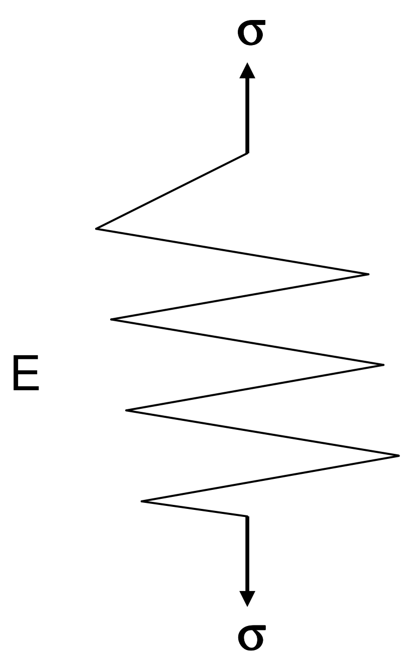
Visco-elastic Layers

Visco-elastic Solutions

- Solutions are obtained by elastic – visco-elastic correspondence principle by applying laplace transform to remove the time variable
- Two methods of characterising visco-elastic materials:
  - Mechanical model
  - Creep compliance curve

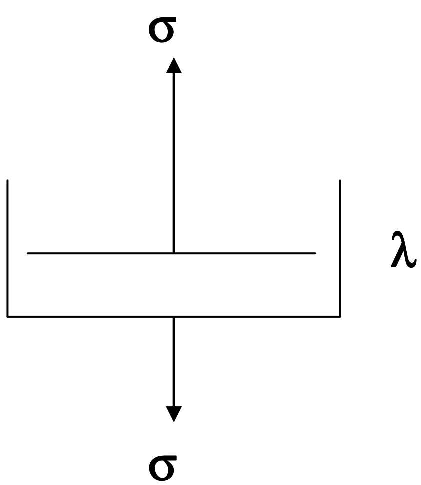
Spring - Elastic



Elastic: Stress α Strain

$\sigma = E\,\varepsilon$

Dashpot - Viscous



Viscous: Stress α time rate of strain

$$\sigma = \lambda \frac{\partial \varepsilon}{\partial t}$$

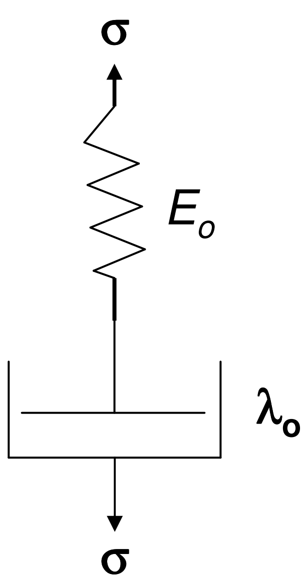
Under constant stress, $\sigma \partial t = \lambda \partial \varepsilon$

i.e., $$\sigma \int_0^t \partial t = \lambda \int_0^\varepsilon \partial \varepsilon$$

$$\therefore \quad \sigma t = \lambda \varepsilon$$

$$\therefore \quad \varepsilon = \frac{\sigma t}{\lambda}$$

Maxwell Model



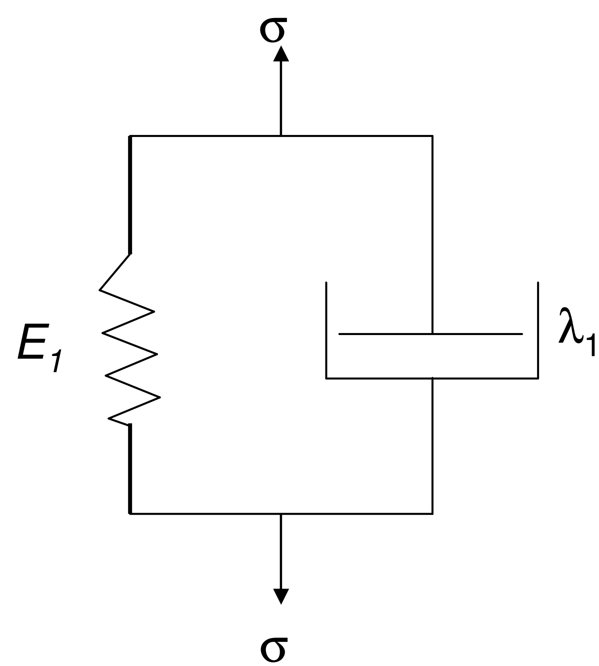
$$\varepsilon = \frac{\sigma}{E_o} + \frac{\sigma t}{\lambda_o}$$

$$= \frac{\sigma}{E_o}\left(1 + \frac{t E_o}{\lambda_o}\right)$$

$$= \frac{\sigma}{E_o}\left(1 + \frac{t}{T_o}\right)$$

$$\text{Where, } T_o = \frac{\lambda_o}{E_o} = \text{relaxation time}$$

Kelvin Model



$$\sigma = E_1\varepsilon + \lambda_1 \frac{\partial \varepsilon}{\partial t}$$

$$\lambda_1 \frac{\partial \varepsilon}{\partial t} = \sigma - E_1\varepsilon$$

if a constant Stress is applied,

$$\int_0^{\varepsilon} \frac{\partial \varepsilon}{\sigma - E_1\varepsilon} = \int_0^{t} \frac{\partial t}{\lambda_1}$$

$$\Big|_0^{\varepsilon} -\frac{1}{E_1}\ln(\sigma - E_1\varepsilon) = \Big|_0^{t} \frac{t}{\lambda_1}$$

$$\ln\left(\frac{\sigma - E_1\varepsilon}{\sigma}\right) = -\frac{E_1 t}{\lambda_1}$$

$$\frac{\sigma - E_1\varepsilon}{\sigma} = e^{-E_1 t/\lambda_1} = e^{-t/T_1}$$

$$\varepsilon = \frac{\sigma}{E_1}\left[1 - \exp\left(-t/T_1\right)\right]$$

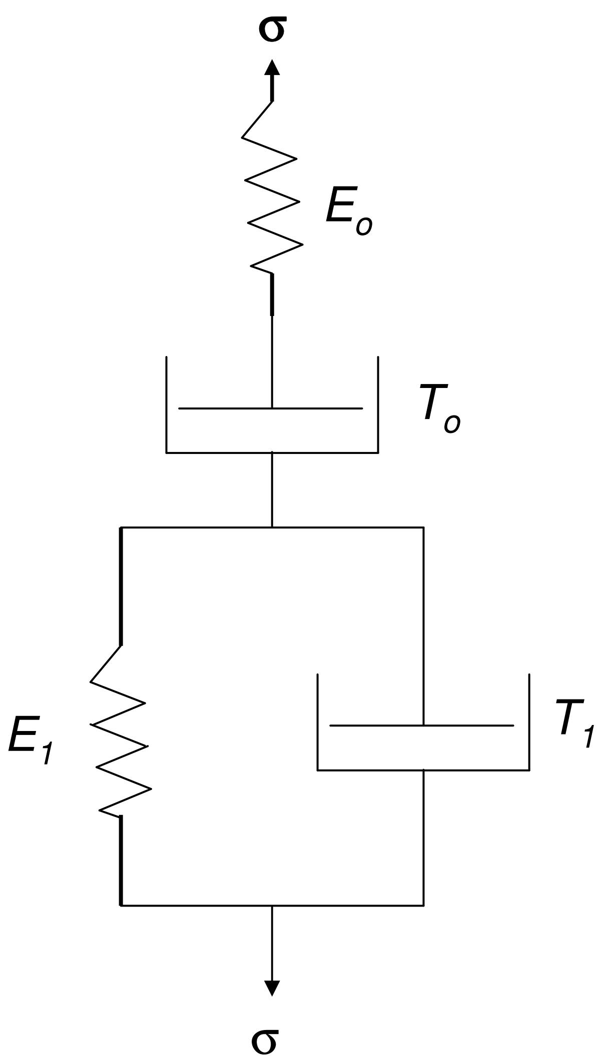
T_1 is retardation time.

When, t = 0, ε = 0; when t = ∞, ε = σ/E_1

When, t = T_1, ε = 0.632 σ/E_1

Therefore, T1 is the time to reach 63.2% of total retarded strain.

Burgers Model

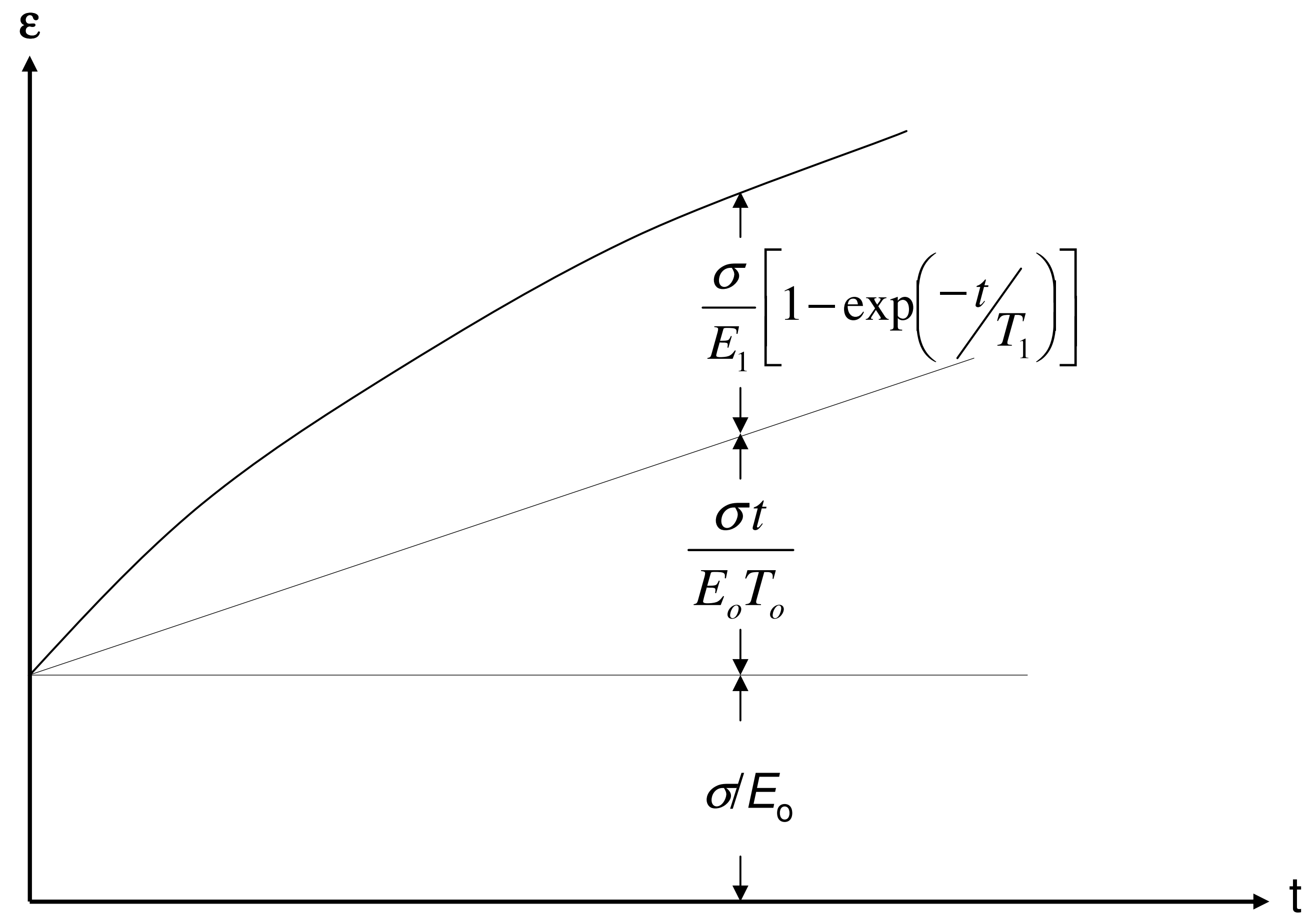


$$\varepsilon = \frac{\sigma}{E_o}\left(1 + {}^{t}\!/_{T_o}\right) + \frac{\sigma}{E_1}\left(1 - \exp\left(-{}^{t}\!/_{T_1}\right)\right)$$

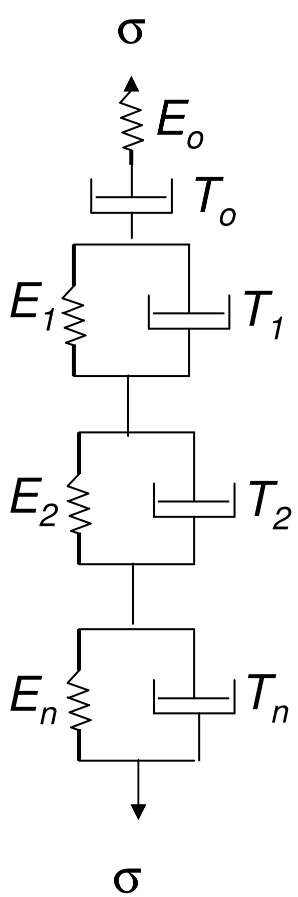
Three components of strain:

1. Elastic strain
2. Viscous strain
3. Retarded elastic strain

Three Components of Strain



Generalised Model



Under a constant stress, the strain in a generalised model is given by

$$\varepsilon = \frac{\sigma}{E_o}\left(1+\frac{t}{T_o}\right)+\sum_{i=1}^{n}\frac{\sigma}{E_i}\left[1-\exp\left(-t/T_i\right)\right]$$

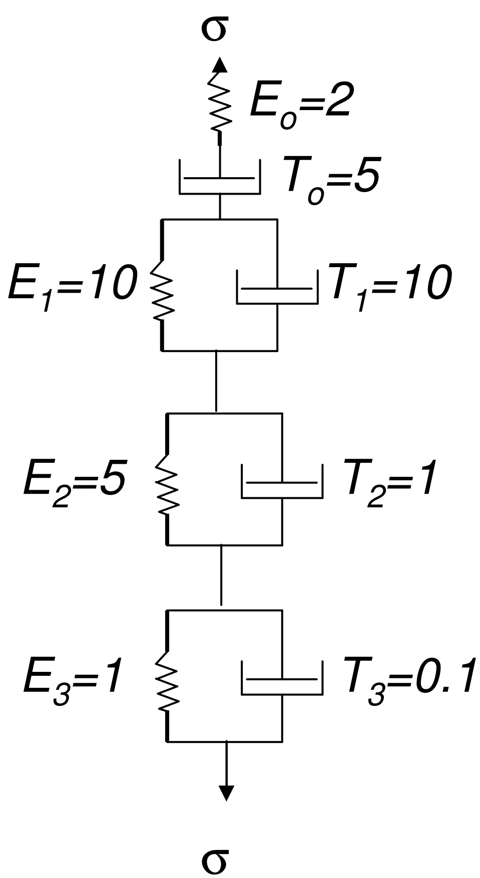
Creep compliance:

$D(t) = \varepsilon(t)/\sigma$

$\varepsilon(t)$ = time dependent strain under constant stress

$$\therefore\ D(t) = \frac{1}{E_o}\left(1+\frac{t}{T_o}\right)+\sum_{i=1}^{n}\frac{1}{E_i}\left[1-\exp\left(-t/T_i\right)\right]$$

Example on Creep Compliance



E is in kN/m^2

T is in second

$D(t)$ is in m^2/kN

Determine the creep compliance at various times and plot the creep compliance.

Time	**D(t)**	**Time**	**D(t)**
0		2	
0.05		3	
0.1		4	
0.2		5	
0.4		10	
0.6		20	
0.8		30	
1.0		40	
1.5		50	

Collocation Method

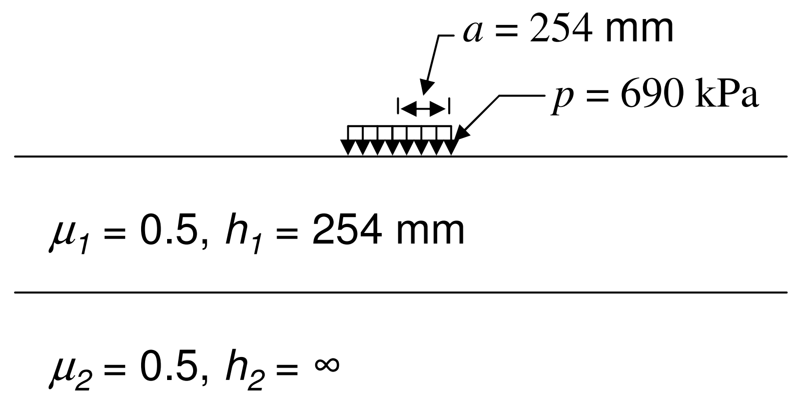
- The creep compliance of visco-elastic materials are determined from creep tests.
- Computed and actual responses are collected at predetermined time responses.
- A 1000 s creep test with compliances measured at 11 different time durations of 0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1, 3, 10, 30 and 100 s is recommended.
- Instead of determining both T_is and E_is, several arbitrary values of T_is are assumed and corresponding E_is are found by solving simultaneous equations.
- Retardation times Ti of 0.01, 0.03, 0.1, 1, 10, 30 and ∞ seconds are specified.

Elastic Solutions

- Given the creep compliance of each visco-elastic material at a given time, the visco-elastic solution at that time can be easily obtained from the elastic solutions:

Visco-elastic solutions from elastic solutions

Given the creep compliance of each visco-elastic material at a given time, obtain the visco-elastic solutions at that time from the elastic solutions using Burmister's two-layer theory.



Creep Compliance Values of the Layers

Time, s	0.01	0.1	1	10	100
D(t) of Layer 1, $\times 10^{-6}$ /kPa)	0.15	0.17	0.39	1.34	2.66
D(t) of Layer 2, $\times 10^{-6}$ /kPa)	0.15	1.06	2.83	10.61	15.94

Time – Temperature Superposition

Time – temperature shift factor is defined as

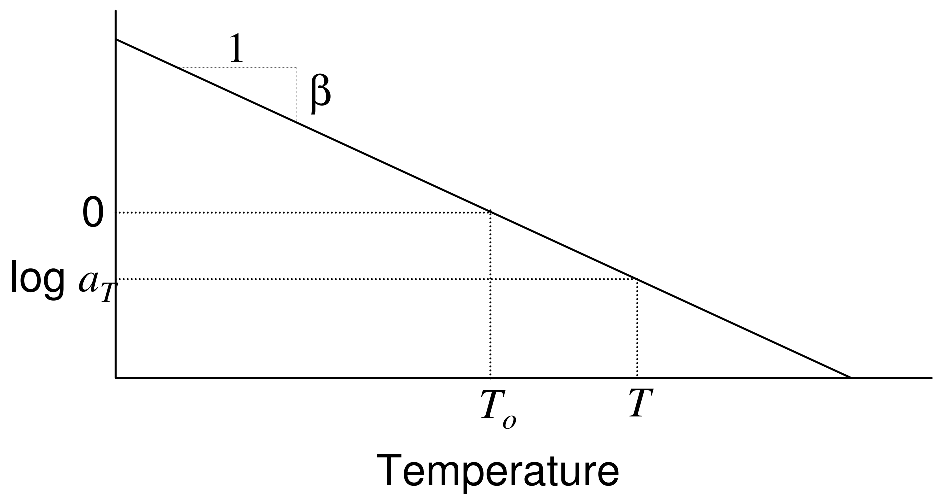
$$a_T = \frac{t_T}{t_{T_o}}$$

Where, t_T = time to obtain creep compliance at temperature T

t_{To} = time to obtain creep compliance at temperature T_o

Time – Temperature Superposition

Laboratory tests on asphalt mixes have shown that a plot of log a_T varies linearly with temperature.



$$\beta = \frac{\log\left(\frac{t_T}{t_{T_o}}\right) - 0}{T - T_o}$$

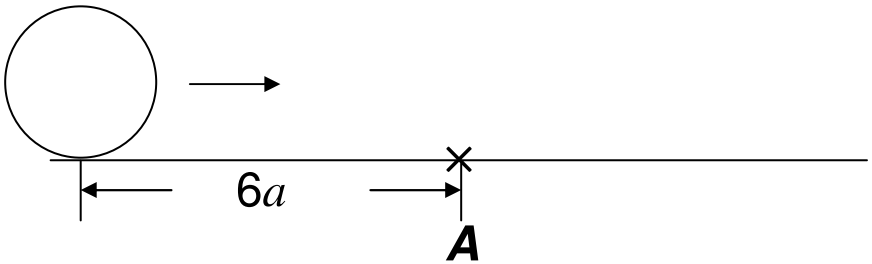
$$\text{or } \log\left(\frac{t_T}{t_{T_o}}\right) = \beta(T - T_o)$$

$$\text{or } \frac{t_T}{t_{T_o}} = 10^{\beta(T - T_o)} = \exp\left[2.3026\beta(T - T_o)\right]$$

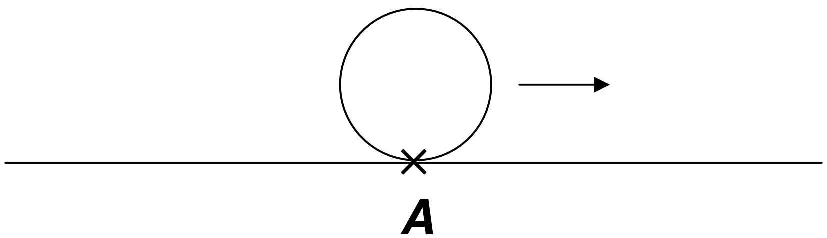
$$\text{or } t_{\mathrm{T}} = t_{T_o} \exp\left[2.3026\,\beta(T - T_o)\right] \quad \text{(A)}$$

By substituting equation (A) for t in the creep compliance equation, creep compliance at temperature T can be obtained.

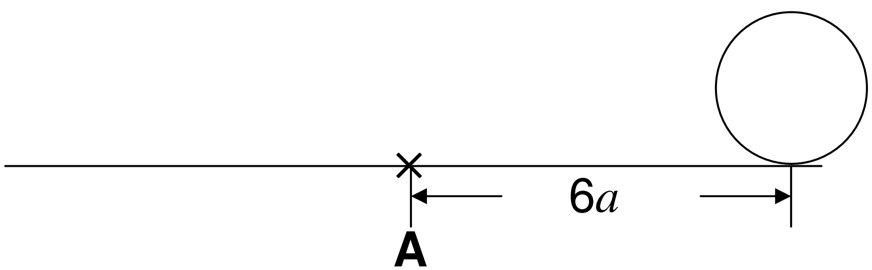
Analysis of Moving Loads



Load has practically no effect at *A* when it is at a distance of $6a$.

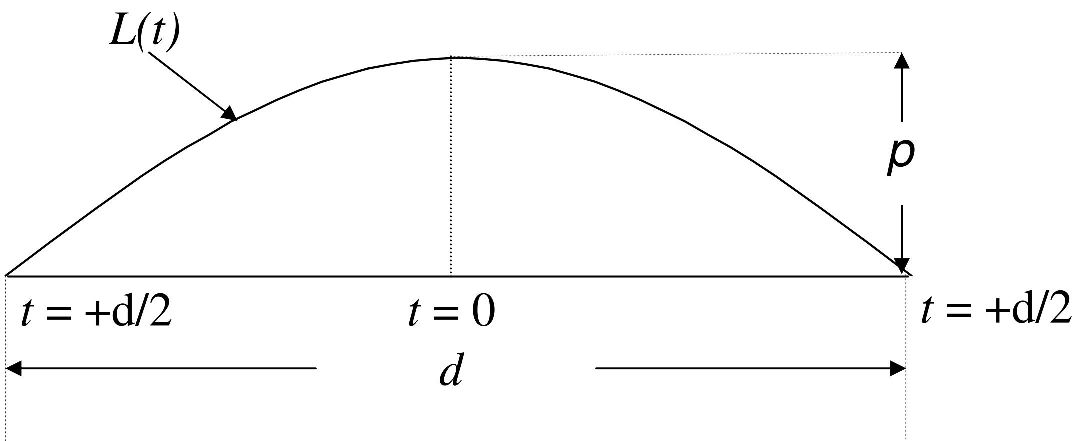


The intensity of load at *A* reaches a maximum value of p (contact pressure) when the wheel is exactly above the point.



The intensity of load at *A* reaches zero again when the wheel is beyond a distance of $6a$.

Analysis of Moving Loads



$$L(t) = p\sin^2\left(\frac{\pi}{2} + \frac{\pi t}{d}\right)$$

When $t = 0$, $L(t) = p$

$t = \pm d/2$, $L(t) = 0$

By taking the speed of vehicle, *v,* as 17.7 m/s (64 km/hr) and the radius of contact, *a,* as 153 mm, the duration of load, *d,* can be computed as

$d = (12\,a)/v = (12 \times 0.153)/17.7 = 0.1\ s$

Non-linear Layers

- Modulus of elasticity (*E*) of non-linear layers depends on the stress level
- The relation between E and the stress level depends on the type of material
- For granular materials the following relation is used:

$$E = K_1 \theta^{K_2}$$

Where,

K_1 and K_2 are the parameters of the material to be calibrated

θ = sum of normal stresses and weight of layered systems

i.e., $\theta = \sigma_x + \sigma_y + \sigma_z + \gamma z(1+2K_o)$

K_o = coefficient of earth pressure at rest = 0.6

Non-linear Layers

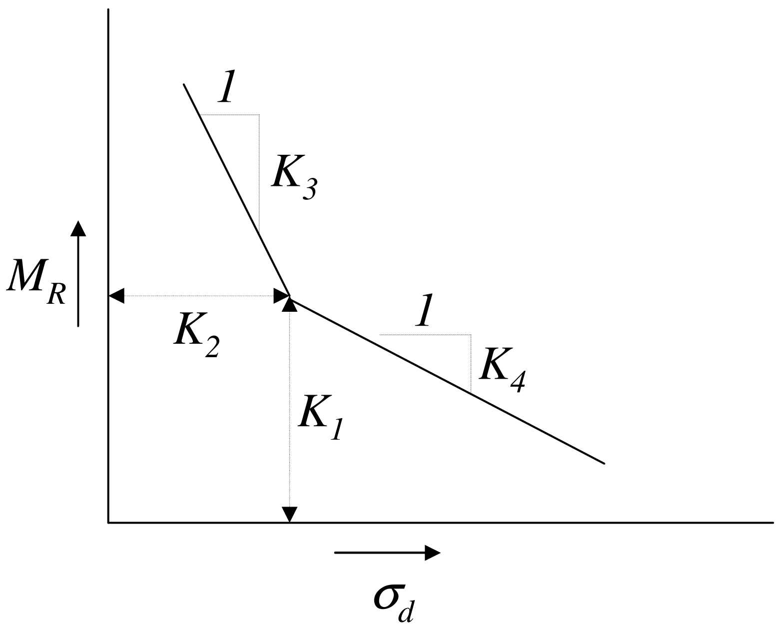
- For fine grained soils the following relation is used:

  $E = K_1 + K_3(K_2 - \sigma_d)$ when $\sigma_d < K_2$

  $E = K_1 - K_4(\sigma_d - K_2)$ when $\sigma_d > K_2$

  The parameters, K_1, K_2, K_3 and K_4 are determined in a tri-axial resilient modulus test by plotting resilient modulus (M_R) versus deviator stress (σ_d) as shown in the figure

  $\sigma_d = \sigma_1 - \sigma_3$ and $\sigma_2 = \sigma_3$ on a tri-axial specimen.



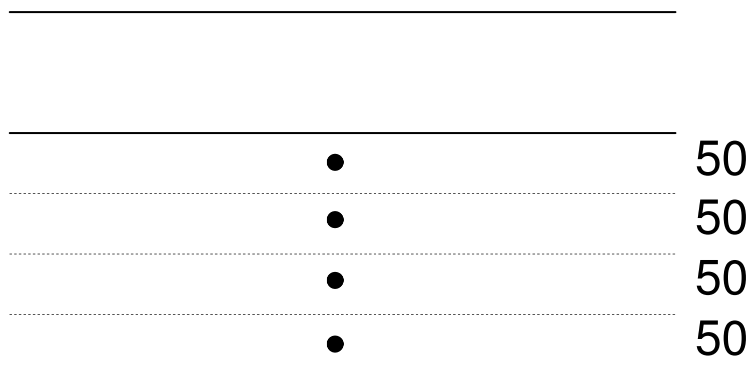
In layered systems,

$\sigma_d = \sigma_1 - 0.5(\sigma_2 + \sigma_3) + \gamma z(1 - K_o)$

$K_o = 0.8$

Approximate Method for Non-linear Elastic Solutions

- Divide the non-linear layer into sub layers of 50 mm thick or less and use the stress at mid height of each layer for computing E value.



- As further approximation, the stress at mid height of a non-linear layer could be used for computing E

Demonstration of KENLAYER