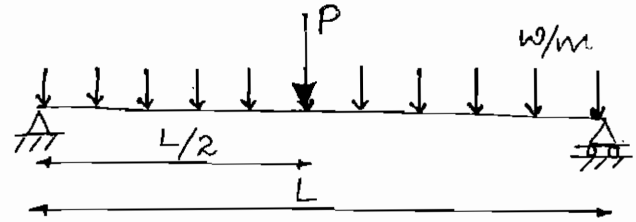
PROBLEM 1 :

- Singularity Approach :



Solution :

We have $EI \dfrac{d^4 v}{dx^4} = q(x)$

The singularity function for the above beam is,

$$EI \frac{d^4 v}{dx^4} = q(x) = -w\langle x-0\rangle^0 - P\langle x-L/2\rangle^{-1}$$

$$EI \frac{d^3 v}{dx^3} = F(x) = -w\langle x-0\rangle^1 - P\langle x-L/2\rangle^0 + C_1$$

$$EI \frac{d^2 v}{dx^2} = M(x) = -\frac{w\langle x-0\rangle^2}{2} - P\langle x-L/2\rangle^1 + C_1 x + C_2$$

Boundary conditions :

At $x = 0$, $M(0) = 0$

$$\Rightarrow EI\, v''(0) = 0$$

$$\Rightarrow C_2 = 0$$

Also, at $x = L$, $M(L) = 0$

$$\therefore\ 0 = -\frac{wL^2}{2} - P\left(\frac{L}{2}\right) + C_1 L$$

$$\Rightarrow C_1 = \frac{wL}{2} + \frac{P}{2}$$

$$EI \frac{dv}{dx} = -\frac{w\langle x-0\rangle^3}{6} - \frac{P\langle x-L/2\rangle^2}{2} + \left(\frac{wL}{2}+\frac{P}{2}\right)\frac{x^2}{2} + C_3.$$

$$EI\, v = -\frac{w\langle x-0\rangle^4}{24} - \frac{P\langle x-L/2\rangle^3}{6} + \left(\frac{wL}{2}+\frac{P}{2}\right)\frac{x^3}{6} + C_3 x + C_4$$

At $x=0, \quad v=0$

$\Rightarrow \quad C_4 = 0$

And at $x=L, \quad v=0$

$$\Rightarrow 0 = -\frac{wL^4}{24} - \frac{PL^3}{48} + \frac{wL^4}{12} + \frac{PL^3}{12} + C_3 L$$

$$\Rightarrow C_3 = -\frac{wL^4}{24} - \frac{PL^2}{16}$$

Maximum deflection occurs at centre, $x = L/2$.

$$EI\,v = -\frac{w\langle L/2\rangle^4}{24} + \left(\frac{wL}{2}+\frac{P}{2}\right)\frac{L^3}{48} + \left(-\frac{wL^3}{24} - \frac{PL^2}{16}\right)\frac{L}{2}$$

$$\boxed{EI\,v = -\frac{5wL^4}{384EI} - \frac{PL^3}{48EI}}$$

<u>SFD</u>

$$F(x) = EI\frac{d^3v}{dx^3} = -w\langle x-0\rangle^1 - P\langle x - L/2\rangle^0 + \frac{wL}{2} + \frac{P}{2}$$

At $x=0$, $F(0) = \frac{wL}{2} + \frac{P}{2}$

At $x=L/2$, $F(L/2) = \frac{P}{2}$

At $x=L$, $F(L) = -\frac{wL}{2} - \frac{P}{2}$

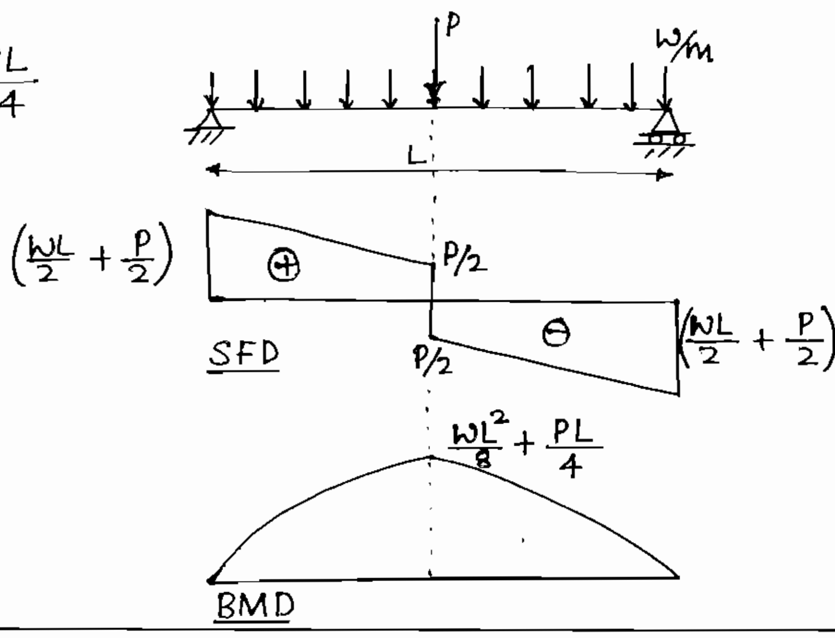
<u>BMD</u>

$$M(x) = \frac{EI\,d^2v}{dx^2} = -\frac{w\langle x-0\rangle^2}{2} - P\langle x-L/2\rangle^1 + \left(\frac{wL}{2}+\frac{P}{2}\right)x$$

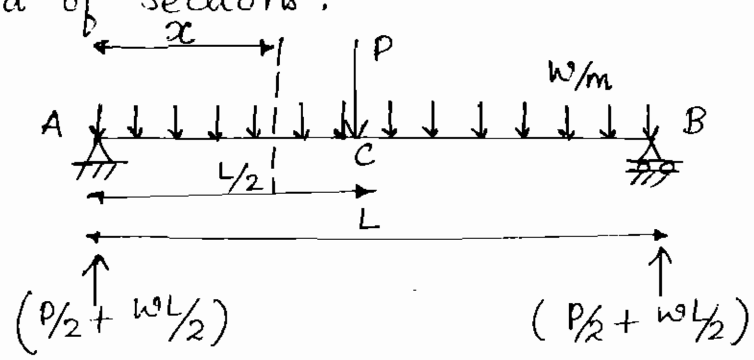
At $x=0$, $M(0) = 0$

At $x=L/2$, $M(L/2) = \frac{wL^2}{8} + \frac{PL}{4}$

At $x=L$, $M(L) = 0$



- Method of Sections :



Solution :

Consider a section at a distance 'x' from A. Since the beam and loading is symmetric, let us consider only part AC.

$$\therefore \quad EI\frac{d^2v}{dx^2} = M(x) = \left(\frac{P}{2} + \frac{wL}{2}\right)x - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \theta(x) = \left(\frac{P}{2} + \frac{wL}{2}\right)\frac{x^2}{2} - \frac{wx^3}{6} + C_1.$$

$$EI\,v = \left(\frac{P}{2} + \frac{wL}{2}\right)\frac{x^3}{6} - \frac{wx^4}{24} + C_1x + C_2$$

Boundary conditions :

At $x = 0$, $v = 0$

$$\Rightarrow C_2 = 0$$

At $x = L/2$, $\theta = 0$

$$\Rightarrow 0 = \left(\frac{P}{2} + \frac{wL}{2}\right)\frac{L^2}{8} - \frac{wL^3}{48} + C_1$$

$$\Rightarrow C_1 = -\frac{PL^2}{16} - \frac{wL^3}{24}$$

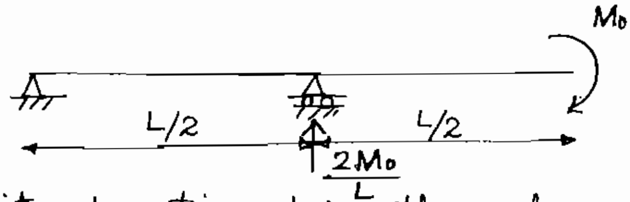
Maximum deflection occurs at centre, $x = L/2$

$$EI\,v = \left(\frac{P}{2} + \frac{wL}{2}\right)\frac{L^3}{48} - \frac{wL^4}{384} + \left(-\frac{PL^2}{16} - \frac{wL^3}{24}\right)\frac{L}{2}$$

$$\boxed{\therefore \quad v = -\frac{PL^3}{48EI} - \frac{5wL^4}{384EI}}$$

PROBLEM 2:

Singularity Approach:



The singularity function for the above beam is,

$$EI\frac{d^4v}{dx^4} = q(x) = \frac{2M_0}{L}\langle x - L/2\rangle^{-1}$$

$$EI\frac{d^3v}{dx^3} = F(x) = \frac{2M_0}{L}\langle x - L/2\rangle^{0} + C_1$$

$$EI\frac{d^2v}{dx^2} = M(x) = \frac{2M_0}{L}\langle x - L/2\rangle^{1} + C_1x + C_2$$

Boundary conditions,

At $x=0$, $M(0) = 0$

$$\Rightarrow \quad C_2 = 0$$

At $x = L$, $M(L) = -M_0$

$$\Rightarrow -M_0 = \frac{2M_0}{L}\left(\frac{L}{2}\right) + C_1L$$

$$\therefore \quad C_1 = -\frac{2M_0}{L}$$

$$M(x) = \frac{2M_0}{L}\langle x - L/2\rangle^{1} - \frac{2M_0}{L}x$$

$$EI\frac{dv}{dx} = \frac{2M_0}{L}\frac{\langle x - L/2\rangle^{2}}{2} - \frac{2M_0}{L}\frac{x^2}{2} + C_3$$

$$EI\,v = \frac{2M_0}{L}\frac{\langle x - L/2\rangle^{3}}{6} - \frac{2M_0}{L}\frac{x^3}{6} + C_3x + C_4$$

At $x = 0$, $v = 0$

$$\Rightarrow C_4 = 0$$

At $x = L/2$, $v = 0$

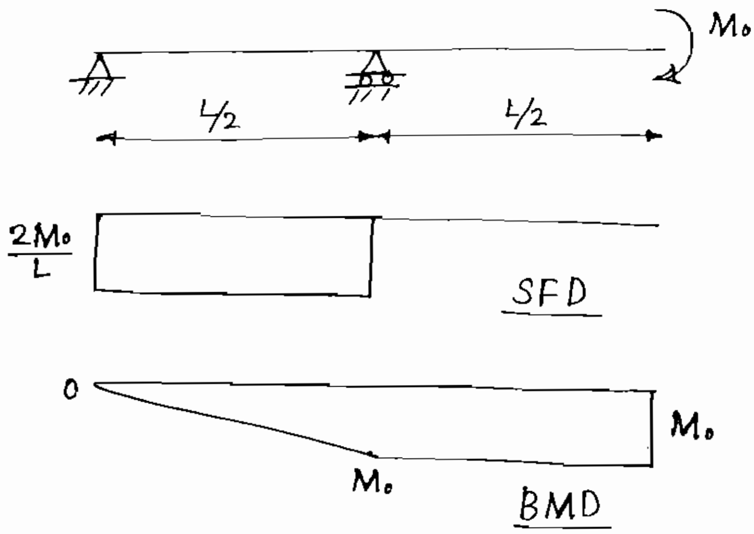
$$0 = \frac{2M_0}{L}(0) - \frac{2M_0}{L}\frac{L^3}{48} + C_3 \cdot \frac{L}{2}$$

$$\Rightarrow C_3 = \frac{M_0L}{12}$$

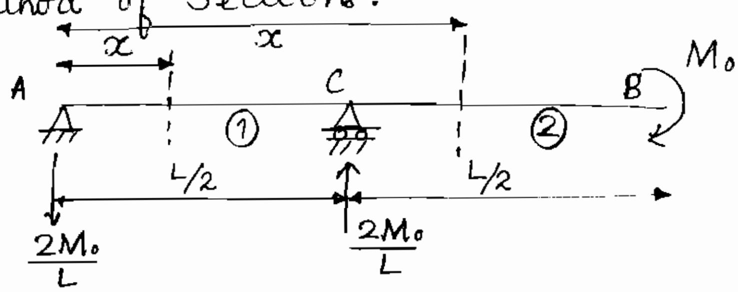
Deflection at $x = L$,

$$EI\, v(L) = \frac{2M_0}{L}\frac{(L/2)^3}{6} - \frac{2M_0}{L}\frac{(L)^3}{6} + \frac{M_0 L}{12} \times L$$

$$\boxed{v(L) = -\frac{5 M_0 L^2}{24\, EI}}$$



• Method of Sections :



Consider sections at distance 'x' from 'A' in part AC and BC of the beam shown above.

$$EI\frac{d^2v}{dx^2} = M(x) = -\frac{2M_0}{L}x \qquad 0 \le x \le L/2$$

$$EI\frac{d^2v}{dx^2} = M(x) = -\frac{2M_0}{L}x + \frac{2M_0}{L}(x - L/2) \qquad L/2 \le x < L$$

For $0 \le x < L/2$,

$$M(x) = -\frac{2M_0}{L}x$$

$$EI\frac{dv}{dx} = v'(x) = -\frac{2M_0}{L}\frac{x^2}{2} + C_1.$$

$$EI\, v = -\frac{2M_0}{L}\frac{x^3}{6} + C_1 x + C_2.$$

For $L/2 \leqslant x < L$,

$$EI\frac{d^2v}{dx^2} = M(x) = -\frac{2M_0}{L}x + \frac{2M_0}{L}(x - L/2)$$

$$EI\frac{dv}{dx} = v'(x) = -\frac{2M_0}{L}\frac{x^2}{2} + \frac{2M_0}{L}\frac{(x-L/2)^2}{2} + C_1.$$

$$EI\,v = -\frac{2M_0}{L}\frac{x^3}{6} + \frac{2M_0}{L}\frac{(x-L/2)^3}{6} + C_1x + C_2$$

Boundary conditions:

- $v(0) = 0$ at $x = 0$

$\Rightarrow C_2 = 0$

- $v(L/2)\big|_1 = 0$ and $v(L/2)\big|_2 = 0$

$$\left.\begin{aligned} &-\frac{2M_0}{L}\frac{L^3}{48} + C_1 x = 0 \\ &\Rightarrow C_1 = \frac{2M_0}{L}\times\frac{L^3}{48}\times\frac{2}{L} = \frac{M_0L}{12} \end{aligned}\right\} \text{For } AC$$

Similarly,

$$\left.\begin{aligned} &-\frac{2M_0}{L}\times\frac{L^3}{48} - \frac{2M_0}{L}\frac{(L/2-L/2)^3}{6} + C_1(L/2) = 0 \\ &\Rightarrow C_1 = \frac{M_0L}{12} \end{aligned}\right\} \text{For } BC$$

- $v'(L/2)\big|_1 = v'(L/2)\big|_2$

$$v'(L/2)\big|_1 = -\frac{2M_0}{L}\frac{L^2}{8} + \frac{M_0L}{12} = -\frac{M_0L}{4} + \frac{M_0L}{12} = -\frac{M_0L}{6}$$

$$v'(L/2)\big|_2 = -\frac{2M_0}{L}\times\frac{L^2}{8} + \frac{2M_0}{L}\frac{(L/2-L/2)^2}{2} + \frac{M_0L}{12} = -\frac{M_0L}{6}$$

- Deflection at $x = L$ is,

$$EI\,v(L) = -\frac{2M_0}{L}\frac{L^3}{6} + \frac{2M_0}{L}\frac{(L-L/2)^3}{6} + \frac{M_0L}{12}\times L + 0$$

$$EI\,v(L) = -\frac{M_0L^2}{3} + \frac{M_0L^2}{24} + \frac{M_0L^2}{12}$$

$$\boxed{v(L) = -\frac{5M_0L^2}{24EI}}$$