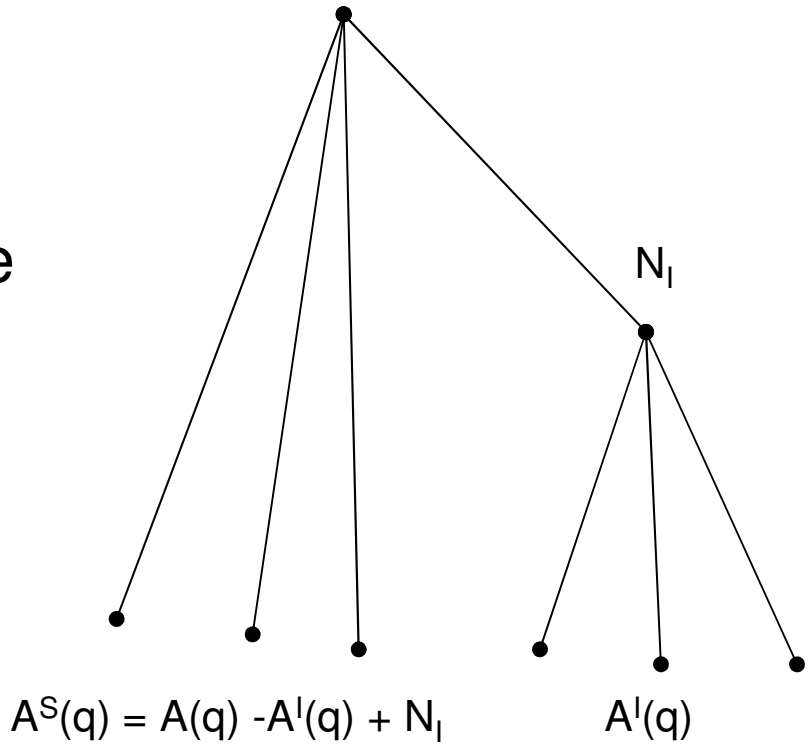
Nested Logit Model

Drawbacks of MNL

- MNL may not work well in either of the following cases due to its IIA property:
  - When alternatives are not independent
    - i.e., when there are groups of alternatives which are more similar than others, such as public transport modes versus the private vehicles
  - When there are taste variations among individuals
    - i.e., perceptions of individuals vary with their socio economic status
    - In such cases we require random coefficient models rather than mean value models as the MNL
- Nested logit or hierarchical logit model addresses to a limited extent the first problem

Nested Logit Model

- Subsets of alternatives [$A^I(q)$] which are similar are grouped in hierarchies or nests.
- Each nest in turn is considered as a composite alternative (N_I) which competes with the other alternatives [$A(q) – A^I(q)$] available to the individual.



Nested Logit Model

- First estimate an MNL for the $A^l(q)$ alternatives of the lower nest, taking care of omitting all those variables (**z**) which take the same value for this subset of options.
- The utility of the composite alternative has two components:
  - One that consists of the expected maximum utility (EMU) of the lower nest options, and
  - Another which considers the vector **z** of attributes which are common to all members of the nest

  EMU has the following expression:

  $EMU = \log \Sigma_j \exp(W_j)$

  Where, W_j is the utility of alternative A_j in the nest

  Therefore, the composite utility of the nest is:

  $V_l = \phi EMU + \alpha \mathbf{z}$

  Where, ϕ *and* α are parameters to be estimated

Nested Logit Model

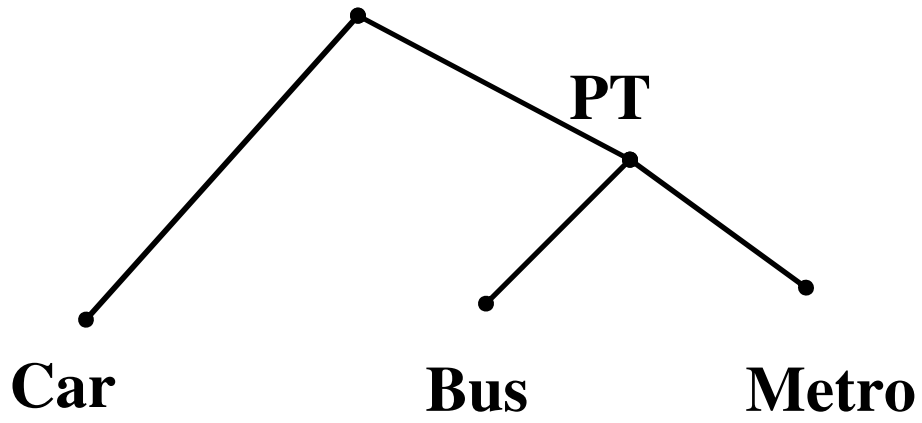
- At the higher nest, an MNL consisting of all composite alternatives representing lower hierarchies and alternatives which are non-nested at that level is estimated.
- The probability that individual q selects option $A_j \in A_l(q)$ is computed as the product of the marginal probability of choosing the composite alternative N_l (in the higher nest) and the conditional probability of choosing option A_j (in the lower nest).

Estimation of NL

- Sequential estimation method
  - The method outlined is sequential method of estimation of NL
- Simultaneous estimation method
  - This is also termed as full information method of estimation.
  - A single maximum likelihood function is formulated and maximised.
  - ALOGIT estimates NL models using the efficient simultaneous estimation method.

NESTED LOGIT MODEL IN A TRINOMIAL MODAL CHOICE SITUATION

- Consider a situation involving choice among car, bus and metro.
- Assuming that bus and metro are correlated, they will be considered at the lower nest. The composite alternative of this nest is **PT.**



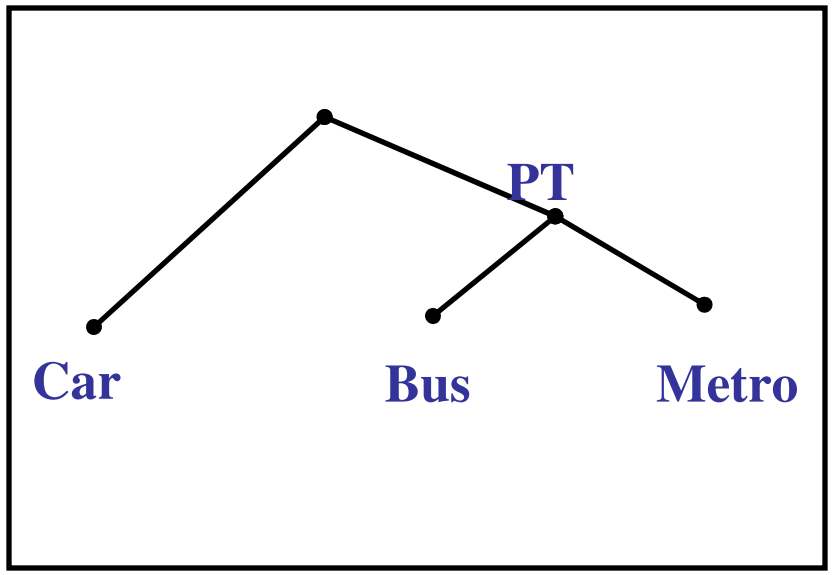
The lower public transport nest would be modelled by a simple binary logit model of the form

$$P(M / PT) = \frac{\exp(W_M)}{\exp(W_M) + \exp(W_B)}$$

and

$$P(B / PT) = 1 - P(M / PT)$$

Where the utilities ***W*** contain only those elements which are not common to both modes (i.e., the cost of travel would not enter if both the modes charged the same fare)



Another binary logit model at the higher nest between car and the composite alternative PT is modelled as:

$$P(C) = \frac{\exp(V_C)}{\exp(V_C) + \exp(V_{PT})}$$

and

$$P(PT) = 1 - P(C)$$

Where V_c incorporates all the attributes of the car option, i.e., it has exactly the same form as in a MNL

The public transport utility is given by

$$V_{PT} = \phi\, EMU + \sum_k \alpha_k Z_k$$

where

$$EMU = \ln\left[\exp(W_B) + \exp(W_M)\right]$$

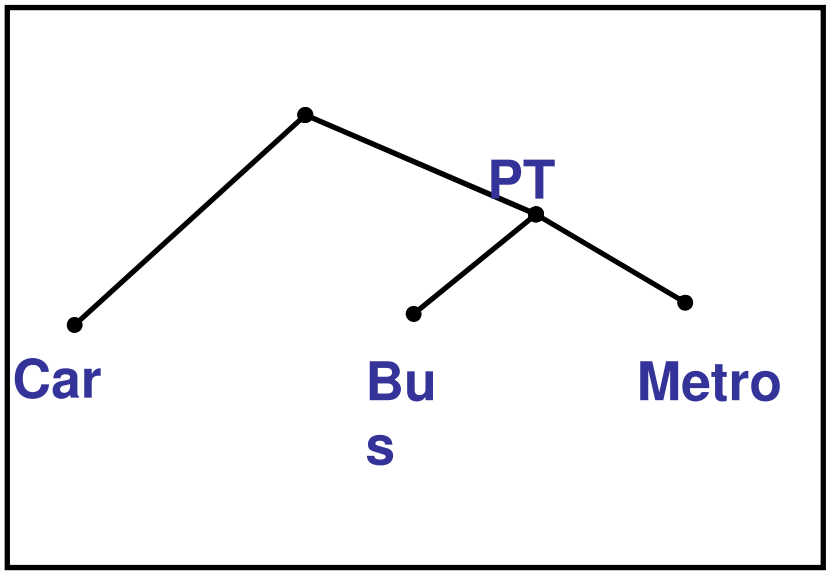
and the summation over k considers all the common elements Z that were taken out to estimate the binary logit model at the lower nest

The modelled choice probabilities of each option are given by

$$P_C = P(C)$$

$$P_B = P(B / PT)\, P(PT)$$

$$P_M = P(M / PT)\, P(PT)$$



Consistency of Structural Parameter

The structural parameter ϕ should satisfy the condition:

$$0 < \phi \leq 1$$

- If $\phi < 0$, an increase in the utility of an alternative in the nest, which should increase the value of EMU, would actually diminish the probability of selecting the nest
- If $\phi = 0$, such an increase would not effect the nest's probability of being selected, as EMU would not effect the choice between car and PT.
- If $\phi > 1$, an increase in the utility of an alternative in the nest would tend to increase not only its selection probability but also those of the rest of the options in the nest.
- If $\phi = 1$, the model becomes equivalent to MNL.

Variable Selection Process

Sign	Significance	Decision	
		Policy	Other
Correct sign	Significant	Include	Include
	Not significant	Include	May reject
Wrong sign	Significant	Big problem	Reject
	Not significant	Problem	Reject