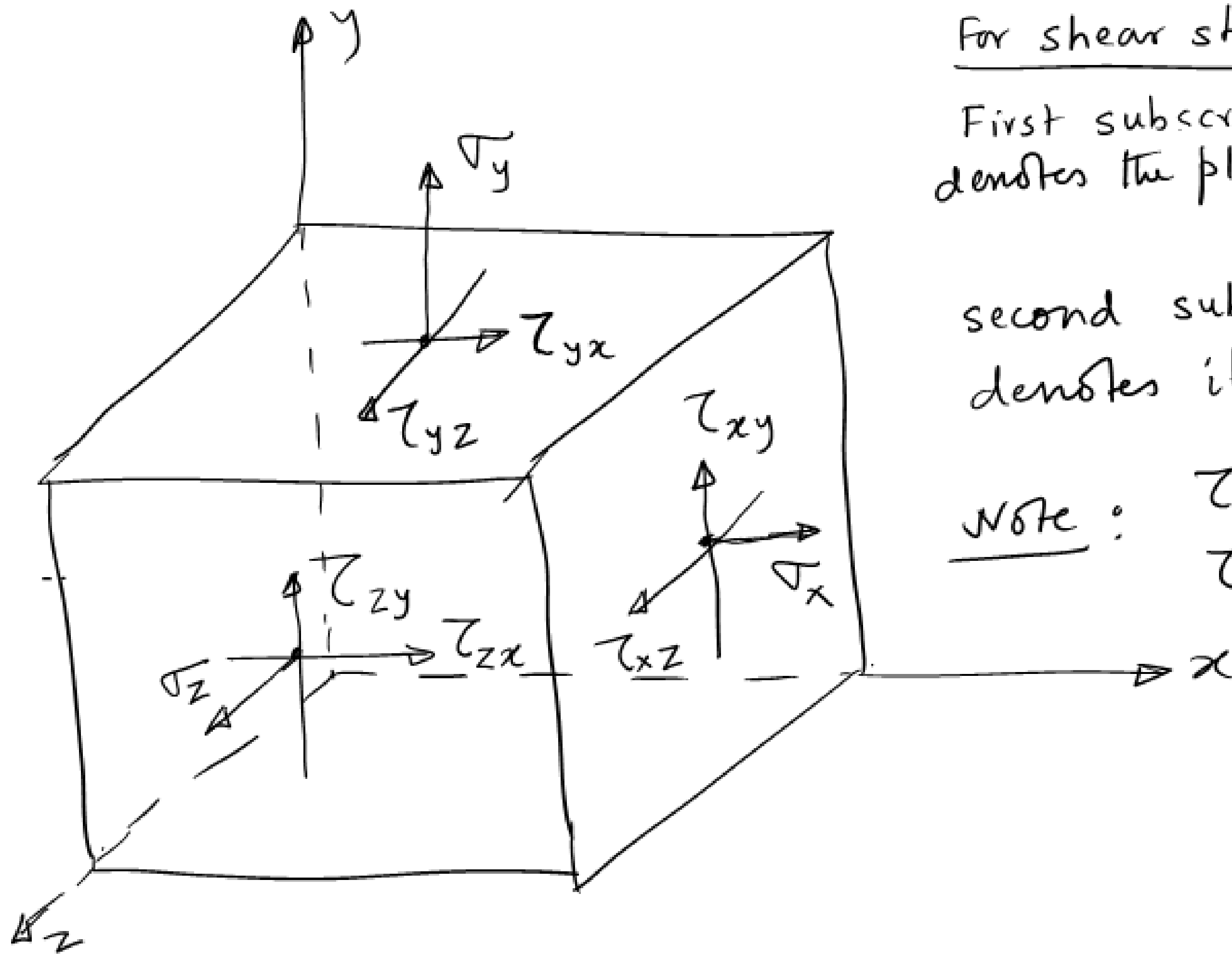
A 3D stress block: Stresses are shown on positive x,y and z faces only



<u>For shear stress</u>

First subscribt denotes the plane it acts

second subscript denotes its direction

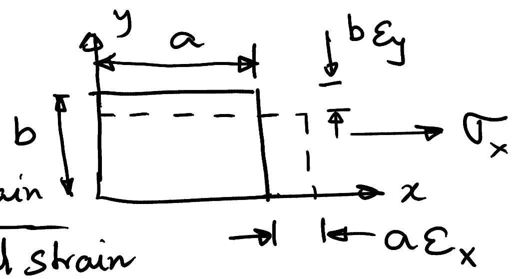
<u>Note</u> : $\tau_{xy} = \tau_{yx}$

$\tau_{xz} = \tau_{zx}$

$\tau_{yz} = \tau_{zy}$

Concept of Generalized Hooke's law

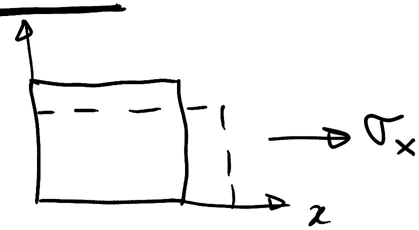
Recall : $\sigma_x = E\,\varepsilon_x$



Poisson's Ratio, $\nu = -\dfrac{\text{lateral strain}}{\text{longitudinal strain}}$

change in length = (strain) × original length

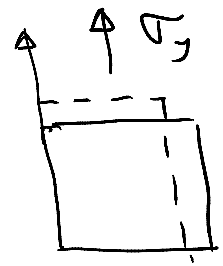
Normal Stress



$\varepsilon_x = \dfrac{\sigma_x}{E}$

$\varepsilon_y = -\nu\,\varepsilon_x = -\nu\dfrac{\sigma_x}{E}$

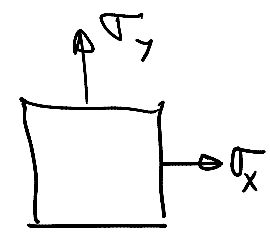
+



$\varepsilon_x = -\nu\,\varepsilon_y = -\nu\dfrac{\sigma_y}{E}$

$\varepsilon_y = \dfrac{\sigma_y}{E}$

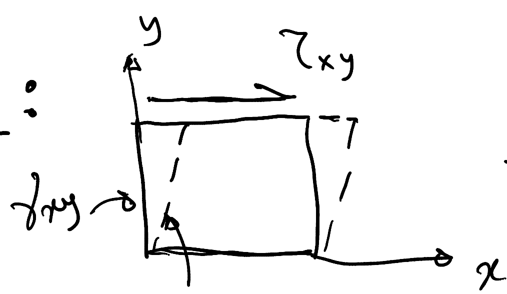
=



$\varepsilon_x = \dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E}$

$\varepsilon_y = \dfrac{\sigma_y}{E} - \nu\dfrac{\sigma_x}{E}$

Shear stress :



$\Rightarrow \quad \tau_{xy} = G\,\gamma_{xy}$

or $\gamma_{xy} = \dfrac{\tau_{xy}}{G}$

Generalized Hooke's law in 3D

(Stress-strain relationships)

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

$$\sigma_x = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_x + \upsilon\left(\varepsilon_y + \varepsilon_z\right)\right]$$

$$\sigma_y = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_y + \upsilon\left(\varepsilon_x + \varepsilon_z\right)\right]$$

$$\sigma_z = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_z + \upsilon\left(\varepsilon_x + \varepsilon_y\right)\right]$$

$$\tau_{xy} = G\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx}$$

E= elastic modulus

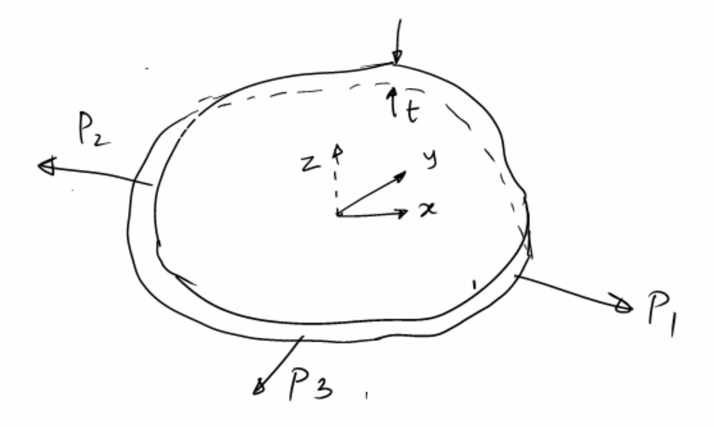
ν = poisson's ratio

G= shear modulus

$$G = \frac{E}{2(1+\upsilon)}$$

Hooke's law in 2D

Plane Stress Problem



A thin plate subject to in plane (x-y) loading

Assumptions:

$$\sigma_z = 0; \tau_{xz} = 0; \tau_{yz} = 0$$

Other stress and strain quantities are non-zero

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon \frac{\sigma_y}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon \frac{\sigma_x}{E}$$

$$\varepsilon_z = -\upsilon \frac{\sigma_x}{E} - \upsilon \frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$\Rightarrow$

$$\sigma_x = \frac{E}{(1-\upsilon^2)}\left[\varepsilon_x + \upsilon \varepsilon_y\right]$$

$$\sigma_y = \frac{E}{(1-\upsilon^2)}\left[\varepsilon_y + \upsilon \varepsilon_x\right]$$

$$\tau_{xy} = G\gamma_{xy}$$

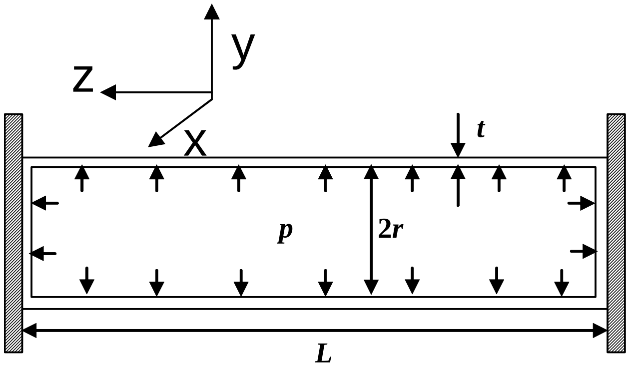


Fig. 1

A cylindrical pressure tube constrained between rigid walls

Hooke's law in 2D

Plane Strain Problem

Assumptions:

$$\varepsilon_z = 0; \gamma_{xz} = 0; \gamma_{yz} = 0$$

Other stress and strain quantities are non-zero

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_z}{E}$$

$$0 = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$\Rightarrow$

$$\sigma_x = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_x + \upsilon\varepsilon_y\right]$$

$$\sigma_y = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_y + \upsilon\varepsilon_x\right]$$

$$\sigma_z = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[\upsilon(\varepsilon_x + \varepsilon_y)\right]$$

$$\tau_{xy} = G\gamma_{xy}$$

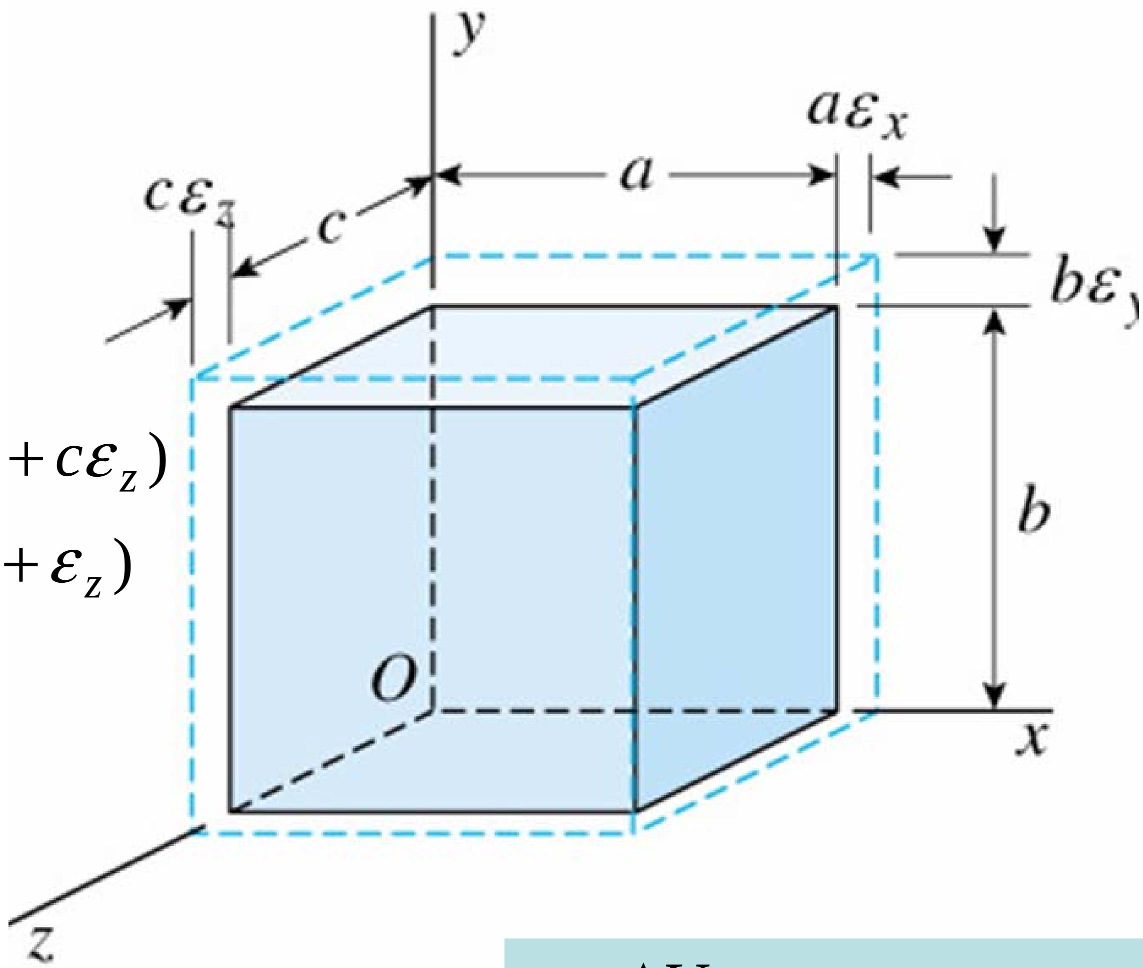
Volume Change

Original Volume

$$V_o = abc$$

Final Volume

$$V_1 = (a + a\varepsilon_x)(b + b\varepsilon_y)(c + c\varepsilon_z)$$
$$= abc(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$$
$$\approx V_o(1+\varepsilon_x+\varepsilon_y+\varepsilon_z)$$



Ignoring square terms of strain, since they are small

Volume change: $\Delta V = V_1 - V_o = V_o(\varepsilon_x + \varepsilon_y + \varepsilon_z)$

Dilatation (e): Change in Unit Volume

$$e = \frac{\Delta V}{V_o} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$
$$= \frac{1-2\upsilon}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$