- **CE 327: Stress Transformation Failure Theories**
  Homework-4

1) The stress at a point is defined by the components $\sigma_{xx} = 0$ MPa, $\sigma_{yy} = 100$ MPa, $\sigma_{xy} = -40$ MPa. Find the principal stresses σ_1 and σ_2 and the inclination of the plane on which the maximum principal stress acts to the x plane. Use both the Mohr circle approach and the matrix method approach.

2) A cylindrical steel shaft of diameter 30 mm is subjected to a compressive axial force $F = 10$ kN, a bending moment $M = 170\, N - m$, and a torque $T = 200\, N - m$. Estimate the factor of safety against using Mises' theory if the steel has a yield stress $S_Y = 300$ MPa.

3) The stress at a point is defined by the components, $\sigma_{xx} = 100$ MPa, $\sigma_{zz} = 100$ MPa, $\sigma_{xy} = -40$ MPa, $\sigma_{yz} = 50$ MPa, $\sigma_{zx} = -20$ MPa.
Find the three principal stresses, and the three principal directions using appropriate functions in *Mathematica*.

4) The principal stress at a point are $\sigma_1 = 10$ MPa, $\sigma_2 = -100$ MPa. Sketch Mohr's circle for this state of stress and determine the normal stress on a plane inclined at an angle θ to the principal plane 1. Hence find the range of values of θ for which the normal stress is tensile.

5) A steel is found to yield in uniaxial tension at a stress $S_Y = 205$ MPa and in torsion at a shear stress $\tau_Y = 116$ MPa. Which of the von Mises' and Tresca's criteria is more consistent with the experimental data.

6) A series of experiments is conducted in which a thin plate is subjected to biaxial tension/compression, σ_1, σ_2, the plane surfaces of the plate being traction free (i.e. $\sigma_3 = 0$). Unbeknown to the experimenter, the material contains macroscopic defects than can be idealized as a sparse distribution of small circular holes through the plate thickness. The hoop stress around the circumference of one of these holes when the plate is loaded in uniaxial tension σ is known to be

$$\sigma_{\theta\theta} = \sigma\,(1 - 2\cos(2\,\theta)),$$

where the angle θ is measured from the direction of the applied stress. Show graphically the relation that will hold at yield between the stresses σ_1 and σ_2 applied to the defective plate if the Tresca criterion applies for the undamaged material.

7) In suitable units, the stress at a particular point in a solid is found to be:

$$\sigma = \begin{pmatrix} 2 & 1 & -4 \\ 1 & 4 & 0 \\ -4 & 0 & 1 \end{pmatrix}.$$

Determine the traction vector on a surface with unit normal (Cos(θ), Sin(θ), 0), where θ is a general angle in the range $0 \le \theta \le \pi$. Plot the variation of the magnitude of the traction vector $|T^n|$ as a function of θ.

8) Using the matrix manipulation techniques of mathematica, show that the von Mises criterion is equivalent to obtaining the shear stress on eight planes (forming an octahedron) with normals to the planes making equal angles with the *principal* axes.

```
<< Graphics`Polyhedra`;

a = {Arrowheads[Large], Arrow[{{0, 0, 0}, {1.5, 1.5, 1.5}}]};
b = {Arrowheads[Large], Arrow[{{0, 0, 0}, {-1.5, 1.5, 1.5}}]};
```

```
Show[Polyhedron[Octahedron, Axes → True, AxesOrigin → {0, 0, 0}, Frame → False],
 Graphics3D[{Thick, a}], Graphics3D[{Thick, b}]]
```