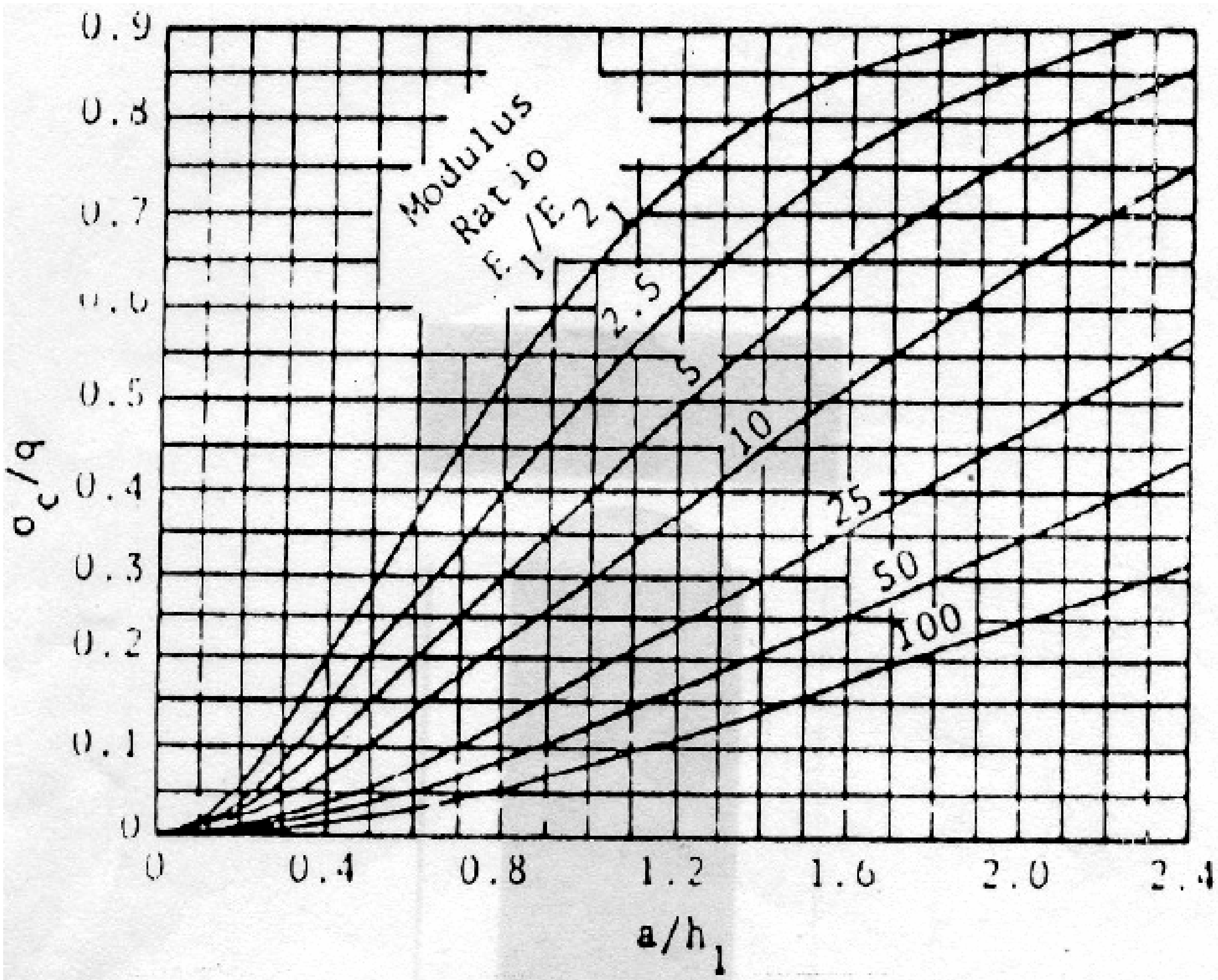
Two-layer Systems

- The effect of layers above subgrade is to reduce the stress and deflections in the subgrade.
- Burmister (1958) obtained solutions for two-layer problem by using strain continuity equations.
- Vertical stress depends on the modular ratio (i.e., E_1/E_2)
- Vertical stress decreases considerably with increase in modular ratio.
- For example,

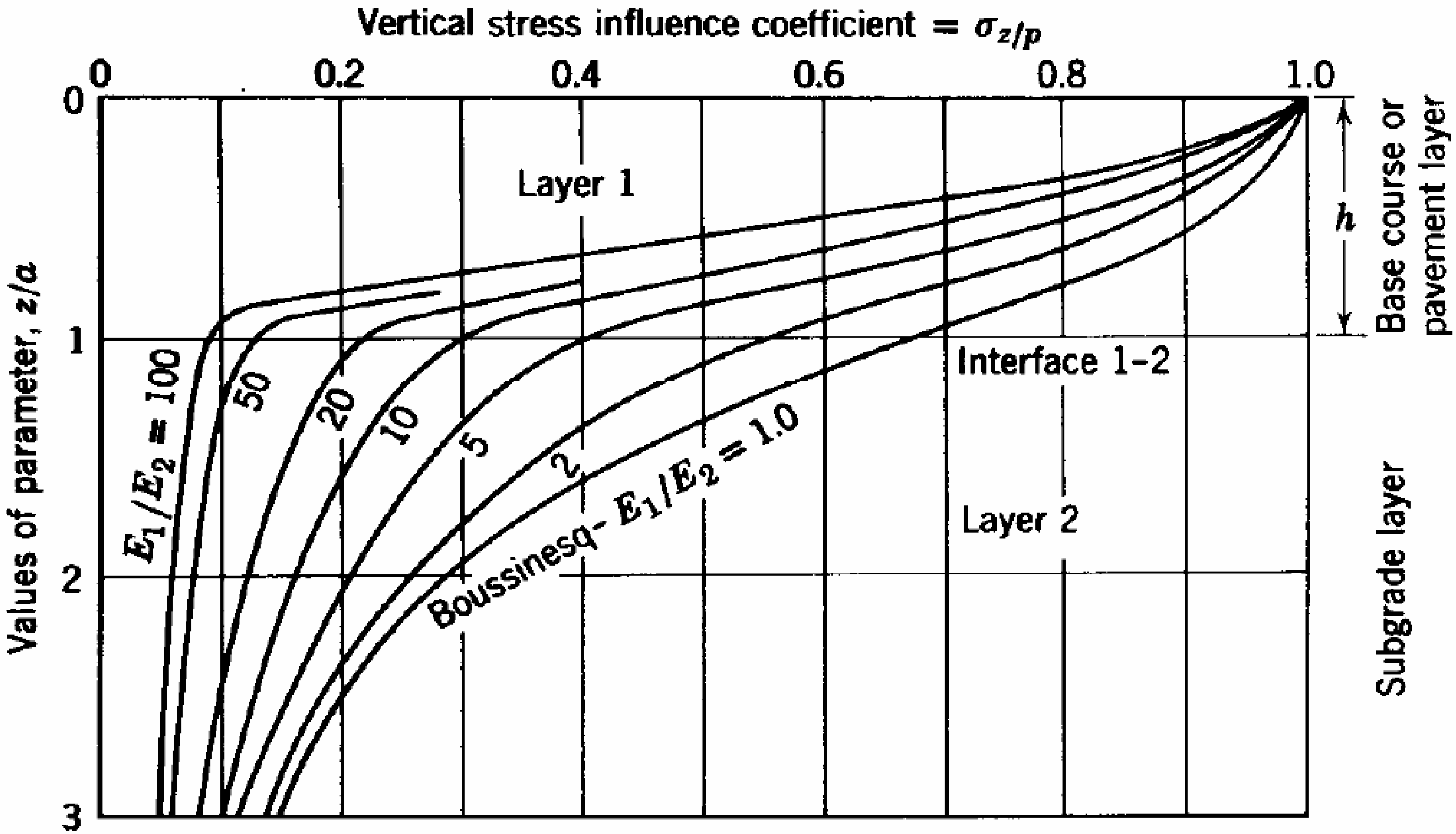
  for $a/h_1=1$ and $E_1/E_2 = 1$, σ_z at interface = 65% of contact pressure

  for $a/h_1=1$ and $E_1/E_2 = 100$, σ_z at interface = 8% of contact pressure

Variation of Subgrade Stress with Modular Ratio



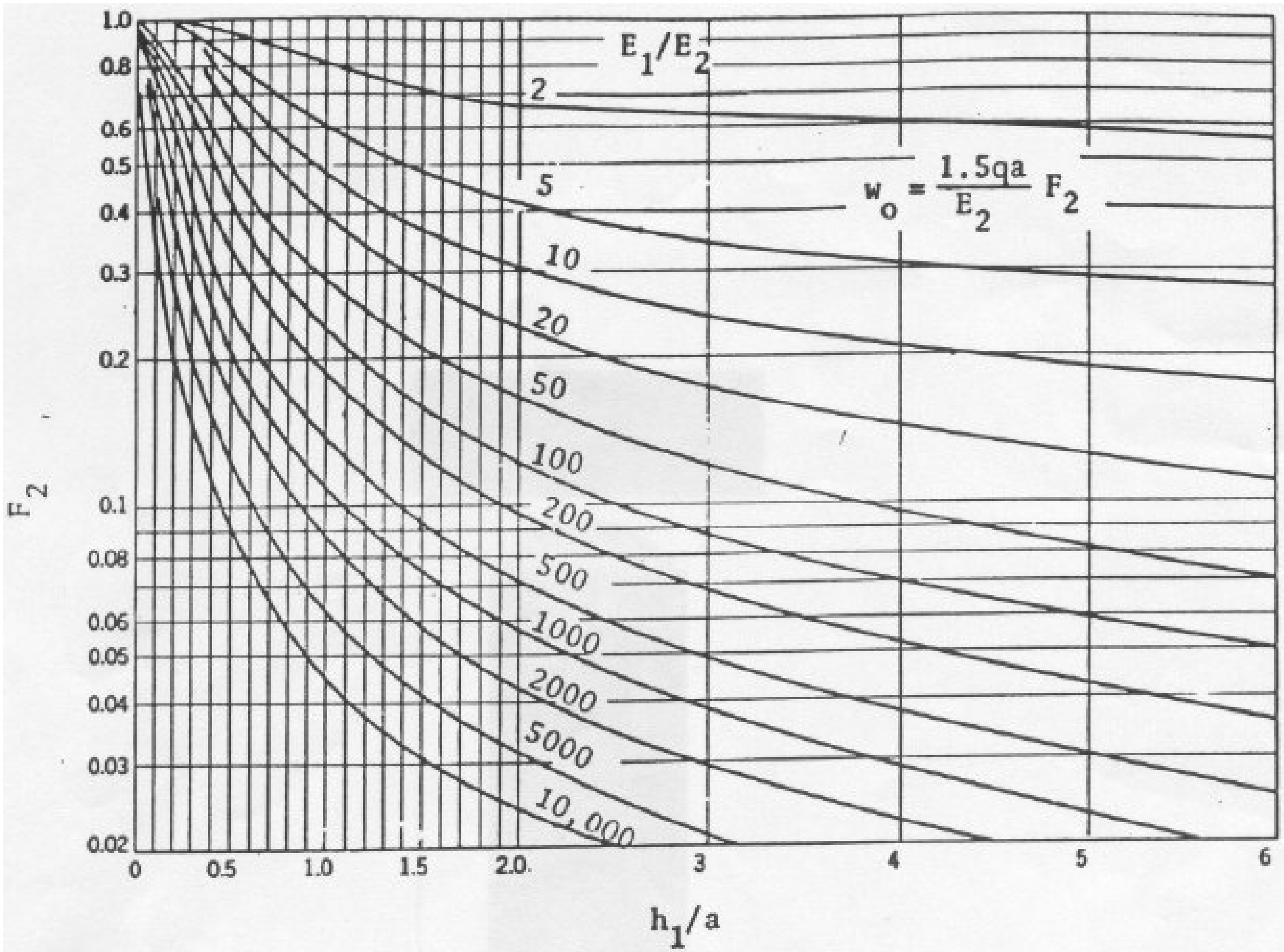
Vertical Stress in a Two-layer System



Vertical Surface Deflection in a Two-layer System

- Burmister (1958) dveloped a chart for computing vertical surface deflection in a two-layer system.
- The deflection factor, F_2, is obtained from the chart based on the values of a/h_1 and E_1/E_2.
- Then the deflection is computed from the following equations:
  - Deflection under a flexible Plate = $\Delta_T = \frac{1.5pa}{E_2} F_2$
  - Deflection under a rigid Plate = $\Delta_T = \frac{1.18pa}{E_2} F_2$

Vertical Surface Deflections for Two Layer Systems (Burmister, 1958)



Interface Deflection in a Two-layer System

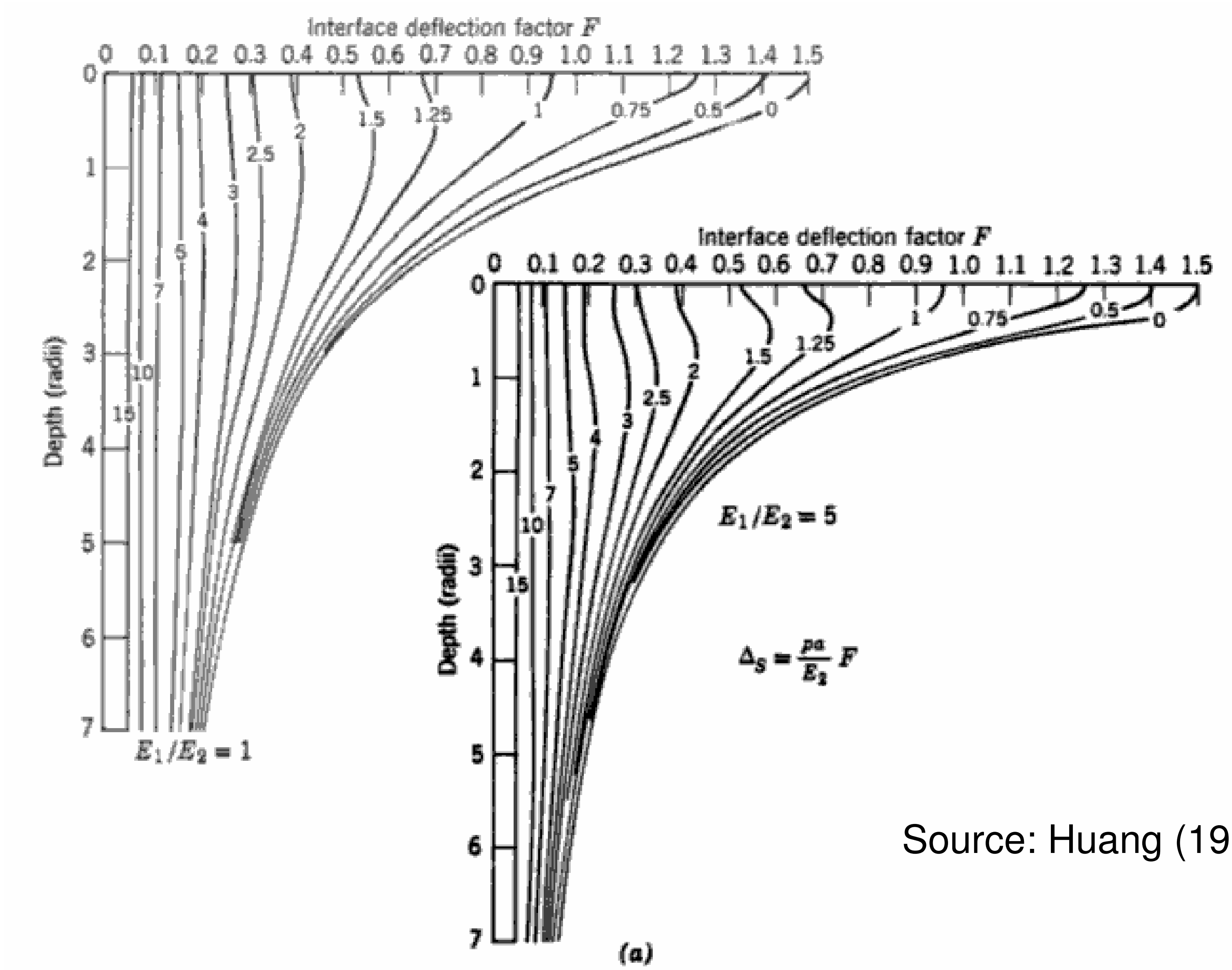
- Huang (1969) developed charts for interface deflection in a two-layer system.
- These charts are prepared for varying E_1/E_2 values.
- The interface deflection factor, F, is obtained from the chart based on the values of E_1/E_2, h_1/a *and* r/a values.
- The interface deflection (Δ_S) is then found from

$$\Delta_s = \frac{pa}{E_2} F$$

- The deflection that takes place within the pavement (Δ_p) is given by

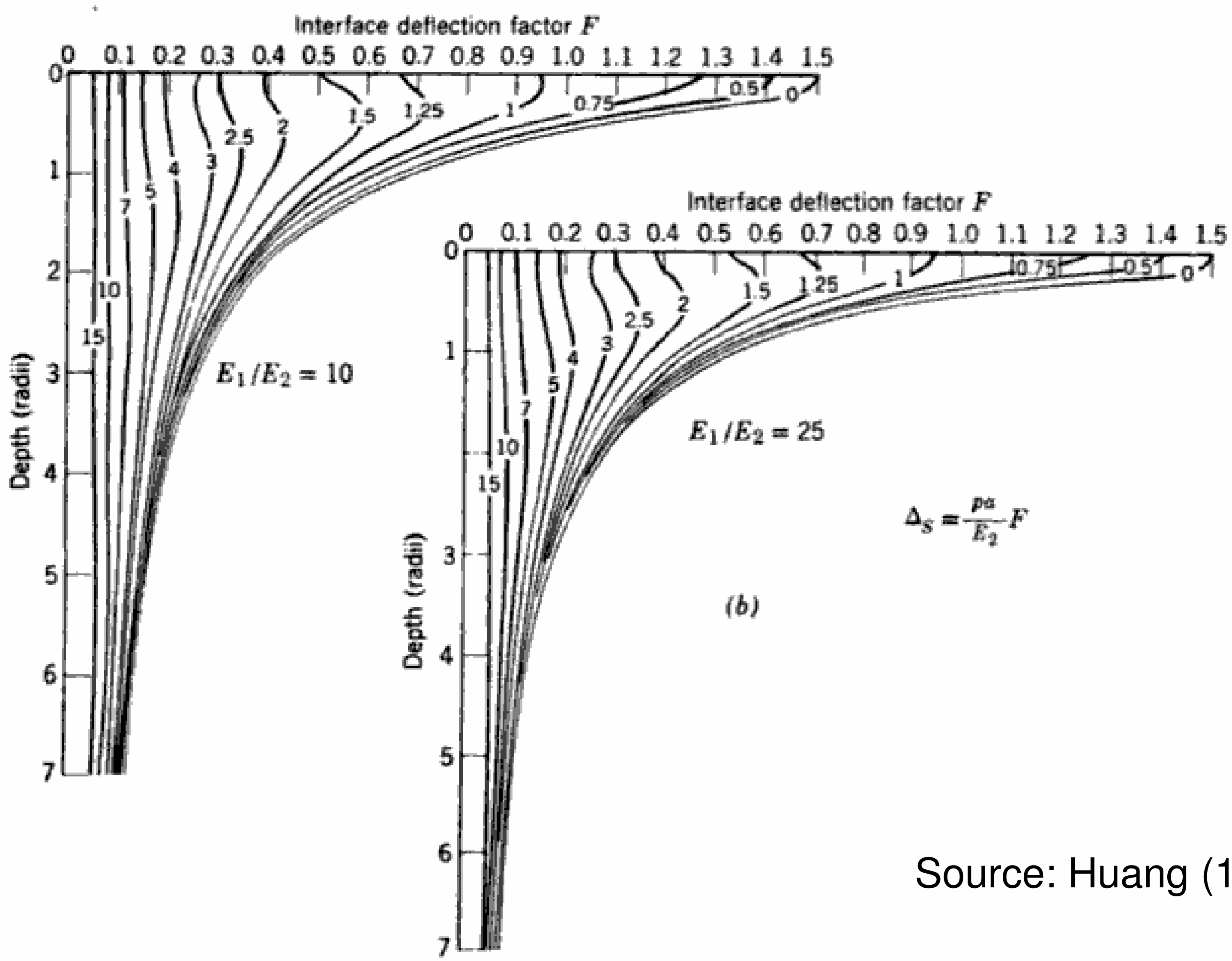
$$\Delta_p = \Delta_T - \Delta_S$$

Interface Deflection in Two-layer Systems



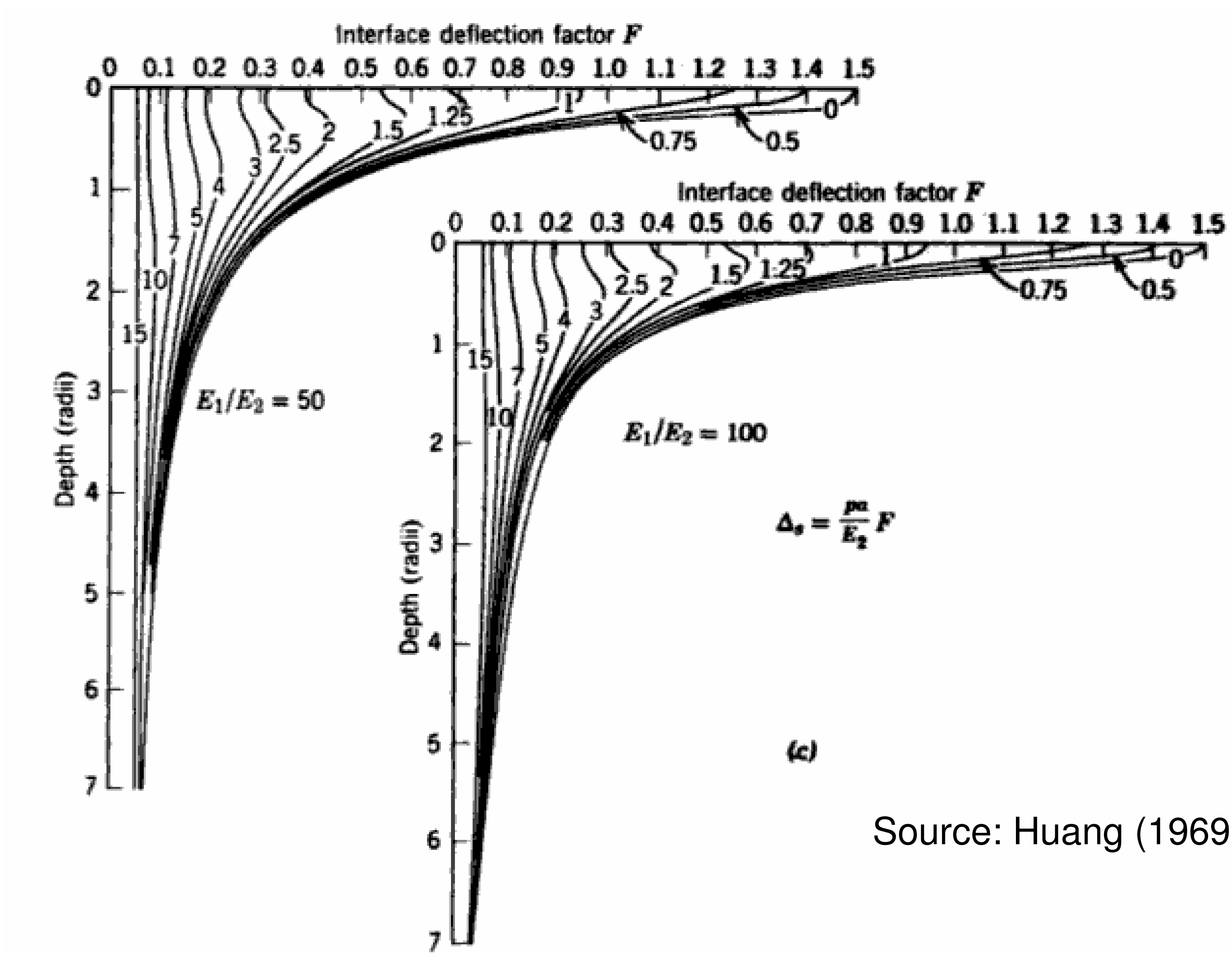
Source: Huang (1969)

Interface Deflection in Two-layer Systems



Source: Huang (1969)

Interface Deflection in Two-layer Systems



Source: Huang (1969)

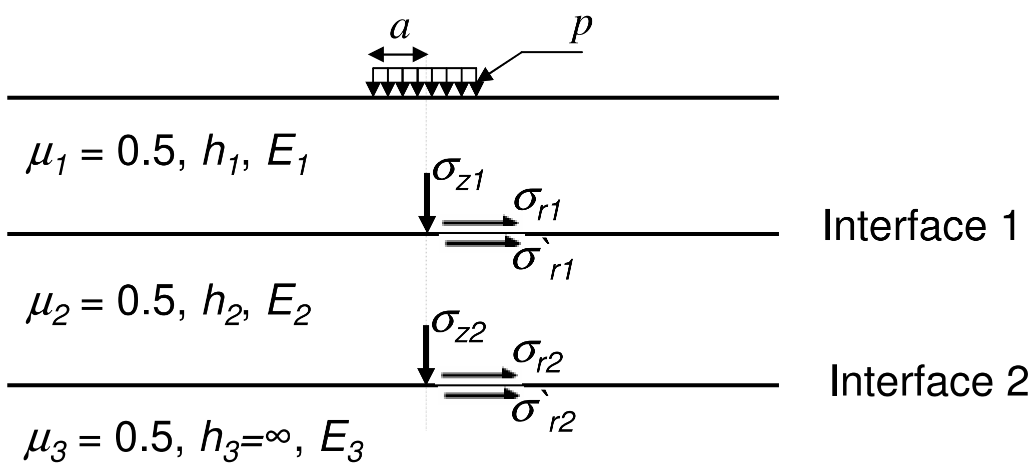
Example Problems

- Calculate the surface deflection under the centre of a tyre (a = 152 mm, p = 552 kPa) for a 305 mm pavement having a 345 MPa modulus and subgrade modulus of 69 MPa from two-layer theory. Also calculate the interface deflection and the deflection that takes place within the pavement layer.
- A circular load with a radius of 152 mm and a uniform pressure of 552 kPa is applied on a two-layer system. The subgrade has an elastic modulus of 35 kPa and can support a maximum vertical stress of 55 kPa. What is the required thickness of full depth AC pavement, if AC has an elastic modulus of 3.45 GPa.

  Instead of a full depth AC pavement, if a thin surface treatment is applied on a granular base (with elastic modulus of 173 MPa), what is the thickness of base course required?
- A plate bearing test using 750 mm diameter rigid plate was made on a subgrade as well as on 254 mm of gravel base course. The unit load required to cause settlement of 5 mm was 69 kPa and 276 kPa, respectively. Determine the required thickness of base course to sustain a 222.5 kN tyre, 690 kPa pressure and maintain a deflection of 5 mm.

Three-layer System

- Fox and Acum produced the first extensive tabular summary of normal and radial stresses in three-layer systems at the intersection of the plate axis with the layer interfaces.
- Jones (1962) and Peattie (1962) subsequently expanded these solutions to a much wider range of solution parameters.



Three-layer system showing location of stresses presented by Jones (1962) and Peattie (1962)

Notation

- σ_{z1} = Vertical stress at interface 1
- σ_{z2} = Vertical stress at interface 2
- σ_{r1} = Horizontal stress at the bottom of layer 1
- $\sigma`_{r1}$ = Horizontal stress at the top of layer 2
- σ_{r2} = Horizontal stress at the bottom of layer 2
- $\sigma`_{r2}$ = Horizontal stress at the top of layer 3
- These stress values are along the axis of symmetry of the load. Therefore, $\sigma_{r1} = \sigma_{t1}$

  For μ = 0.5, $\varepsilon_z = \frac{1}{E_1}(\sigma_{z1} - \sigma_{r1})$ and $\varepsilon_r = \frac{1}{2E_1}(\sigma_{r1} - \sigma_{z1})$

  Therefore, radial strain = one half the vertical strain

  i.e., $\varepsilon_r = -0.5\ \varepsilon_z$

Parameters in Jones Tables

- Stresses in a three layer system depend on the following ratios:

  $K_1 = E_1/E_2$; $K_2 = E_2/E_3$

  $A = a/h_2$; $H = h_1/h_2$

- Jones (1962) presented a series of tables for determining σ_{z1}, $\sigma_{z1} - \sigma_{r1}$, σ_{z2}, $\sigma_{z2} - \sigma_{r2}$.
- His tables also include values of $\sigma_{z1} - \sigma`_{r1}$ *and* $\sigma_{z2} - \sigma`_{r2}$*. But these can be readily obtained from those at bottom of layer 1 and 2.*
- *For continuity;*

$$(\sigma_{z1} - \sigma_{r1})/E_1 = (\sigma_{z1} - \sigma`_{r1})/E_2$$

i.e.,

$$\sigma_{z1} - \sigma`_{r1} = (\sigma_{z1} - \sigma_{r1})/K_1$$

$$\sigma_{z2} - \sigma`_{r2} = (\sigma_{z2} - \sigma_{r2})/K_2$$

Computing Stresses from Jones Tables

- Tables presented by Jones (1962) consist of four values of K_1 and K_2 i.e., 0.2, 2, 20 and 200.
- Therefore, interpolation of stress factors is necessary for many problem solutions. No extrapolation is allowed.
- Four sets of stress factors i.e., *ZZ1*, *ZZ2*, *ZZ1-RR1* and *ZZ2-RR2*, are shown. The product of contact pressure and the stress factor gives the stress.
  - $\sigma_{z1} = p(ZZ1)$
  - $\sigma_{z2} = p(ZZ2)$
  - $\sigma_{z1} - \sigma_{r1} = p(ZZ1-RR1)$
  - $\sigma_{z2} - \sigma_{r2} = p(ZZ2-RR2)$

Jones (1962) Tables

Example Problem

- Given the three layer system shown in figure, determine all the stresses and strains at the two interfaces on the axis of symmetry.