Optimisation of IDA-based fragility curves

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ABSTRACT: The seismic fragility of a structure is the conditional probability of failure for a given seismic hazard level. It is measured as the probability of exceedance of a particular limit state of the selected damage measure (DM) for a given intensity measure (IM). Over the last decade, the incremental dynamic analysis or 'IDA' has become the preferred choice of obtaining the seismic fragility of a structure. An IDA consists of a series of nonlinear response-history analyses (NLRHA) of the mathematical model of a structure subjected to incremented intensity measures of a ground acceleration data. A multi-IDA, where a multitude of ground acceleration records are used to obtain multiple IM vs. DM 'IDA curves', are typically used in a seismic fragility analysis. For a selected IM, the variation in DM are treated as random samples in calculating fragility. Typically, lognormal distributions are used to model the distribution of DM at each hazard level. The parameters of these lognormal distributions vary over hazard levels. Previous researchers used trendlines to express these parameters as functions of the intensity measure/hazard level. These trendlines were obtained by minimising the error in estimating the sample mean and the sample standard deviation values at each hazard level. In current study, a simple 2D stick model idealisation of a nuclear (primary) containment structure is selected. The multi-IDA is performed using a set of 25 ground acceleration records of similar seismo/geological origins. An optimised solution obtaining for the trendlines is proposed, by minimising the error in estimating the probability of failure at each hazard level. Fragility curves are obtained from the multi-IDA data, using both the conventional fit to the parameters and the proposed one. It is observed that proposed approximation provides a better fit for the IDA-based data. In a multi-IDA-based fragility analysis, it is a common requirement to obtain fragility curves for more than one limit state, from the same set of IDA curves, which becomes an issue with the proposed method. Considering this, the proposed method is extended further, where the regression coefficients are obtained by minimising the error in estimating probabilities of failure across different limit states of performance. The fragility curves obtained using the proposed method for the different limit states are found to be closer (individually) to the IDA-based data, compared to the fragility curves obtained using the conventional approximation method.

1 INTRODUCTION

The objective of seismic probabilistic safety assessments (PSA) for nuclear power plants (NPP) is to examine the existence of vulnerabilities against postulated earthquake hazards (Hari Prasad et al. 2006). It involves numerically assessing the plants (or, its components) safety in a probabilistic framework, so that appropriate measures can be taken to enhance a NPPs safety level. One of the major components in the seismic PSA of a NPP, is the seismic fragility evaluation. The seismic fragility of a structure is defined as its conditional probability of failure given a specific intensity of the hazard. Typically, the fragility of a component or a system is represented by a conditional probability of failure versus seismic hazard curve. The probability of failure can be defined for any selected limit state. For example, while traditional fragility estimations used force-based ground acceleration capacity, recent works in structural earthquake engineering commonly defined displacement-based limit states in terms of interstorey drift ratio. The seismic hazard is most commonly described by the pseudo spectral acceleration corresponding to the fundamental mode (S_a), or by the peak ground acceleration (PGA) while assessing fragilities system-wide. In the parlance of performance-based earthquake engineering, the limit state is typically described as the 'performance level' defined for a specific response quantity or 'damage measure' (DM), and the seismic 'hazard level' is defined by an 'intensity measure' (IM). This way, the fragility of a structure may be defined as

$$F_r = \Pr(\text{DM} \geq \text{DM}_l \mid \text{IM}) \tag{1}$$

where, DM_l is the threshold response quantity.

The main objective of this paper is to optimise the fragility curves obtained using the incremental dynamic analysis (IDA) approach. The primary containment (PC) structure of a typical Indian 700 MWe PHWR (pressurised heavy water reactor) is selected for the case study. The proposed method is aimed primarily at increasing the accuracy level in IDA-based fragility calculations, which is typically practised using a nonlinear regression technique on the IDA data (Ellingwood et al. 2007). Results based on the 'Proposed' method are compared with this 'Regressed' method of analysis for fragility evaluations of the selected containment structure for both single limit state based fragility curve and multiple limit states based fragility curves.

2 FRAGILITY ANALYSIS OF STRUCTURES: A BRIEF REVIEW

Over the last 30 years, many researchers contributed to the development of fragility studies in the field of seismic safety of structures. One of the pioneering works in this area was by Kennedy et al. (1980) who presented a methodology for the estimation of the median ground acceleration capacity and associated uncertainties for the estimation of fragility curves of an existing NPP. A detailed procedure for estimating the fragility of a NPP based on the selection of components, identification of failure modes and evaluation of uncertainties using factors of safety was presented by Kennedy and Ravindra (1984). In more recent years, the use of empirical factors of safety have been substituted by the use of computation-intensive analytical approaches, or by the use of detailed information gathered from post-earthquake damage surveys. Also, fragility evaluations have been extended from the very important/hazardous structures to more common structures, such as, highway bridges (Shinouzuka et al. 2000), buildings (Ellingwood et al. 2007) and water tanks (Bhargava et al. 2002).

A statistical study of structural fragility curves was conducted by Shinouzuka et al. (2000), where two-parameter lognormal distribution functions were used to represent fragility curves. The distribution parameters were estimated by the maximum likelihood method. A simulation based method involving nonlinear dynamic analyses was presented by Lupoi et al. (2006) for the evaluation of seismic fragility functions for a realistic structural system. Cho and Joe (2005) proposed an improved method of seismic fragility analysis based on response spectrum shape factors. Ellingwood et al. (2007) used a multi-incremental dynamic analysis (multi-IDA) and regression based approach to estimate fragility functions. Details of this IDA-based approach is discussed in detail in Section 3. Porter et al. (2007) used Bayes' theorem to revise the parameters of existing fragility functions based on available empirical data. A response surface based approach of fragility evaluation was used by De Grandis et al. (2009). Zentner (2010) advanced the method proposed by Shinouzuka et al. (2000), by adopting a bootstrap technique for the set of ground motion records to be considered. Together, these works represent the variety of statistical/probabilistic analytical techniques applied to seismic fragility analysis of structures.

3 IDA-BASED FRAGILITY ANALYSIS

Incremental dynamic analysis (Vamvatsikos and Cornell 2002) has emerged, over the last decade, as an efficient and rigorous tool for seismic demand analysis, specifically in its probabilistic domain. An IDA consists of a series of nonlinear response history analysis (NLRHA) performed on a structure using a range of scaled ground acceleration records. The basic objective of an IDA is to cover the whole range of response from the linear elastic to the nonlinear behaviour, and finally to the collapse/instability of the structure. The results are typically expressed as an IM vs. DM plot for a selected record. A multi-IDA involves multiple IM vs. DM curves for a suit of acceleration records, which is used commonly in probabilistic seismic demand analysis (PSDA).

The multi-IDA based PSDA can be easily incorporated into seismic fragility analysis of structures. This methodology of fragility analysis was used by various authors, including Ellingwood et al. (2007), who used IDA-based fragility analyses to obtain fragility functions for steel and RC moment framed buildings in the Central and Eastern United States (CEUS). Using performance-based seismic design concepts, as in FEMA-356 (FEMA 2000), limit states are defined in terms of the maximum interstorey drift ratio (θ_{max}) as the DM parameter, for 'immediate occupancy' (IO), 'life safety' (LS) and 'collapse prevention' (CP) limit states. Spectral acceleration (S_a) is considered as the IM parameter. For a selected IM level, the multi-IDA based values of DM are modelled using a lognormal distribution. One distribution parameter, namely the median (m_{DM}), is obtained at each IM level. Using nonlinear regression, the median is then expressed as a function of the IM level:

$$m_{\text{DM}} = a(\text{IM})^b \epsilon \tag{2}$$

where a and b are regression parameters. The dispersion in DM is modelled with ϵ which follows a lognormal distribution with a median equal to one and standard deviation $\sigma_{\ln \epsilon}$:

$$\beta_{\text{DM}} = \sigma_{\ln \epsilon} \tag{3}$$

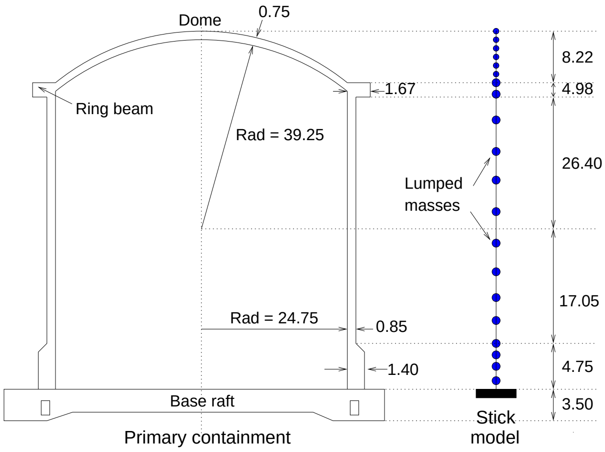

Figure 1: Schematic of the primary containment structure and its stick model (dimensions in m).

The regression is based on minimising an error in estimating the median value. The total squared error in this case (E_{reg}), cumulative over N levels of IM, is

$$\begin{aligned} E_{reg} &= \sum_{i=1}^{N} \left[\overline{m}_{\mathrm{DM}i} - m_{\mathrm{DM}i}\right]^2 \\ &= \sum_{i=1}^{N} \left[\overline{m}_{\mathrm{DM}i} - a(\mathrm{IM}_i)^b\right]^2 \end{aligned} \tag{4}$$

where $\overline{m}_{\mathrm{DM}i}$ is the sample median based on the IDA data and $m_{\mathrm{DM}i}$ is the fitted median. Finally, fragility values are expressed as a function of the IM level, for a selected limit state defined by DM_l:

$$\begin{aligned} F_r &= \Phi\left[\frac{\ln(m_{\mathrm{DM}}/\mathrm{DM}_l)}{\beta_{\mathrm{DM}}}\right] \\ &= \Phi\left[\frac{\ln\left\{a(\mathrm{IM})^b/\mathrm{DM}_l\right\}}{\beta_{\mathrm{DM}}}\right] \end{aligned} \tag{5}$$

where, Φ is the standard normal CDF operator. Since the fragility function depends on parameters that are obtained by regression, this method of fragility analysis is termed as the 'Regressed' method.

The 'Regressed' method shows considerable differences from the probability of failure values that can be computed in a frequentist way as the proportion of IDA curves at a selected IM level that exceed the damage measure (DM_l) for a selected limit state. This fact motivates the present work in redefining the error in the 'Regressed' method and thus optimising the fragility curves further.

4 THE STUDY STRUCTURE AND ITS COMPUTATIONAL MODEL

The primary containment (PC) structure of a 700 MWe Indian PHWR is considered for testing the proposed enhancement of the IDA-based 'Regressed' method of seismic fragility analysis. A schematic of the primary containment shell is provided in Figure 1, along with its 2D 'stick model' idealisation. The stick model idealisation is very common to the Indian nuclear industry, specifically for a seismic analysis of the containment structure (Reddy et al. 1996). How the containment is idealised to a 2D nonlinear beam-column cantilever structure is discussed in brief here, but the details are available in the dissertation by Mandal (2012).

The nonlinear response-history analyses, as part of the multi-IDA, are performed in the OpenSees platform (Mazzoni et al. 2006), The structure is modelled using the *nonlinearBeamColumn* element, and is assumed to be fixed at the base on the raft foundation. This element is a force-based element and it considers the spread of plasticity along the length of the member and five integration points are considered along the length of an element for this purpose. The cross-section is modelled with a *FiberSection*, where concrete is modelled as an annular patch of the *concrete02* material and reinforcing steel as a circular layer of *steel01* material. A damaged plasticity model (for both compression and tension behaviour) is considered for concrete, while the steel has an elastic-1% strain hardening plasticity behaviour. The shear deformation behaviour is modelled using the *sectionAggregator* approach. For simplicity, an elastic-perfectly plastic (EPP) type of shear force-deformation model is considered. This model requires two parameters: slope of the elastic curve and yield strength of the material. The slope of this curve is GA_s, where, G is the shear modulus of concrete and A_s is the shear area of the particular annular section. The yield shear strength of each section is calculated as per ACI-318 (ACI 2005).

5 INCREMENTAL DYNAMIC ANALYSES

Nonlinear response-history analyses of the idealised 2D stick model is performed for 25 ground acceleration records. These are real (recorded) acceleration time-histories of intra-plate earthquakes of similar seismo-/geological origins as in peninsular India. The details of these acceleration records are provided in Table 1. A multi-IDA is performed by scaling these 25 records (GM-01 to GM-25) to a desired IM level keeping in mind the desired accuracy in fragility. The DM parameter adopted in this work is the maximum interstorey drift ratio ($\theta_{\max}$). Three limit states are considered for this damage parameter: LS1, LS2 and LS3, corresponding to the IO, LS and CP performance levels of FEMA-356, respectively. $\theta_{\max}$ = 0.4%, 0.6% and 0.75% for these limit states, respectively. The IM parameter adopted here is the peak ground acceleration (PGA), considering the general practise in the nuclear industry where fragility is evaluated and compared for various components of the whole NPP system. The scaled PGA vs. $\theta_{\max}$ IDA plots for all the 25 records are shown in Figure 2. The vertical lines labelled LS1, LS2 and LS3 represent the three limit states of performance.

Table 1: Details of the ground motion data considered for the study.

Record	Event	Station	Component	R (km)	PGA (g)
GM-01	Bhuj, 2001	Ahmedabad	Radial	239.00	0.106
GM-02	Bhuj, 2001	Ahmedabad	Transverse	239.0	0.080
GM-03	Koyna, 1967	Koyna Dam	Radial	35.30	0.474
GM-04	Saguenay, 1988	St.-Ferreol	Radial	117.23	0.121
GM-05	Saguenay, 1988	St.-Ferreol	Transverse	117.23	0.097
GM-06	Saguenay, 1988	Quebec	Radial	149.40	0.051
GM-07	Saguenay, 1988	Quebec	Transverse	149.40	0.051
GM-08	Saguenay, 1988	Tadoussac	Radial	163.03	0.027
GM-09	Saguenay, 1988	Tadoussac	Transverse	163.03	0.002
GM-10	Saguenay, 1988	Baie-St-Paul	Radial	106.34	0.125
GM-11	Saguenay, 1988	Baie-St-Paul	Transverse	106.34	0.174
GM-12	Saguenay, 1988	La Malbaie	Radial	125.70	0.124
GM-13	Saguenay, 1988	La Malbaie	Transverse	125.70	0.060
GM-14	Saguenay, 1988	St.-Pascal	Radial	167.00	0.046
GM-15	Saguenay, 1988	St.-Pascal	Transverse	167.00	0.056
GM-16	Saguenay, 1988	Riviere-Ouelle	Radial	150.20	0.040
GM-17	Saguenay, 1988	Riviere-Ouelle	Transverse	150.20	0.057
GM-18	Saguenay, 1988	Ste.-Lucie-de-Beauregard	Radial	136.36	0.014
GM-19	Saguenay, 1988	Ste.-Lucie-de-Beauregard	Transverse	136.36	0.023
GM-20	Saguenay, 1988	Chicoutimi-Nord	Radial	45.69	0.107
GM-21	Saguenay, 1988	Chicoutimi-Nord	Transverse	45.69	0.131
GM-22	Saguenay, 1988	St-Andre-du-Lac-St-Jean	Radial	92.96	0.156
GM-23	Saguenay, 1988	St-Andre-du-Lac-St-Jean	Transverse	92.96	0.091
GM-24	Saguenay, 1988	Les Eboulements	Radial	114.31	0.125
GM-25	Saguenay, 1988	Les Eboulements	Transverse	114.31	0.102

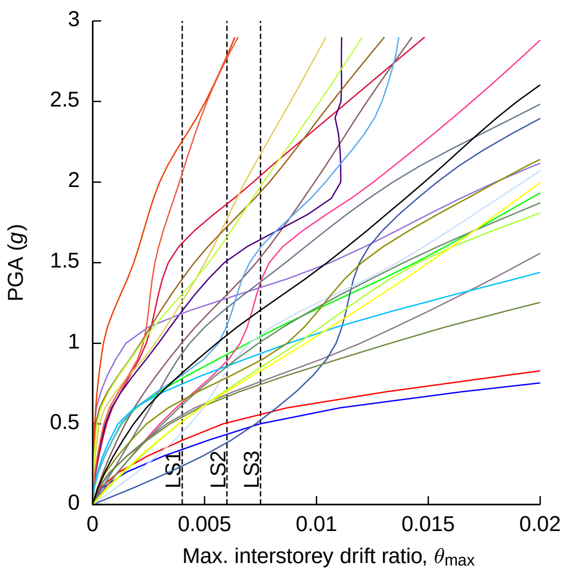

Figure 2: IDA plots for the selected 25 ground records.

6 PROPOSED FRAGILITY ANALYSIS PROCEDURE

From the IDA data, the probability of failure can be calculated at each IM level by counting the number of IDA curves that cross the vertical line corresponding to the limit state under consideration. The ratio of these IDA curves to the total number of IDA curves (25 in this test case) is the probability of failure at that level of IM, which is also the fragility. Given the available and selected information, these are the 'actual' fragility values based on the multi-IDA. However, it is advantageous to express fragility as a continuous function of the intensity measure, such as in Equation 5. This format is accepted for both PGA-based and S_a-based fragility analyses. The proposed method follows this format and develops the fragility functions using a lognormal model similar to the studies by Kennedy and Ravindra (1984) and Ellingwood et al. (2007). The variation in the seismic demand (DM) data is modelled using a two-parameter lognormal model, similar to Ellingwood et al., at each IM level. However, it differs from the method adopted by them in that the regression is not based on minimising the error between the sample median and the regressed median. Instead, it minimises the error between the sample probability of failure calculated from the IDA data and the probability of failure based on the fitted lognormal distribution. The cumulative squared error that is minimised in the 'Proposed' regression is

$$\begin{aligned} E_{prop} &= \sum_{i=1}^{N} \left[\overline{P}_{fi} - P_{fi}\right]^2 \\ &= \sum_{i=1}^{N} \left[F_{\mathrm{DM}_l i} - \overline{F}_{\mathrm{DM}_l i}\right]^2 \end{aligned} \tag{6}$$

where, $\overline{P}_{fi}$ is the probability of failure based on the sample IDA data, and P_{fi} is the probability of failure based on the fitted lognormal distribution. Considering the relation between the probability of failure and the cumulative distribution function (CDF) at a given value, this error is also expressed in terms of the CDF values for the sampled ($\overline{F}_{\mathrm{DM}_l i}$) and the fitted ($F_{\mathrm{DM}_l i}$) distributions at the limit state of DM_l. The fitted CDF is obviously a function of the distribution parameters m_{DM} and β_{DM}.

The regression format selected for the median value remains the same as in the work of Ellingwood et al. (2007), however the standard deviation (σ_{DM}) is fitted to a straight line with zero intercept:

$$\begin{aligned} m_{\mathrm{DM}} &= a(\mathrm{IM})^b \\ \sigma_{\mathrm{DM}} &= c(\mathrm{IM}), \text{ where,} \\ (\beta_{\mathrm{DM}})^2 &= \ln\left[(\sigma_{\mathrm{DM}}/\mu_{\mathrm{DM}})^2 + 1\right] \end{aligned} \quad (7)$$

μ_{DM} is the mean of the fitted distribution. After obtaining the regression coefficients (a, b and c), the fragility curve is obtained using Equation 5. Optimum values of a, b and c are provided in Table 2.

Table 2: Optimum values of regression coefficients.

	Regressed	Proposed			Modified Proposed
		LS1	LS2	LS3	
a (10^{-3})	5.70	5.10	6.80	8.30	7.70
b	1.47	1.25	1.03	0.958	1.25
c (10^{-3})		4.70	4.30	4.30	4.30

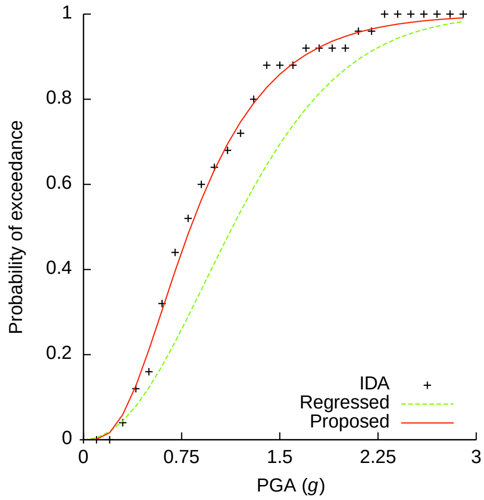

Figure 3: Fragility curves for LS1, along with IDA data.

The fragility curves for LS1 obtained using the 'Regressed' and the 'Proposed' methods are shown in Figure 3, along with the IDA-based fragility data. This figure very clearly shows that the Proposed method gives significantly better fragility estimates. It also maintains the format of fragility expressed as a continuous function of the intensity level. When extended to the other two limits states selected here, the Proposed method provides better fragility estimates at all limit states of failure (Figure 4). However, it should be noted here that the Proposed method obtains a different set of optimum a, b and c values for each limit state, whereas, the Regressed method uses only one set of regression coefficients. This means that the computation involved in the Proposed method will be roughly three (or, whatever be the total number of limit states of failure) times of that in the Regressed method.

Considering that multiple limit states may be considered simultaneously in a fragility analysis, and that the Proposed method becomes computationally demanding in such cases, a modified version of the proposed method ('Modified Proposed' method) is tested

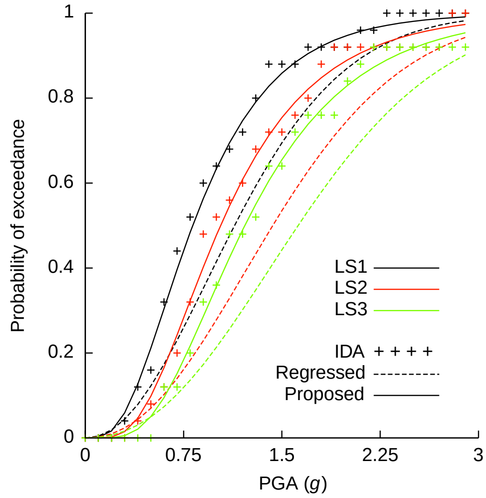

Figure 4: Fragility curves and IDA data for the three limit states.

here. In this modified method, only one set of optimum a, b and c values are obtained for all the limit states considered in a specific fragility analysis. For this, the cumulative squared error is redefined as

$$E_{modprop} = \sum_{i=1}^{N} \sum_{j=1}^{L} \left[F_{\mathrm{DM}_l j,i} - \overline{F}_{\mathrm{DM}_l j,i}\right]^2 \quad (8)$$

where L is the total number of limit states considered. The modified method remains the same as the original Proposed method, except for this change. This single error defined for all limit states together reduces computation close to the level of the Regressed method. For the three limit states considered here, one set of regression coefficients are obtained using the 'Modified Proposed' method (Table 2). Using these, the three fragility plots are obtained based on Equation 5. These curves are shown in Figure 5, which shows that the modified method provides better fragility estimates compared to the Regressed method for every limit state of failure considered.

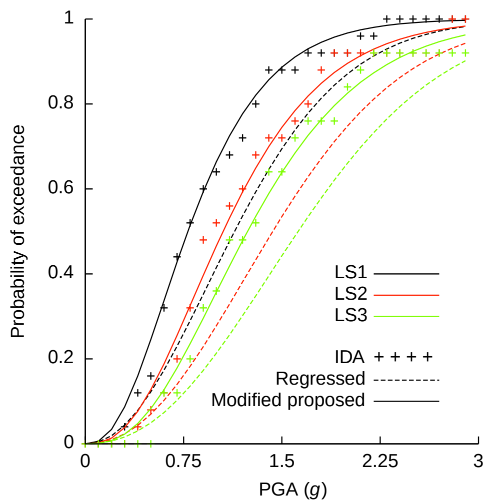

Figure 5: Comparison of 'Regressed' and 'Modified Proposed' fragility curves.

Table 3: Error/accuracy in fragility estimations.

	Limit states	Regressed	Proposed	Modified Proposed
	LS1	0.520	0.0160	0.0260
E	LS2	0.625	0.0270	0.0400
	LS3	0.581	0.0210	0.0340
	LS1	0.972	0.998	0.998
ρ_P	LS2	0.967	0.997	0.995
	LS3	0.974	0.997	0.996

A quantitative comparison of the fragility estimations based on the three methods tested here is performed using a cumulative squared error, where the error is defined in terms of the difference with the IDA data:

$$E = \sum_{i=1}^{N} [F_{ri} - F_{r\text{-IDA}i}]^2 \tag{9}$$

In addition, the closeness of the fragility curves to the IDA-based data is also measured using the Pearson's product-moment correlation (ρ_P). Values of E and ρ_P for each method at different limit states are presented Table 3. These numbers clearly show that the 'Proposed' method provide better fragility estimates that the 'Regressed' method in all cases considered here. The 'Modified Proposed', too, provides better fragility estimates than the Regressed method for all limit states. However, the modified fragility curves are not as good as the original proposed ones. This is expected, as the 'Modified Proposed' method reduces computation at the cost of accuracy in the regression.

7 CONCLUSIONS

Seismic fragility analysis of structures using data from multi-IDA provides a very practical approach. In this study, we have proposed modifications to the existing method of obtaining the parameters for the log-normal fragility model. Proposed modifications are based on how the regression is performed on the sample (IDA-based) response data. By minimising the error in estimating the fragility values, the regression coefficients are optimised to provide better fragility estimates, while maintaining the format of the fragility expression given by earlier researchers. A sample case study for a nuclear primary containment structure shows that the proposed method provide much better fragility estimates than what is practised now. Since the proposed method involves more computation for multiple limit states based fragility estimations, a modified version of this is suggested as well. The modified method, while reducing computational costs to the level of the method practised currently, provides better estimates of fragility at all limit states. This is shown through both quantitative and illustrative comparisons.

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