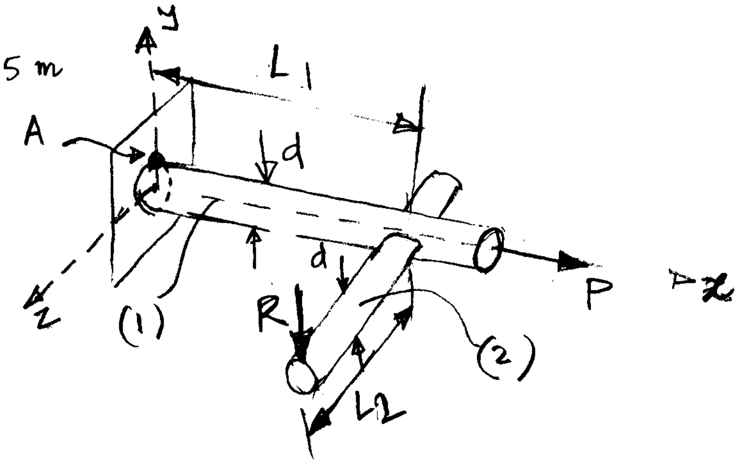
Problem – 1

Calculate the principal stresses generated at A

Given, $L_1 = 1.5\ m$, $L_2 = 1.0\ m$ } $d = 0.05\ m$
$R = 500\ N$
$P = 100\ kN$



Solution

- We need to create the stress block at A

(1) Consider the effect of load P at A

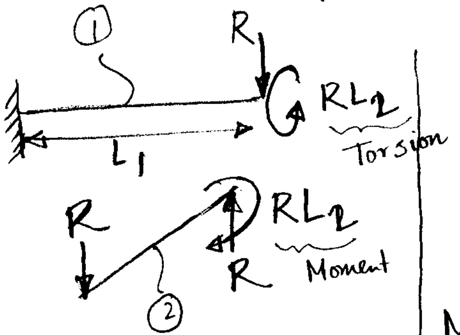
It will produce only axial stress $\sigma_x = P/A$

i,e $\sigma_x^{(a)} = \dfrac{P}{A} = \dfrac{100(10)^3}{\frac{\pi(0.05)^2}{4}} = 50.93(10)^6\ N/m^2$
$= \underline{50.93\ MPa}$

(2) Conside the effect of load R at A now,

It has two effects (a) bending (b) twisting!

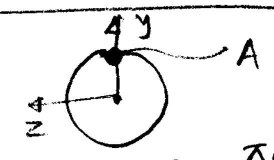
Draw the free bodies of beam (1) & (2)



This was discussed in the class before.
At the junction Newton's 3rd law follows. Moment (RL_2) in beam (2) is transferred as a twist of same magnitude (RL_2) in beam (1)

Hence, moment at A, $M = -RL_1$
Torsion at A, $T = RL_2$

(a) Moment will induce bending (flexural) stress $\sigma_x^{(b)} = -\dfrac{My}{I_z} = -\dfrac{(-RL_1)\, d/2}{\frac{\pi d^4}{64}}$



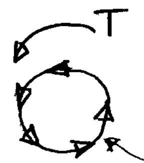
$I_z = \dfrac{\pi d^4}{64}$

$= \dfrac{32RL_1}{\pi d^3} = \dfrac{32(500)\,1.5}{\pi(0.05)^3} = 61.12(10^6)\ N/m^2$
$= \underline{61.12\ MPa}$

Note: It's a tensile stress. since beam bends downwards, top fiber stretches! Reasonable

(b) Torsion will induce shear stress

$$\tau = \frac{Tr}{I_p}$$



flow of shear stress on the cross section [This determines the direction of τ in the stress block]

$$= \frac{(RL_2)(d/2)}{\left(\pi d^4/32\right)} = \frac{16 R L_2}{\pi d^3}$$

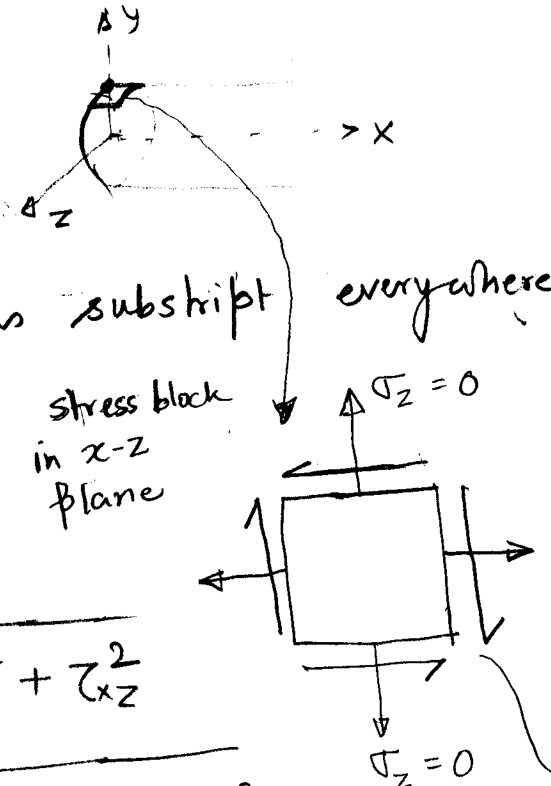
$$= \frac{16(500)(1)}{\pi (0.05)^3}$$

$$= 20.37 (10)^6 \ N/m^2$$

$$= 20.37 \ MPa$$

Note: in the stress block it is negative according to the sign convention

We need to draw the stress block at A in the x-z plane, but problem remains 2D. We will just use 'z' as subscript everywhere instead of y.



Superposition

$$\sigma_x = \sigma_x^{(a)} + \sigma_x^{(b)}$$

$$= 50.93 + 61.12$$

$$= 112.05 \ MPa$$

$\tau_{xz} = -\tau = -20.37$ MPa

Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \frac{112.05}{2} \pm \sqrt{\left(\frac{112.05}{2}\right)^2 + (-20.37)^2}$$

$$= 56.025 \pm 59.613 \ , \quad \sigma_1 = 115.6 \ MPa$$

$$\sigma_2 = -3.6 \ MPa$$

* You could also calculate τ_{max}, ε_1, ε_2, γ_{max}, show them in properly oriented elements.

* You can also check the safety at point A, i.e., whether,

$$|\sigma_1| \ \& \ |\sigma_2| \le \frac{\sigma_Y}{f_s} \quad \& \quad |\tau_{max}| \le \frac{\tau_Y}{f_s}$$

σ_Y → yield stress of material in tension/compression

f_s → factor of safety

τ_Y: Yield stress in shear