Forecasting of Population

- The study of the characteristics of a population and of their evolution through time and space constitutes the field of demography
- Population projection can be done by
  - Trend models:
    - treat the population as a whole i.e, without disaggregation with respect to age, sex or other characteristics
    - These methods take only time as independent variable
    - Examples: linear model, exponential model, modified exponential model, logistic model
  - Composite Models:
    - Treat the population as an aggregate of various groups. The evolution of population results from the interactions from these groups.
    - Example: cohort survival model

Linear Trend Models

- The increase in equal time periods is constant. i.e., the growth of population is linear with time.
  - Let the constant increase in population every year is a and the population for year n is P_n
  - then, $P_1 = P_o + a$; $P_2 = P_1 + a = P_o + 2a$;

    …. …

    $$\boldsymbol{P_n = P_o + na}$$
  - The practical method of calibration of a and P_o is by least square regression.

Exponential Trend Model

- **Exponential**
  - The increase in population in equal time periods is not constant. The increase in population, $(P_{n+1} - P_n)$, is proportional to the present population, P_n
  - *i.e.*, $P_{n+1} - P_n = \gamma P_n$; where, γ is the growth rate.

    $P_1 - P_0 = \gamma P_0 \text{ or } P_1 = (1+\gamma)P_0$

    $P_2 - P_1 = \gamma P_1 \text{ or } P_2 = (1+\gamma)P_1 = (1+\gamma)^2 P_0$

    …………

    $\boldsymbol{P_n = (1+\gamma)^n P_0}$
  - γ and $\boldsymbol{P_0}$ can be calibrated by least square regression method after linearisng the above equation.

Modified Exponential Model

- Characteristic of linear or exponential model is that the population level continues to grow indefinitely. As this is unrealistic, models are proposed based on the assumption of a finite limit to the population level.
- Premise: The remaining growth in population, i.e., the difference between the final population level and the existing population level, is a constant fraction of what it was at the previous time period.

i.e., $(P_\infty - P_n)/(P_\infty - P_{n-1}) = \upsilon$

υ is a constant smaller than 1

$P_\infty - P_1 = \upsilon(P_\infty - P_0)$

$P_\infty - P_2 = \upsilon(P_\infty - P_1) = \upsilon^2(P_\infty - P_0)$

.

.

$P_\infty - P_n = \upsilon^n(P_\infty - P_0)$

i.e., $Pn = P_\infty - \upsilon^n(P_\infty - P_0)$

Double Exponential Model

Assumption: The growth of the population is proportional to the population level with a proportionality factor which instead of being constant increases exponentially with time

$$P_t = P_\infty a^{b^t} \quad \text{--} \quad \text{(A)}$$

$$\log P_t = \log P_\infty + b^t \log a$$

$$\log\left(\frac{P_\infty}{P_t}\right) = b^t \log\left(\frac{1}{a}\right)$$

at $t = 0$, from (A), $a = P_0 / P_\infty$

at $t = \infty$, $P_\infty = P_\infty \, a^{b^\infty}$

$$\therefore a^{b^t} = 1, \, i.e., b^\infty = 0$$

$$\therefore b < 1$$

Logistic Trend Model

- The population initially grows moderately, picks up when the economic base reaches a certain minimum level, and ultimately reaches a saturation level. This cycle of population growth pattern is best depicted by a logistic model.
- The rate of growth (γ_t), as in exponential model, is not constant, but is a linearly decreasing function of the demand level (P_t).
  - *i.e.,* $\boldsymbol{\gamma_t = a - b\,P_t}$ (1)

but, $\gamma_t = \frac{1}{P_t}\frac{dP_t}{dt}$ (2)

therefore, $\frac{dP_t}{dt} = P_t(a - bP_t)$ (3)

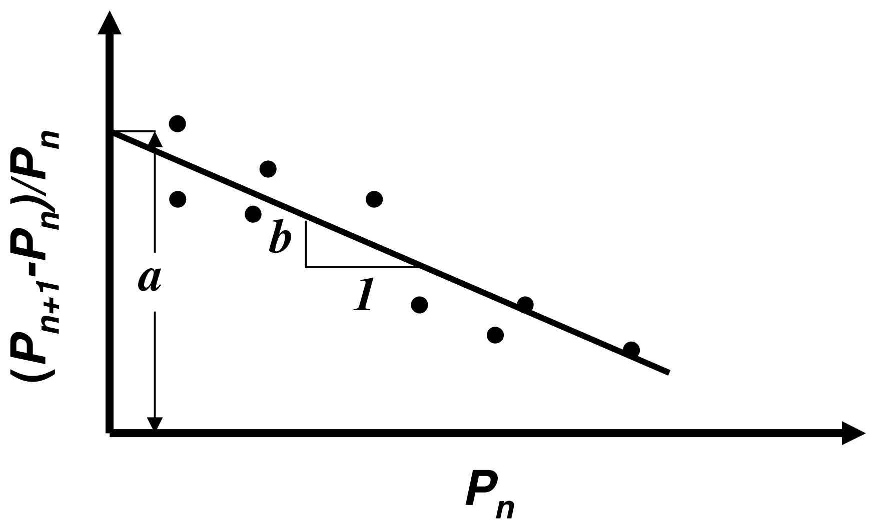
The solution to the above differential equation is the logistic model

$$P_t = \frac{1}{\left(\frac{1}{P_0} - \frac{b}{a}\right)e^{-at} + \frac{b}{a}} \qquad (4)$$

From (1), at $t = \infty$, $\gamma_t = 0$ and therefore, $P_\infty = a/b$

Calibration of Logistic Trend Model

- Generalized least squares or maximum likelihood method can be used to calibrate the logistic model.
- Alternatively, the parameters a and b can be estimated by plotting between $(P_{n+1} - P_n)/P_n$ and P_n. The intercept is a and the absolute value of the slope is b.



Comparative Models

- *These models are also known as*
  - ***Ratio methods***
  - ***Market share models***
- *Typical Comparative Models*

$$P_T^{\ C} = K P_T^{\ S}$$

$$P_T^{\ C} = K(P_T^{\ S} - b)$$

$$P_T^{\ A} = K\, P^B{}_{T-t}$$

$$P_T^{\ A} = K\left(P^B{}_{T-t} - b\right)$$