Transferability of Discrete Choice Model

Test of equality for two model parameters

To evaluate the absolute difference between coefficients of a given model estimated in two different contexts, the t*statistic may be used (Galbraith and Hensher, 1982). This statistic is computed as

$$t^* = \frac{\theta_i - \theta_j}{\sqrt{\left[\left(\frac{\theta_i}{t_i}\right)^2 + \left(\frac{\theta_j}{t_j}\right)^2\right]}}$$
(1)

Where, θ denotes coefficients, t their t-values, *i* stands for the original context, and *j* for the new context.

If *t** is less than the table value then the null hypothesis that the difference between the parameters is 0 cannot be rejected at the chosen confidence level

Variable	RP1 Model	RP2 Model	t-statistic ^a	Relevance of Variable
BA	0.5746(4.4)	0.6704(2.2)	-0.289	Specific to 1 car
NPEMSSC	0.2376(3.0)	-	-	Specific to 1 car
NCLH	1.4960(7.7)	2.8180(9.1)	3.616	Specific to 1 car
HHINC	0.5057(7.8)	-	1.814	Specific to 1 car
FS	-0.2870(3.8)	-0.1864(0.9 *)	-1.800	Specific to 1 car
NBPHH	0.2262(1.5 [*])	-	-	Specific to 1 car
HHINC	0.8367(7.9)	2.0580(2.0)	-	Specific to 2 car
BA	0.2738 (1.3 [*])	0.8206 (1.0 [*])	0.646	Specific to 2 car
HOL	-1.7410(3.0)	-9.8570 (1.6*)	1.312	Specific to 2 car
FS	-0.2802(2.4)	-0.6655(1.6*)	0.892	Specific to 2 car
NBPHH	0.3427(1.8)	1.7140(2.1)	1.636	Specific to 2 car
Constant	4.4030(10.6)	5.0820(4.4)	-0.553	Specific to 0 car
Statistical Para	meters			
L(0)	-1014.0191	-392.205	-	-
L(c)	-685.0074	-200.510	-	-
L(θ)	-426.2742	-75.149	-	-
ρ ² (0)	0.5796	0.8084	-	-
ρ ² (c)	0.3777	0.6252	-	-
Adj. ρ^2	0.5687	0.7879	-	-
Sample Size	923	357	-	

t-statistic for evaluating the equality of parameters of two models

Transferability of Discrete Choice Model

The transfer of a previously estimated model to a new application context can reduce or eliminate the need for a large data collection and model development effort in the application context.

Past studies also suggested some effective measures to evaluate the ability of a transferred model in explaining the decisions of individuals in the new context.

Three Measures of transferability of a model indices are

Transferability Test Statistic (TTS) - Atherton and Ben-Akiva (1976)

Transfer Index (TI) - Koppelman and Wilmott (1982)

The transfer rho-square (ρ *2) - McFadden (1973)

Measures of transferability

$$TTS_{j}(\boldsymbol{\theta}_{i}) = -2\{l_{j}^{*}(\boldsymbol{\theta}_{i}) - l_{j}^{*}(\boldsymbol{\theta}_{j})\}$$
⁽²⁾

$$TI_{j}(\theta_{i}) = \frac{l_{j}^{*}(\theta_{i}) - l_{j}^{*}(C)}{l_{j}^{*}(\theta_{j}) - l_{j}^{*}(C)}$$
(3)

(4)

$$\rho_j^{*2}(\boldsymbol{\theta}_i) = 1 - \frac{l_j^{*}(\boldsymbol{\theta}_i)}{l_j^{*}(\boldsymbol{C})}$$

Measures of transferability

 $I_j^*(\theta_i) = \text{log-likelihood of the observed data in the application context$ *j*generated by the transferred model estimated in context*i*.

 $I_j^*(\theta_j) =$ log-likelihood of the model estimated in the application context *j*.

 $I_j^*(C) =$ log-likelihood of the model estimated in the application context *j* with constants only.

 $TTS_j(\theta_i) =$ Transferability Test Static of the model estimated in context *i* applied to context *j*.

 $TI_j(\theta_i)$ = Transfer Index of the model estimated in context *i* applied to context *j*.

 $\rho_j^{*2}(\theta_j)$ = The transfer rho-square of the model estimated in context *i* applied to context *j*.

Measures of transferability - Discussion

The *TTS* value follows χ^2 distribution with degrees of freedom equal to the number of parameters if the two contexts are same.

Therefore if *TTS* value is less than the table χ^2 value at 95% significance level, it is reasonable to accept the transferability of the model to the new context.

71 has an upper bound of one (which is obtained when the transferred model is as accurate as the local one), but does not have a lower bound;

Negative values imply only that the transferred model is worse than the local reference model.

Measures of transferability - Discussion

The transfer rho-square (ρ^{*2}) describes the degree to which the log likelihood of the transferred model exceeds that of the base model relative to the degree of improvement in log likelihood achieved with a perfect local model. This measure is analogous to the commonly used rho-square (McFadden, 1973). This measure is related to *TI* by

$$\boldsymbol{\rho}_{j}^{*2}(\boldsymbol{\theta}_{i}) = TI_{j}(\boldsymbol{\theta}_{i})\boldsymbol{\rho}_{j}^{2}(\boldsymbol{\theta}_{j})$$
⁽⁵⁾

accordingly, it is upper bounded by the local rho-square measure, has no lower bound, and negative values are interpreted as for the TI.

Measures of transferability - Discussion

The three measures are interrelated by their dependence on the difference in log likelihood between the transferred and local models.

These provide different perspectives on model transferability.

Transfer rho-square - an absolute measure of disaggregate transferability,

Transfer index - provides a relative measure, and

Transfer test statistic - provides a statistical test measure (Koppelman and Wilmott, 1982).