.029	1.884	29.971
4.5	**-1.0**	17.5	102.9	0.670	16.344	72.144	1.156	30.731
5.0	**-1.0**	17.0	111.5	0.817	16.678	80.399	0.322	31.101
5.5	**-1.0**	16.5	119.9	0.489	17.087	88.841	-0.587	31.034
6.0	**0.0**	16.0	128.0	0.134	17.332	97.445	-1.332	30.555
6.5	**0.0**	16.0	136.0	-0.246	17.399	106.128	-1.399	29.872
7.0	**0.0**	16.0	144.0	-0.567	17.276	114.797	-1.276	29.203

General Motors: Car Following Model

dt = 0.5 α = 13 l=0, m=0, T=1

t	L-a(t)	L-v(t)	L-x(t)	F-a(t)	F-v(t)	F-x(t)	dv	dx
0.0	**0.0**	**16.0**	**28.0**	**0.000**	**16.000**	**0.000**	0.000	28.000
0.5	**0.0**	16.0	36.0	0.000	16.000	8.000	0.000	28.000
1.0	**0.0**	16.0	44.0	0.000	16.000	16.000	0.000	28.000
1.5	**0.0**	16.0	52.0	0.000	16.000	24.000	0.000	28.000
2.0	**1.0**	16.0	60.0	0.000	16.000	32.000	0.000	28.000
2.5	**1.0**	16.5	68.1	0.000	16.000	40.000	0.500	28.125
3.0	**1.0**	17.0	76.5	0.000	16.000	48.000	1.000	28.500
3.5	**1.0**	17.5	85.1	0.231	16.000	56.000	1.500	29.125
4.0	**-1.0**						1.884	29.971
4.5	**-1.0**						1.156	30.731
5.0	**-1.0**						0.322	31.101
5.5	**-1.0**						-0.587	31.034
6.0	**0.0**						-1.332	30.555
6.5	**0.0**						-1.399	29.872
7.0	**0.0**						-1.276	29.203



Micro-Simulation

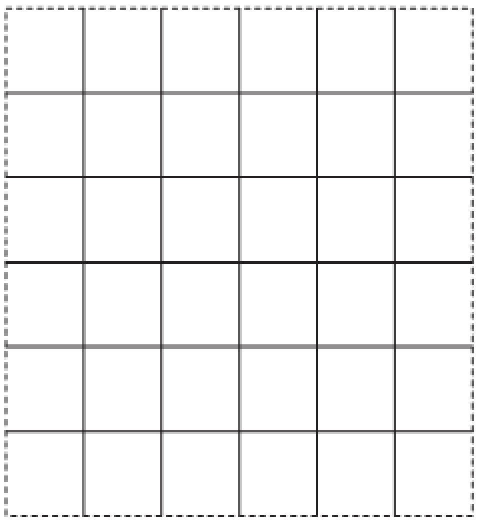
- **Limitations**
  - Complex modeling
  - High computational time
  - Large data required for calibration
  - Practical application require less accuracy

Cellular Automata

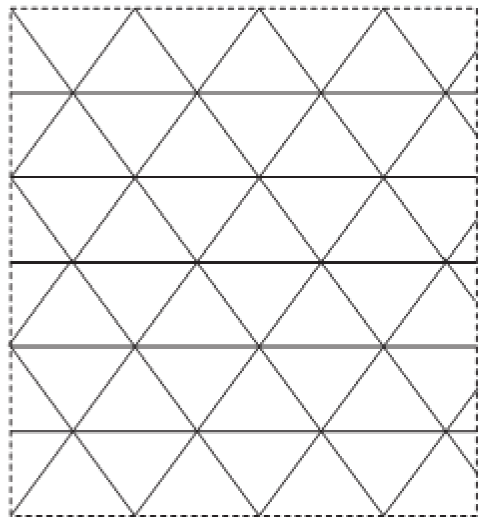
- **Features**
  - Dynamical systems
    - Space and time discrete
  - Consists of grid of cells
    - They interact with their neighbors
    - Grid can be of any dimensions (1, 2, … n)
    - Each cell has its own state variable
  - Wolfram's CA Rule 184
    - Similar to a single lane traffic flow

Cellular Automata

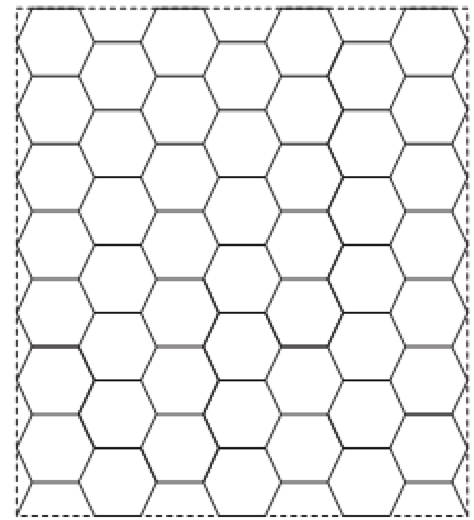
- **Lattice topologies for a 2D CA**



Rectangular



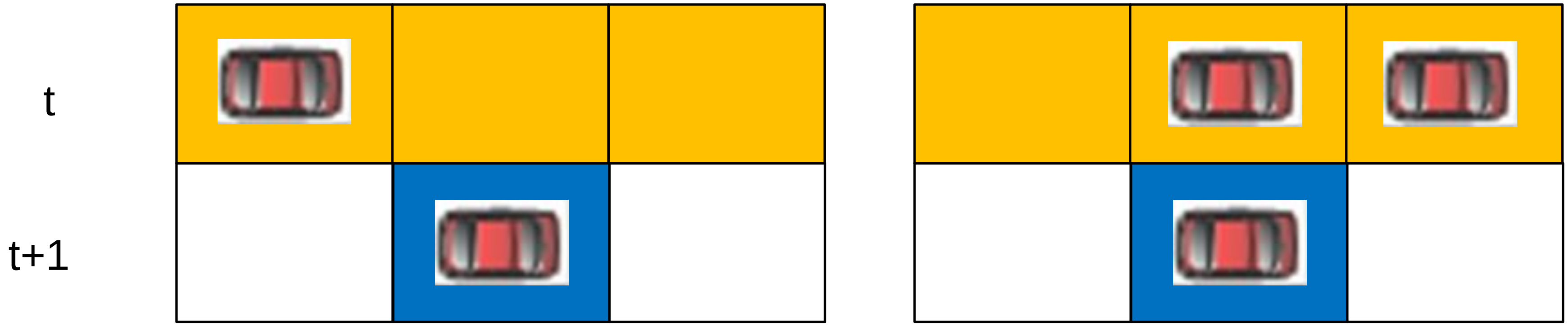
Triangular



Hexagonal

Cellular Automata

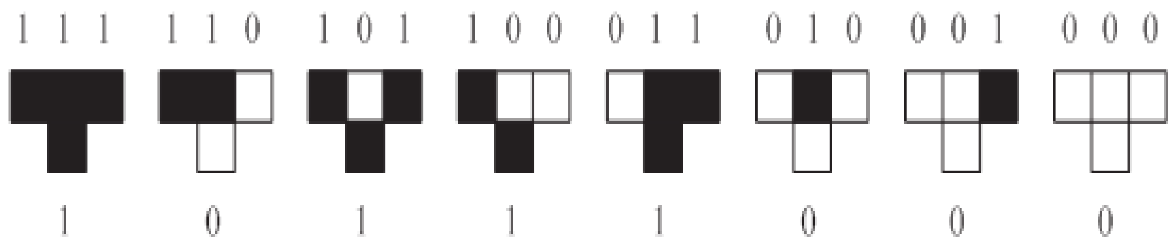
- **Features**
  - Wolfram's CA Rule 184
    - Similar to a single lane traffic flow



Cell state = 1 if vehicle present
= 0 if empty

Cellular Automata

- **Features**
  - Wolfram’s CA Rule 184
    - 8 Possible scenario



Wolfram 184 Rule

$= 1*2^7 + 0*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 0*2^2 + 0*2^1 + 0*2^0$

$= 128 + 0 + 32 + 16 + 8 + 0 + 0 + 0$

$= 184$

Why Cellular Automata ?

- CA has become more popular for computer implementation and simulation due to its discrete nature
- Discretization results in high computing efficiency
- It also proved ideal for large-scale computer simulation
- Due to discrete nature it has limitations in predicting the acceleration and deceleration

1 D Cellular Automata

Nagel Schreckenberg Model

Traffic flow Modelling using CA

- **1 D Cellular automata**
  - Discrete road of length L with cell size d
  - States : Velocity of cars $v \in \{1,2,\ldots.Vmax\}$
- **Nagel and Schreckenberg (1992) Model**

Acceleration

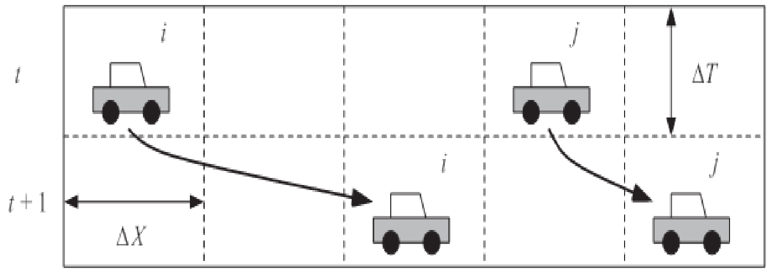
Deceleration

Randomization

Updation

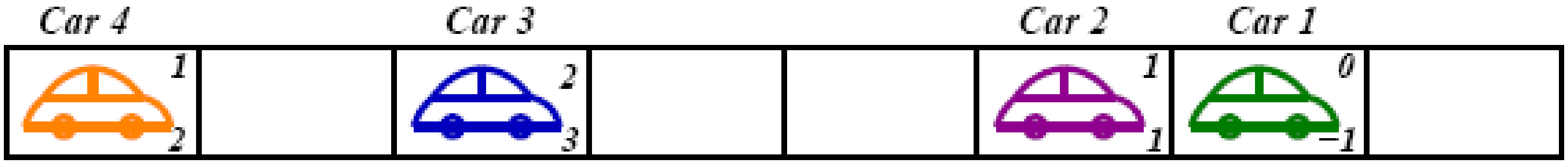
Nagel Schreckenberg Model

- **Scheme**
  - Two vehicles i & j travelling in a 1-D lattice
  - Time discretization: ΔT= 1 sec
    - Based on driver reaction time
  - Space discretization: ΔX=7.5 m
    - Based on vehicle spacing in jam condition
    - Speed increments of ΔV= 7.5m/sec = 27k m/hr



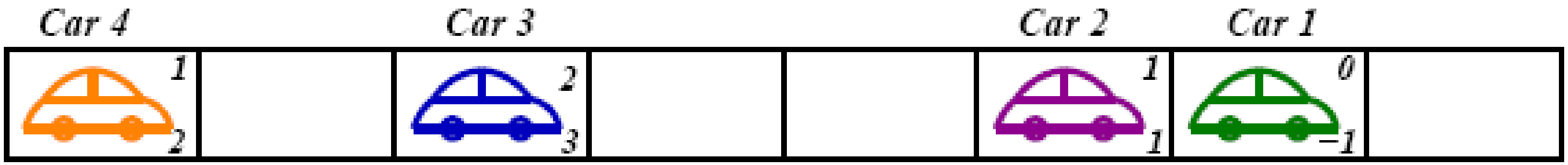
Nagel Schreckenberg Model

- **Conversions for the Macroscopic traffic stream**
  - $k = K * 1000/\Delta X$
  - $q = Q * 3600/\Delta T$
  - $v = V * 3.6 * \Delta X/\Delta T$
    - K, Q, V are the values of in CA units of CA
    - k, q, v are the real world values
  - E.g.



Nagel Schreckenberg Model

- **Conversions for the Macroscopic traffic stream**
  - k = K *1000/ΔX
  - E.g.
    - K = 4/8 => k = 0.5 * 1000 / 7.5 = 66.7 veh/km
    - V = 2 cells/1 sec => v = 2*7.5/1=15 m/s=54 kmph



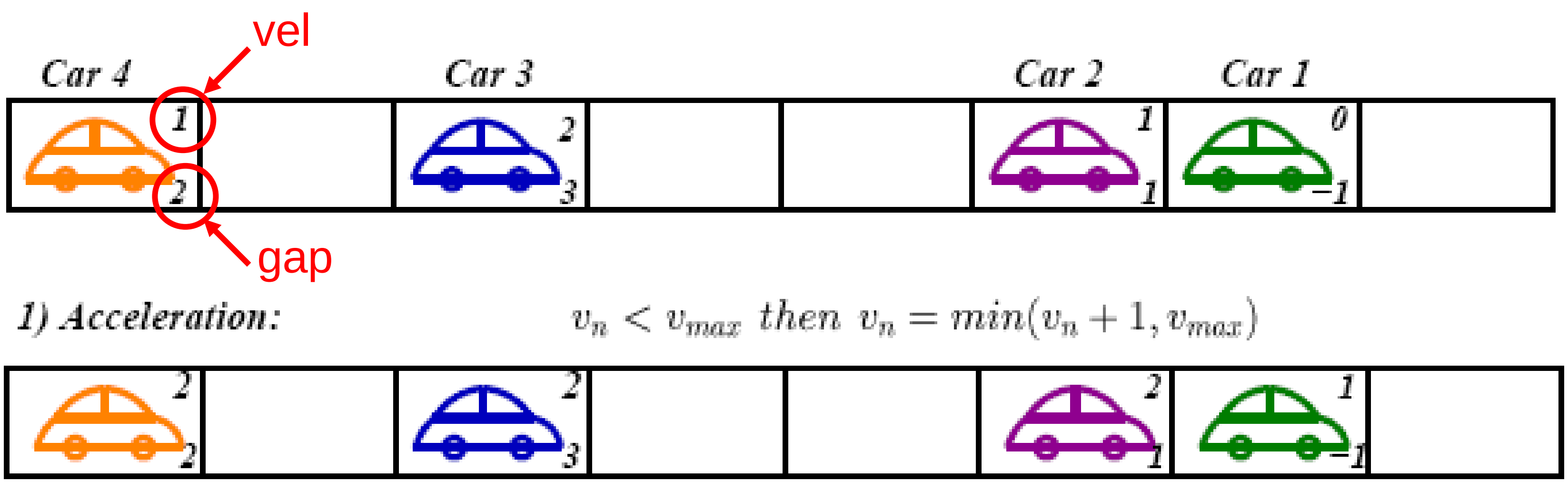
Nagel Schreckenberg Model

Acceleration

if $v_n < v_{max}$

then $v_n = \min(v_n + 1, v_{max})$

$V_{max} = 2$



For car 4: $v_n=1$ **so** $v_n=2$

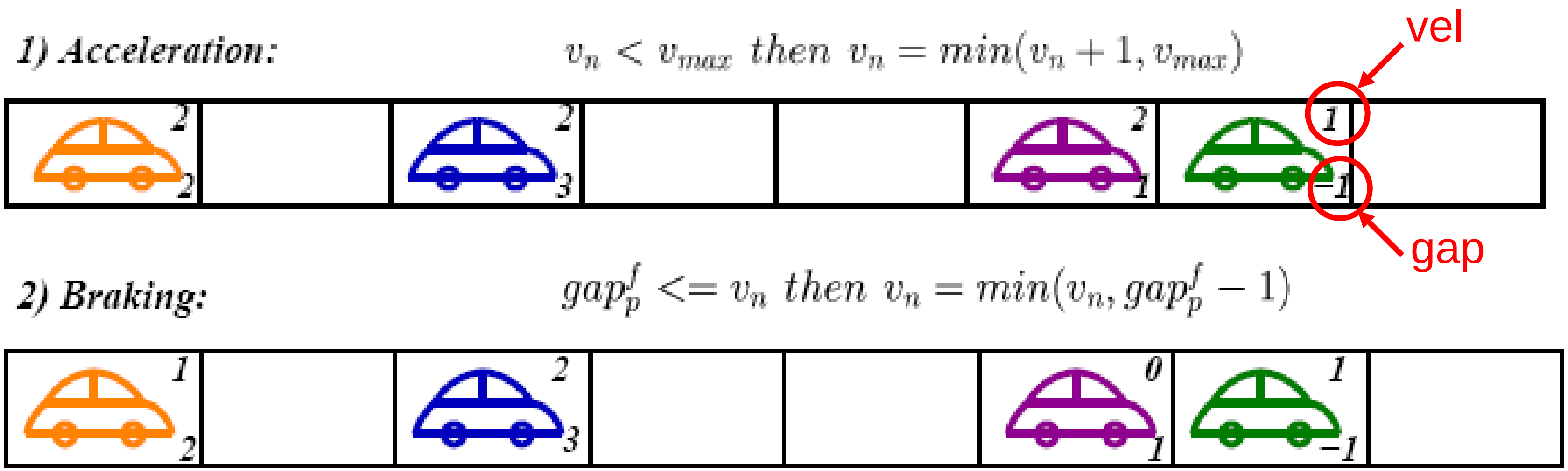
For car 3: $v_n=2$ **so** $v_n=2$

Nagel Schreckenberg Model

Deceleration

if **gap <= v_n**

then **v_n = min (v_n, gap-1)**



For car 4: gap<2, so v_n=1

For car 3:gap=3, so v_n=2

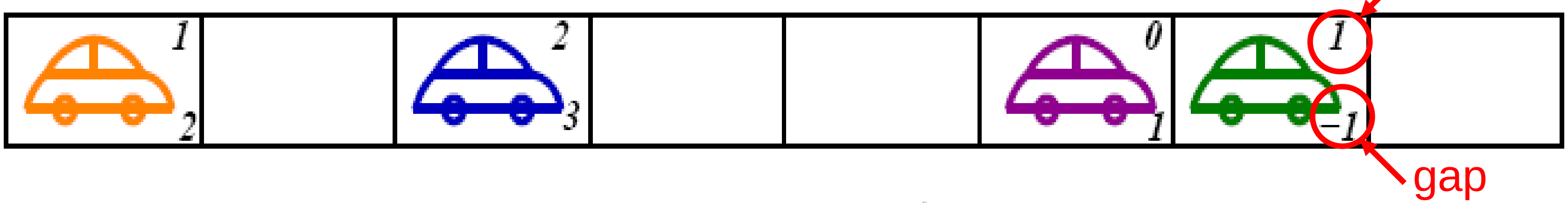
Nagel Schreckenberg Model

Randomization

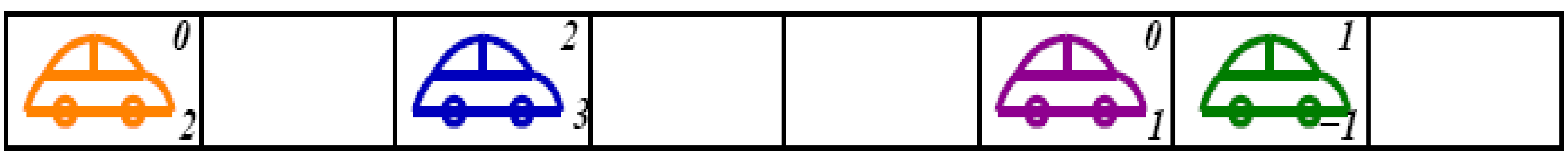
if $v_n > 0$

then $v_n = \max(v_n - 1, 0)$ **with probability** p

2) Braking: $gap_p^f <= v_n \ then \ v_n = min(v_n, gap_p^f - 1)$



3) Randomisation (p = 1/3): $v_n > 0, \ then \ v_n = max(v_n - 1, 0) \ with \ probability \ p <= 1/3$



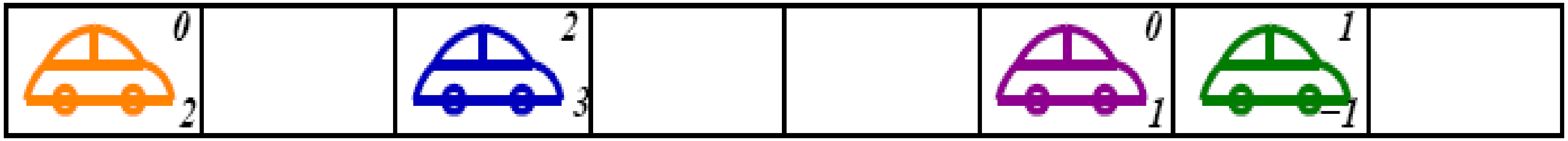
For car 4: $v_n = 1$ **, so** $v_n = 1 - 1 = 0$

Nagel Schreckenberg Model

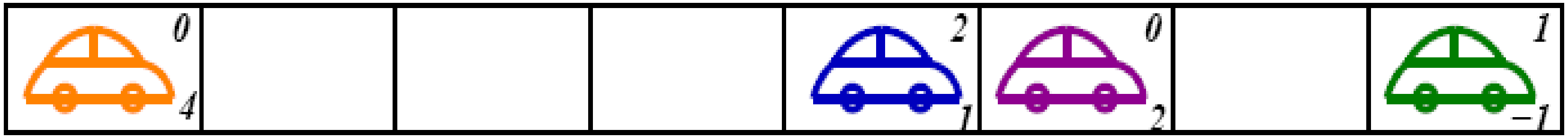
Updation

$$X_n = X_n + V_n$$

3) Randomisation (p = 1/3): $v_n > 0,\ then\ v_n = max(v_n - 1, 0)\ with\ probability\ p <= 1/3$

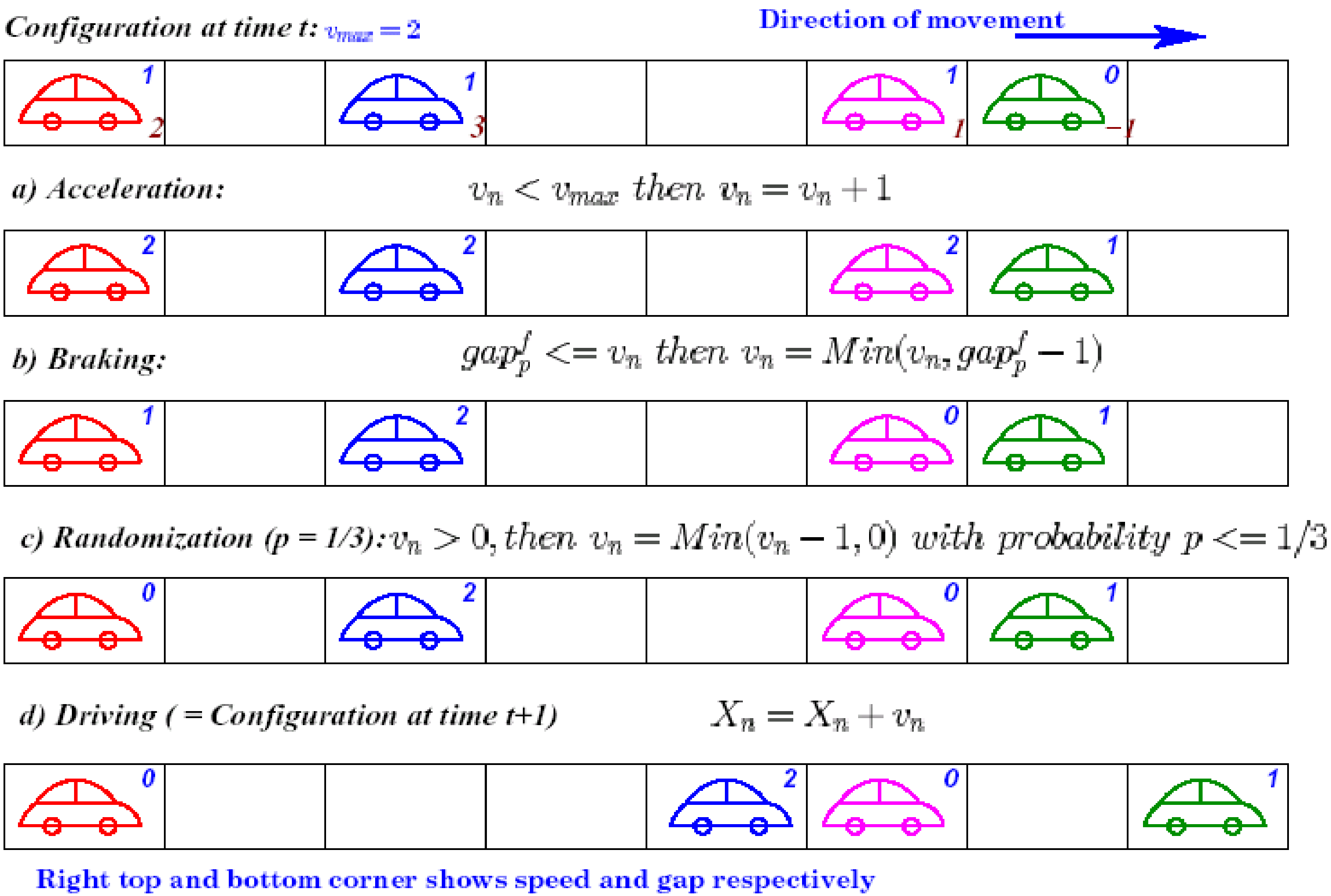


4) Updation (= Configuration at time t+1) $x_n = x_n + v_n$



Car 1: Updated by 1 cell

Nagel Schreckenberg Model



NaSch Model: Performance

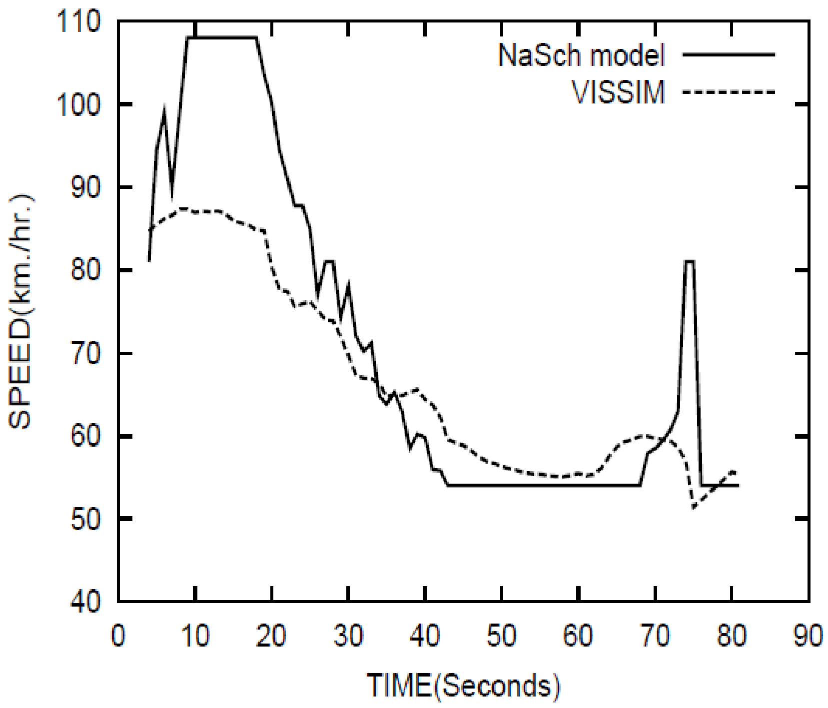
Speed in c/ts	NaSch(
	Speed (kmph)
(1)	(2)
1	27
2	54
3	81
4	108
5	135

Speed in c/ts	NaSch(CA-7.5)		CA-5	
	Speed (kmph)	Headway (meters)	Speed (kmph)	Headway (meters)
(1)	(2)	(3)	(4)	(5)
1	27	7.5	18	5
2	54	15	36	10
3	81	22.5	54	15
4	108	30	72	20
5	135	37.5	90	25
6	-	-	107	30
7	-	-	126	35
8	-	-	144	40

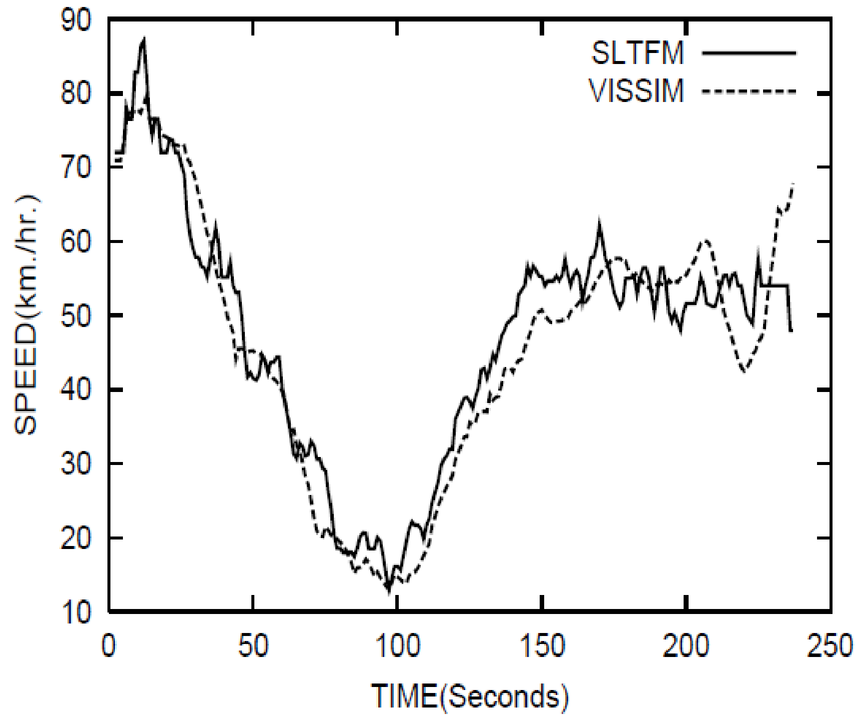
Cell size 7.5 m

Cell size 5 m

NaSch Model: Performance

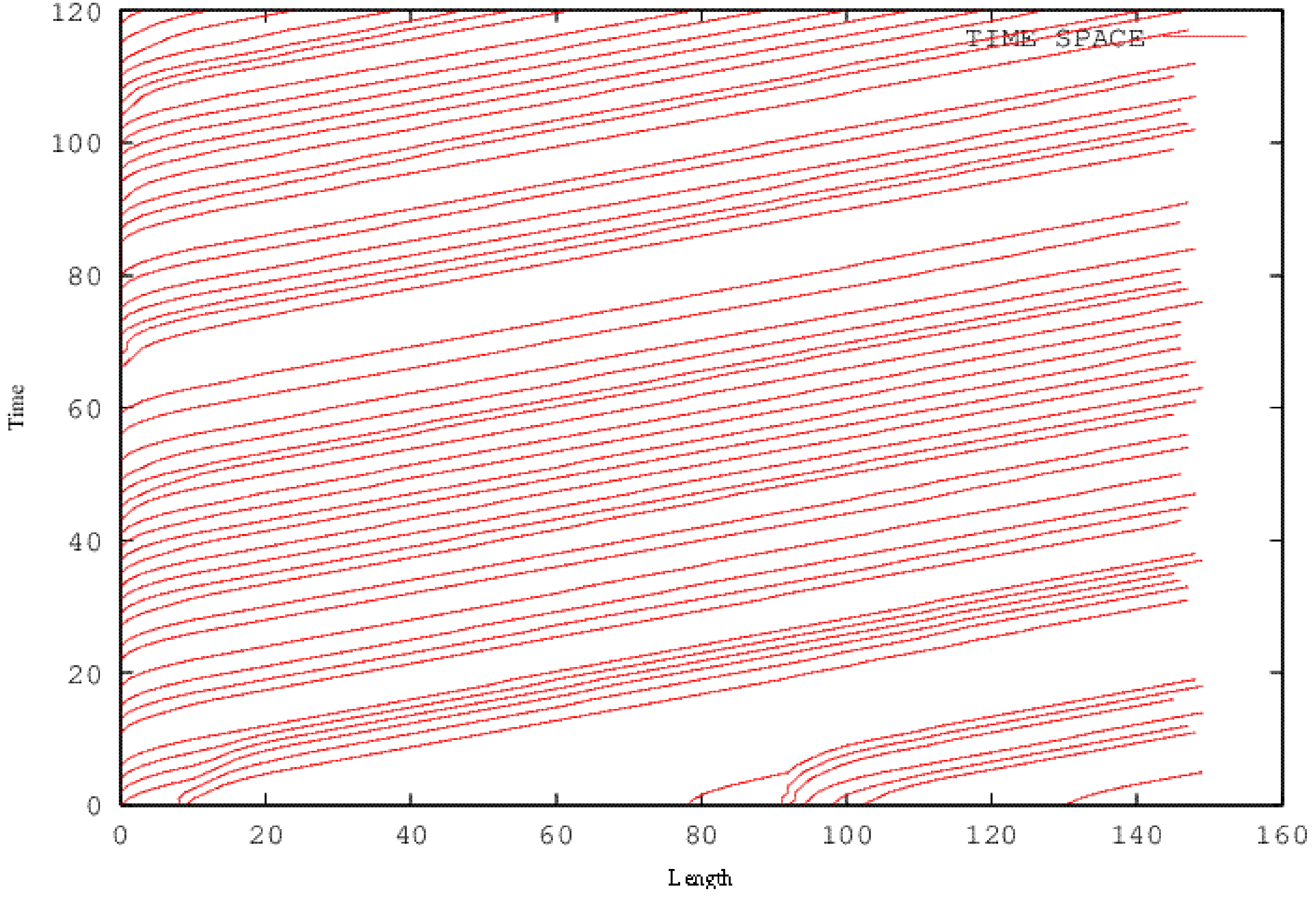


Cell size 7.5 m



Cell size 5 m

CA Model: Simulation



Lane Changing

Proposed Lane Changing

Trigger criteria

1. **Spacing in the present lane less than the spacing required to maintain the maximum speed of the lane**
2. **Sufficient spacing available in the next lane *to maintain the maximum speed***
3. ***Lane changing probabilities***
4. ***Vehicle probability***

Safety criteria

5. **Back-gap should be more than maximum speed allowed in the other lane**
6. **Adjacent cell must be empty**

– Trigger Criterion

1. $gap_C^f \leq v_{max}^{l1}$ *or* v_{max}^{l2}
2. $gap_T^f > v_{max}^k$
3. $p_l \leq r_n$
4. $p_{lv} \leq r_n$

– Safety criterion

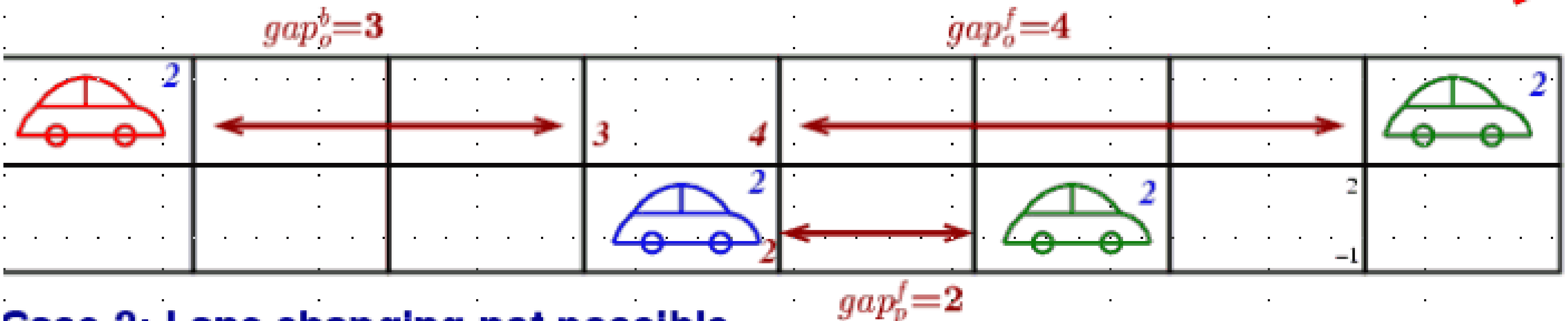
5 $X_n^T = 0(Empty)$

6 $gap_T^b > v_{max}^{l2}$ *or* v_{max}^{l1}

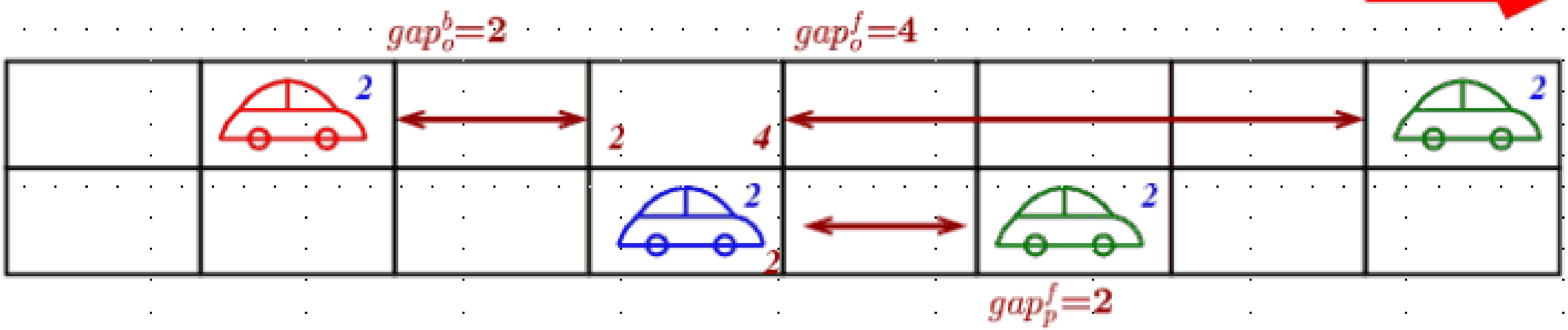
Lane Changing

Case 1: Lane changing Possible

$V_{max} = 2$



Case 2: Lane changing not possible



1) TRIGGER CRITERIA

$gap_p^f < V_n + 1$

$gap_o^f > V_{max}$

2)SAFETY CRITERIA

Neighbour cell on other lane should be empty

$gap_n^b > V_{max}$

Two lane traffic flow model

Heterogeneous traffic

Homogeneous Model

- Input
- Initialization
- Application of CA rules
  - Acceleration
  - Deceleration
  - Randomization
  - Lane changing
  - Updation
- Vehicle Generation

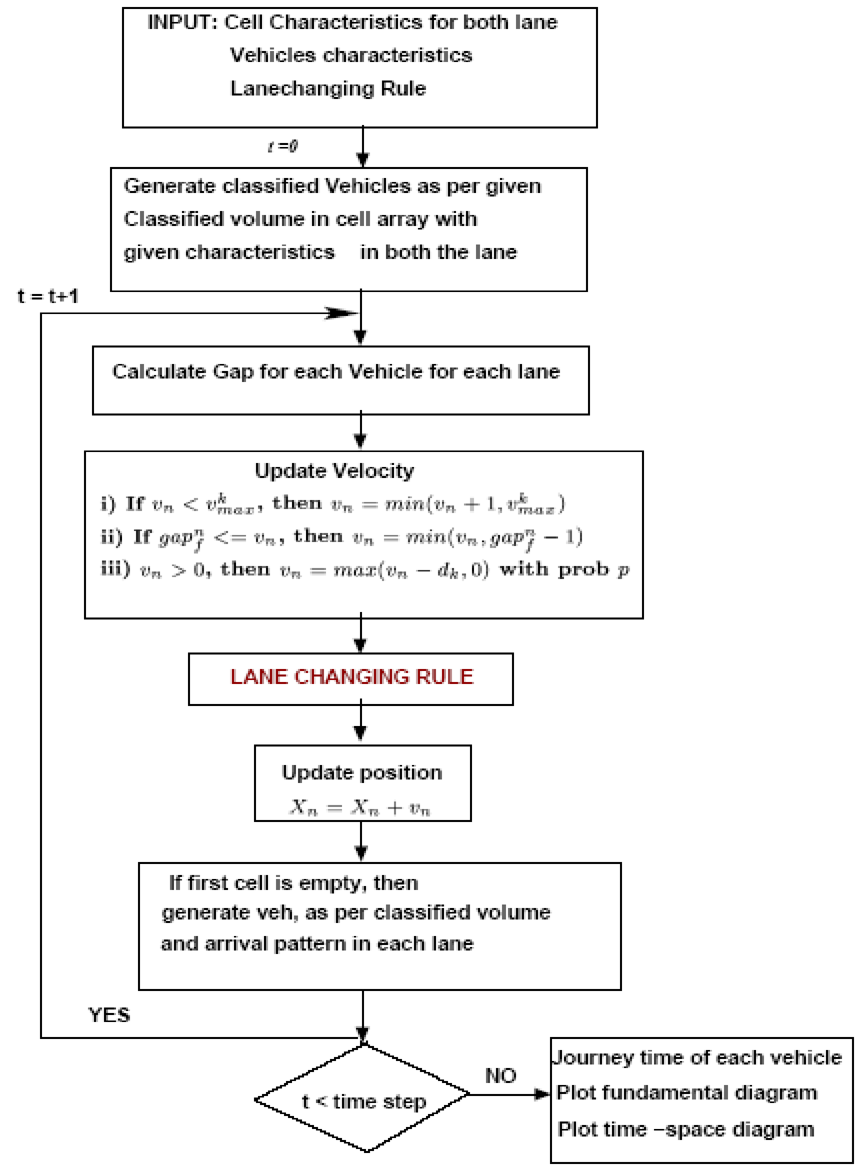
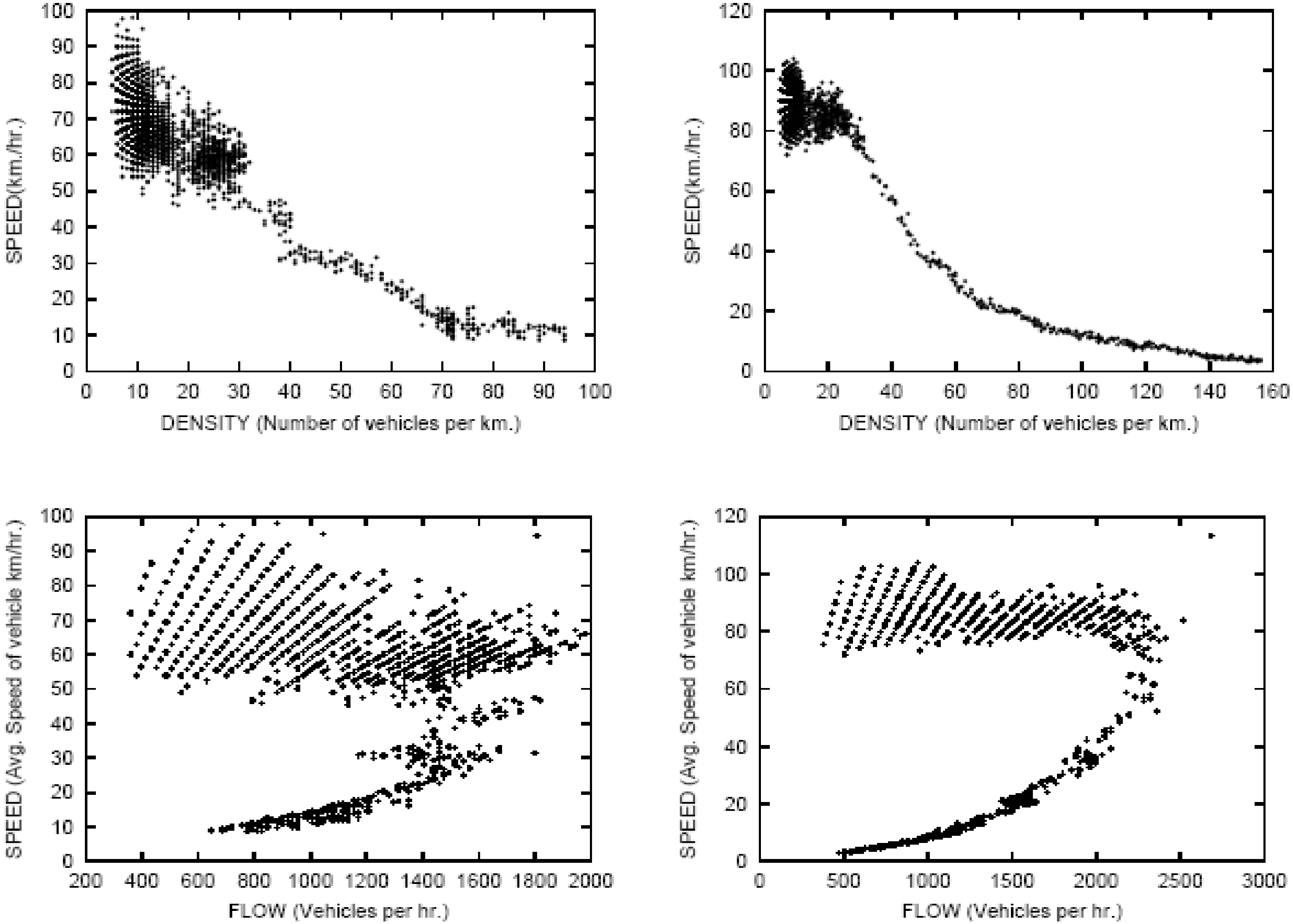


Figure 6.1: Flowchart for two-lane traffic flow CA Model

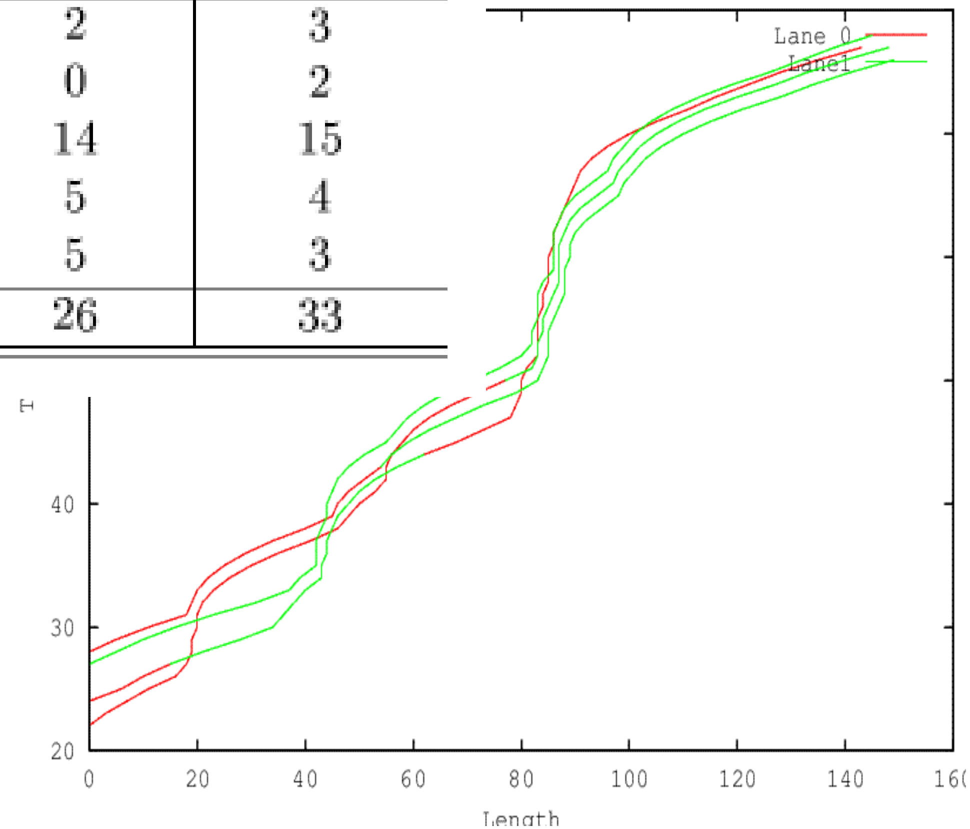
Fundamental diagrams: (hetero and homogeneous)



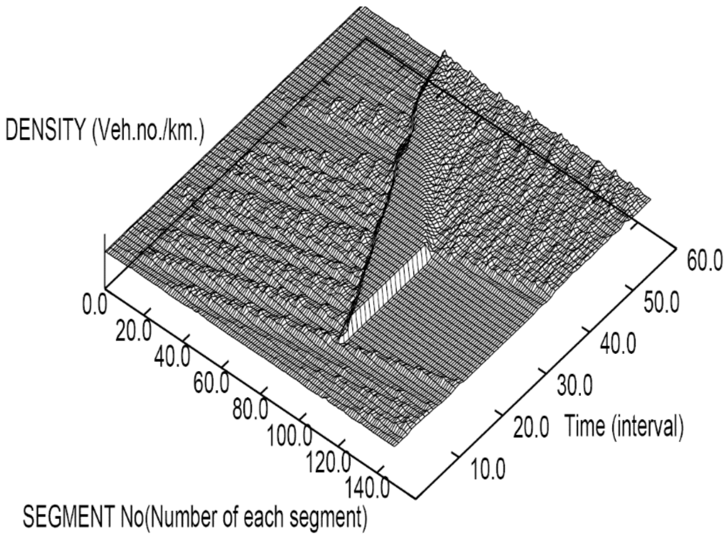
Validation of Lane changing

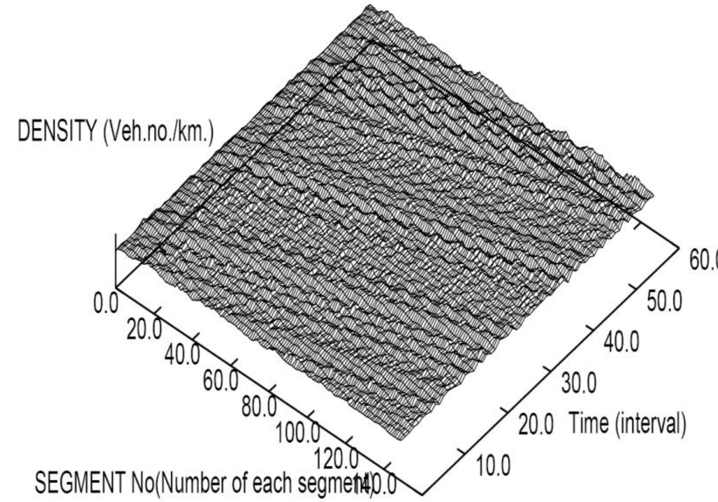
Table 6.5: Lane changing (Rule 3) between observed and simulated values

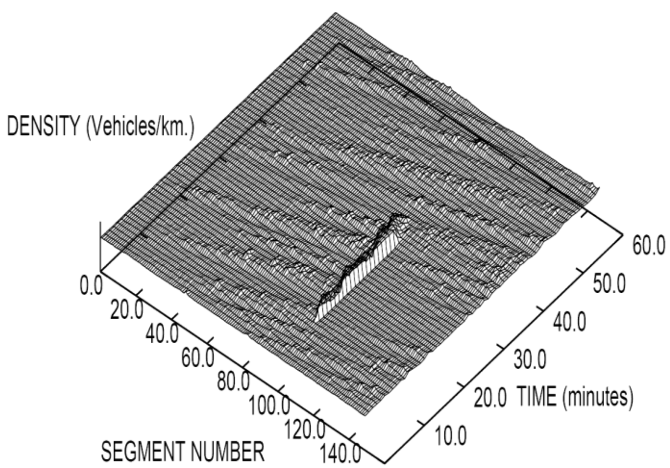
Sr. no.	Type of Vehicle	lane 1 to lane 2		lane 2 to lane 1	
		Observed	Simulated	Observed	Simulated
(1)	(2)	(3)	(4)	(5)	(6)
1	2W	6	7	2	3
2	3W	1	1	0	2
3	Car	32	39	14	15
4	LCV1	1	3	5	4
5	HCV1	7	2	5	3
	Total	47	52	26	33



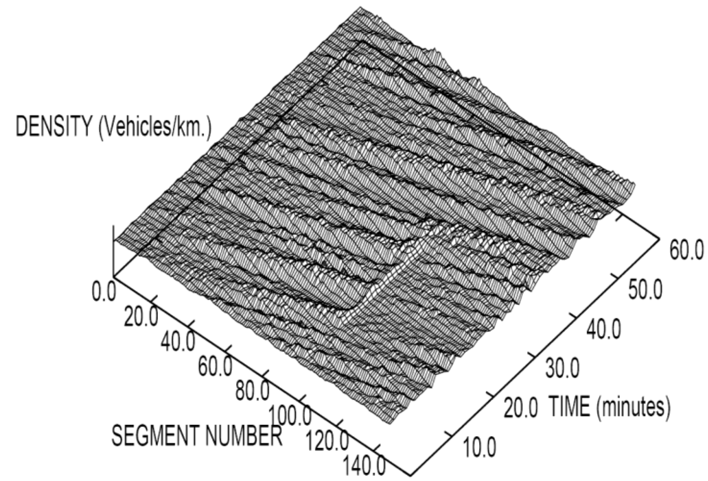
Validation of Lane changing







C: Incident effect on lane 1 in case 2



D: Incident effect on lane 2 in case 2

Grid based traffic flow model

Modified CA for Heterogeneity

Grid Based Model

- Input
- Initialization
- Application of CA rules
  - Acceleration
  - Deceleration
  - Randomization
  - Lane changing
  - Updation
- Vehicle Generation

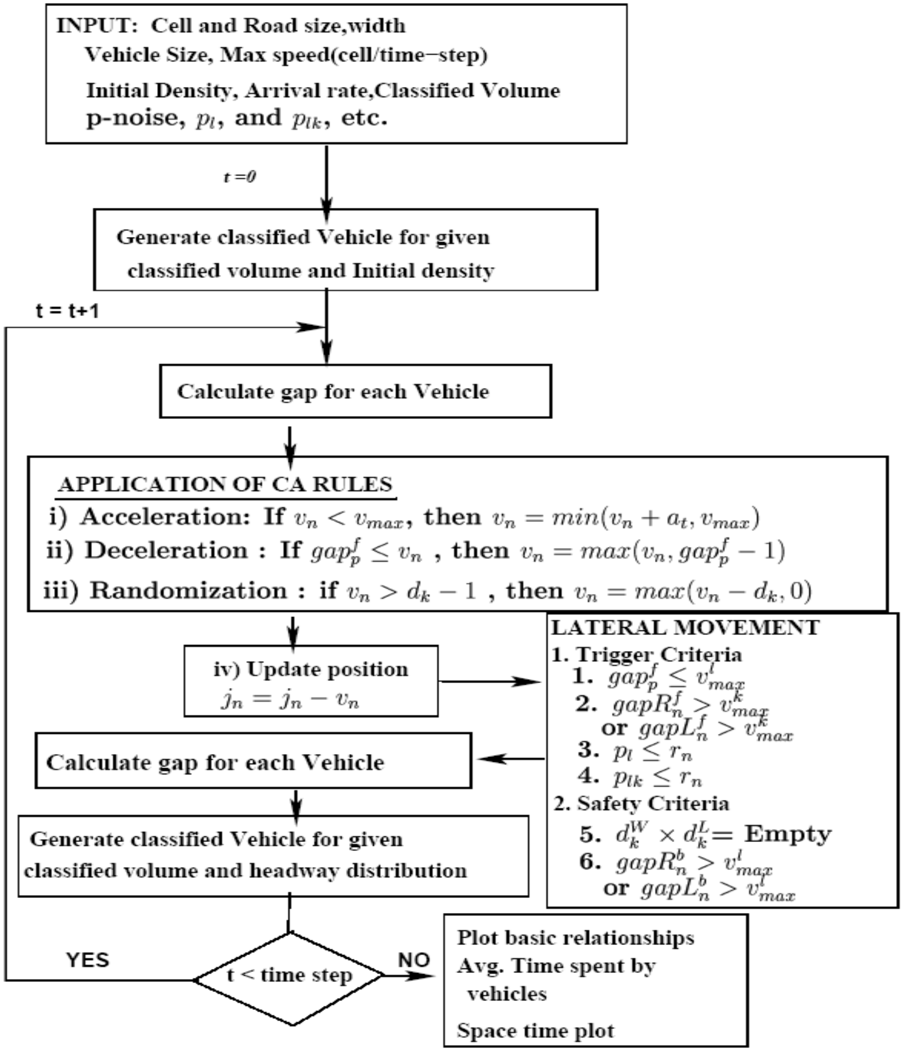


Figure 7.1: Flow chart for the Heterogeneous traffic flow model

Vehicle representation

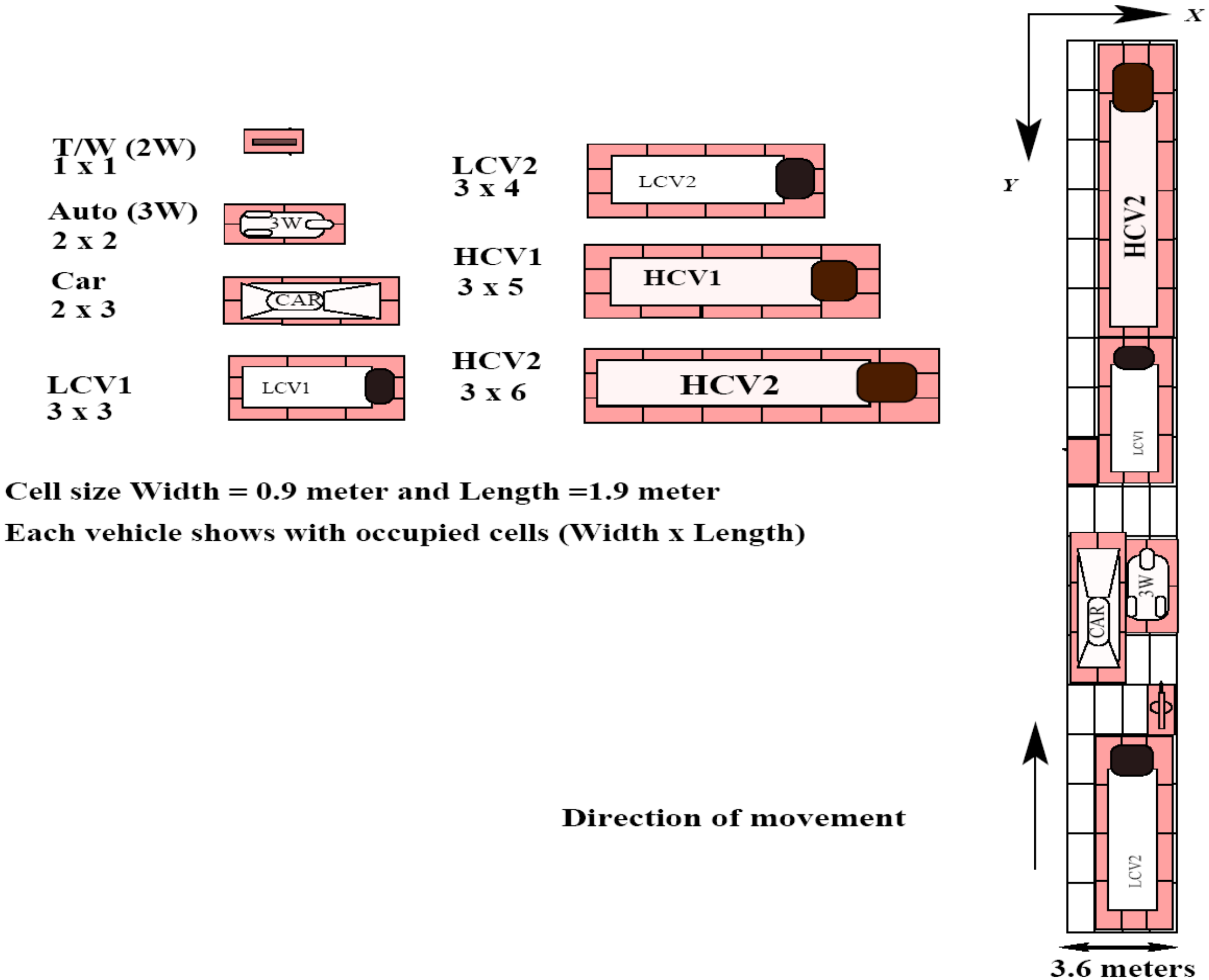


Figure 7.3: Physical representation of vehicles on single lane road

Vehicle representation

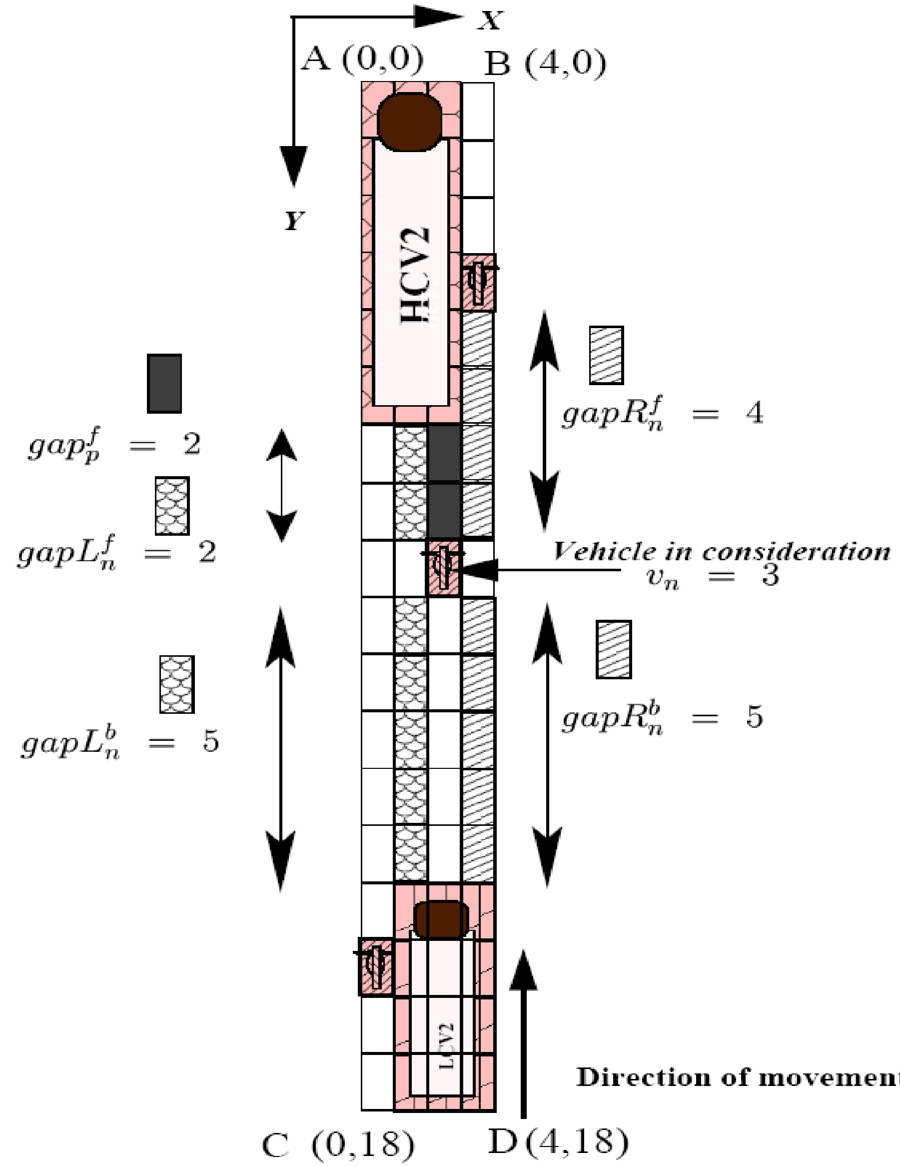


Figure 7.4: Illustration of lateral movement

Vehicle dimensions details

Table 7.2: Vehicle dimensions details

Sr. No	Vehicle Type	Actual		Taken in model		Taken in model		Clearance	
		Width meters	Length meters	Width meters	Length meters	Width cells	Length cells	Width meters	Length meters
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	2W	0.6	1.8	0.9	1.9	1	1	0.3	0.1
2	3W	1.4	2.6	1.8	3.8	2	2	0.4	1.2
3	Car	1.7	4.7	1.8	5.7	2	3	0.1	1.0
4	LCV1	1.9	5.0	2.7	5.7	3	3	0.8	0.7
5	LCV2	2.2	6.8	2.7	7.6	3	4	0.5	0.8
6	HCV1	2.5	8.5	2.7	9.5	3	5	0.2	1.0
7	HCV2	2.5	10.3	2.7	11.4	3	6	0.2	1.1

Speed - Headway for CA and Grid based model

Table 7.1: Speed and distance headway for different discrete models

Speed in c/ts	NaSch(CA-7.5)		CA-5		GBTFM	
	Speed (kmph)	Headway (meters)	Speed (kmph)	Headway (meters)	Speed (kmph)	Headway (meters)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	27	7.5	18	5	6.84	1.9
2	54	15	36	10	13.68	3.8
3	81	22.5	54	15	20.52	5.7
4	108	30	72	20	27.36	7.6
5	135	37.5	90	25	34.2	9.5
6	-	-	107	30	41.04	11.4
7	-	-	126	35	47.88	13.3
8	-	-	144	40	54.72	15.2
9	-	-	-	-	61.56	17.1
10	-	-	-	-	68.4	19
11	-	-	-	-	75.24	20.9
12	-	-	-	-	82.08	22.8
13	-	-	-	-	88.92	24.7
14	-	-	-	-	95.76	26.6
15	-	-	-	-	102.6	28.5
16	-	-	-	-	109.44	30.4
17	-	-	-	-	116.28	32.3
18	-	-	-	-	123.12	34.2
19	-	-	-	-	129.96	36.1
20	-	-	-	-	136.8	38

Microscopic Validation in case 1: (Without incident)

CA- 7.5 | CA-5 | GBTFM

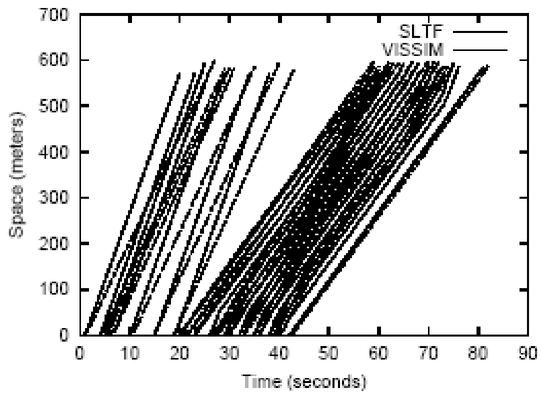


Figure 5.5: Trajectory plot of VISSIM versus SLTFM in case 1 for CA-7.5

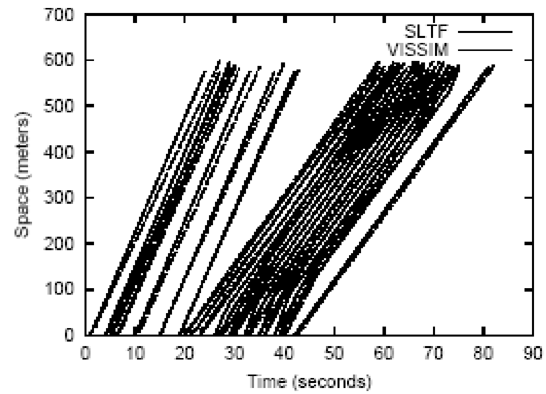


Figure 5.6: Trajectory plot of VISSIM versus SLTFM in case 1 for CA-5

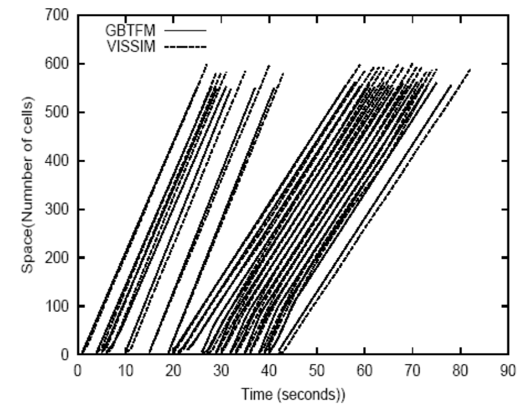


Figure 7.5: Trajectory plot of VISSIM versus GBTFM in case 1

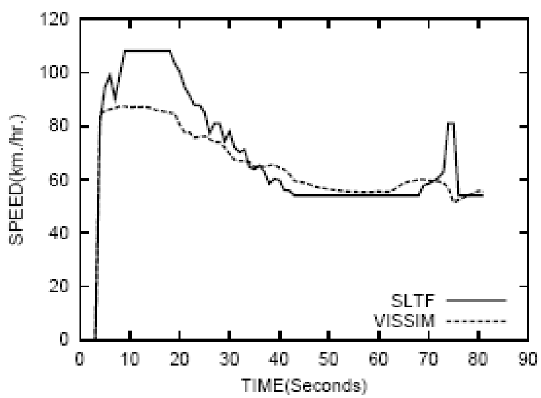


Figure 5.7: Speed variation - VISSIM versus SLTFM in case 1 for CA-7.5

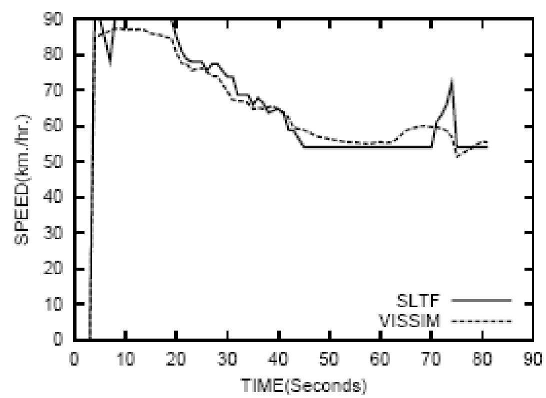


Figure 5.8: Speed variation - VISSIM versus SLTFM in case 1 for CA-5

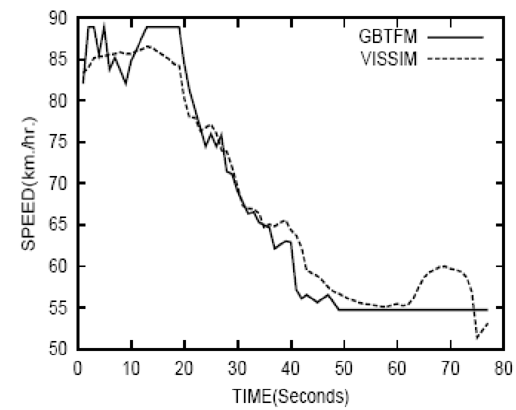


Figure 7.6: Comparisons of speed variation of l VISSIM versus GBTFM in case 1

Computational comparisons

Model	CA 5	GBTFM	VISSIM
Time (sec.)	5.44	103.5	1530

- A 2.5 km arterial of single lane with the 2.54 mean rate, one hour simulation

Conclusion

- **Novel attempt**
  - Precise vehicle type representation
  - Retains simplicity and efficiency of CA
- **Model is generic**
  - handle any vehicle type and road width
- **A new incident rule**
  - model stop-go behavior
- **Scalable**

Thank You

?