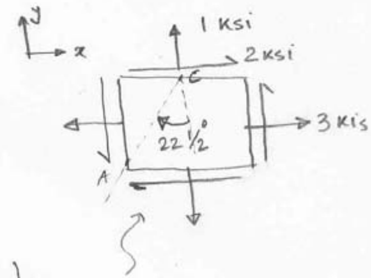
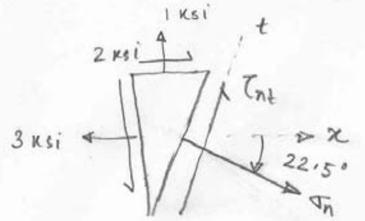
Replace n by x′ and t by y′ !

Example

(a) Find σ_n and τ_{nt} for $\theta = -22.5°$





Note

$\sigma_x = 3$

$\sigma_y = 1$

$\tau_{xy} = 2$

In the element they are positive according to the sign convention

Use,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_n(-22.5°) = \frac{3+1}{2} + \frac{3-1}{2} \cos(-45°) + 2 \sin(-45°)$$

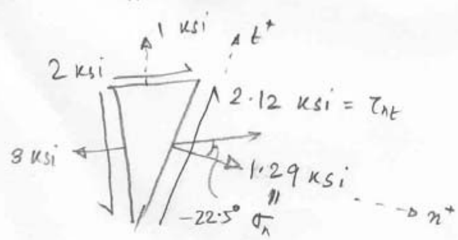
$$= 2 + 1(0.707) - 2(0.707) = \underline{\underline{1.29 \text{ ksi}}}$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\left(\frac{3-1}{2}\right) \sin(-45°) + 2 \cos(-45°)$$

$$= 1(0.707) + 2(0.707) = \underline{\underline{2.12 \text{ ksi}}}$$

Both values are positive, so they act in the positive n and t directions, as shown below



(b) Find principal stresses (σ_1 & σ_2) and show on stress block w.r.t x - axis

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3+1}{2} \pm \sqrt{\left(\frac{3-1}{2}\right)^2 + 2^2} = 2 \pm \sqrt{5}$$

$$\sigma_1 = \underline{\underline{4.25 \text{ ksi}}} \quad \text{and} \quad \sigma_2 = \underline{\underline{-0.24 \text{ ksi}}}$$

Orientation. $\tan 2\theta_p = \dfrac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} = \dfrac{2}{\left(\frac{3-1}{2}\right)} = 2$

$$\Rightarrow 2\theta_p = 63.43^\circ \text{ and } 243.43^\circ$$

Therefore $\theta_p = 31.7^\circ$ and 121.7° (both positive, measured CC from x - axis)

(*) Which angle is θ_{p1} (gives σ_1 plane) ?
& " " is θ_{p2} (gives σ_2 plane) ?

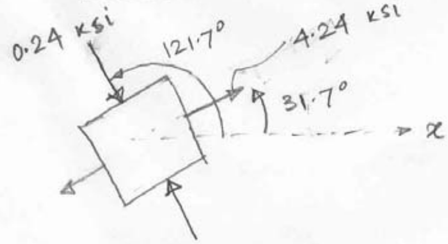
Try 31.7° in $\sigma_n(31.7^\circ) = \frac{3+1}{2} + \frac{3-1}{2}\cos(63.43^\circ) + 2\sin(63.43^\circ)$

$$= \underline{\underline{4.24}} = \sigma_1 \;!$$

↑ we obtained this already using direct formula above

Remark – This is a good check also

Hence, $\boxed{\begin{array}{l} \theta_{p1} = 31.7^\circ \\ \theta_{p2} = 121.7^\circ \end{array}}$



(c) Find τ_{max} and show its plane w.r.t x axis

$$\tau_{max} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \pm \sqrt{\left(\frac{3-1}{2}\right)^2 + 2^2} = \pm\sqrt{5} = \pm 2.24 \text{ ksi}$$

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -0.5$$

$$2\theta_s = \tan^{-1}(-0.5) = -26.56^\circ \text{ and}$$
$$-26.56^\circ + 180^\circ = 153.4^\circ$$

Therefore $\theta_s = -13.28^\circ$ (measured clockwise) and 76.7° (measured CC)

[observe they differ by 45° from θ_p !]

Which angle is θ_{s1} (gives $+\tau_{max}$ plane) ?

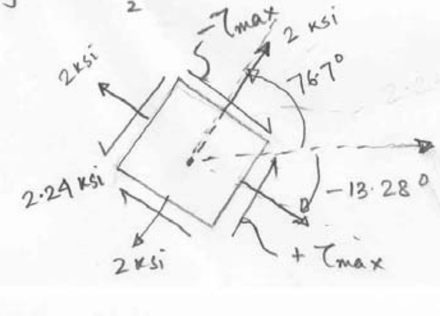
& " " is θ_{s2} (gives $-\tau_{max}$ plane) ?

Try 76.7° in $\tau_{nt}(76.7^\circ) = -\left(\frac{3-1}{2}\right)\sin(153.4^\circ)$
$+ 2\cos(153.4^\circ)$
$= -2.24 = -\tau_{max}$
(great !)

Hence $\theta_{s1} = -13.28^\circ$

$\theta_{s2} = 76.7^\circ$

Note, $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 2$ ksi (presents in max^m shear stress planes)



{ Note for 76.7° τ_{max} is negative, means must act along $(-t)$ directi.