Fabrication Uncertainty in $B_2$ for Nuclear Pipe Bends Subjected to In-Plane Opening Moment

For reliability-based design of pipe bends and elbows, a probabilistic characterization of the primary stress indices ($B_1$ and $B_2$) is essential. This paper aims at the characterization of the fabrication/geometry uncertainty in $B_2$, for thin stainless steel long radius pipe bends, subjected to in-plane opening moment. This characterization is performed in a framework based on Monte Carlo simulation and nonlinear finite element analysis. A revision of the code-based expression for $B_2$ is proposed where a random variable $K$ replaces the constant numerator in this expression. The statistics for $K$ are provided for different pipe nominal dimensions, which indicates that the existing provision gives a very conservative estimate of the plastic collapse moment for pipe bends subjected to in-plane opening flexure. [DOI: 10.1115/1.4007470]

Keywords: nuclear pipe bends, stress index $B_2$, Monte Carlo simulation, in-plane opening moment, probabilistic characterization

1 Introduction

In recent years, various researchers working in the area of nuclear piping emphasized the need to move from a deterministic and working/allowable stress design (WSD/ASD) approach to a more rational reliability-based design approach following the load and resistance factor design (LRFD) format. Gupta and Choi [1] presented an exploratory case study with these objectives for the design of nuclear piping using the ASME B&PV Code, Section III [2]. They considered a “cold” straight pipe section subjected to pressure and seismic moment at Service Level D for their sample code calibration. The CRTD-86 report [3] was a comprehensive effort in reliability-based code calibration for Class 2 and 3 pipes. This report provided LRFD design equations for design against “pipe burst” and “plastic collapse” (at all the four service levels) with sample load factors ($\gamma$) and resistance factors ($\phi$), considering target reliability index ($\beta$) of 2 and 3. Latter works by Avrithi and Ayyub [4–8] focused on various related topics, such as: modeling uncertainty in design against pipe burst, LRFD equation for Class 1 pipes subjected to pressure only, code calibration for Class 2 and 3 pipes subjected to internal pressure and moment, fatigue reliability for pipes, etc.

However, none of the works on the LRFD/reliability-based design of pipes, so far, dealt with pipe bends and elbows. This paper presents a part of a research project on the development of reliability-based design guidelines for nuclear pipe bends. The design of pipe bends and elbows is critical because the plastic collapse behavior of these components, subjected to internal pressure and bending moments, is typically marked with significant geometric nonlinearity (ovalization of the cross-section, etc.). As per Section III (NB-, NC-, and ND-3600 for “design by rule”) of the ASME B&PV Code, the general design equation against the plastic collapse of a nuclear pipe component, for all service levels, can be written as

$$B_1 \frac{PD}{2t} + B_2 \frac{M}{Z} \leq k S_m$$  \hspace{1cm} (1)

where, $B_1 =$ stress index for pressure, $B_2 =$ stress index for bending, $P =$ internal pressure, $D_o =$ nominal outer diameter of the pipe, $t =$ nominal wall thickness, $M =$ bending moment at the pipe cross-section, $Z =$ elastic section modulus, $k =$ a constant multiplier for $S_m$, and $S_m =$ permissible stress limit of the pipe material. Values of $k$ and $S_m$ depend on the service level. It should be noted that by using the two primary stress indices ($B_1$ and $B_2$), the designer is able to use this equation with the same basic allowable stress levels for both straight and bend pipes. When a simplified method of analysis is used, these indices act as scalar multipliers for nominal stresses in Eq. (1). Detailed and complex analysis techniques can thus be avoided by using these indices, although detailed finite element analyses (FEA) give more accurate results.

As per the ASME B&PV Code, for an elbow or a pipe bend, these stress indices can be calculated based on the pipe’s bend factor ($h$). For example

$$B_2 = \frac{1.30}{h^{2/3}} \geq 1.0$$  \hspace{1cm} (2)

where

$$h = \frac{\varepsilon R}{R_m}$$  \hspace{1cm} (3)

$R$ is the nominal bend radius, and $r_m = (D_o - t)/2$ is the mean pipe radius. The plastic collapse behavior of pipe bends changes depending on the orientation of the flexure. Pipe bends and elbows are subjected to bending moments, which may be in the in-plane closing, in-plane opening, or out-of-plane orientation. The nonlinear behavior is also affected by the pipe geometry, defined by its $D_o$, $t$, and $R$. The stress index $B_2$ can only approximately incorporate these effects, and Eq. (2) cannot accurately represent all pipe bends with varying geometry and different flexure modes.

To arrive at an equivalent of Eq. (1) following the LRFD format for pipe bends and elbows, as indicated by Gupta and Choi [1]
2 Overall Methodology in Brief

The fabrication uncertainty in \( B_2 \) is determined using Monte Carlo simulation techniques and nonlinear FEA. As mentioned earlier, the fabrication uncertainty arises due to the variation in pipe geometry. Using the Monte Carlo simulation technique, various values of the pipe geometry parameters \( D_{op}, t, \) and \( h \) are generated. Each set of randomly generated pipe geometries is then used to create a detailed three-dimensional FEA model of the pipe. This model is applied with a monotonically increasing rotation to create in-plane opening bending moment in the pipe bend. Based on the maximum bending moment versus rotation plot for each model, the plastic collapse moment \( (M_{pl}) \) is obtained using the “twice-elastic-slope method.” From the statistics of \( M_{pl} \) for all the simulated pipe geometries, \( B_2 \) is expressed statistically as

\[
B_2 = \frac{K}{R^{2/3}}
\]

where \( K \) is a random variable, which replaces the constant numerator in the code-based expression for \( B_2 \) (Eq. (2)). The fabrication uncertainty in \( B_2 \) is, thus, finally expressed in terms of the statistics of \( K \).

3 Generation of Random Values for \( D_{op}, t, \) and \( h \)

For simulating random values of the pipe geometry and bend geometry parameters, \( D_{op}, t, \) and \( h \), the probabilistic/uncertainty characteristics of each of these parameters are needed. The ASME Special Working Group on Probabilistic Methods in Design reported detailed statistical data on various parameters related to the pipe geometry, pipe material, and different types of load acting on the pipe at different design conditions [3]. The required statistics for the pipe geometry parameters (quantifying fabrication uncertainties) are adopted based on their recommendations. Uncertainty in pipe material properties are not considered here, because the statistical variation in \( B_2 \) is precisely due to the variation in the pipe bend geometry. Similar to in sample code calibrations in the CRTD-86 report [3], uncertainties due to initial imperfection and in the variation of thickness for a selected cross-section (due to manufacturing/extrusion process) are not included in this study, as well.

The scope defined in Sec. 1 limits the pipe geometry, more specifically the relation between different parameters defining the pipe and the bend geometry. For example, a pipe is considered to be “thin” only when \( D_{op}/t \geq 20.0 \), where \( D_{op} = D_o - 2t \). Class 2 and 3 stainless steel (Type 304) pipes require to satisfy that \( 6.0 < D_{op}/t < 40.0 \). The pipe bend is considered to be a “long radius” one, only when \( R/r_{mm} \) is greater than 3.0. These limits are applied to the pipe geometry simulated using a “crude” Monte Carlo technique. A variety of pipe diameters and schedules (defining the nominal thickness) are used in nuclear piping systems. However, considering the limits described here, four different nominal diameter-nominal thickness/schedule cases are considered in this study. These cases are described in Table 1. The fabrication uncertainty in \( B_2 \), in terms of the statistics of \( K \), is first obtained for each case separately, and the suitability of combining the statistics for different cases are discussed later.

As per CRTD-86, diameter tolerances are quite tight and tolerance levels vary with the pipe size \( D_{op} \). The diameter variations corresponding to nominal values of the outer diameter are provided in Table 2 [3]. \( D_{op} \) is assumed to follow a normal distribution with the maximum and the minimum values in this table set at \( \mu + 2\sigma \) and \( \mu - 2\sigma \), respectively (where, \( \mu \) denotes the mean and \( \sigma \) denotes the standard deviation (SD) of a distribution). The same assumption was also made in the CRTD-86 report to arrive at a probability distribution for \( D_{op} \). Based on this assumption, the distribution parameters (mean, SD, and coefficient of variation/CoV) are calculated for the four nominal diameters in Table 1. Table 3 presents the \( D_{op} \) statistics for the four cases. For the four nominal thickness values considered in this study, the statistical characteristics are again based on the suggestions in the CRTD-86 report. A bias value of 0.925 is considered for the pipe thickness \( (t) \). The mean values for the four selected schedules are obtained from the nominal values using this information (Table 4). \( t \) is also assumed to follow a normal distribution, with a CoV = 0.03.

CRTD-86, however, did not provide any information on pipe bend characteristics. The bend factor \( (b) \) is an important random variable in the work presented here, and the probabilistic characteristics for this variable should be adopted carefully. Chattopadhyay [11] reported the range of values for the bend factor based on his study of PHWR, which are reproduced in Table 5. Similar
to the assumption for $D_o$, the maximum and the minimum values of $h$ are set to $\mu + 2\sigma$ and $\mu - 2\sigma$, respectively, and a normal distribution is assumed for $h$. The corresponding distribution parameters for $h$ are presented in Table 5.

Following the distributions adopted for $D_o$, $t$, and $h$ (Tables 3–5), random values for these parameters are generated using the open-source scientific computation package SCILAB [12]. Each simulated set corresponds to a unique pipe geometry. The mean radius ($r_m$) and the bend radius ($R$) for a simulated geometry are calculated from the simulated $D_o$, $t$, and $h$. However, simulated pipe geometries that do not satisfy the requirements in terms of $D_i/t$, $D_o/t$, and $R/r_m$ as mentioned earlier, are discarded from the Monte Carlo simulation procedure. For example, Table 6 shows the range of $R/r_m$ values for the accepted simulated geometries only. The accepted simulations are considered for the FEA and final inclusion in the statistical quantification of $K$.

4 Nonlinear FEA of Pipe Bends Subjected to In-Plane Opening Moment

The objective of conducting the nonlinear FEA of the pipe bend subjected to in-plane opening is to obtain the plastic collapse moment, $M_c$, for a simulated geometry. The generic pipe bend specimen that is considered for these FEA, is shown in Fig. 1. The pipe specimen has a 90 deg bend part in the middle connected to two straight parts at both ends of the bend. The pipe is fixed against all degrees of freedom at one end, while the other end of the pipe is kept absolutely free. The length of each straight part is kept exactly at $5D_o$ to avoid flange effects from the supports on the bend part [13–15]. The pipe cross-section has a uniform geometry for the whole length of the pipe. For each simulated geometry, with specific $D_o$, $t$, and $R$, a finite element model of the pipe is created for analysis. Each of these simulated geometries has different dimensions, but the configuration and the support conditions of the specimen remain the same.

Type 304 stainless steel is the selected pipe material, which is a “ductile material”. Following the assumption made in various previous research works in this area [3,5], its stress–strain behavior is considered to be elastic-perfectly plastic for the pipe bend under consideration. However, inelasticity is considered only in the 90 deg bend part. The material for the straight parts is considered to be linear elastic, so that the plastic collapse behavior is contained within the bend part. Based on measured responses at room temperature [15], the following material properties are used to model this material: Young’s modulus ($E$) = 193.0 GPa, yield stress ($\sigma_y$) = 271.9 MPa, and Poisson’s ratio ($\nu$) = 0.2642.

Two types of pipe element are used to model this specimen using the FE package ABAQUS [16]: ELBOW31B for the bend part and PIPE31 for the straight parts. Although both the elements appear as two-noded linear beams in space, the ELBOW31B is basically a degenerated shell element which can incorporate effects such as, ovalization of the cross-section under flexure and pressure stiffening. Tan et al. [17] showed that analysis of pipe elbows with ELBOW31 elements was equivalent to analysis using thin shell elements. The number of elements (12 in the bend and 12 in each straight part) and the number of integration points (through the wall thickness and around the cross-section) are arrived at through sample convergence studies for a few simulated pipe geometries. It should be noted that these analysis options may not constitute the optimum model for the thousands of simulated geometries considered in this study.

The finite element model of the pipe is subjected to in-plane opening moment without any internal pressure or shear force. For this, an in-plane opening moment is applied at the free end of the model as shown in Fig. 1. An incremental nonlinear static analysis is performed in order to obtain the collapse moment. Since both material and geometric nonlinearities are expected to be significant, the Static, Riks option is used in ABAQUS [17] for this. Although the location and the direction of the external moment are specified, it is not a load-controlled analyses. Instead, the analysis is controlled by a constant arc length at each increment [17], which can successfully follow a load-deformation curve with a negative slope. The maximum moment on the bend part of the pipe is monitored along with the rotation at the free end. This analysis is repeated for all the simulated pipe geometry sets.

5 Determination of Collapse Moment and $K$

Several methods of determining the collapse moment ($M_c$) are available in the literature [18]. The most common methods for determining the plastic collapse moment in a pipe are the tangent-intersection method, twice-elastic-deformation method, twice-elastic-slope method, 1%-plastic-strain method, and proportional-limit method. The ASME B&PV Code, Section III [2] has preferred the twice-elastic-slope method over others, for it gives a clear definition of the collapse moment resulting in greater consistency. Like the other methods mentioned here, the twice-elastic-slope method also obtains the plastic collapse moment from the moment versus rotation plot. In this graphical method, $M_c$ is obtained from the intercept of the moment versus end rotation plot (obtained ideally from an experiment or a detailed finite element analysis) and a straight line which starts from the origin and has twice the elastic slope of the moment-end rotation curve. Figure 2 illustrates how $M_c$ is obtained using this method for a simulated moment versus end rotation plot of one simulated pipe geometry.

The moment-end rotation curve is plotted for every simulation and the collapse moment is calculated for that simulated geometry using the twice-elastic-slope method as described here. Figures 3–6 show sample moment-end rotation plots for each different nominal diameters considered in this study. Each plot represents a unique...
(simulated) pipe geometry. It should be noted here that an intercept between the moment-rotation curve and the twice-elastic-slope line is not reached in every simulated geometry, as the analysis stops (due to numerical instability) prior to reaching the “intercept.” The primary reason for this is that the selections for the number of elements, number of integration points, rotation increment for each step, and other Static, Riks analysis options, do not work for all the varied simulated geometries. Only the successful simulations (in terms of obtaining \( M_c \) using the twice-elastic-slope method) are used to generate the statistics for \( K \). From \( M_c \), the stress index \( B_2 \) for a particular geometry/simulation is calculated using the following equation, derived considering \( P = 0 \) in Eq. (4):

\[
B_2 = \frac{S_y Z_p}{M_c}
\]

which, can also be obtained from Eq. (1) by putting the limit state requirements of replacing \( Z \) with \( Z_p \) and \( S \) with \( S_y \) (or, \( S_u \)), as recommended by Avrithi and Ayyub [6] and Gupta and Choi [1]. The plastic section modulus is calculated from the simulated pipe geometry \( (D_o, t) \). The random variable \( K \), expressing \( B_2 \) in terms of the pipe bend factor, is calculated using Eq. (5). These realizations of \( K \) for all the successful simulations constitute the statistics for \( K \), which essentially quantifies the fabrication uncertainty in \( B_2 \).

6 Results and Discussion

Applying the principles of the basic Monte Carlo simulation procedure, the distribution parameters for \( K \) are obtained. These parameters include the mean (\( \mu_K \)), standard deviation (\( \sigma_K \)) and CoV. Also, the suitability of modeling the probability distribution of \( K \) with “common” probability distributions (namely, normal, lognormal, gamma, extreme type I, and extreme type II distributions) are checked with the Pearson’s chi-square test, the Kolmogorov–Smirnov test and the Anderson-Darling test. Table 7 provides the calculated distribution parameters for each of the four cases individually. It also provides the \( K \) statistics parameters combining all four cases together. These calculations are based on
1000 simulations for each case. This required number of simulations is arrived at considering the convergence of these parameters. Figure 7 shows the change in $\mu_K$ with the increasing number of simulations for each case, which clearly illustrates that 1000 simulations are adequate for this Monte Carlo simulation.

Out of the four cases considered, all except case 4 show almost the same $\mu_K$, ranging from 0.7869 to 0.7887. For case 4, $\mu_K = 0.8448$. These values are much smaller compared to the code-recommended value for the constant numerator, 1.3 [2]. These $\mu_K$ values signify $B_2$ values much lower than its code-specified value, which in turn result in higher values of plastic collapse moment capacity for a given section. On the basis of this large statistics, the recommendation of the code is deemed very conservative. The coefficient of variation for each individual case is very large, the overall statistics of $K$, for a large range of nominal diameters and schedules, can be expressed with a mean of 0.8021 and a coefficient of variation of 4.5%. However, this statistics cannot be described using any common probability distribution.

The overall statistics of $K$ very clearly indicates that the existing ASME provisions give a very conservative estimate of the plastic collapse moment ($M_c$) for the in-plane opening mode of flexure.

### 7 Conclusions

The following significant conclusions are drawn on the basis of the results of the detailed simulation study presented in this paper:

1. To incorporate the fabrication uncertainty in the primary stress index $B_2$, a modified version of the code-based expression is suggested, where a random variable $K$ replaces the constant numerator. The modified expression retains the familiarity of the code-based equation.

2. For thin stainless steel long radius pipe bends subjected to in-plane opening moments, the overall statistics of $K$, for a large range of nominal diameters and schedules, can be expressed with a mean of 0.8021 and a coefficient of variation of 4.5%. However, this statistics cannot be described using any common probability distribution.

3. The overall statistics of $K$ very clearly indicates that the existing ASME provisions give a very conservative estimate of the plastic collapse moment ($M_c$) for the in-plane opening mode of flexure.

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### References


