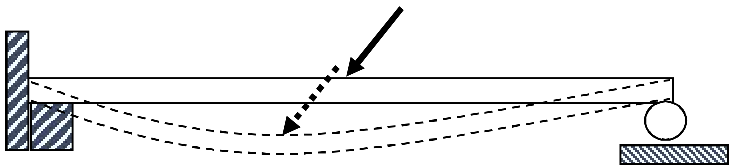
Engineers Mechanics- Introduction

Definitions, assumptions

- **Mechanics:**
  - ***Statics* – body at rest under action of forces. All quantities time independent**
  - ***Dynamics* – body in motion under action of forces. All quantities time dependent**

- **Rigid Body Mechanics:**

  ***Assume rigid body*– no deformation, original geometry**



A '*particle*' idealizes a body by placing its mass at its center and neglecting its dimensions.

Introduction

- For a rigid body in static equilibrium, external forces and moments are balanced and will impart no translational or rotational motion to body.

- Condition for static equilibrium of a body is that resultant force and resultant couple due to all external forces are zero,

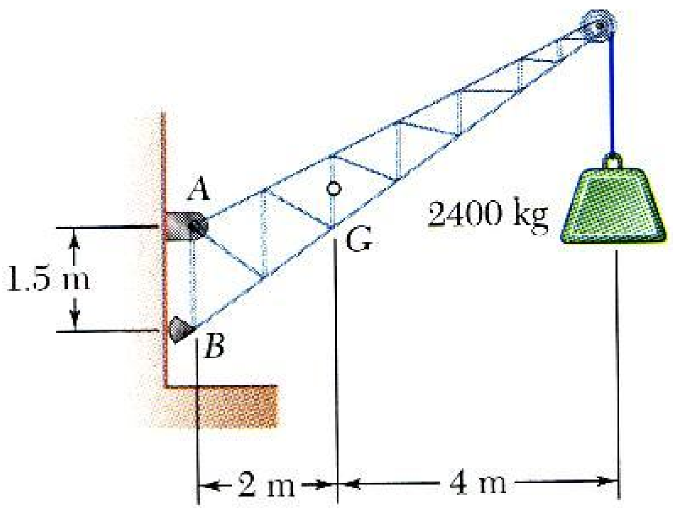
$$\sum \vec{R} = 0 \; ; \qquad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

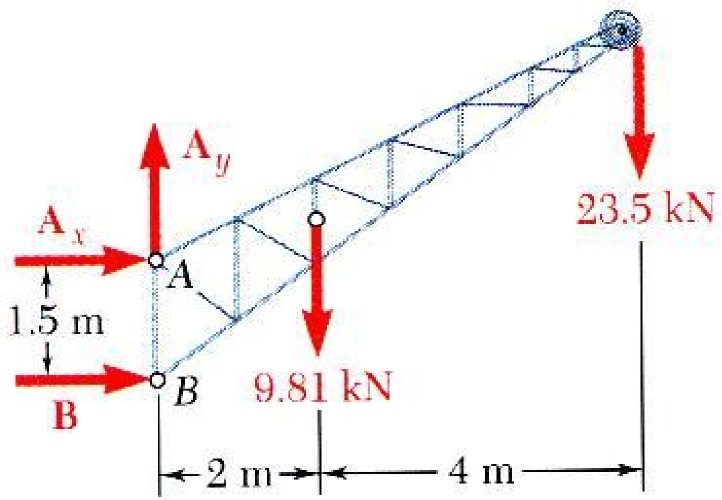
- Resolving into rectangular Cartesian components leads to 6 scalar equations for static equilibrium,

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

Free-Body Diagram (**Most Important Step**)





Show all forces acting on the body in a *free-body* diagram.

- Select entire body, **isolate it from ground and from other bodies.**
- Show applied external forces/moments (magn. & dir.) at their points of application (include self weight of body).
- Show unknown forces/moments (with assumed direction) at their points of application. *These usually consist of reactions by which the ground and other bodies oppose the possible motion of the body.*
- Show dimensions necessary to compute moments due to forces.

Reactions from supports and connections

Supports and connections hold body in position (equilibrium) under action of externally applied forces

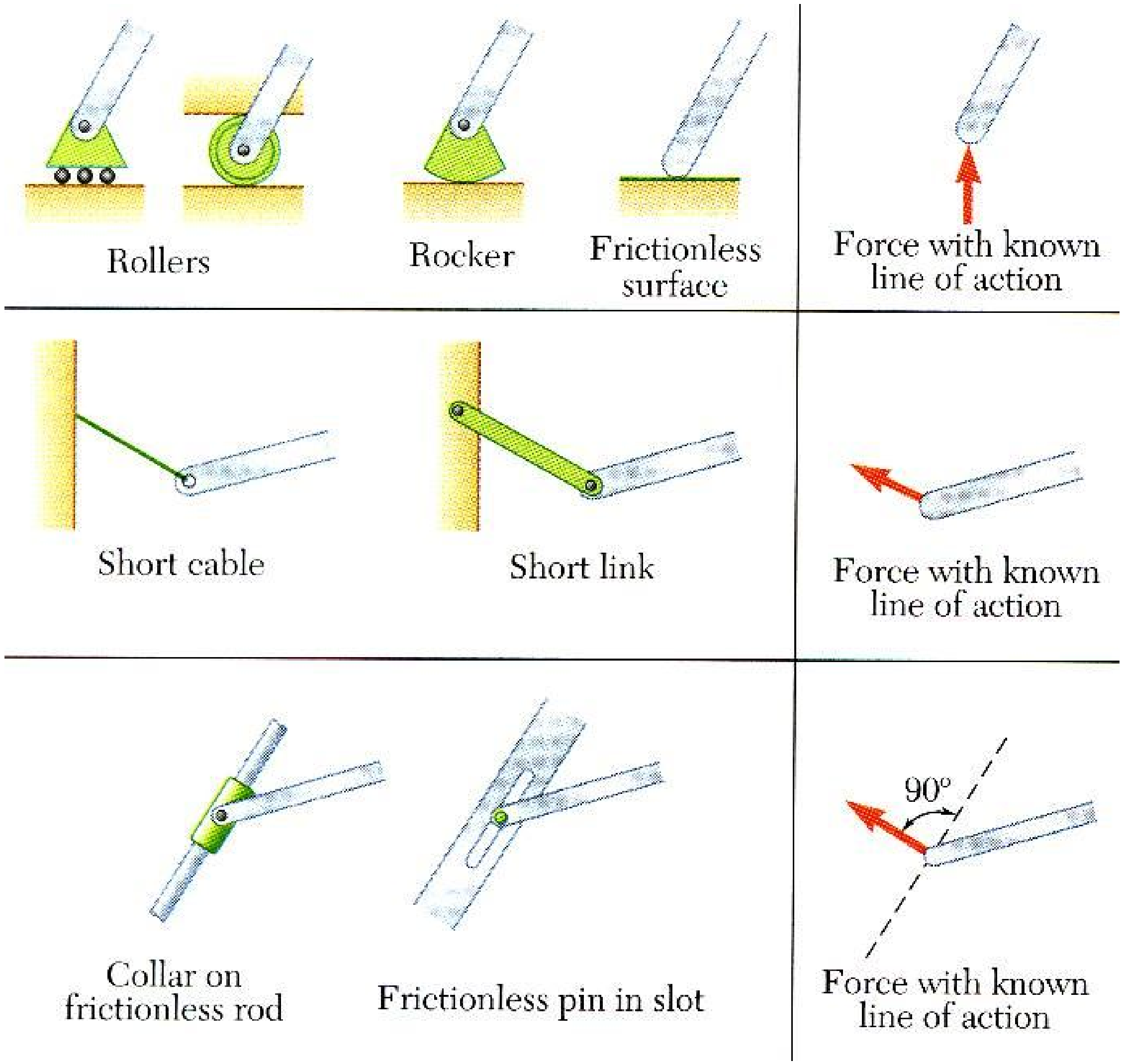
Body exerts force on support / connection. From Newton's third law, support / connection exerts equal and opposite reaction force on body

The reaction force on body opposes the motion of the body.

For a 2D body, 2 translational motions and 1 rotational motion possible

For a 3D body, 3 translational motions and 3 rotational motions possible

Reactions at Supports / Connections for 2-D Structure



- Reactions equivalent to a force with known direction.

In case of a guide roller, short link, collar and frictionless pin in slot, motion opposable in both vertical directions. So no need to judge a-priori the correct direction, will come out thru sign.

Reactions at Supports / Connections for 2-D Structure

Hinge support opposes movement in vertical and horizontal direction

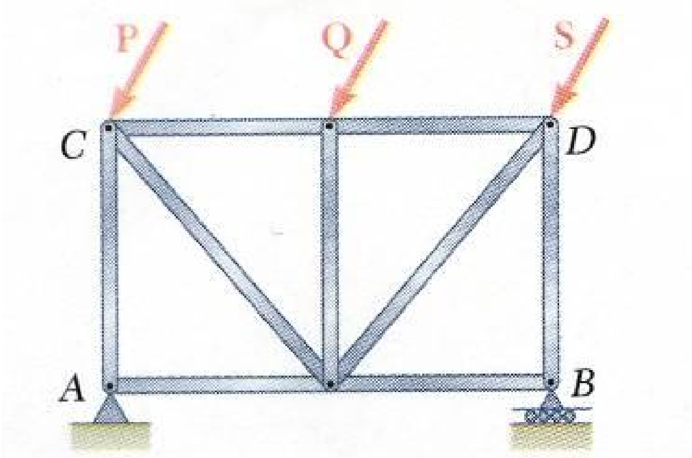
- Reactions equivalent to a force of unknown direction and magnitude, which is eqvt. to two forces of unknown magnitude.

Fixed support opposes movement in vertical and horizontal direction as well as rotation

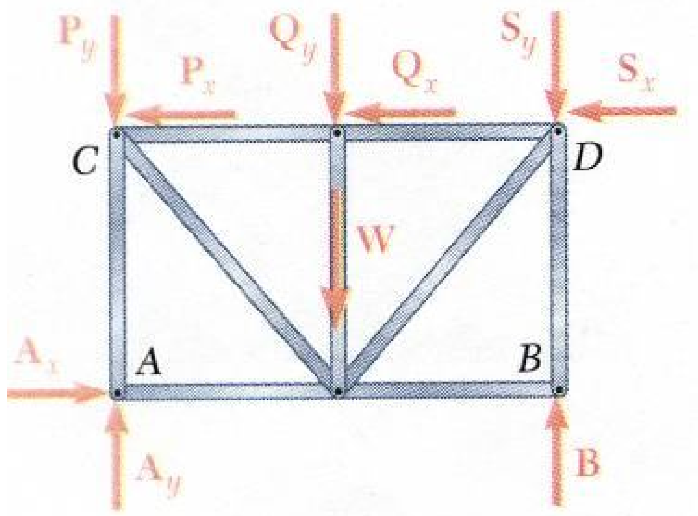
- Reactions equivalent to a force of unknown direction and magnitude and a couple.of unknown magnitude

For pin or fixed support, no need to a-priori judge the correct direction (i.e., + or -). It will come out thru the sign in the result.

2-D Equilibrium



(a)



(b)

- For 2-D structure in x-y plane, with applied forces/moments in in x-y plane, following identically satisfied

$$F_z = 0 \quad M_x = M_y = 0$$

- Useful equations of equilibrium are

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum (M_z)_A = 0$$

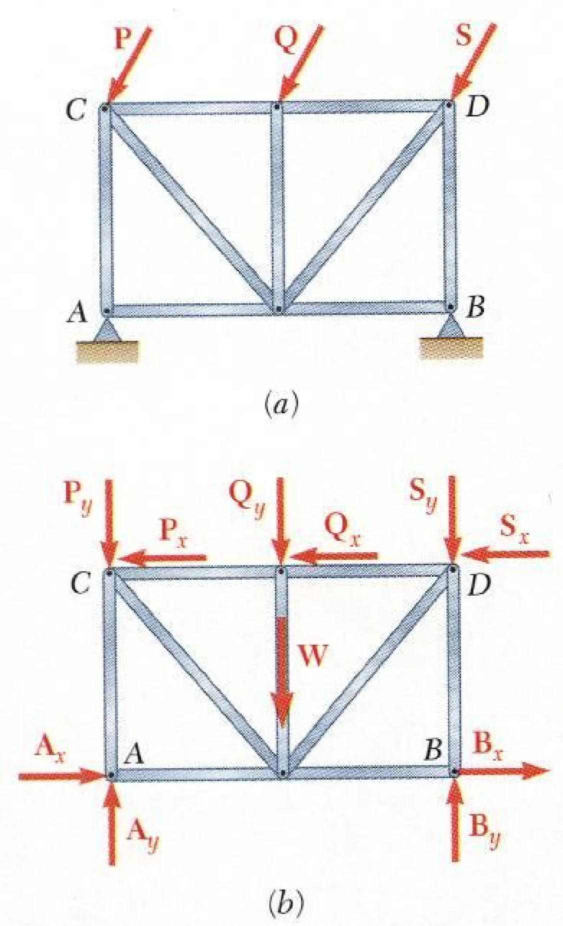
where A is any point in plane of structure.

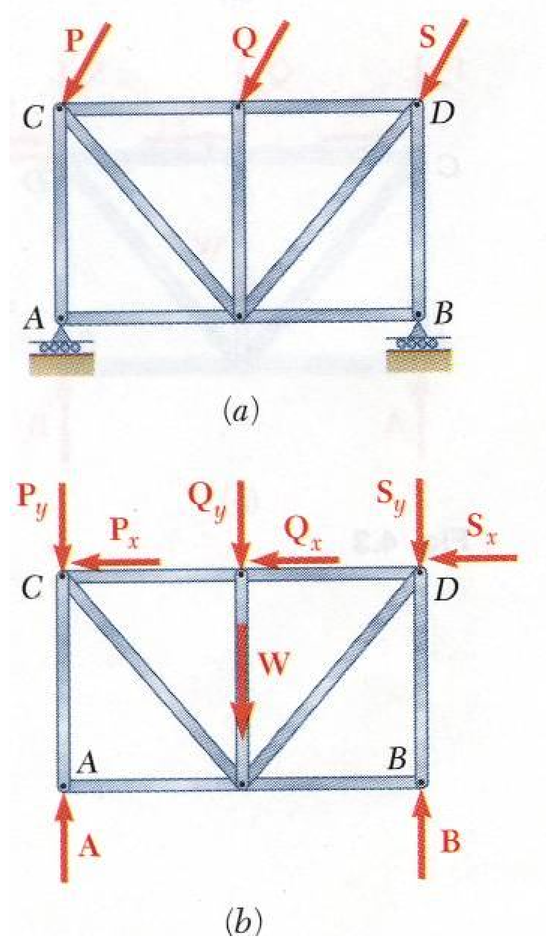
- The 3 equations can be solved for at most 3 unknowns. Concept of Static Determinacy.

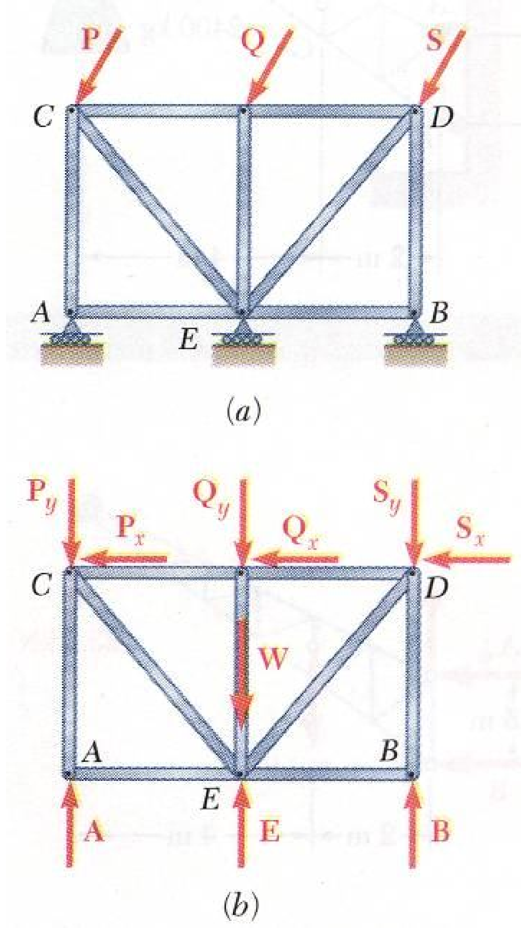
- Due to linear dependence, the 3 equations can not be augmented with additional equations, but can be replaced

$$\sum F_x = 0 \quad \sum (M_z)_A = 0 \quad \sum (M_z)_B = 0$$

Statically Indeterminate Reactions







- **More unknowns than equations**

- Fewer unknowns than equations, partially constrained

- Equal number unknowns and equations but improperly constrained

3-D Equilibrium

- For general 3-D body, six scalar equations required to express equilibrium.

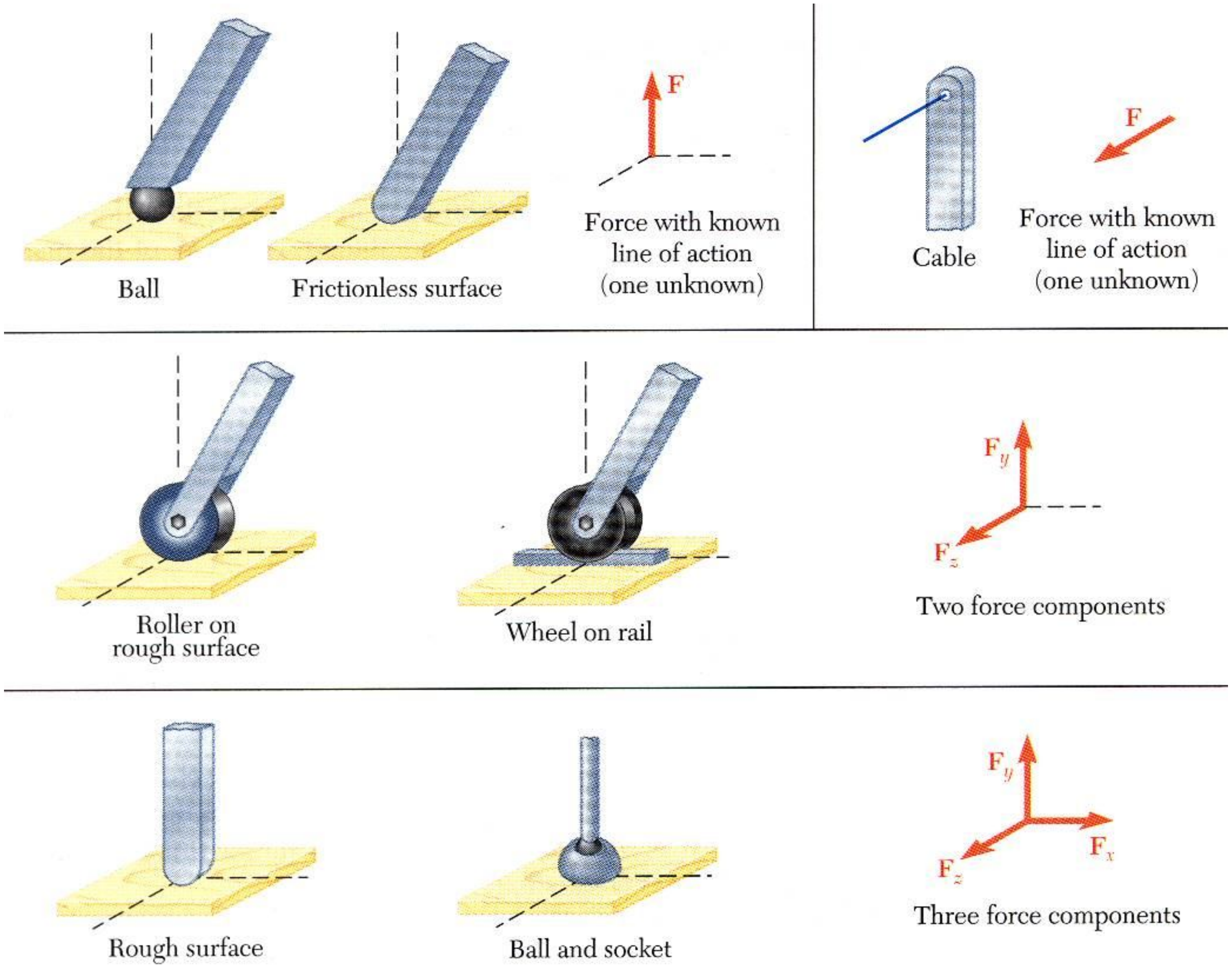
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- Equations can be solved for at most 6 unknowns which generally are reactions.

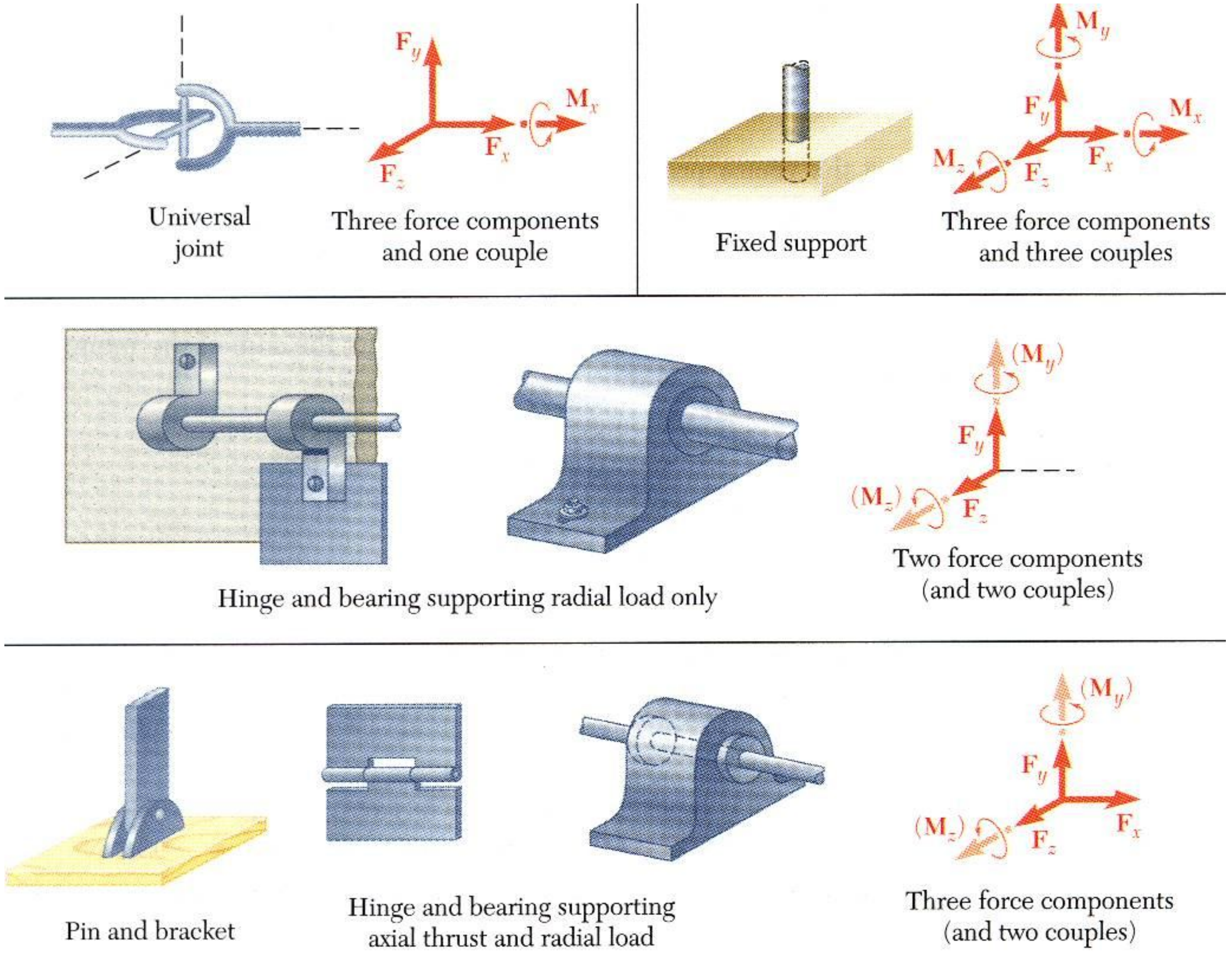
- Scalar equations conveniently obtained from vector equations of equilibrium,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

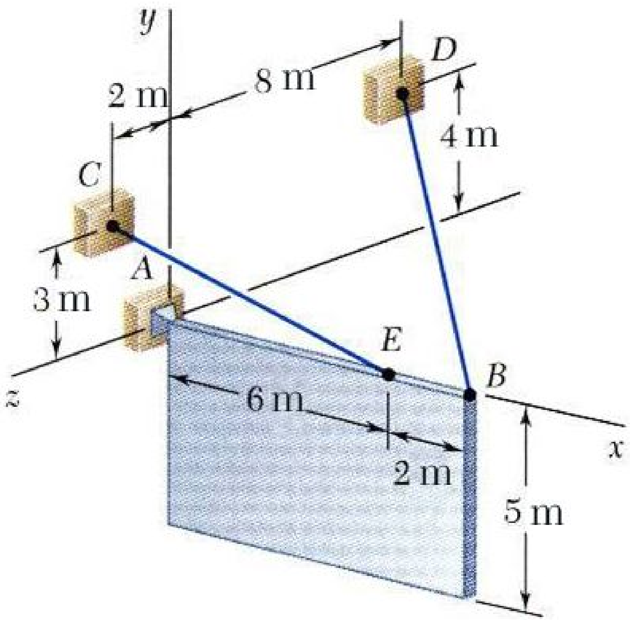
Reactions at Supports / Connections for 3-D Structure



Reactions at Supports / Connections for 3-D Structure



3-D Equilibrium example

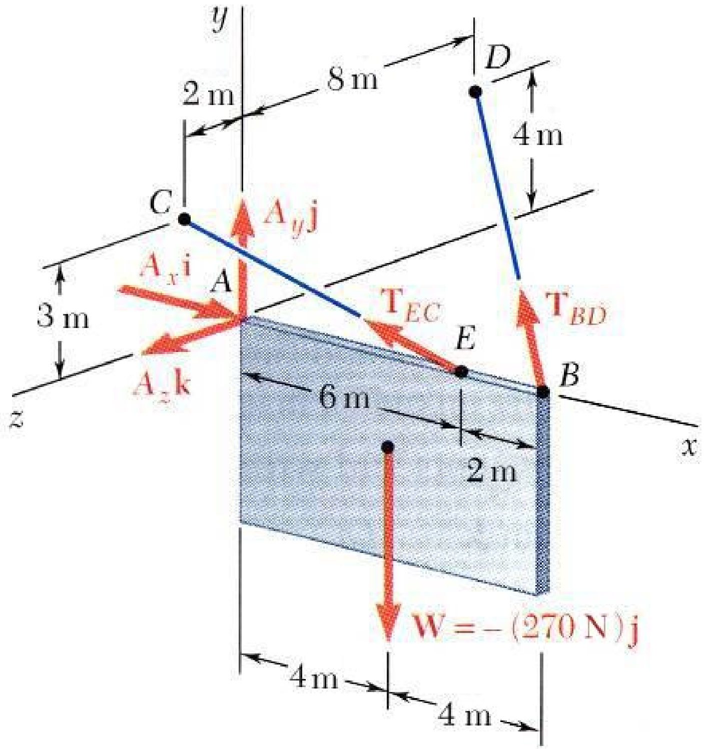


Signpost of uniform density weighs 270 N, supported by ball-and-socket joint at *A* and by two cables.

Determine tension in each cable and reaction at *A*.

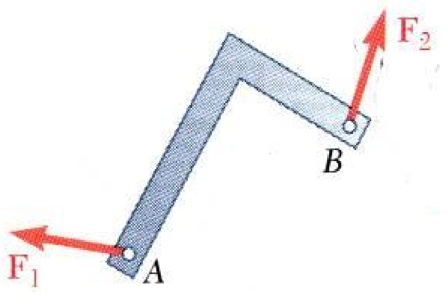
SOLUTION:

- **Create a free-body diagram for the sign.**

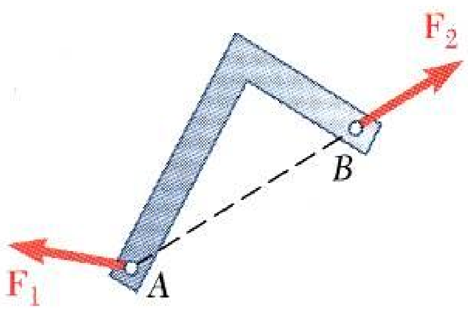


- **Apply scalar or vector eqns of equil. to find 5 reactions/unknons.**
- **But 6 equil eqns?** $\sum M_x = 0$ **?**

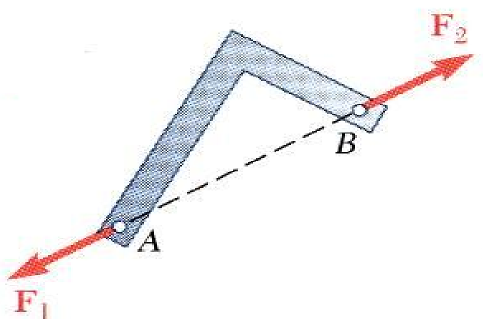
Equilibrium of Two-Force Body



- Applied forces only $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ **.** No moments applied.

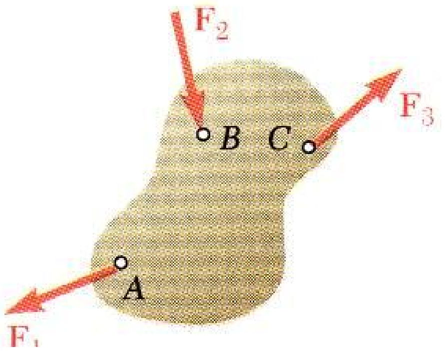


- For equilibrium, moment of $\boldsymbol{F_2}$ about *A* must be zero. Thus line of action of $\boldsymbol{F_2}$ must pass through *A*.
- Similarly, line of action of $\boldsymbol{F_1}$ must pass through *B* for sum of moments about *B* to be zero.

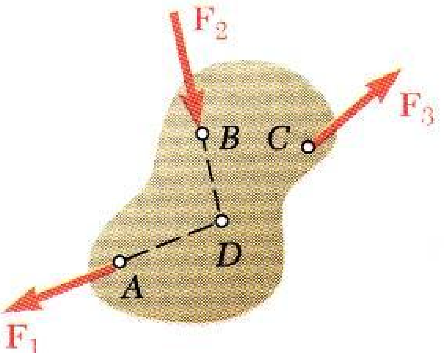


- From above, and from sum of forces in any direction being zero, we conclude that F_1 and F_2 must have equal magnitude and opposite sense, and be directed along line joining their points of application.

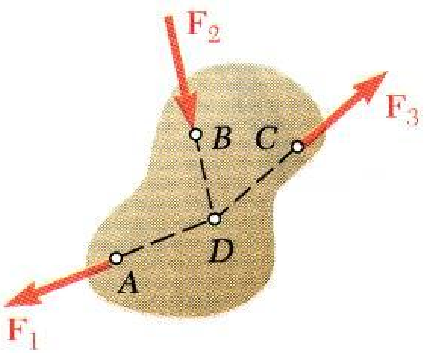
Equilibrium of Three-Force Body



- Forces acting at only 3 points.

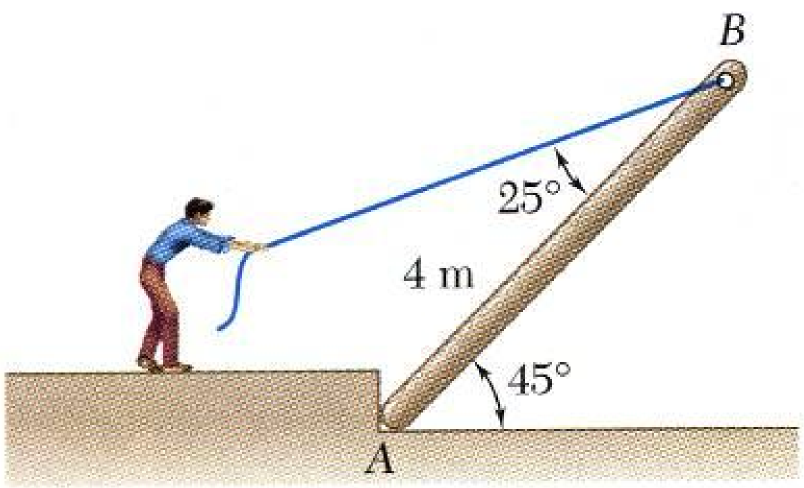


- Assume their lines of action intersect. Moment of $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ about intersection *D* is zero.
- But, for equilibrium, sum of moments of $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, $\boldsymbol{F_3}$, about any point is zero. Thus moment of $\boldsymbol{F_3}$ about *D* is zero, i.e., line of action of $\boldsymbol{F_3}$ must pass through *D*.



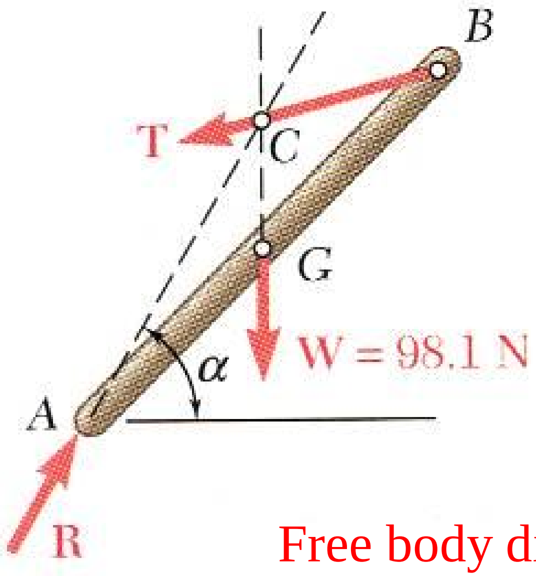
- The lines of action of the three forces must be concurrent.

Three-Force Body example

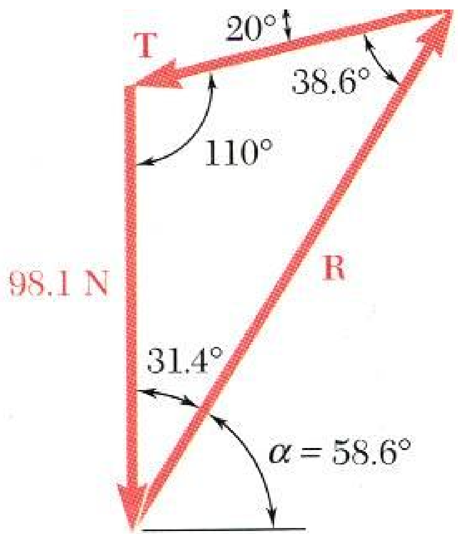


Man raises 10 kg joist, of length 4 m, by pulling rope.

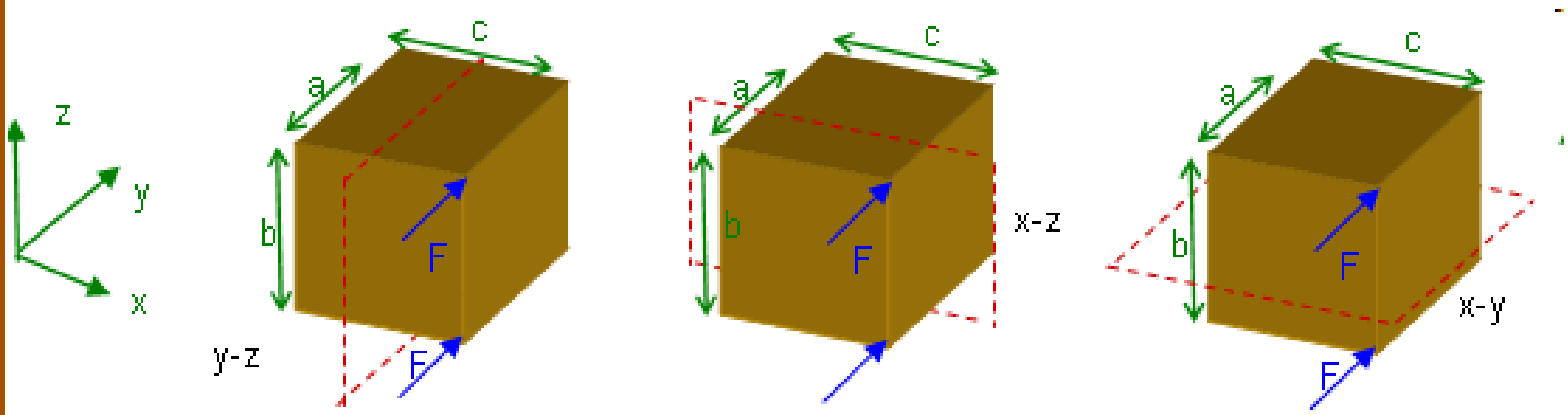
Find tension in rope and reaction at *A*.



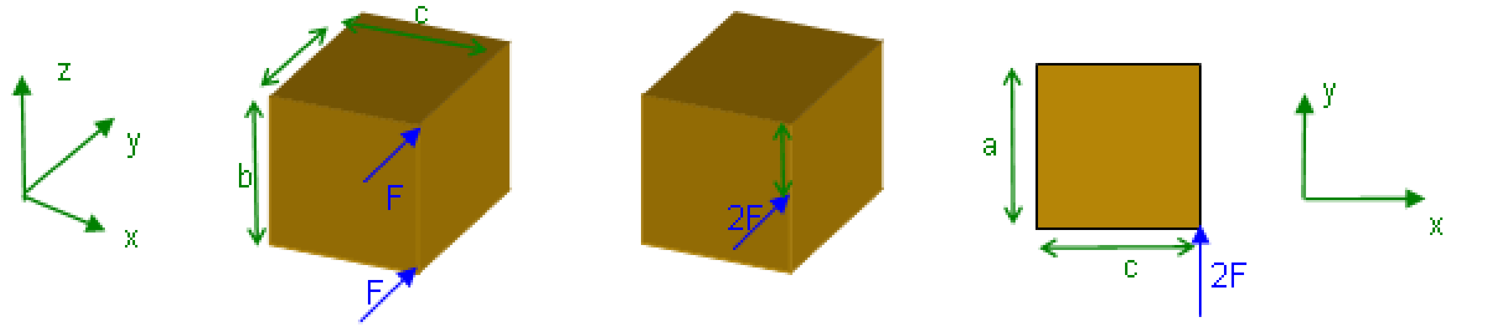
Free body diagram



Using Symmetry to convert 3D problems to 2D

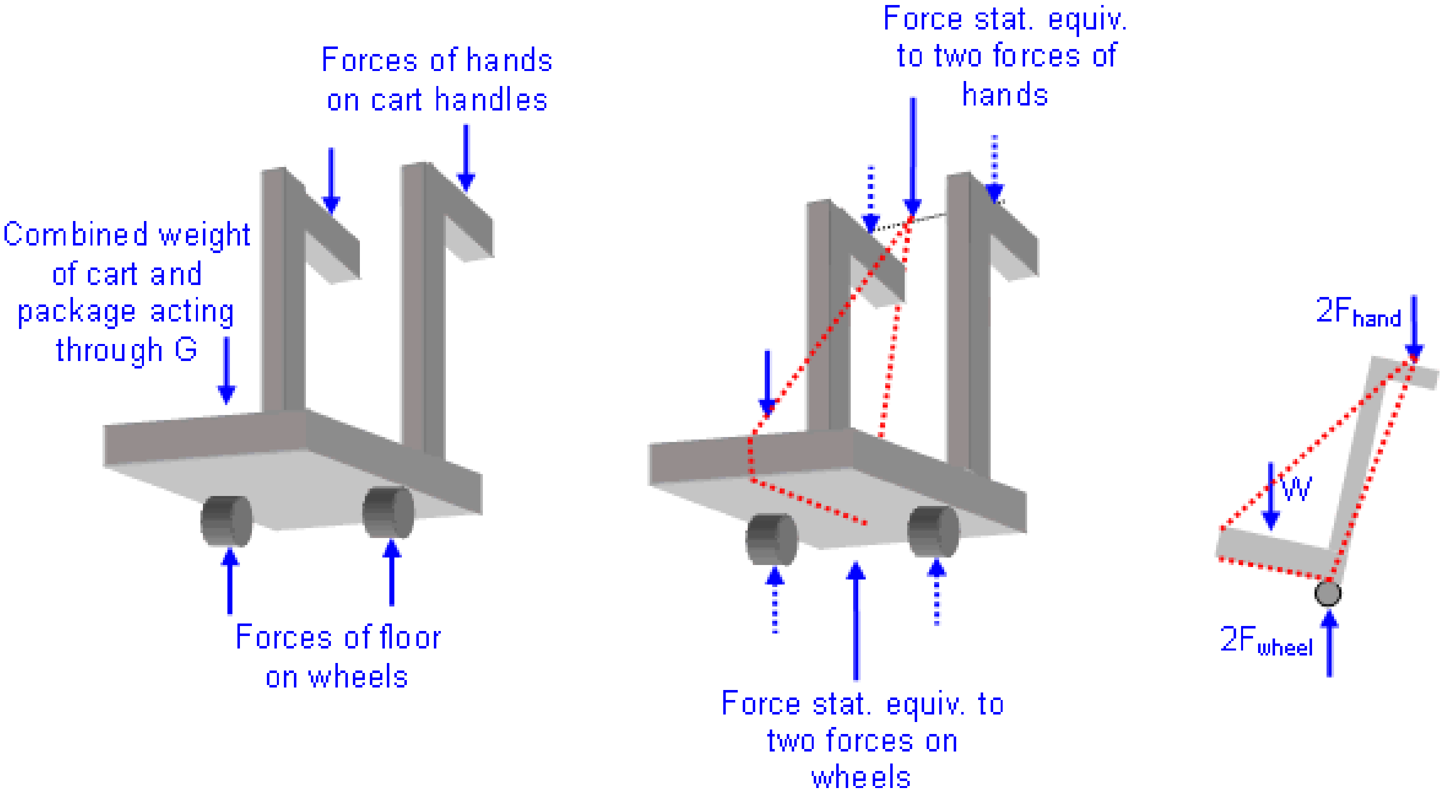


- Box has 3-planes of symmetry.
- Loading had only one plane of symmetry
- Using symmetry and static equivalence, the problem can be converted into a 2D problem

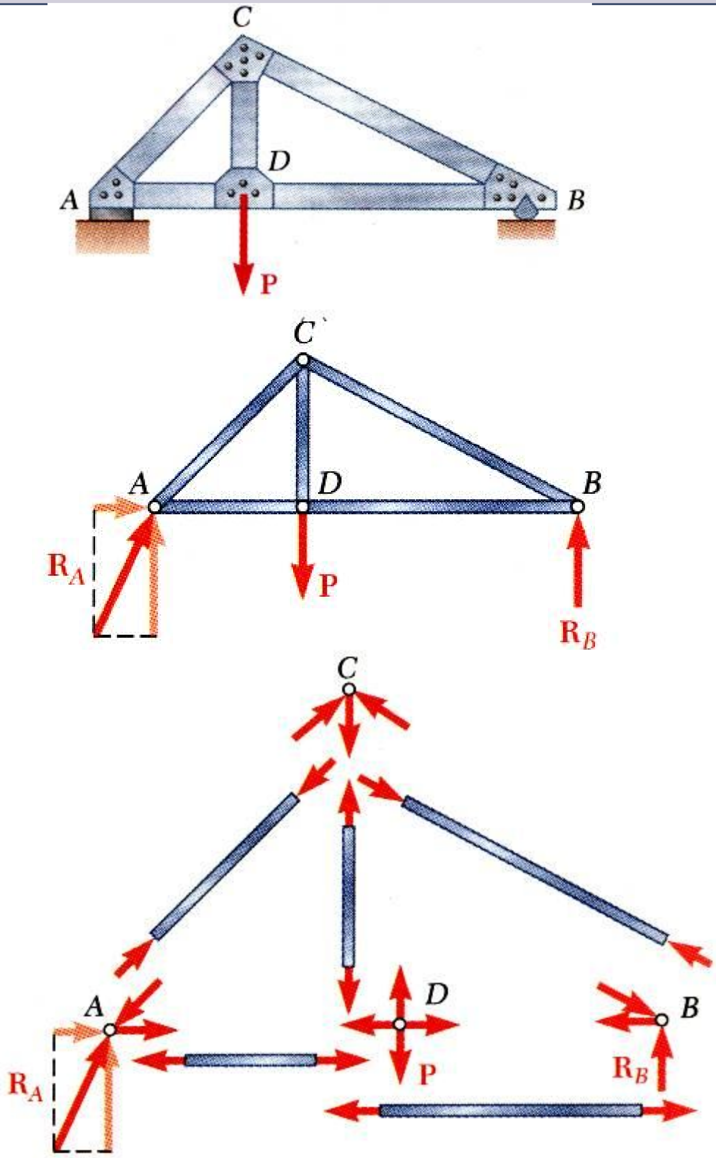


© IIT Bombay , CE Dept, Lecture Notes for CE201

Example: 3D problem as 2D problem

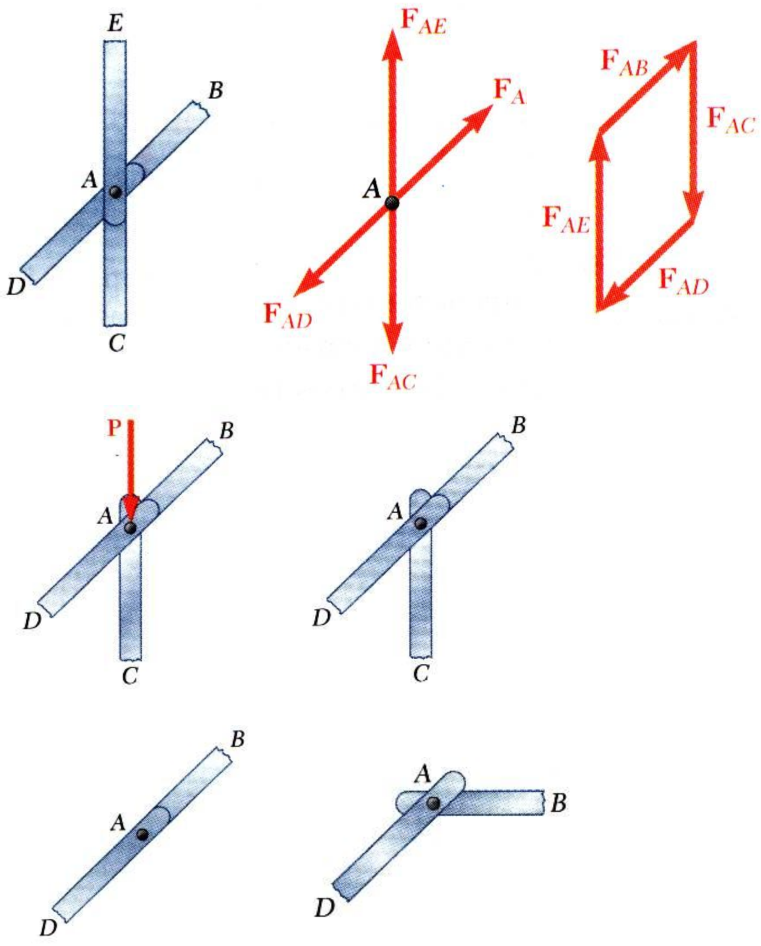


Truss - Method of Joints

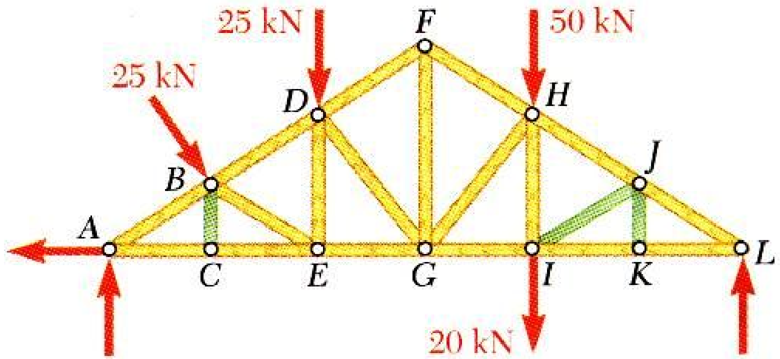


- Loading at joints only and joints are pins. So member are two force members, i.e., they possess equal and opp. forces, directed along member, at each end.
- Create FBD for each (member and) pin joint.
- So forces exerted by member on the pin joints at its ends are directed along member (equal and opposite to coresponding member force).
- Equilibrium of pins provide 2*n* equations for 2*n* unknowns for plane truss. For simple truss, 2*n* = *m* + *r*. Solve *m* member forces and *r* reactions. ***r* = 3** for a simple truss. **Simple truss is statically determinate.**

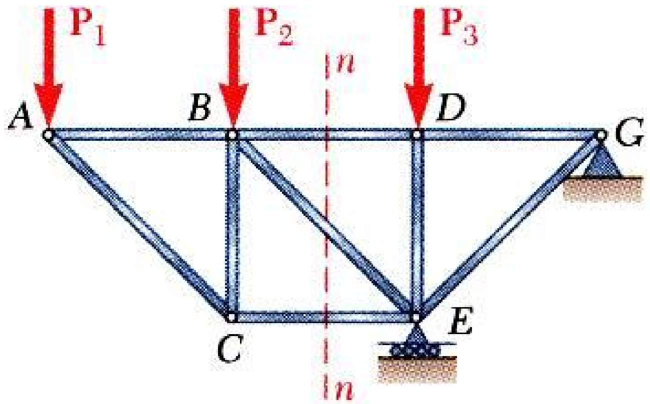
Special cases – zero force members, etc.

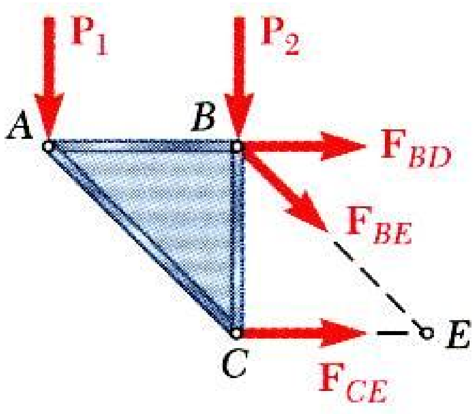


- Only four members intersect at joint as two str. lines, forces in colinear. mem's equal.
- Only 3 mem's intersect, two of them colinear, force in two col. mem's equal if load aligned with third mem. Third mem. force equals load (including zero load).
- Only two members connected at unloaded joint. Forces in mem's equal if mem's colinear, else zero.
- Recognizing these simplifies truss analysis.



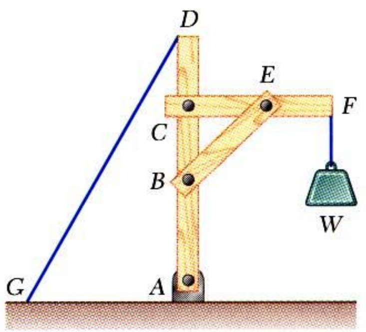
Truss - Method of Sections

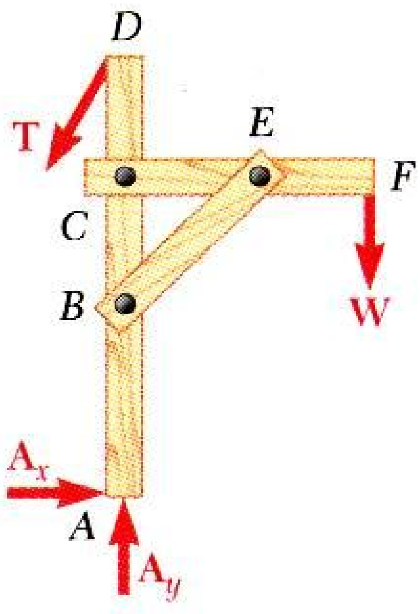




- When forces in few members are required.
- To find force in *BD*, *pass section n-n*, create FBD for left side.
- Apply equilibrium to FBD. Only three unknowns (exposed member forces), so can solve for them.
- Choose section so that at most three unknown member forces get exposed.

Analysis of Frames



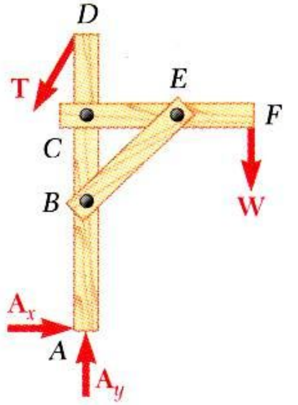


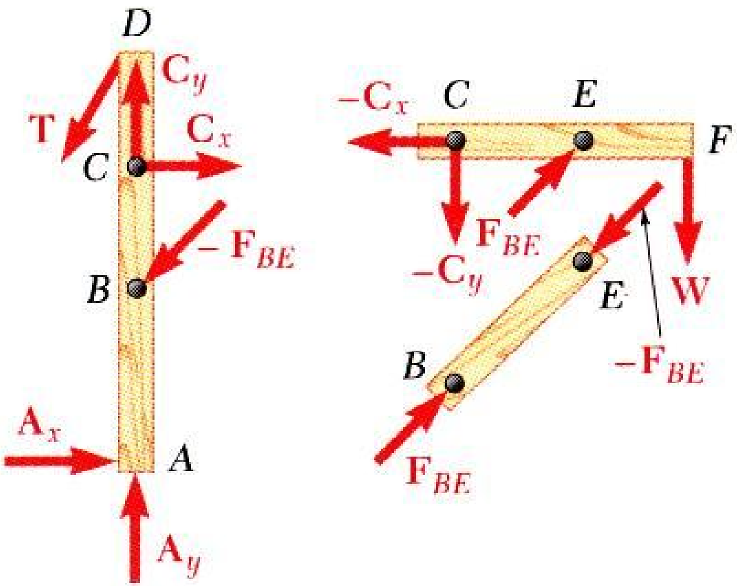
- Joints may not be only pins, loading may not be only at joints.
- At least one member *multiforce* member.

This is Rigid Frame , i.e., its internal members do not collapse if supports are removed

- FBD of **complete frame** used to determine unknown external forces coming from supports or connected bodies). This is external equilibrium analysis.

Analysis of Frames





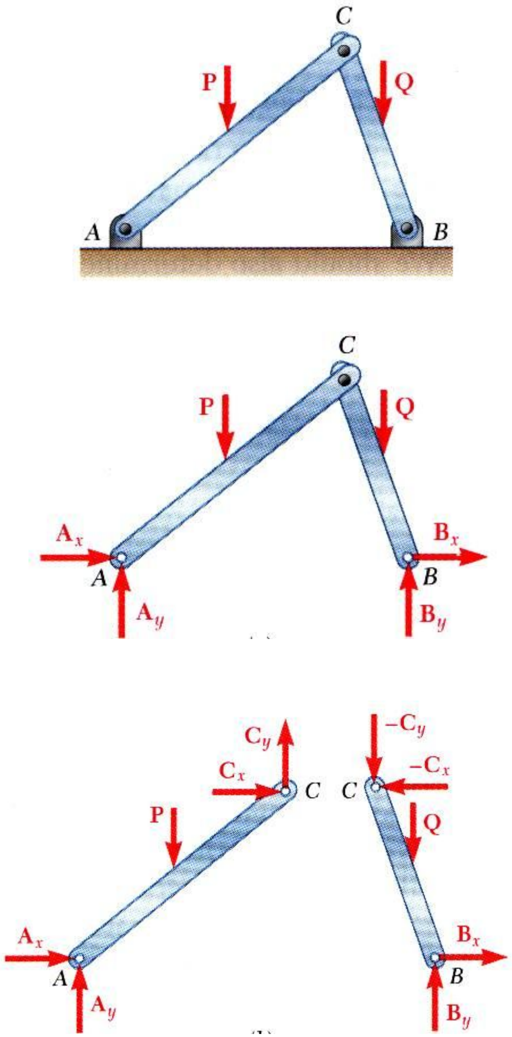
- Forces on two force members have known lines of action but unknown magnitude and sense.

- Forces on multiforce members have unknown magnitude and line of action. They must be represented with two unknown components.

- Forces between connected components are equal and opposite.

- 2 planar FBD's give 2*3=6 equilibrium equations. We have 3 unknowns, after solving reactions from external equilibrium. FBD of *BE* useless since two force member concept used, so equilibrium identically satisfied for *BE*.

Non-rigid frames



- Some frames collapse if removed from supports. Such frames can not be treated as rigid bodies.
- External FBD shows 4 reaction components. Cannot be determined from 3 external equilibrium equations.
- **Must** dismember frame. Draw component FBD's.
- 2 FBD's, 2*3=6 equil. eqns., 6 unknown forces.
- Nothing special about this method. Can use it for rigid frames also.
- In previous example of rigid frame, we have 2 planar FBD's give 2*3=6 equilibrium equations. We have 6 unknowns. FBD of *BE* useless since two force member concept used, so equilibrium identically satisfied for *BE*.