

Climate change impact assessment: Uncertainty modeling with imprecise probability

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[1] Hydrologic impacts of climate change are usually assessed by downscaling the General Circulation Model (GCM) output of large-scale climate variables to local-scale hydrologic variables. Such an assessment is characterized by uncertainty resulting from the ensembles of projections generated with multiple GCMs, which is known as intermodel or GCM uncertainty. Ensemble averaging with the assignment of weights to GCMs based on model evaluation is one of the methods to address such uncertainty and is used in the present study for regional-scale impact assessment. GCM outputs of large-scale climate variables are downscaled to subdivisional-scale monsoon rainfall. Weights are assigned to the GCMs on the basis of model performance and model convergence, which are evaluated with the Cumulative Distribution Functions (CDFs) generated from the downscaled GCM output (for both 20th Century [20C3M] and future scenarios) and observed data. Ensemble averaging approach, with the assignment of weights to GCMs, is characterized by the uncertainty caused by partial ignorance, which stems from nonavailability of the outputs of some of the GCMs for a few scenarios (in Intergovernmental Panel on Climate Change [IPCC] data distribution center for Assessment Report 4 [AR4]). This uncertainty is modeled with imprecise probability, i.e., the probability being represented as an interval gray number. Furthermore, the CDF generated with one GCM is entirely different from that with another and therefore the use of multiple GCMs results in a band of CDFs. Representing this band of CDFs with a single valued weighted mean CDF may be misleading. Such a band of CDFs can only be represented with an envelope that contains all the CDFs generated with a number of GCMs. Imprecise CDF represents such an envelope, which not only contains the CDFs generated with all the available GCMs but also to an extent accounts for the uncertainty resulting from the missing GCM output. This concept of imprecise probability is also validated in the present study. The imprecise CDFs of monsoon rainfall are derived for three 30-year time slices, 2020s, 2050s and 2080s, with A1B, A2 and B1 scenarios. The model is demonstrated with the prediction of monsoon rainfall in Orissa meteorological subdivision, which shows a possible decreasing trend in the future.

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1. Introduction

[2] Climate change refers to any systematic change in the long-term statistics of climate elements (such as temperature, pressure, or winds) sustained over several decades or longer time periods. Water resources are inextricably linked with climate, so the prospect of global climate change has serious implications for water resources and regional development [Intergovernmental Panel on Climate Change (IPCC), 2001]. Increased evaporation (resulting from higher

temperatures), combined with regional changes in precipitation characteristics (e.g., total amount, variability, and frequency of extremes), has the potential to affect mean runoff, frequency and intensity of floods and droughts, soil moisture, and water supplies for irrigation and hydroelectric generation. Assessing the impact of climate change on hydrology essentially involves projections of climatic variables (e.g., temperature, humidity, mean sea level pressure etc.) at a global scale, downscaling of global-scale climatic variables to local-scale hydrologic variables and computations of risk of hydrologic extremes in future for water resources planning and management. Projections of climatic variables globally can be performed with General Circulations Models (GCMs), which provide projections at large spatial scales. Such large-scale climate projections must then be downscaled to obtain smaller-scale hydrologic projections with appropriate linkages between the climate

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and hydrologic variables. Climate change impact assessment on hydrology, based on statistical downscaling from GCM output is characterized by uncertainty caused by inadequate information and understanding about the underlying geophysical process of global change, leading to limitations in the accuracy of GCMs. This may lead to a mismatch between the future projections of GCMs and can also be termed as GCM uncertainty. It is widely acknowledged that disagreements between different GCMs over regional climate change represent significant sources of uncertainty [Wilby and Harris, 2006]. Downscaled output of a single GCM represents a single trajectory among a number of possible realizations derived with various GCMs. Such a single trajectory alone cannot represent a future hydrologic scenario. Therefore overreliance on a single GCM could lead to inappropriate planning or adaptation responses.

[3] During the last decade, research on modeling uncertainty in assessment of climate change impact has advanced on several fronts. New and Hulme [2000] presented an approach to quantify uncertainties associated with climate change within a probabilistic framework. A hierarchical impact model with Bayesian Monte Carlo simulation was developed for addressing uncertainty about future greenhouse gas emissions, the climate sensitivity, and limitations and unpredictability in GCMs. Allen et al. [2000] assessed the range of warming rates over the coming 50 years that are consistent with the observed near-surface temperature record as well as with the overall patterns of response predicted by several GCMs. Raisanen and Palmer [2001] developed a probabilistic approach to model the inherent uncertainties in the computational representation of climate and the unforced chaotic climate variability and expressed climate change projections in probabilistic form. In that study, 17 Coupled Model experiments sharing the same gradual increase in atmospheric CO₂ were treated as a probabilistic multimodel ensemble projection of future climate. Giorgi and Mearns [2002, 2003] developed a Reliability Ensemble Averaging (REA) method for estimating probability of regional climate change exceeding given thresholds based on ensembles of different model simulations. The method takes into account two reliability criteria: the performance of the model in reproducing present-day climate ("model performance criterion") and the convergence of the simulated changes across the models ("model convergence criterion"). The method was applied to a set of transient experiments for the A2 and B2 IPCC emission scenarios with 9 different Atmosphere-Ocean General Circulation Models (AOGCMs). Weights were assigned to the GCMs on the basis of model performance and model convergence which were evaluated on the basis of simulated global mean climate scenarios obtained for 10 regions of subcontinental scale. Local-scale projections and temporal variability of climate variables were not considered in that study. Murphy et al. [2004] reported a systematic attempt to determine the range of climate change projections consistent with the uncertainties due to intramodel variation, based on a 53-member ensemble of model versions constructed by varying model parameters. They estimated a probability density function for the sensitivity of climate to a doubling of atmospheric carbon dioxide levels. Tebaldi et al. [2004, 2005] presented a Bayesian approach to determine proba-

bility density functions of temperature change, from the output of a multimodel ensemble, run under the same scenario of future anthropogenic emissions. A main feature of the method was the formalization of the two criteria of bias and convergence that the REA method [Giorgi and Mearns, 2003] first quantified as a way of assessing model reliability. Thus the GCMs of the ensemble were combined in a way that accounts for their performance with respect to current climate and a measure of each model's agreement with the majority of the ensemble. For illustration purpose, Tebaldi et al. [2005] considered the output of mean surface temperature from nine GCMs, run under the A2 emission scenario, for winter and summer, aggregated over 22 land regions and into two 30 year averages representative of current and future climate conditions. Like REA, the studies by Tebaldi et al. [2004, 2005] also did not consider the temporal variability of the climate variables and local hydrologic scenarios. More recently, Wilby and Harris [2006] developed a framework for assessing uncertainties in climate change impacts in projecting low flow scenarios of River Thames, UK. This model considers the predictions of the local-scale hydrologic scenario with multiple GCMs. A probabilistic framework is developed for combining information from an ensemble of four GCMs, two greenhouse gas emission scenarios, two statistical downscaling techniques, two hydrologic model structures and two sets of hydrologic model parameters. GCMs are weighted on the basis of the bias calculated with Impact Relevant Climate Prediction Index (IRCPI). A limitation of the model is that it does not consider model convergence, which is a measure of each model's agreement with the majority of the ensemble.

[4] Ghosh and Mujumdar [2007] used nonparametric approach for modeling uncertainty in drought assessment incorporating climate change. Standardized Precipitation Index-12 (SPI-12) has been used as a drought indicator in that study and computed from GCM projections using statistical downscaling and equiprobability transformation. The probability density function of SPI in each year has been computed with nonparametric methods, viz., kernel density estimation and orthonormal series method. The model assigns equal weights to all the GCMs, which may not be valid. To assign weights to the GCMs and scenarios, a possibilistic approach for modeling uncertainty has been developed by Mujumdar and Ghosh [2008], where possibilities of GCMs and scenarios are computed with their performances in the near past (after 1990) under climate forcing. The limitations of these models are as follows.

[5] 1. Evaluation of GCMs based on model convergence is not performed in either of the studies performed by Ghosh and Mujumdar [2007] and Mujumdar and Ghosh [2008].

[6] 2. GCM and scenario uncertainty are treated on equal ground, although the latter is not really an uncertainty in a mathematical sense but a control influenced by socioeconomic behavior. Hence the analysis should be performed per forcing scenario, to demonstrate the different future hydrologic conditions the region could expect under various global mitigation policies.

[7] 3. Not for all combinations of scenarios and GCMs outputs are available or, in other words, the IPCC data distribution center, which is the source of GCM data used in the study, does not provide outputs for all GCMs with all

the scenarios. For example, the output of the GCM, CM 3.0 developed by Institute for Numerical Mathematics, Russia, is not available for the A2 scenario. Such missing output imposes another source of uncertainty, which contributes to “partial ignorance” in climate change impact assessment. To date, such uncertainty has not been considered in any research studies on hydroclimatology. Furthermore, the existing GCMs are few in number. In fact they are a small finite set from an infinite space of possible models, which introduces a source of uncertainty which should be taken care.

[8] Therefore from hydrologic point of view there is a need to develop an uncertainty model, for climate change impact assessment on local-scale hydrology, which evaluates GCMs on the basis of Reliability Ensemble Averaging (REA), considers temporal variability of hydrologic variable and incorporates uncertainty caused by partial ignorance. For modeling GCM uncertainty in climate change impact assessment, *Giorgi and Mearns* [2002, 2003] proposed Reliability Ensemble Averaging (REA) method. The method takes into account two reliability criteria: the performance of the model in reproducing present-day climate (“model performance criterion”) and the convergence of the simulated changes across the models (“model convergence criterion”). The first criterion is based on the ability of GCMs to reproduce present-day climate: the better the model performance, the higher the reliability of the GCM. The second criterion is based on the convergence of simulations by different models for a given forcing scenario for future. As the observed climate time series is not available for future, a factor is used as reliability indicator of a GCM, which measures the model reliability in terms of the deviation of simulations by that GCM from the REA average (weighted mean) simulations. High deviation denotes low model reliability. The philosophy underlying the REA approach is to minimize the contribution of simulations that either perform poorly in the representation of present-day climate over a region or provide outlier simulations for future with respect to the other models in the ensemble. In the present study, the deviation of the simulated variable (Rainfall) with respect to the observed or REA average variable is computed with the deviation of CDFs. Uncertainty caused by partial ignorance resulting from missing GCM output is also modeled. The present study aims to achieve this using the concept of imprecise probability, i.e., probability expressed as interval gray number (closed and bounded interval with known lower and upper bounds but unknown distribution information [*Huang et al.*, 1995]). The flowchart of the model is presented in Figure 1. GCM outputs of large-scale climate variables (downloaded for AR4 [*Intergovernmental Panel on Climate Change (IPCC)*, 2007], from IPCC data distribution center, <http://www.ipcc-data.org>) are first down-scaled to subdivisional monsoon rainfall using Principal Component Analysis (PCA) and linear regression. The underlying uncertainty is modeled with reliability ensemble averaging, where weights are assigned to GCMs on the basis of model performance and model convergence, computed with the deviation of simulated CDF from that of target (for observed period) and weighted mean CDF (for future). Uncertainty caused by missing outputs of GCMs is then modeled with imprecise probability, where an imprecise

CDF is fitted to the future rainfall scenario. Imprecise CDF is fitted with interval regression, which consists of two parts: least square fitting and determination of optimum interval. Least square fitting is achieved with the weighted mean CDF and the optimum interval is obtained considering the CDFs derived from all available GCMs. The methodology is then validated and used to model GCM uncertainty for Orissa meteorological subdivision under A1B, A2 and B1 scenarios. The following section presents a brief overview of the data used in the study and the statistical downscaling method.

2. Data Extraction and Statistical Downscaling

[9] The Orissa meteorological subdivision (Figure 2), located on the eastern coast of India, extends from 17°N to 22°N in latitude, and 82°E to 87°E in longitude. The monthly area weighted precipitation data of Orissa meteorological subdivision in India, from January 1950 to December 1999, is obtained from Indian Institute of Tropical Meteorology, Pune (<http://www.tropmet.res.in>). This data set is used in the downscaling as predictand. The predictors used for downscaling [*Wilby et al.*, 1999; *Wetterhall et al.*, 2005] should be (1) reliably simulated by GCMs, (2) readily available from archives of GCM outputs, and (3) strongly correlated with the surface variables of interest (rainfall in the present case). Considering these criteria, the predictors selected for the present study are Mean Sea Level Pressure (MSLP), surface specific humidity, near-surface air temperature, zonal wind speed and meridional wind speed. Monsoon rainfall in Orissa is caused by high temperature in the land area and subsequent generation of low-pressure zone. This results in wind flow with moisture from Bay of Bengal to the land area. This is considered in selection of predictors for the downscaling model. Correlation coefficient of these predictor variables with monsoon rainfall is also observed to be high. It has been reported in literature [*Wilby et al.*, 1999; *Wetterhall et al.*, 2005; *Wilby and Harris*, 2006] that these variables can be simulated well at a larger scale by a GCM. In the present study the predictors are selected on the basis of literature. However, a statistical skill test, based on *Johnson and Sharma* [2009], will add credibility in selection of predictors for downscaling models. Statistical downscaling (Figure 3) involves development of statistical relationship between large-scale climate variables and local-scale hydrologic variable and use of the statistical relationship with the GCM output for future projections. Training (calibration) of the statistical downscaling model requires observed climate data. In the absence of adequate observed climatological data, the data from the National Center for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) reanalysis project [*Kalnay et al.*, 1996] may be used as a proxy to the observed data. In the present study NCEP/NCAR reanalysis data is used for calibration of the downscaling model. Monthly average climate variables from January 1950 to December 1999 were obtained for a region spanning 10.00°N–27.50°N in latitude and 77.50°E–95.00°E in longitude (constituting 64 grid points) that encapsules the study region. Figure 2 shows the NCEP grid points superposed on the map of Orissa meteorological subdivision. The output (MSLP, surface specific humidity, near-surface air

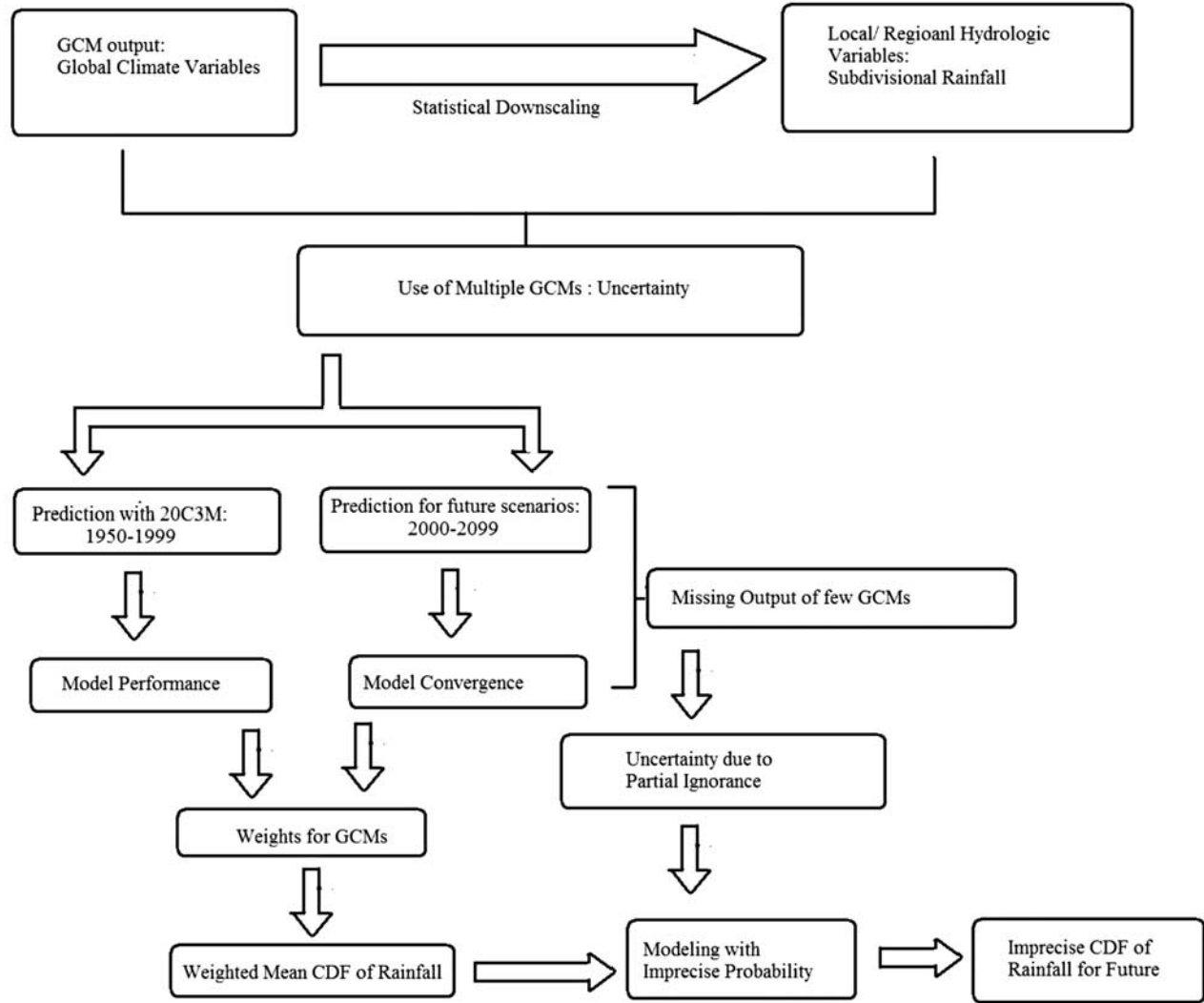


Figure 1. Overview of the proposed model.

temperature, zonal wind speed, meridional wind speed) of GCMs are downloaded from IPCC data distribution center for AR4 [IPCC, 2007]. The GCMs considered, on the basis of the availability of the output in IPCC Data, are given in Table 1.

[10] Five climate variables at 64 grid points are used as predictors, and hence the dimension of the predictors is 320. Furthermore, predictors at a grid point are expected to be highly correlated with those of neighboring grid points. Therefore direct use of the predictor variables, in statistical regression, may lead to multicollinearity and may be computationally unstable. Principal Component Analysis (PCA) is performed to reduce the dimensionality of the predictor variables. Principal components obtained from PCA are uncorrelated and therefore can be used directly in the regression. It is observed that first 35 principal components represent 98% variability of the original data set and hence are used in the study. Standardization [Wilby *et al.*, 2004] is performed prior to principal component analysis and downscaling to remove systematic bias in mean and standard deviation of the GCM simulated climate variables. Principal

components are used as regressors to predict the monthly monsoon rainfall of Orissa meteorological subdivision in the linear regression model (equation (1)).

$$Rain_t = b_0 + \sum_{i=1}^K b_i \times pc_{it} \quad (1)$$

where, $Rain_t$ is the monthly monsoon rainfall during month t , $b_0, \dots, b_K$ are the coefficient of linear regression model, K is the number of principal components considered, and pc_{it} is the i th principal component at time t , obtained from standardized reanalysis data. Two-third of the data set is used in training and rest of the data set is used in testing of the model. The training and testing R values (correlation coefficient between observed and predicted rainfall) are obtained as 0.8540 and 0.7911 respectively. The predicted and observed monsoon rainfalls are presented in Figure 4. The statistical relationship developed is then used on the standardized output of the GCMs as mentioned in Table 1, with the scenarios 20C3M (20th century forcing), COM-MIT (maintaining CO_2 concentration of future as that of

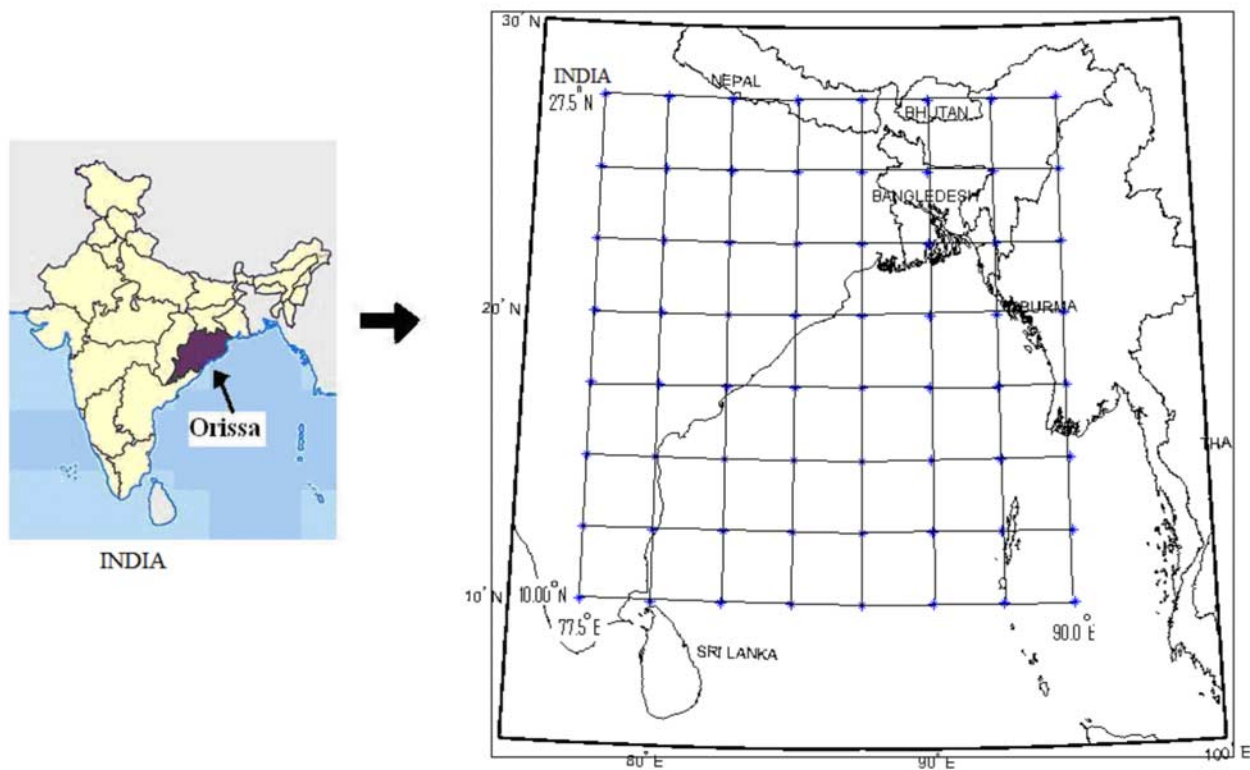


Figure 2. Case-study area with NCEP grid points.

year 2000), A1b (future changed scenario), A2 (future changed scenario) and B1 (future changed scenario). Figures 5, 6, 7, and 8 represent the CDFs (computed empirically with Weibull's Plotting Position Formula) of total monsoon rainfall derived with multiple GCMs under COMMIT, A1B, A2 and B1 scenario respectively for 30 year time slices 2020s, 2050s and 2080s. It should be noted that the parameters of CDFs are not stable over the next century and therefore the CDFs are computed separately for the three time slices. Outputs of GCM GISSAOM are not available for COMMIT and A2 scenario, outputs of GCM GISSER are not available for B2 scenario, and outputs of GCM INM are not available for A2 scenario. Figures 5–8 show that significant dissimilarity exists between the projections derived with multiple GCMs, which leads to GCM uncertainty. Projection of monsoon rainfall generated from a single GCM represents a single trajectory among a number of realizations derived using different GCMs and therefore cannot by itself represent the future hydrologic condition. For COMMIT scenario, as the CO_2 concentration is kept constant at the level of year 2000, significant change is not expected for future, and is not observed for all the GCMs except for GISSER. For other scenarios (A1B, A2 and B1), with majority of the GCMs, the value of rainfall at which the CDF reaches the value of 1 reduces with time (and also with respect to the simulated rainfall for 20C3M), which shows reduction in probability of occurrence of extremely high monsoon rainfall in future. Another interesting feature observed, is the increased dissimilarity between the GCMs with time. The amount of uncertainty in 2080s is higher

than that at earlier time slices. This may point to an increasing ignorance about the geophysical processes with increase in the signals of climate forcing. For all the scenarios, the GCMs GISSER and GISSAOM deviate significantly from the majority of ensembles. Such characteristics should be considered in assigning weights to the GCMs in modeling GCM uncertainty with reliability ensemble averaging. The next section presents reliability ensemble averaging method used for assigning weights to GCMs.

3. Reliability Ensemble Averaging

[11] Reliability Ensemble Averaging (REA), for modeling uncertainty resulting from the use of multiple GCMs, was proposed by *Giorgi and Mearns* [2003]. The procedure takes into account two reliability criteria: performance of the model in reproducing present-day climate (“model performance” criterion) and convergence of the simulated changes across models (“model convergence” criterion). The REA method was applied by *Giorgi and Mearns* [2003] to mean seasonal temperature and precipitation changes, over 22 land regions of the world, at continental scales, for A2 and B2 scenarios. In the present study, the objective is to model monsoon rainfall at subdivisional scale with an estimate of the temporal variation along 30 years time slices. Therefore the earlier developed REA model is slightly modified here and performed with respect to the Cumulative Distribution Function (CDF) of the monsoon rainfall and not only with respect to the mean condition.

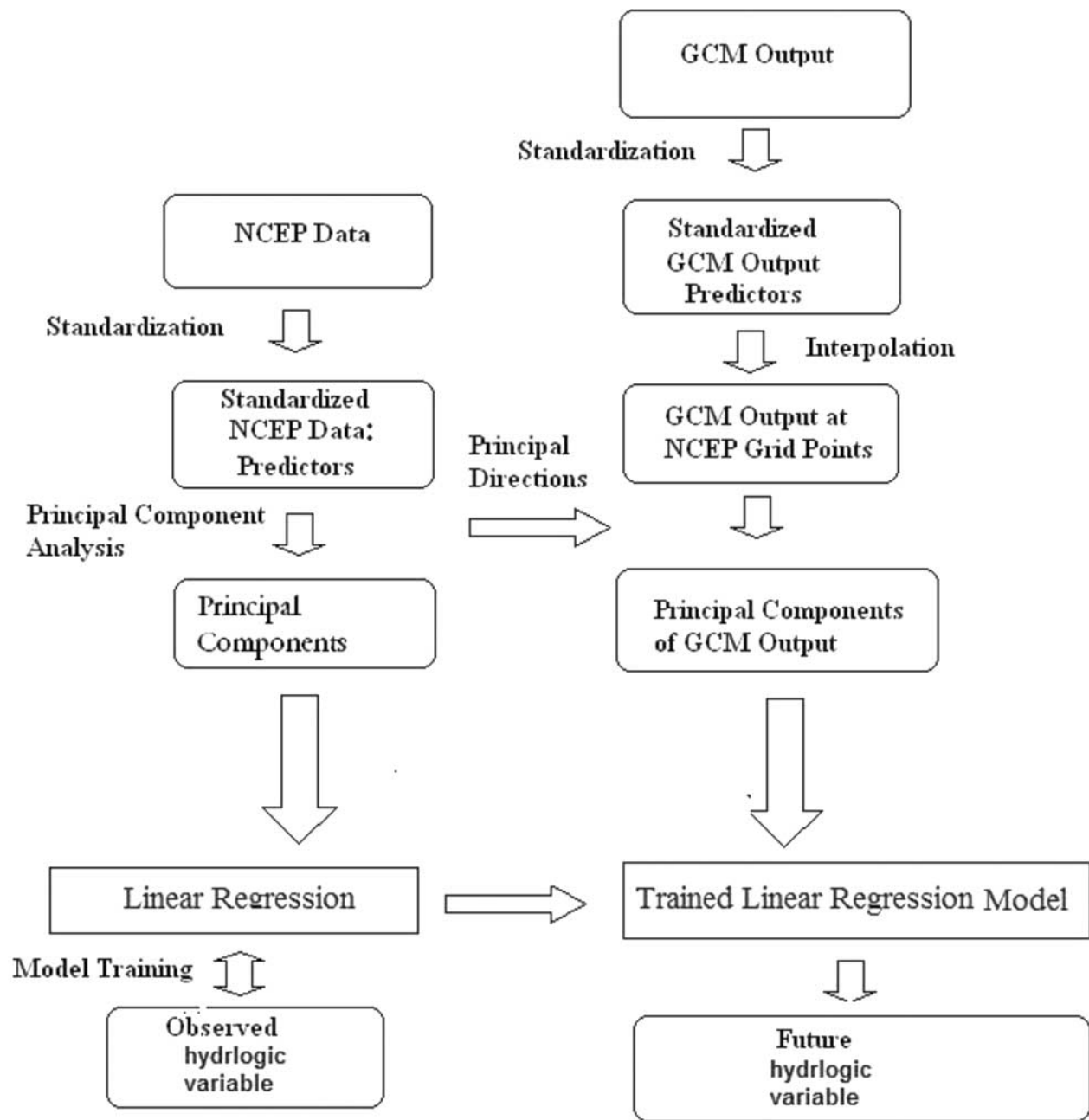


Figure 3. Algorithm for statistical downscaling.

Table 1. GCMs Used in the Study

Sl. No.	GCM	Institute	Abbreviation Used in the Study
1	BCCR	Bjerknes Center for Climate Research, Norway	BCCR
2	CNRM	Center National de Recherches Meteorologiques, France	CNRM
3	AOM	Goddard Institute for Space Studies, USA	GISSAOM
4	E-R	Goddard Institute for Space Studies, USA	GISSER
5	CM 3.0	Institute for Numerical Mathematics, Russia	INM
6	CM4	Institut Pierre Simon Laplace, France	IPSL
7	MIROC3.2 medres	National Institute for Environmental Studies, Japan	NIES
8	CGCM2.3.2	Meteorological Research Institute, Japan	MRI

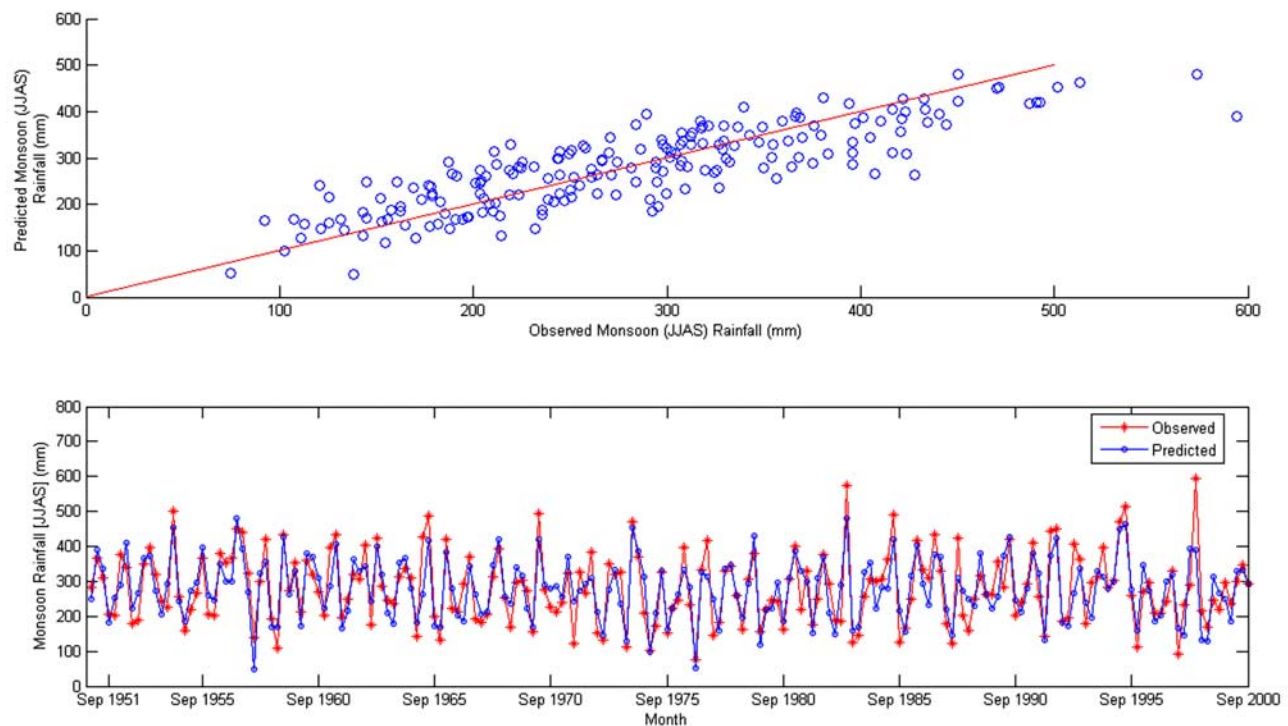


Figure 4. Observed and predicted monsoon rainfall.

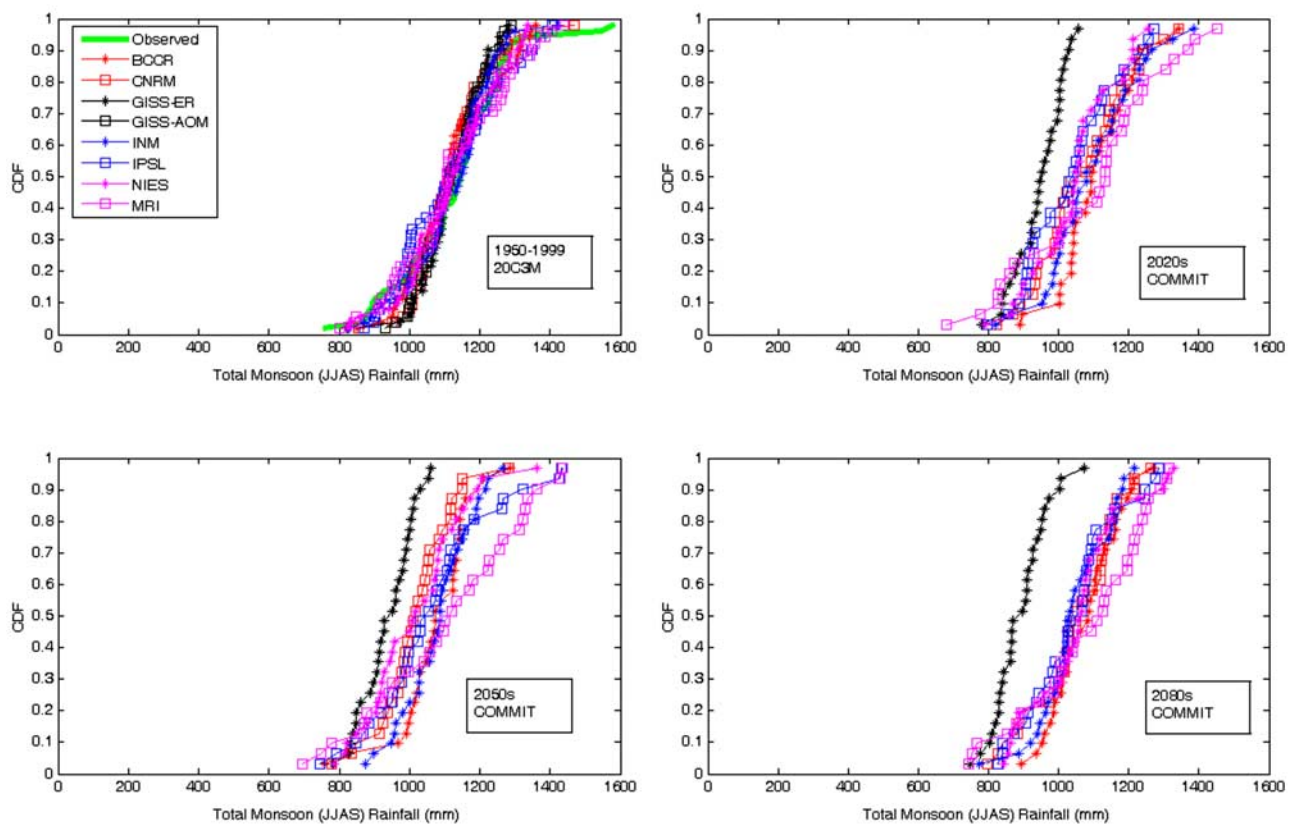


Figure 5. Prediction of monsoon rainfall with COMMIT scenario.

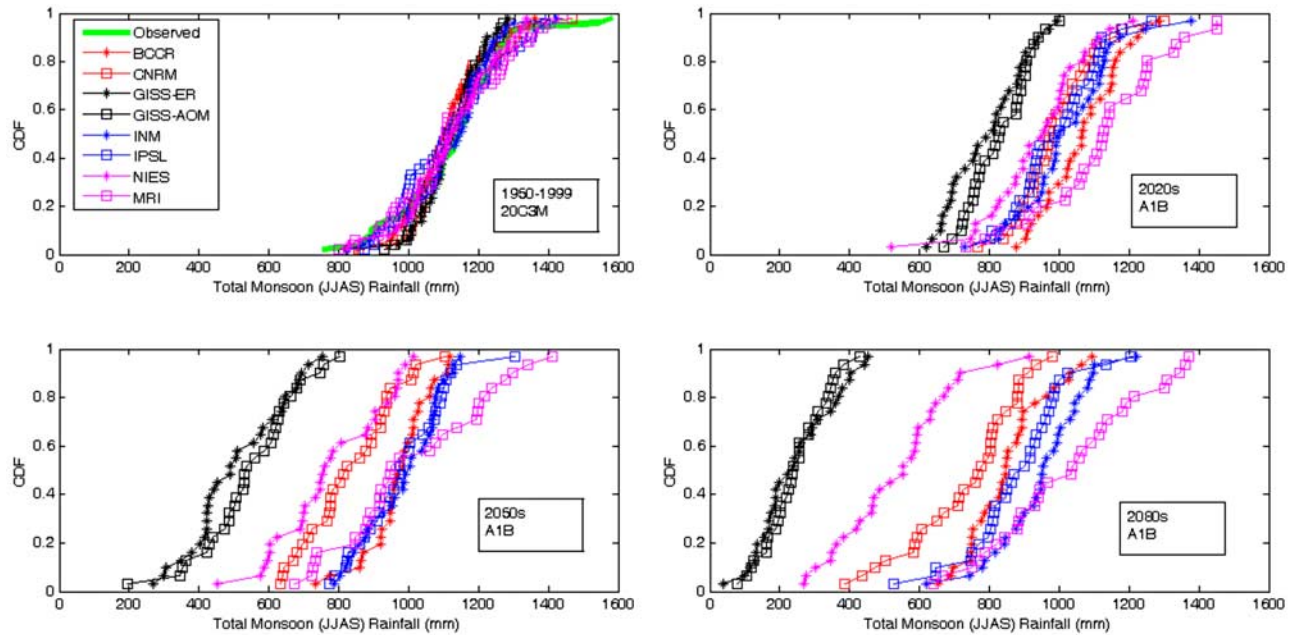


Figure 6. Prediction of monsoon rainfall with A1B scenario.

Model performance measure was evaluated by determining the deviation of CDF of GCM-simulated downscaled rainfall for 20C3M (duration, 1950–1999) with respect to the CDF derived with observed rainfall. Model convergence measure is evaluated on the basis of the deviation of CDFs derived with individual GCMs for future with respect to weighted mean CDF (derived with the weighted projections of multiple GCMs). The deviation of CDF is computed with Root Mean Square Error (RMSE), at 10 points, picked up at equal intervals covering the full range of monsoon rainfall.

It should be noted that Akaike Information Criteria (AIC) or Bayesian Information Criteria (BIC) can also be used for the same and sometimes perform better than RMSE for comparing monotone function like CDF. As weights are unknown and to be determined using REA, the algorithm used is an iterative method. The algorithm for the proposed approach is as follows.

[12] 1. Weights are assigned to GCMs on the basis of the model performance. The deviation of the CDF of GCM projected rainfall (downscaled), for 20C3M (duration,

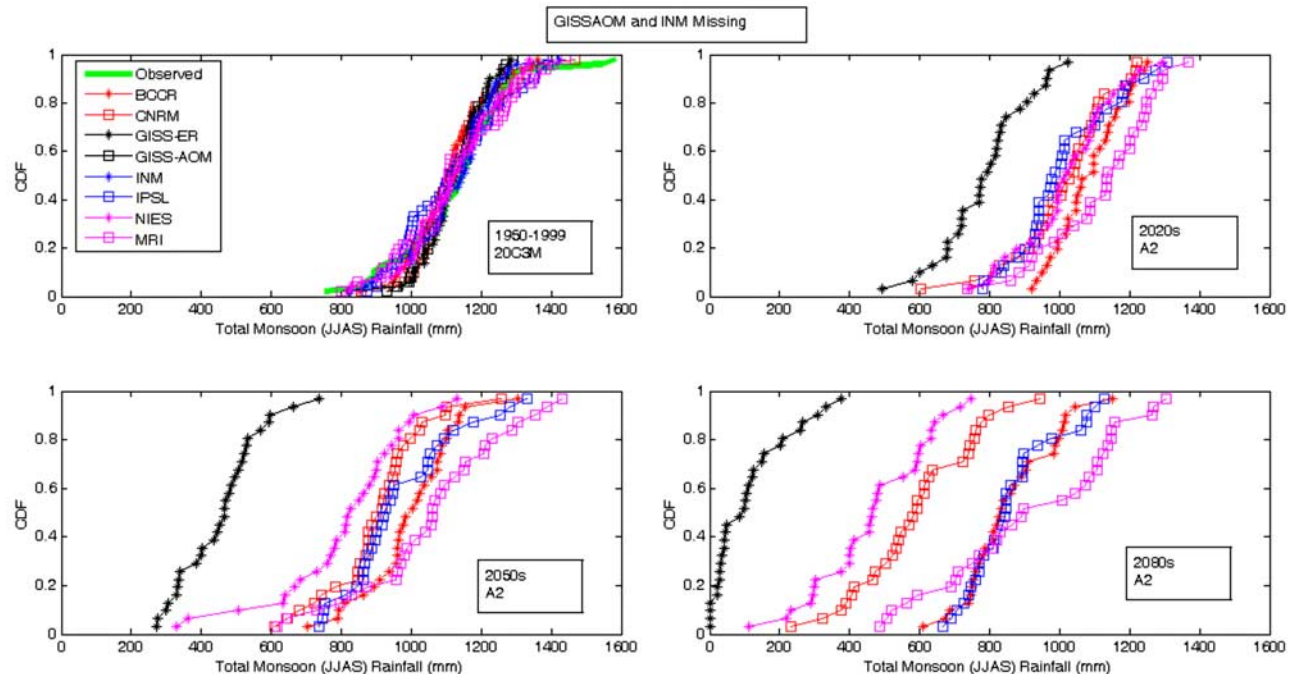


Figure 7. Prediction of monsoon rainfall with A2 scenario.

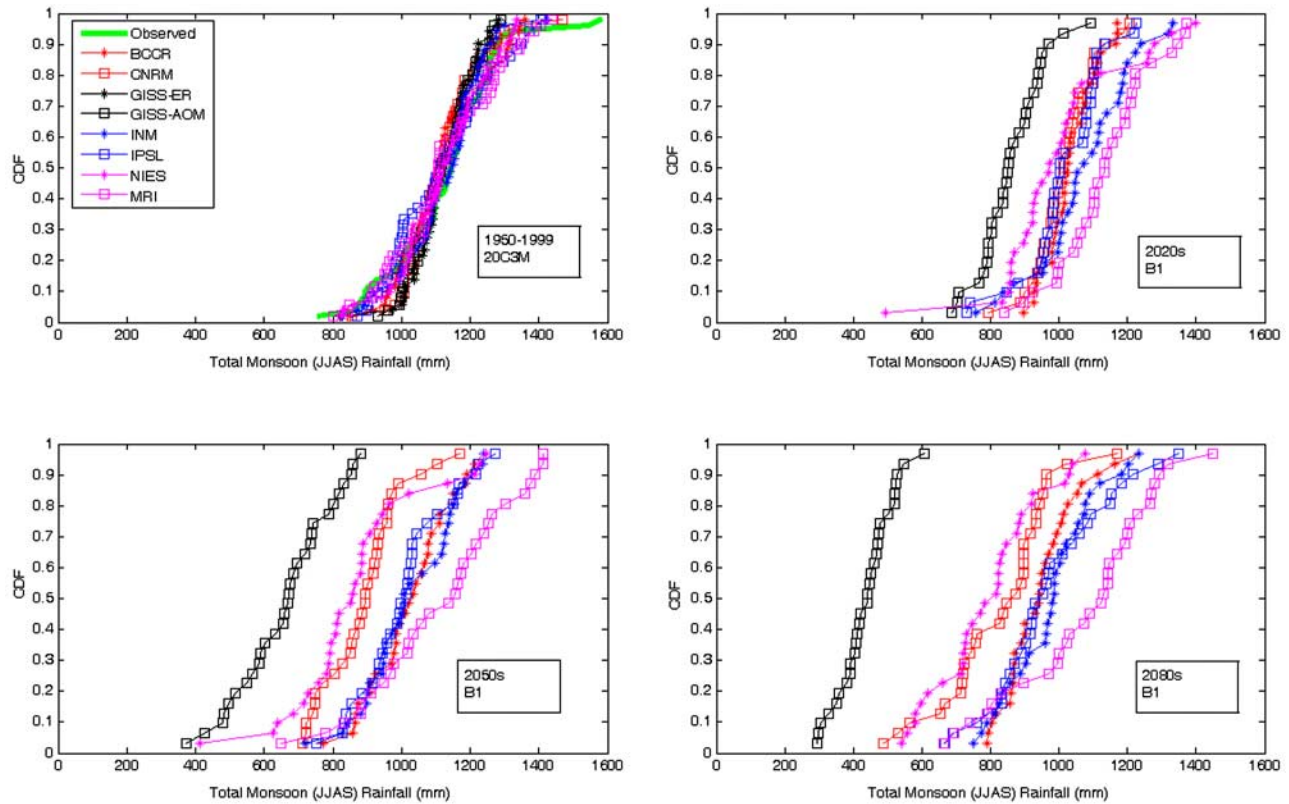


Figure 8. Prediction of monsoon rainfall with B1 scenario.

1950–1999), from that of observed data for the same duration (years, 1950–1999) is computed in terms of RMSE. The inverse values of RMSE are proportionately used as weights so that the sum of weights across all the GCMs is equal to 1.

[13] 2. The weights, thus computed, are used as initial weights assigned to the GCMs.

[14] 3. With the weights and the CDFs derived for the downscaled GCM predictions, the weighted mean CDF ($F_{X_{wm}}$) of future monsoon rainfall is computed.

$$F_{X_{wm}} = \sum_k w_k \times F_{X_{GCM_k}} \quad (2)$$

where, w_k is the weight assigned to k th GCM, $F_{X_{GCM_k}}$ is the corresponding CDF, and $F_{X_{wm}}$ is the weighted mean CDF.

[15] 4. The deviation of the CDFs (future) for all the GCMs is computed individually from the weighted mean CDF in terms of RMSE.

[16] 5. The average of the inverse of RMSE (derived from steps 1 and 4) is computed and proportionately (maintaining

the same ratio among the weights) used as new weights so that the sum of new weights across all the GCMs is equal to 1.

[17] 6. Steps 3 to 5 are repeated until convergence of the weights is achieved.

[18] It is observed that for all the scenarios, weights associated with the GCMs GISSER and GISSAOM are very low, around, 0.02. This is because the ensemble of CDFs derived with other GCMs are farther from these two GCMs. Consideration of these GCMs further, will only increase the uncertainty of the final outcome without a significant contribution toward weighted mean CDF. Furthermore, in COMMIT scenario the CDF derived with GISSER deviates significantly from that of 20C3M, which is unrealistic (as COMMIT GHG concentration is constant and equal to that of year 2000). Therefore these two GCMs (viz., GISSER and GISSAOM) are excluded from the ensemble and then REA is carried out. For demonstration of the convergence of weights, the results obtained from the iterations are given in Table 2 for A1B scenario. The weights obtained for A1B, A2 and B1 are presented in Table 3. For all the scenarios the weights for the GCMs, INM and IPSL are higher than those for others. These

Table 2. Convergence of Weight for A1B Scenario

GCM	Iteration 1	Iteration 2	Iteration 3	Iteration 4
BCCR	0.1499	0.1254	0.1173	0.1173
CNRM	0.1019	0.0883	0.0866	0.0866
INM	0.3368	0.3572	0.3629	0.3630
IPSL	0.1911	0.2210	0.2264	0.2263
NIES	0.0861	0.0812	0.0807	0.0807
MRI	0.1342	0.1269	0.1261	0.1261

Table 3. Weights Assigned to GCMs for Various Scenarios

GCM	A1B	A2	B1
BCCR	0.1173	0.1430	0.1684
CNRM	0.0866	0.2143	0.0971
INM	0.3630	Missing	0.2144
IPSL	0.2263	0.2105	0.3053
NIES	0.0807	0.2625	0.0851
MRI	0.1261	0.1697	0.1296

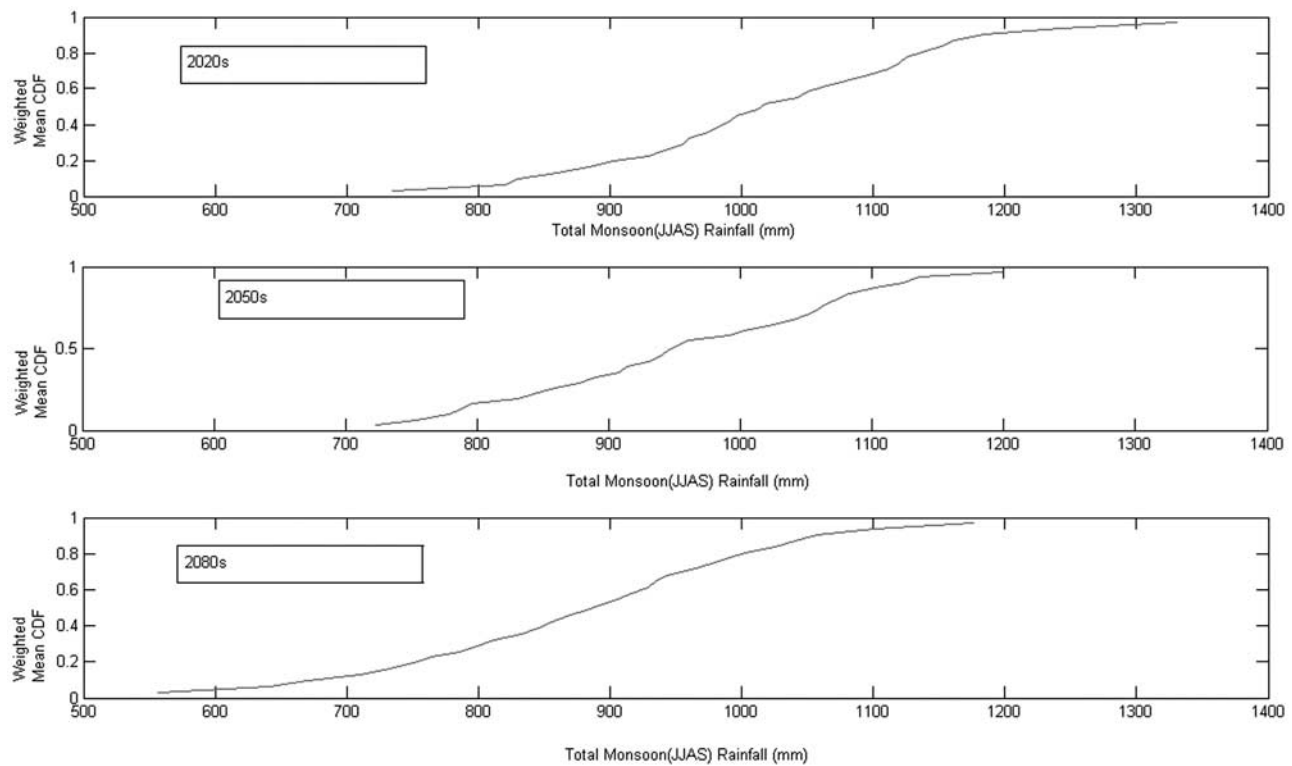


Figure 9. Weighted mean CDF for A1B scenario.

weights are used to derive the weighted mean CDF for future. The weighted mean CDFs (for years 2020s, 2050s and 2080s) are presented with A1b, A2 and B1 scenarios in Figures 9, 10 and 11 respectively. For A2 and A1B scenarios,

the value of rainfall at which the CDF reaches 1, decreases with time which signifies a possible decreasing trend in the high monsoon rainfall events in Orissa meteorological subdivision. Although REA, used here, presents the weight-

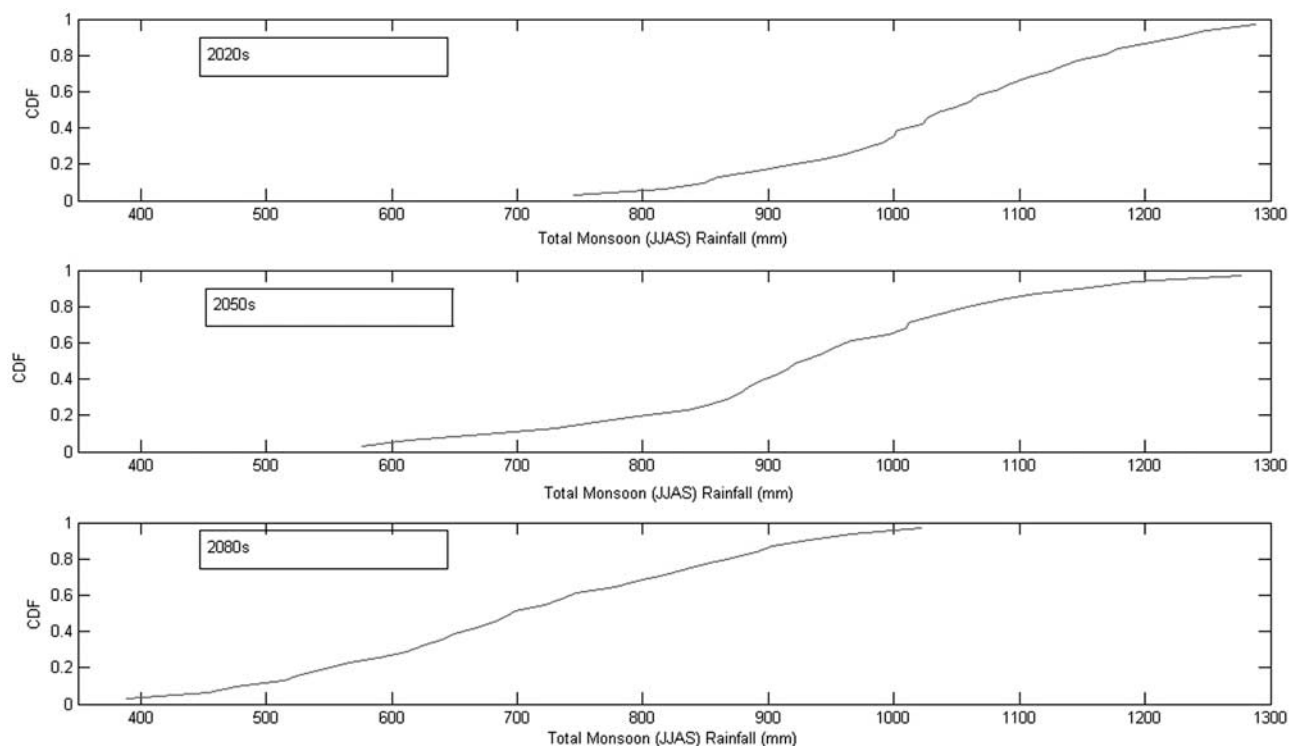


Figure 10. Weighted mean CDF for A2 scenario.

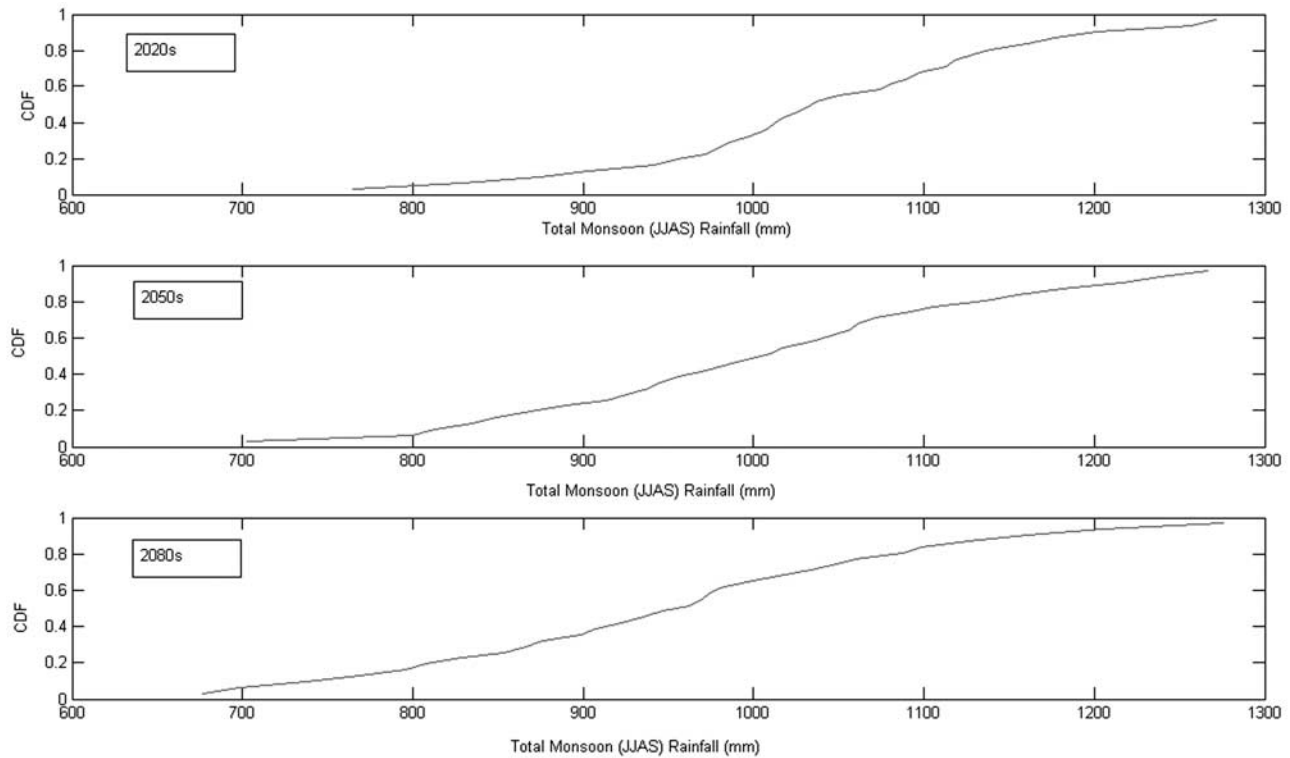


Figure 11. Weighted mean CDF for B1 scenario.

ed mean CDF considering uncertainty resulting from the use of multiple GCMs, the following limitations still need to be addressed.

[19] 1. For A2 scenario, the outputs of the GCM, INM is missing in IPCC data distribution center. The weighted CDF for A2 scenario does not account for the uncertainty caused by partial ignorance resulting from the missing GCM (which has highest weight for A1B scenario) output. Therefore the result represented by weighted mean CDF for A2 scenario may not fully represent the future condition.

[20] 2. On the basis of the availability of selected predictor variables at IPCC data distribution center, only 8 GCMs out of 23, have been used, which also leads to partial ignorance. Inclusion of all GCMs may lead to a different weighted mean CDF. For more reliable prediction uncertainty caused by partial ignorance should be considered in climate change impact studies.

[21] 3. A band of CDFs has been derived using multiple GCMs, and representing such bands with a single curve may not be realistic. A more reliable approach can be representation of such band using an envelope or intervals, using the concept of imprecise probability.

[22] To overcome these limitations the concept of imprecise probability is used in present study with interval regression. As a prerequisite a brief overview of imprecise probability is presented in Appendix A.

4. Uncertainty Modeling With Imprecise Probability

[23] The theory of imprecise probability is devoted to address several types of uncertainty including that caused by partial ignorance, vague or qualitative judgements of

uncertainty, models for complete ignorance or partial ignorance, random sets and multivalued mapping, and partial information about an unknown probability measure [Walley, 1991, 2000]. Applications of imprecise probability in climate change impact assessment may be found in the work of Krieglner and Held [2005] and Hall *et al.* [2007], but in a different context. Fuzzy green house emission scenarios have been constructed in these studies and the resulting uncertainty has been propagated through a low-dimensional climate model. Finally the results are presented in terms of imprecise CDFs which contain all the CDFs derived with multiple scenarios. Uncertainty resulting from use of multiple GCMs and from the missing GCM outputs, have not been considered in these studies. The present study focuses on these two sources of uncertainty and models them with imprecise probability.

[24] Missing outputs of GCMs lead to uncertainty caused by partial ignorance in climate change impact studies. Furthermore, reducing the present knowledge about climate sensitivity to a single probability distribution would clearly misrepresent the scientific disagreement [Hall *et al.*, 2007]. It is therefore more appropriate to deal with interval probability distributions that include the distributions derived with multiple GCMs. Uncertainty resulting from multiple GCMs can be represented by the variation on the collections of CDFs, and modeling the bands of imprecise CDF is a way to represent this variation. Williamson and Downs [1990] suggested construction of an outer envelope on the cumulative probability distributions to deal with such cases. A methodology is presented in this section to construct the outer envelopes considering the weighted mean CDF, derived with REA. The advantage of the proposed model is that it considers the weights assigned

to the GCMs through weighted mean CDF and at the same time the uncertainty caused by partial ignorance is also modeled. The methodology is based on interval regression, which, being highly nonlinear, is solved with nonlinear search algorithm, Probabilistic Global Search Laussane (PGSL) [Raphael and Smith, 2003]. The imprecise distribution of predicted monsoon rainfall is assumed to be Gaussian with mean and standard deviation being interval gray numbers (closed and bounded interval with known lower and upper bounds but unknown distribution information [Huang *et al.*, 1995]). Interval resulting from an interval regression absorbs the uncertainty lying in the assumption of the type of the fitted curve, and therefore the uncertainty resulting from the assumption of Gaussian distribution will be taken care by the interval regression.

[25] For computing a normal or Gaussian cumulative distribution function, the usual procedure is to convert the data into its standard normal deviate and then use it in deriving the CDF. When the distribution is imprecise, i.e., mean (μ) and standard deviation (σ) are both interval gray numbers, denoted by $\mu^\pm$ and $\sigma^\pm$, then the original data (monsoon rainfall in the present case) (x_i) can be converted to imprecise standard normal deviate ($z_i^\pm$) by

$$z_i^\pm = \frac{x_i - \mu^\pm}{\sigma^\pm} \quad (3)$$

[26] If $A^\pm = \frac{1}{\sigma^\pm}$ and $B^\pm = \frac{-\mu^\pm}{\sigma^\pm}$, then equation (3) can be modified to

$$z_i^\pm = A^\pm x_i + B^\pm \quad (4)$$

[27] The involvement of lower and upper bounds of $A^\pm$ and $B^\pm$ in deriving the bounds of $z_i^\pm$ depends on the sign of x_i . Here x_i denotes rainfall and therefore it is nonnegative (≥ 0). For nonnegative x_i , the upper bounds of both $A^\pm$ and $B^\pm$ will result in the upper bound of $z_i^\pm$, and the lower bounds of $A^\pm$ and $B^\pm$ will result in lower bound of $z_i^\pm$, i.e.

$$z_i^+ = A^+ x_i + B^+ \quad x_i \geq 0 \quad (5)$$

$$z_i^- = A^- x_i + B^- \quad x_i \geq 0 \quad (6)$$

[28] The imprecise CDF ($F_X^\pm(x)$) can be computed from standard normal deviate using

$$F_X^\pm(x_i) = H(z_i^\pm) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_i^\pm} e^{-\frac{z^2}{2}} dz \quad (7)$$

[29] Since the function $F_X^\pm(x_i)$ increases continuously with $z_i^\pm$, z_i^+ and z_i^- will be involved in computing $F_X^+(x_i)$ and $F_X^-(x_i)$ respectively. The imprecise Gaussian CDF is fitted to the weighted mean CDF (F_{Xwm}) with interval regression, to account for the uncertainty caused by partial ignorance, so that all the CDFs derived with multiple GCMs will lie within the bounds.

[30] Interval regression, where a model is assumed to have interval coefficients, is regarded as the simplest version of possibilistic regression analysis [Tanaka and Lee, 1998]. Tanaka and Lee [1998] developed a model for

interval regression where the objective is to minimize the squared spread of the estimated output. Hong and Hwang [2005] used support vector machine for interval regression where the objective is to minimize the squared distance between the whitened mid value (average of upper and lower bounds of an interval gray number) of the estimated output and the observed output along with the minimization of the squared spread of the estimated output.

[31] The imprecise CDF, $F_X^\pm(x)$, of monsoon rainfall, can be determined by computing the parameters of the imprecise normal distribution in terms of $A^\pm$ and $B^\pm$. The interval regression used for computing $A^\pm$ and $B^\pm$, has three objectives: (1) to minimize of the squared distance between the whitened mid value (average of upper and lower bounds of an interval gray number) of the imprecise CDF to be estimated, and the weighted mean CDF ($F_{Xwm}(x_i)$); (2) to accommodate the CDFs derived with multiple GCMs within the bounds of imprecise CDF; and (3) to minimize the squared spread of the imprecise CDF to be estimated.

[32] The following optimization model of interval regression is developed for fitting the imprecise normal distribution to the monsoon rainfall.

$$\text{Minimize } \sum_i \left\{ F_{Xwm}(x_i) - \frac{F_X^+(x_i) + F_X^-(x_i)}{2} \right\}^2 + \sum_i Gd(F_X^\pm(x_i)) \quad (8)$$

subject to

$$z_i^\pm = A^\pm x_i + B^\pm \quad (9)$$

$$F_X^\pm(x_i) = H(z_i^\pm) \quad (10)$$

$$F_X^-(x_i) \leq F_{XGCMk}(x_i) \leq F_X^+(x_i) \quad \forall k \quad (11)$$

$$A^\pm > 0 \quad (12)$$

[33] Equation (8) is the objective function of the nonlinear optimization model, which comprises of two parts. The first part minimizes the squared distance between the whitened mid value (average of upper and lower bounds of an interval gray number) of the estimated CDF and the weighted mean CDF computed with REA. The second part minimizes the gray degree of the estimated CDF ($Gd(F_X^\pm(x_i))$) to restrict the spread. gray degree is a measure of uncertainty of the output [Huang *et al.*, 1995; Karmakar and Mujumdar, 2006], which, for an interval gray number ($a^\pm$) is defined by the ratio of its width ($a^+ - a^-$) to the whitened mid value [$\frac{1}{2}(a^+ + a^-)$] and is expressed as

$$Gd(a^\pm) = \frac{a^+ - a^-}{\frac{1}{2}(a^+ + a^-)} \quad (13)$$

[34] By minimizing the gray degree of the estimated CDF, the model minimizes the uncertainty in the regression output. Equations (9) and (10) present the functional relationship between $F_X^\pm(x_i)$ and x_i . Constraint (11) ensures that the CDFs ($F_{XGCMk}(x_i)$) obtained from multiple GCMs will lie in the p-box (details of p-box are in Appendix A) with

Table 4. Parameters of Imprecise CDF for A1B Scenario^a

Parameters		2020s	2050s	2080s
Mean	UB	1190.81	1100.52	1082.96
	LB	832.34	734.26	444.80
Standard deviation	UB	169.49	169.49	222.22
	LB	169.49	151.52	204.08

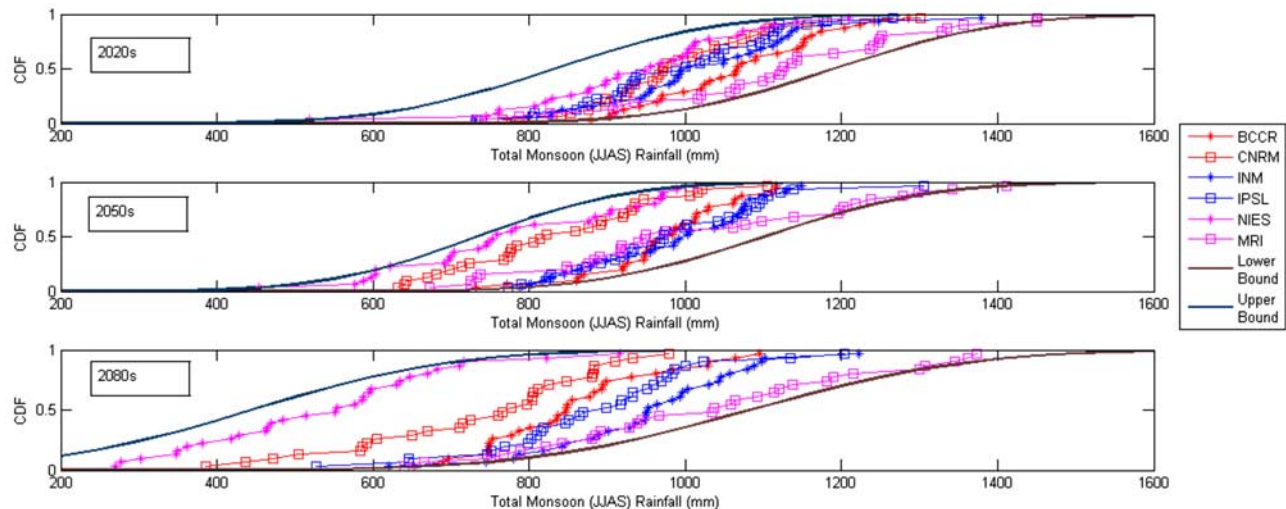
^aUB, upper bound; LB, lower bound.

the bounds $F_X^-(x_i)$ and $F_X^+(x_i)$. The subscript i and k denote the serial number of data and GCM respectively. $A^\pm$ is the inverse of $\sigma^\pm$ which is a positive quantity (constraint (12)). The bounds of $A^\pm$ and $B^\pm$ are the decision variables of the optimization model. The model is solved for three time slices 2020s, 2050s and 2080s. As the model is nonlinear, Probabilistic Global Search Laussane (PGSL), a global search algorithm for nonlinear optimization is used. Tests on benchmark problems having multiparameter nonlinear objective function have revealed that PGSL performs better than Genetic Algorithm and advanced algorithms for Simulated Annealing [Raphael and Smith, 2003]. The algorithm is based on the assumption that better sets of points are more likely to be found in the neighborhood of good sets of points and therefore intensifying the search in the regions that contain good solutions. Details of the algorithm may be found in the work of Raphael and Smith [2003].

[35] The optimization model (equations (8)–(12)) is solved for three time slices 2020s, 2050s and 2080s, first with A1B scenario. The parameters of imprecise CDFs are computed from the decision variables of the optimization model and are presented in Table 4. The resulting imprecise CDF is presented in Figure 12. The lower bound CDF does not show any significant change with time whereas the value, at which the upper bound CDF reaches 1, decreases with time. Consideration of imprecise probability helps the water resources planners to consider both the possible cases, “no change” in rainfall and decrease in rainfall corresponding to lower and upper envelope of the CDF respectively, whereas REA method does not allow to consider the “no

change” cases. Because of the ignorance about future scenario, the “no change” case cannot be ignored and there lies the advantage of using imprecise probability. Constraint (opt4) ensures that the CDFs obtained from multiple GCMs will lie in the p-box, and that is reflected in Figure 12. However, it is not ensured that the missing outputs of GCMs will also lie within the bounds and therefore it is required to validate the model. For validation, the GCM, INM (which is missing for A2 scenario) is not considered for A1B scenario, and the analysis is reformed (REA and subsequent solving of optimization model) for A1B scenario. The imprecise CDFs along with the CDFs for the GCM INM (for years 2020s, 2050s and 2080s) are presented in Figure 13. For 2020s, the CDF derived with INM is well within the bounds of p-box. For 2050s and 2080s, there are few points which are not within the bounds but almost at the border of the p-box. The imprecise CDF derived without considering INM is quite similar to that considering all the GCMs (“no change” for lower bound, and decrease in upper bound).

[36] The optimization model is also solved for A2 and B1 scenarios and the results are presented in Figures 14 and 15 respectively. For A2 scenario, the result is quite similar to that of A1B scenario with “no change” for lower bound CDF and decrease in rainfall corresponding to upper bound CDF. For B1 scenario, the change is not as significant as that of A1B or A2 scenario. The result shows the possibility of most significant reduction of monsoon rainfall for A2 scenario. The B1 scenario presents the most favorable condition (with minimum change) among all the scenarios. The results presented in Figures 12–15 indicate that there will be a reduction in monsoon rainfall for future if at all there is a change. Earlier study [Ghosh and Mujumdar, 2007] on Orissa meteorological subdivision has also projected an extremely dry condition for the case study area. The results obtained from the present study resemble that of the earlier study. Climate change impact study [Mujumdar and Ghosh, 2008] on Mahanadi River (at Orissa meteorological subdivision) streamflow has also indicated similar dry conditions.

**Figure 12.** Imprecise CDF for A1B scenario.

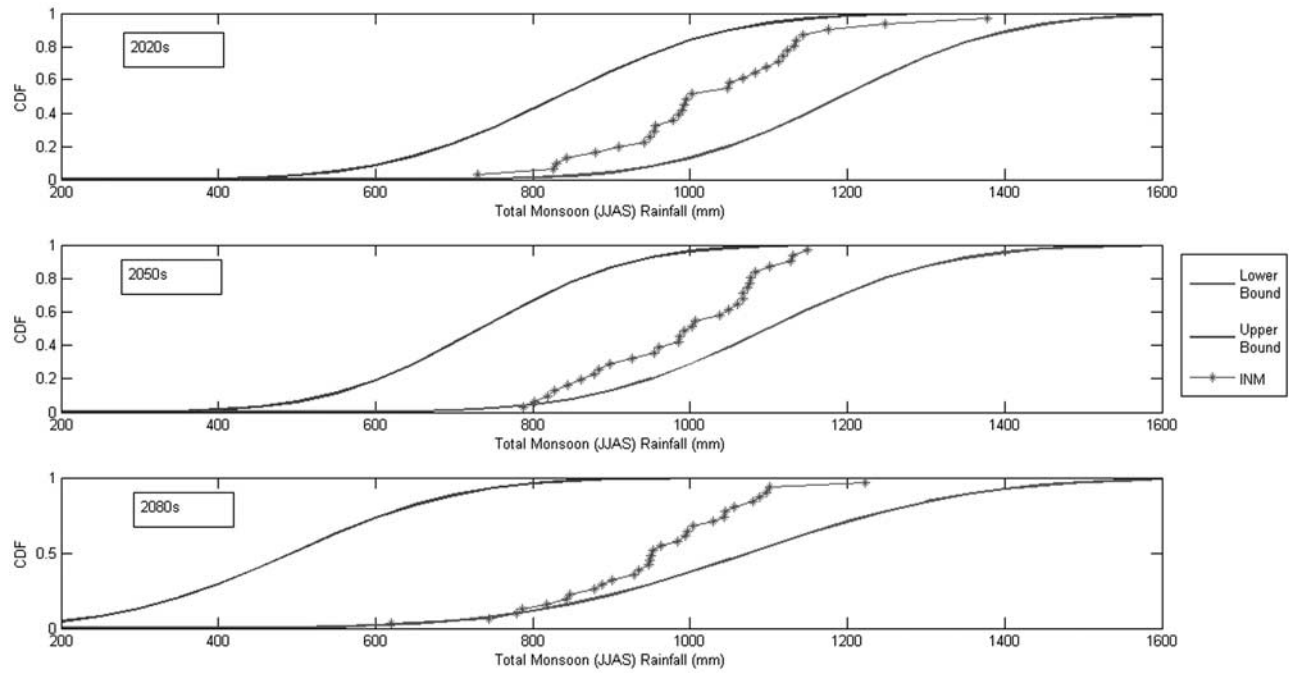


Figure 13. Validation for A1B scenario with GCM INM.

[37] As indicated by these results, impact of climate change may be more severe for the Orissa meteorological subdivision because of its position at the coast of the Bay of Bengal. A slight change in the pressure anomaly of the sea can have a severe impact on the precipitation of Orissa, which results in increase of hydrologic extremes in that region. In a recent analysis, *Dai et al.* [2004] observed from the monthly data set of global PDSI (Palmer Drought Severity Index), that most parts of Eurasia, Africa, Canada, Alaska, and eastern Australia became drier from 1950 to 2002 as large surface warming has occurred since 1950 over these regions. Surface warming caused by climate change and green house gas effect may lead to a severe dry condition in the case study area of Orissa. *Dai and Wigley* [2000] found that a decrease in precipitation occurs mainly

over El Nino Southern Oscillation (ENSO) sensitive regions. There is an established evidence of climatic teleconnection between ENSO and Indian rainfall and thus the impact is severe for India. Therefore global warming with high surface warming in Orissa, sensitivity of precipitation to ENSO and coastal position are possible reasons for the decreasing trend in monsoon rainfall as obtained in this study.

[38] The probabilities obtained from the analysis will be useful in computing the expected future risk, in meeting water demands, in terms of interval gray numbers, and to prepare the policy makers in generating appropriate responses. It is worth mentioning that use of bounds of imprecise probability in computation of expected risk is not straightforward. To compute the bounds of expected risk, the probabilities (of

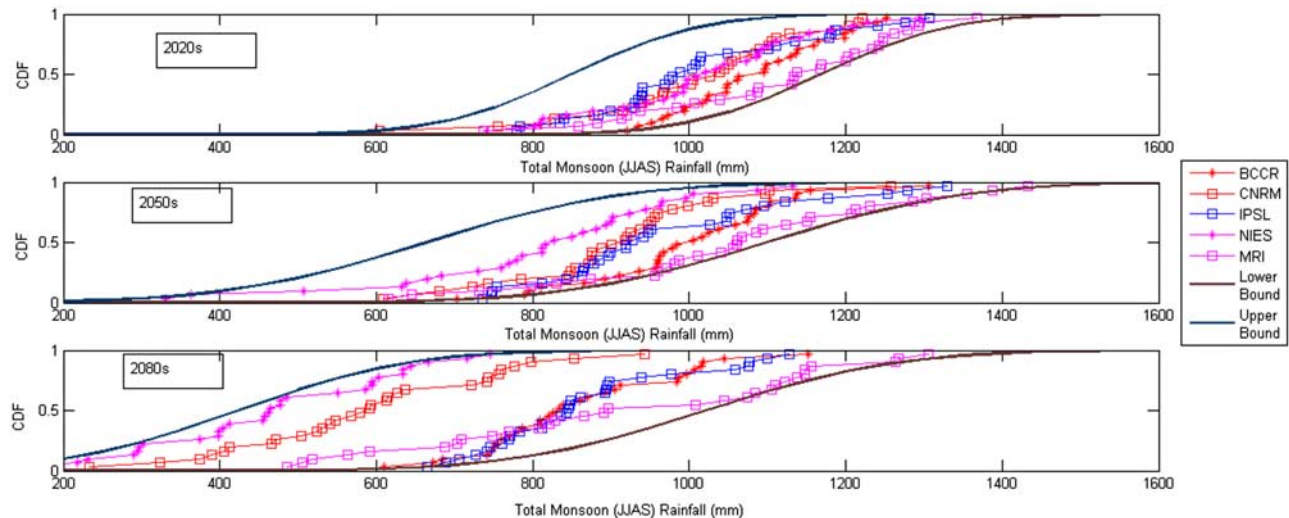


Figure 14. Imprecise CDF for A2 scenario.

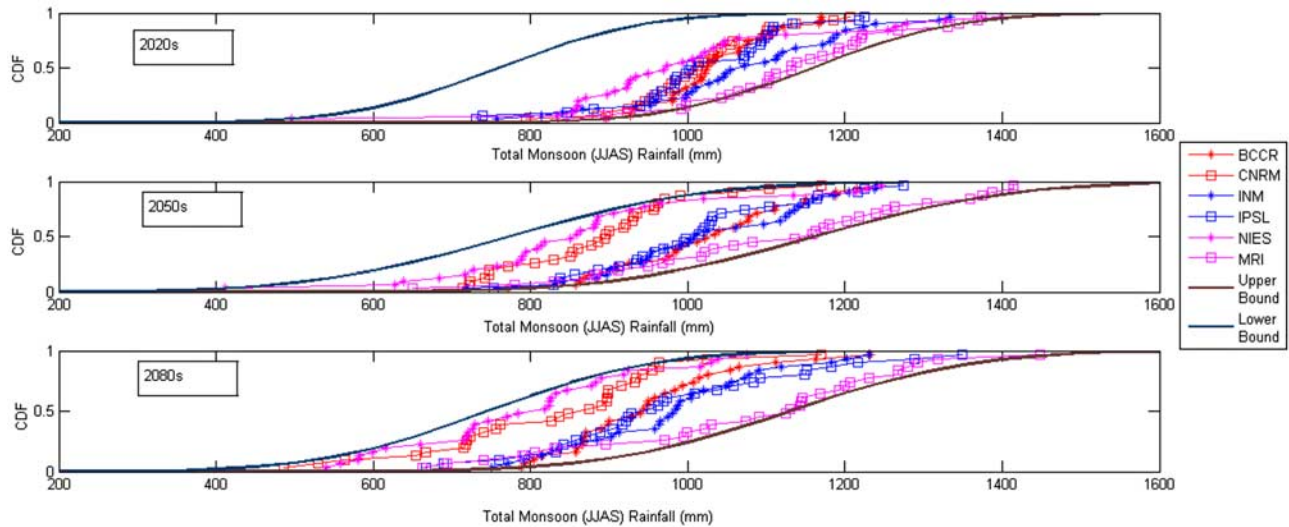


Figure 15. Imprecise CDF for B1 scenario.

different levels of dry and wet conditions) should be selected from their intervals in such a way that their sum will be 1 and they will lead to maximum/minimum values of risk resulting upper/lower bounds of risk in meeting future water demands (S. Ghosh et al., Waste load allocation: An imprecise fuzzy risk approach, paper presented at International Conference on Civil Engineering in the New Millennium: Opportunities and Challenges (CENeM-2007), Bengal Engineering and Science University, Shibpur, Howrah, West Bengal, India, 11–14 January, 2007). Minimization of imprecise risk can be performed using gray optimization model [Karmakar and Mujumdar, 2006] for water resources management.

5. Concluding Remarks

[39] A methodology for modeling GCM uncertainty with imprecise probability for prediction of monsoon rainfall is presented in this paper. REA method has been modified to model uncertainty resulting from the use of multiple GCMs at subdivisional scale. Missing outputs of GCM in IPCC data distribution center result in uncertainty caused by partial ignorance. This uncertainty is modeled with imprecise probability which results in imprecise CDF. Least square interval regression along with the objective of minimization of gray degree of the p-box is performed to fit a imprecise normal distribution considering both mean and standard deviation as interval gray numbers. The results show a possible decrease of monsoon rainfall corresponding to upper bound of CDF and “no change” corresponding to lower bound of CDF. From the resulting imprecise CDF it may be concluded that there is a possibility of decrease in monsoon rainfall if there is any hydrologic change at all. Imprecise CDFs of monsoon rainfall can be used in computation of imprecise risk for meeting water demands in the meteorological subdivision, in terms of upper and lower bounds. Gray optimization models may be used for water resources decision making and water management to minimize the imprecise risk, with the imprecise CDFs as input. The limitation of the model presented is that, if the output of a missing GCM deviates largely from the majority of the ensemble, imprecise probability may not capture the uncer-

tainty associated with the missing information. The proposed methodology has the limitation of not considering uncertainty caused by parameterization and the structure of the impact model itself. The other two sources of uncertainty not considered here are those caused by starting conditions used in GCM simulations and the downscaling techniques. It should be noted that the relative weights assigned to the GCMs using REA are not absolute and might change with the changes in the structure of the downscaling model used and changes in the predictor variable set. Therefore there is a need to model the uncertainty associated with multiple downscaling methods also. Modeling downscaling uncertainty along with GCM uncertainty can be considered as a potential area for future research. The downscaling model considered in the present study may have the limitations for use in assessments of water resources systems, because (1) they result in rainfall at a monthly timescale and not a daily timescale, and (2) they do not invoke any persistence between rainfall in adjacent months. These can be overcome using the methodology suggested in recent literature [e.g., Charles et al., 2004; Mehrotra and Sharma, 2005, 2006]. For validation of the imprecise probability model, the GCM developed by INM is selected as the output of A2 scenario is missing for this GCM in IPCC data distribution center. More exhaustive analysis can be performed with cross-validation or leave-one-out study, but the validation will be computationally more involved with the increase in the number of GCMs. It is true that imprecise probability introduces an extra dimension or degree of freedom into the formal expression of uncertainty, i.e., probability models. This inevitably introduces some indeterminism into any subsequent decision analysis as the dimensionality of the problem is increased while dealing with less information. This indeterminism, rather than being a shortcoming, can be viewed as a strength, more faithfully reflecting the reality of the situation [Caselton and Luo, 1992].

Appendix A: Imprecise Probability

[40] Theory of imprecise probability is considered as the generalized version of probability theory for an event

characterized by partial ignorance. It is also considered as the generalized form of other two uncertainty theories: (1) theory of possibility and necessity measures [Dubois and Prade, 1988], in which the model is characterized by qualitative knowledge and intuitions in the absence of precise measurement, and (2) theory of belief and plausibility functions following Dempster-Shafer structure [Dempster, 1967; Shafer, 1976; Caselton and Luo, 1992], when evidence is available in terms of sets or intervals and not as precise values. In the present work, uncertainty caused by partial ignorance resulting from missing GCM outputs is modeled with imprecise probability.

[41] The essence of imprecise probability is the assignment of ranges of probabilities to events. Therefore uncertainty about an event, E , would be expressed by an interval $[P^-(E), P^+(E)]$, where, $0 \leq P^-(E) \leq P^+(E) \leq 1$. The two bounds $P^-(E)$ and $P^+(E)$, called respectively, the lower and upper probabilities of E , are chosen so that, given the evidence, one can be reasonably sure that probability of E is neither less than $P^-(E)$ nor greater than $P^+(E)$. The following illustration (following the work of Tonn [2005]) is designed to show how imprecise probability can be more expressive in the highly uncertain hydrologic data modeling characterized with missing information. Let us assume that a hydrologic event has two states 1 and 2 and the data set of that event has 100 data points. The probability of hydrologic event to be in state 1 and 2 are defined by $P(1)$ and $P(2)$, respectively. The following four cases are considered to explain the concept of imprecise probability.

[42] 1. If all the data points lie in state 1, then $P(1) = 1$ and $P(2) = 0$, and the probability model is considered to be a “certain or deterministic model”, which is quite uncommon in hydrology.

[43] 2. If 50 data points are in state 1 and rest are in state 2, then $P(1) = 0.5$ and $P(2) = 0.5$, and the probability model is considered to be a uncertain (because of randomness) model with “complete knowledge”.

[44] 3. If 40 data points are in state 1, 40 data points are in state 2, and the rest are missing, then the two extreme possible cases are: (1) 60 data points are in state 1, and 40 data points are in state 2, i.e., $P(1) = 0.6$ and $P(2) = 0.4$; and (2) 40 data points are in state 1, and 60 data points are in state 2, i.e., $P(1) = 0.4$ and $P(2) = 0.6$. From these two possible cases it can be inferred that $P(1)$ and $P(2)$ can vary between 0.4 and 0.6, which results in imprecise probabilities, probabilities denoted by interval gray numbers. In such a case, the upper and lower bounds of $P(1)$ will be 0.6 and 0.4. Similarly, the bounds of $P(2)$ will also be 0.6 and 0.4. Such a probability model can be categorized as an uncertain model with “partial ignorance”.

[45] 4. If all the data points are missing, then $P(1)$ and $P(2)$ can vary between 0 and 1 and the upper and lower bounds of both the probabilities will be 1 and 0. Such a model is termed as uncertain model with “complete ignorance”.

[46] Imprecise probability thus addresses uncertainty caused by partial ignorance with interval gray numbers and the interval between the bounds of the probability reflects the incomplete nature of knowledge or partial ignorance. It may be noted that the application presented in this appendix is entirely different from the proposed methodology of modeling uncertainty because of missing GCM output. For the proposed model imprecise probability

is used to capture to the best possible extent, the missing realizations of multiple ensemble generated with multiple GCMs.

[47] The properties of imprecise probability are as follows.

[48] 1. For universal sample set (Ω) and null set (Φ) both the bounds of imprecise probability follow axioms of probability.

$$P^-(\Phi) = P^+(\Phi) = 0 \quad (A1)$$

$$P^-(\Omega) = P^+(\Omega) = 1 \quad (A2)$$

[49] 2. For any event E , both the bounds $P^-(E)$ and $P^+(E)$ are monotonically increasing, i.e.

$$E_1 \subseteq E_2 \Rightarrow P^-(E_1) \leq P^-(E_2) \quad (A3)$$

$$E_1 \subseteq E_2 \Rightarrow P^+(E_1) \leq P^+(E_2) \quad (A4)$$

[50] 3. Bounds of imprecise probability follow properties of complementarity, i.e.

$$P^-(E) + P^+(E') = 1 \quad \forall E \subseteq \Omega \quad (A5)$$

where, E' is the complement set of E .

[51] 4. For two mutually exclusive sets E_3 and E_4 , imprecise probability follows.

$$E_3 \cap E_4 = \Phi \Rightarrow P^-(E_3 \cup E_4) \geq P^-(E_3) + P^-(E_4) \quad (A6)$$

$$E_3 \cap E_4 = \Phi \Rightarrow P^+(E_3 \cup E_4) \leq P^+(E_3) + P^+(E_4) \quad (A7)$$

where Φ denotes null set. If E_3 and E_4 are mutually exclusive and exhaustive sets, then, from equations (A6) and (A7)

$$E_3 \cup E_4 = \Omega \Rightarrow P^-(E_3) + P^-(E_4) \leq 1 \quad (A8)$$

and

$$E_3 \cup E_4 = \Omega \Rightarrow P^+(E_3) + P^+(E_4) \geq 1 \quad (A9)$$

Following, equation (A9), for any set E

$$P^+(E) + P^+(E') \geq 1 \quad (A10)$$

It should be noted that the difference between $(P^+(E) + P^+(E'))$ and 1 reflects the uncertainty caused by incomplete knowledge. If U denotes the total uncertainty, then

$$P^+(E) + P^+(E') - 1 = U \quad (A11)$$

Following equation (A5), equation (A11) can be rewritten as

$$P^+(E) + 1 - P^-(E) - 1 = U \Rightarrow P^+(E) - P^-(E) = U \quad (A12)$$

Therefore the interval between $P^-(E)$ and $P^+(E)$ reflects the total uncertainty resulting from partial ignorance.

[52] A natural model of an imprecise probability measure is obtained by considering a pair (F^-, F^+) of CDFs, generalizing an interval (C. Baudrit and D. Dubois, Comparing methods for joint objective and subjective uncertainty propagation with an example in a risk assessment, paper presented at 4th International Symposium on Imprecise Probabilities and Their Applications, Pittsburgh, Pa, 20–23 July, 2005). The interval $[F^-, F^+]$ is called a probability box or p-box. Following equation (A12), the interval between F^- and F^+ reflects the incomplete nature of knowledge, thus picturing the extent of what is ignored.

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