# Stresses and Strains in flexible Pavements



#### Assumptions in Multi Layered Elastic Systems

- The material properties of each layer are homogeneous property at point  $A_i$  is the same at  $B_i$
- Each layer has a finite thickness except for the lower layer, and all are infinite in lateral directions.
- Each layer is isotropic, that is, the property at a specific point such as A<sub>i</sub> is the same in every direction or orientation.
- Full friction is developed between layers at each interface.
- Surface shearing forces are not present at the surface.
- The stress solutions are characterized by two material properties for each layer, i.e., (μ, *E*).

## Stresses in Layered Systems

- At any point, 9 stresses exist. They are 3 normal stresses ( $\sigma_z$ ,  $\sigma_r$ ,  $\sigma_t$ ) and 6 shearing stresses ( $\tau_{rz} = \tau_{zr}$ ;  $\tau_{rt} = \tau_{tr}$ ;  $\tau_{tz} = \tau_{zt}$ )
- At each point in the system there exists a certain orientation of the element such that the shearing stresses acting on each face are zero.
  - The normal stresses under this condition are principal stresses and are denoted by  $\sigma_1(major)$ ,  $\sigma_2$  (intermediate) and  $\sigma_3$  (minor).

### **Computation of Strains**

$$\varepsilon_{z} = \frac{1}{E} (\sigma_{z} - \mu(\sigma_{r} + \sigma_{t}))$$
$$\varepsilon_{r} = \frac{1}{E} (\sigma_{r} - \mu(\sigma_{t} + \sigma_{z}))$$
$$\varepsilon_{t} = \frac{1}{E} (\sigma_{t} - \mu(\sigma_{r} + \sigma_{z}))$$

## One –Layer Systems

- If, one-layer system is assumed as a homogeneous half space, Boussinesq equations can be applied.
- Half space is an infinite large area with infinite depth with a top plane on which loads are applied.
- Boussinesq equations are developed for computing stresses in a homogeneous, isotropic and elastic media due to a point load at the surface.

$$\sigma_z = K \frac{P}{z^2}$$

$$K = \frac{3}{2\pi} \frac{1}{\left[1 + (r/z)^2\right]^{5/2}}$$

- Stress is independent of the properties of the transmitting medium.
- Maximum stress occurs on the vertical plane passing through the point of load application, on a particular horizontal plane.
- Pressure is maximum at shallow depths, theoretically becoming zero at infinite depth. But, for all practical purposes,  $\sigma_z$  is taken as zero when z is sufficiently large.

## One –Layer Systems

- Load is not a point load it is distributed over an elliptical area. This contact area can be approximated to a circular shape.
- Variation of stress follows the same general pattern
- Vertical stress resulting from uniformly distributed circular load may be obtained by integration of Boussinesq equation.
- Love has obtained the following closed from equation for the vertical stress beneath the centre of the loaded area:

$$\sigma_{z} = p \left[ 1 - \frac{z^{3}}{\left(a^{2} + z^{2}\right)^{3/2}} \right]$$

Newmark has developed charts for foundation work for computing stresses

## One –Layer Systems

- Foster and Ahlvin (1954) developed charts for computing vertical, tangential and radial stresses. The charts were developed for  $\mu = 0.5$ .
- This work was subsequently refined by Ahlvin and Ulery (1962) allowing for evaluation of stresses and strains at any point in the homogenous mass for any  $\mu$ .
- Due to axis symmetry, there are only three normal stresses,  $\sigma_z$ ,  $\sigma_r$  and  $\sigma_t$  and one shear stress  $\tau_{rz}$ .
- One-layer theory can be applied as an approximation for a conventional flexible pavement with granular base/subbase with a thin asphaltic layer on a stiff subgrade comparable to the base/subbase. (i.e.,  $E_1/E_2 \cong 1$ )
- The deflection that occurs within the pavement  $(\Delta_p)$  is neglected and therefore, the pavement surface deflection  $(\Delta_T)$  is equal to the deflection on the top of subgrade  $(\Delta_s)$

$$\Delta_{T} = \Delta_{s} + \Delta_{p}$$
  
$$\Delta_{p} = 0$$
  
Therefore,  $\Delta_{T} = \Delta_{s}$ 



# Charts for One-layer Solutions by Foster and Ahlvin (1954)

These are applicable for  $\mu = 0.5$ 











# Tables for One-layer Solutions by Ahlvin and Ulery (1962)

Parameter	General Case	Special Case ( $\mu = 0.5$ )
Vertical stress	$\sigma_z = p[A + B]$	(same)
Radial horizontal stress	$\sigma_r = p[2\mu A + C + (1 - 2\mu)F]$	$\sigma_r = p[A + C]$
Tangential horizontal stress	$\sigma_t = p[2\mu A - D + (1 - 2\mu)E]$	$\sigma_t = p[A - D]$
Vertical radial shear stress	$\tau_{rz} = \tau_{zr} = pG$	(same)
Vertical strain	$\epsilon_z = \frac{p(1 + \mu)}{E_1} [(1 - 2\mu)A + B]$	$\epsilon_z \qquad \frac{1.5p}{E_1} B$
Radial horizontal strain	$\epsilon_r = \frac{p(1+\mu)}{E_1} \left[ (1-2\mu)F + C \right]$	$\epsilon_r = \frac{1.5p}{E_1} C$
Tangential horizontal strain	$\epsilon_t = \frac{p(1+\mu)}{E_1} \left[ (1-2\mu)E - D \right]$	$\epsilon_t = -\frac{1.5p}{E_1} D$
Vertical deflection	$\Delta_z = \frac{p(1+\mu)a}{E_1} \left[ \frac{z}{a} A + (1-\mu)H \right]$	$\Delta_{z} = \frac{1.5pa}{E_1} \left( \frac{z}{a} A + \frac{H}{2} \right)$
Bulk stress	$\theta = \sigma_z + \sigma_r + \sigma_t$	
Bulk strain	$\epsilon_{\theta} = \epsilon_z + \epsilon_r + \epsilon_t$	
Vertical tangential shear stress	$\tau_{zt} = \tau_{tz} = 0$ $\therefore$ $[\sigma_t(\epsilon_t)$ is principal st	ress (strain)]
Principal stresses	$\sigma_{1,2,3} = \frac{(\sigma_z + \sigma_r) \pm \sqrt{(\sigma_z - \sigma_r)^2 + \sigma_r^2}}{2}$	$-(2 au_{rz})^2$
Maximum shear stress	$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$	

TABLE 2.1. Summary of One-Layer Elastic Equations<sup>a</sup> (after Ahlvin and Ulery)

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### Tables for Functions A to H

(after Ahlvin and Ulery)

#### Example problems on One-layer Systems

- 1. A homogeneous half space is subjected to a circular load, 254 mm in diameter. The pressure on the circular area is 345 kPa. The half space has an elastic modulus of 69 MPa and a Poisson's ratio of 0.5. Determine the vertical stress, strain and deflection at point A, which is located 254 mm below and 508 mm away from the centre.
- 2. A homogeneous half space is subjected to two circular loads, each 254 mm in diameter and spaced at 508 mm on centres. The pressure on the circular area is 345 kPa. The half space has an elastic modulus of 69 MPa and a Poisson's ratio of 0.5. Determine the vertical stress, strain and deflection at point A, which is located 254 mm below the centre of one of the wheels.

## **Example Problems Continued**

3. Given the following data for the pavement shown in the figure, compute the deflection at point *m*,  $\sigma_z$  and  $\varepsilon_r$  at point *o* and  $\sigma_{1,2,3}$  and  $\tau_{max}$  at point *p*.

$$h_1 = 254 \text{ mm}; E_1 = 173 \text{ MPa}; \mu_1 = 0.35$$



### Solutions at Axis of Symmetry



#### Solutions at Axis of Symmetry – Flexible Plate

Normal stress at axis symmetry:

$$\sigma_{z} = p \left[ 1 - \frac{z^{3}}{\left(a^{2} + z^{2}\right)^{3/2}} \right]$$

Radial stress at axis symmetry:  $\sigma_r = \frac{p}{2} \left[ 1 + 2\mu - \frac{2(1+\mu^2)}{(a^2+z^2)^{0.5}} + \frac{z^3}{(a^2+z^2)^{3/2}} \right]$ 

Strains can be computed from the following equations:

$$\varepsilon_{z} = \frac{1}{E} (\sigma_{z} - \mu(\sigma_{r} + \sigma_{t}))$$
  

$$\varepsilon_{r} = \frac{1}{E} (\sigma_{r} - \mu(\sigma_{t} + \sigma_{z}))$$
  

$$\varepsilon_{t} = \frac{1}{E} (\sigma_{t} - \mu(\sigma_{r} + \sigma_{z}))$$

At the axis of symmetry,  $\tau_{rz}$ =0 and  $\sigma_r = \sigma_t$ 

#### Solutions at Axis of Symmetry – Flexible Plate

Vertical deflection  $\Delta_z$  can be determined by

$$\Delta_{z} = \int_{\infty}^{z} \mathcal{E}_{z} dz$$
$$\Delta_{z} = \frac{(1+\mu)pa}{E} \left\{ \frac{a}{(a^{2}+z^{2})^{0.5}} + \frac{1-2\mu}{a} \left[ (a^{2}+z^{2})^{0.5} - z \right] \right\}$$



For 
$$\mu = 0.5$$
,  $\Delta_z = \frac{3pa^2}{2E(a^2 + z^2)^{0.5}}$   
At  $z = 0$ ,  $\Delta_z = \frac{2(1 - \mu^2)pa}{E}$ 

At z = 0 and 
$$\mu$$
 = 0.5,  $\Delta_z = \frac{1.5 \, pa}{E}$ 

### **Deflection of Rigid Plate**



over the contact area of the plate it can be shown that  $\pi(1 - u^2)$  by

$$\Delta_o = \frac{\pi (1 - \mu^2) pa}{2E}$$

If 
$$\mu = 0.5$$
, then,  $\Delta_o = \frac{1.18 \, pa}{E}$ 

## **Two-layer Systems**

- The effect of layers above subgrade is to reduce the stress and deflections in the subgrade.
- Burmister (1958) obtained solutions for two-layer problem by using strain continuity equations.
- Vertical stress depends on the modular ratio (i.e.,  $E_1/E_2$ )
- Vertical stress decreases considerably with increase in modular ratio.
- For example,

for  $a/h_1=1$  and  $E_1/E_2=1$ ,  $\sigma_z$  at interface = 65% of contact pressure for  $a/h_1=1$  and  $E_1/E_2=100$ ,  $\sigma_z$  at interface = 8% of contact pressure

#### Variation of Subgrade Stress with Modular Ratio



#### Vertical Stress in a Two-layer System



#### Vertical Surface Deflection in a Twolayer System

- Burmister (1958) dveloped a chart for computing vertical surface deflection in a two-layer system.
- The deflection factor,  $F_2$ , is obtained from the chart based on the values of  $a/h_1$  and  $E_1/E_2$ .
- Then the deflection is computed from the following equations:

- Deflection under a flexible Plate =  $\Delta_T = \frac{1.5 \, pa}{E_2} F_2$ 

Deflection under a rigid Plate =

$$\Delta_T = \frac{1.18\,pa}{E_2}F_2$$

#### Vertical Surface Deflections for Two Layer Systems (Burmister, 1958)

