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Modelling bivariate rainfall distribution and generating bivariate correlated rainfall data in neighbouring meteorological subdivisions using copula

Subimal Ghosh*
Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India

Abstract:

The rainfall patterns of neighbouring meteorological subdivisions of India are similar because of similar climatological and geographical characteristics. Analysing the rainfall pattern separately for these meteorological subdivisions may not always capture the correlation and tail dependence. Furthermore, generating the multivariate rainfall data separately may not preserve the correlation. In this study, copula method is used to derive the bivariate distribution of monsoon rainfall in neighbouring meteorological subdivisions. Different Archimedean copulas are used for this purpose and the best copula is selected based on nonparametric test and tail dependence coefficient. The fitted copula is then applied to derive the bivariate distribution, joint return period and conditional distribution. Bivariate rainfall data is generated with the fitted copula and it is observed with the increase of sample size, the generated data is able to capture the correlation as well as tail dependence. The methodology is demonstrated with the case study of two neighbouring meteorological subdivisions of North-East India: Assam and Meghalaya meteorological subdivision and Nagaland, Manipur, Mizoram and Tripura meteorological subdivision. Copyright © 2010 John Wiley & Sons, Ltd.

KEY WORDS rainfall; copula; multivariate analysis

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INTRODUCTION

Indian monsoon rainfall is characterized by significant spatial heterogeneity in terms of arrival of monsoon, monthly rainfall intensity, maximum daily rainfall intensity, trend and so forth (Ghosh *et al*., 2009). To facilitate meteorological analysis, India is divided into 36 meteorological subdivisions (http://www.tropmet.res.in/IITM/india-subdiv-rev1.png). However, these subdivisions are not based on regionalization (Satyanarayana and Srinivas, 2008) with rainfall data, rather they are based on mostly state level political boundaries. Therefore, rainfall patterns in neighbouring meteorological subdivisions are often found to be similar, having significant correlation. Individual analysis of rainfall for neighbouring meteorological subdivision with probabilistic theory may not capture such correlations and there is a need of multivariate statistical analysis to capture this correlation along with modelling the rainfall in neighbouring subdivisions.

It is very important to capture the rainfall correlation between the neighbouring subdivisions, with probabilistic analysis, for simulating runoff in a river basin, where both the subdivisions belong to. As, for example, Assam and Meghalaya (AM) meterological subdivision and Nagaland, Manipur, Mizoram, Tripura (NMMT) subdivision partially belong to Brahmaputra river basin of North-East India, and rainfalls in both the subdivisions contribute to the river flow. High rainfall correlation of these two subdivisions points that high rainfall in AM is associated with high rainfall in NMMT resulting high streamflow value in Brahmaputra. If the correlation is not captured properly, such association is not guaranteed and individual data generation of these two subdivisions may lead to a lower streamflow value than the actual case.

A possible methodology may be the use of multivariate normal, lognormal or extreme value distribution; however, the limitation is that the individual behaviour of the two variables must then be characterized by the same parametric family of univariate distribution (Genest and Favre, 2007).

A copula is a tool for modelling multivariate distribution considering the marginal distributions as input and avoids such restrictions mentioned in the preceding text. It joins or 'couples' multivariate distribution functions to their corresponding marginal distribution functions (Poulin *et al*., 2007; Salvadori *et al*., 2007). Following the definition given by Sklar (1959), a p-dimensional distribution function F can be written in the form:

$$F(x_1, x_2, \ldots, x_p) = C(F_1(x_1), F_2(x_2), \ldots, F_p(x_p)) \quad (1)$$

where $F_1, \ldots, F_p$ = marginal distribution functions. If $F_1, \ldots, F_p$ are continuous then the copula C is unique and has the representation (Poulin *et al*., 2007):

$$C(u_1, \ldots, u_p) = F(F_1^{-1}(u_1), \ldots, F_p^{-1}(u_p)), \quad 0 \le u_1, \ldots, u_p \le 1 \quad (2)$$

* Correspondence to: Subimal Ghosh, Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India. E-mail: subimal@civil.iitb.ac.in

where the quantile function F_i^{-1} is defined by $F_i^{-1}(u_i) = \inf\{x \in \Re | F_i(x) \geq u_i\}$. Conversely, if C = copula and $F_1, \ldots, F_p$ are distribution functions, then F described in Equation (1) is a p-dimensional distribution function with marginals $F_1, \ldots, F_p$ (Zhang and Singh, 2006).

The copula method has been developed by Sklar (1959), Genest and MacKay (1986), Genest and Rivest (1993) and Nelsen (1999) but did not get much attention by hydrologists and water resources engineers in 20th century. The merits of copula in hydrologic applications have been discussed in Favre *et al.* (2004), Genest and Favre (2007) and Salvadori and De Michele (2007). The idea of copula has been applied in modelling joint distribution of flow and volume. It has also been reported that choice of copula is a crucial step for such modelling. Zhang and Singh (2006) have applied copula for deriving bivariate distributions of flood peak and volume, volume and duration, flood peak and duration. A detailed overview on the selection of Archimedean copula has been presented. Dupuis (2007) has presented an overview of use of copulas in hydrology with the benefits and cautions. The methodology has been demonstrated for derivation of bivariate distribution of low flow volume and its duration. Poulin *et al.* (2007) have shown the importance of tail dependence in selecting copula for hydrologic applications, which is important specifically for extreme values. However, there are only limited numbers of studies till now, which have considered tail dependence. Tail dependence has its importance in capturing extreme events. High tail dependence between rainfall values of two neighbouring stations suggests simultaneous occurrences of extremes in those two stations. Analysis without tail dependence cannot simulate simultaneous occurrences of extremes in those stations, resulting failure in modelling extreme streamflow events, to which both the stations are contributing. Use of trivariate copula in hydrologic applications has been demonstrated by Zhang and Singh (2007) for rainfall frequency analysis. Genest *et al.* (2007) have demonstrated the use of metaelliptical copulas in hydrologic applications, for modelling flood peak, volume and duration. A review of goodness of fit test for copulas may be found in Genest *et al.* (2009). Maity and Nagesh Kumar (2008) have used Archimedean copula for hydroclimatic teleconnection for prediction of hydrologic extremes; however, tail dependence test has not been performed which is an essential step related to modelling extremes. In a very recent study, Karmakar and Simonovic (2009) have shown the importance of selection of marginal distribution for copula-based models. Archimedean copulas have been used for bivariate modelling flood peak and volume, volume and duration, flood peak and duration. Tail dependence is also not performed in this study. This work demonstrates use of copula in deriving bivariate distribution and conditional distribution of monsoon rainfall in neighbouring meteorological subdivisions of India with statistical selection of marginal distribution, tail dependence test and generation of bivariate data from derived joint distribution. Such a methodology will be useful in generating bivariate rainfall for hydrologic modelling, downscaling rainfall for neighbouring subdivisions, where it is important to maintain the correlation structure. Two meteorological subdivisions of North-East India, 'AM' and 'NMMT' are used as case studies.

CASE STUDY, DATA AND MARGINAL DISTRIBUTIONS

'AM' and 'NMMT', the two neighbouring meteorological subdivisions of North-East India, extend from 90 °E

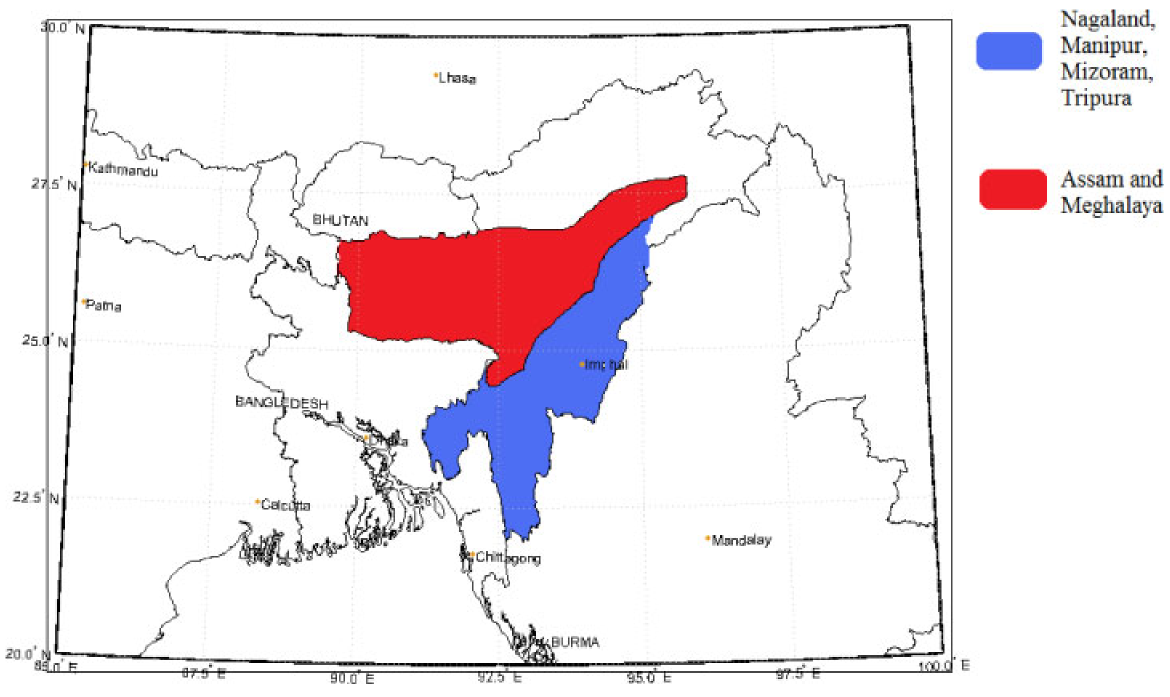

Figure 1. Case study area

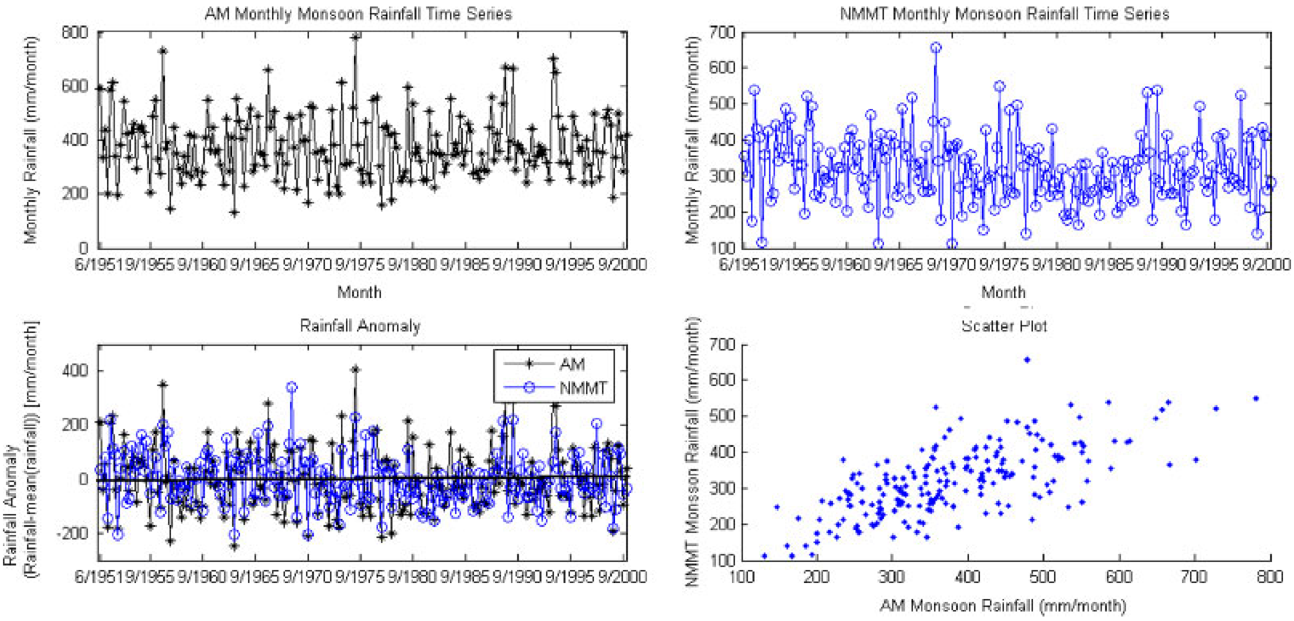

Figure 2. Rainfall in AM and NMMT meteorological subdivisions

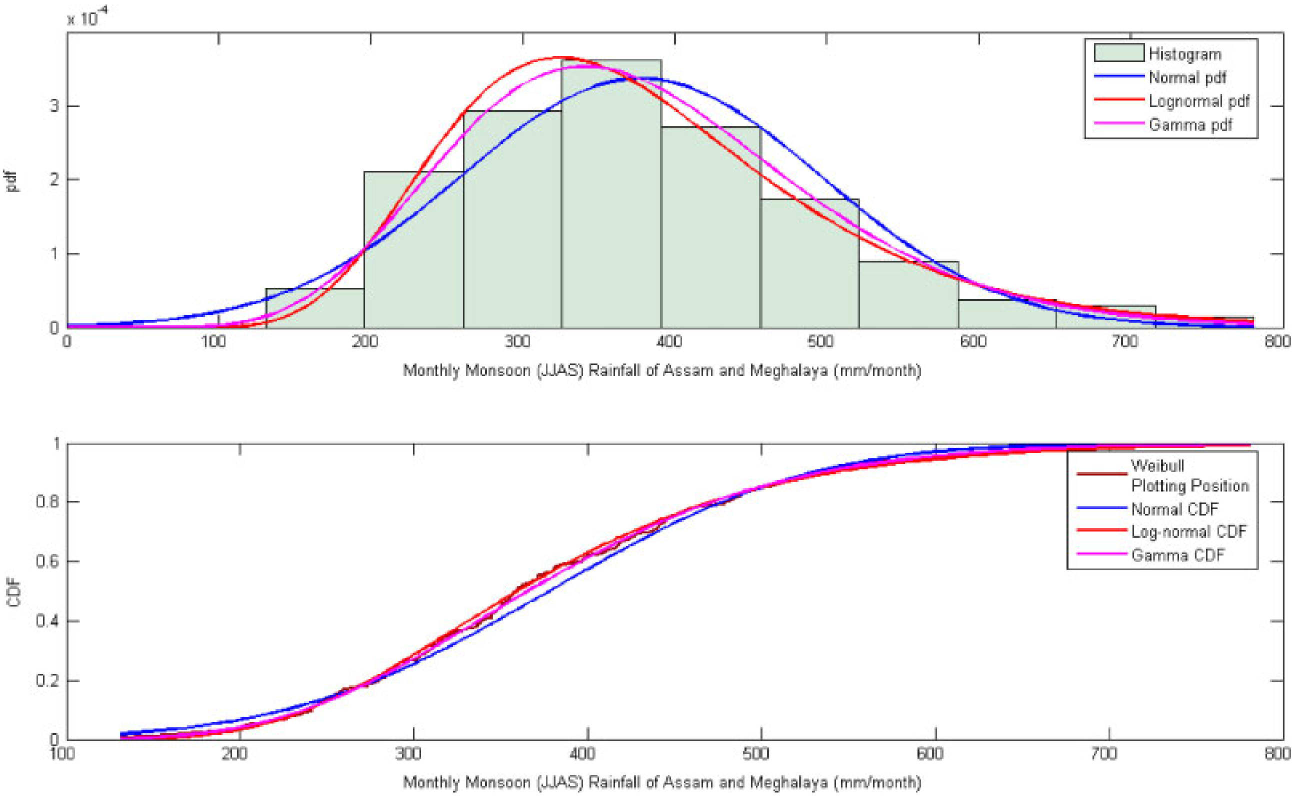

Figure 3. Selection of marginal distribution for AM

to 95 °E and 22 °N to 28 °N (Figure 1). The monthly monsoon (June, July, August and September) data for these two meteorological subdivisions from 1951 to 2000 are obtained from Indian Institute of Tropical Meteorology, Pune, India (www.tropmet.res.in). Data of 50 years (1951–2000) for 4 months (June, July, August and September) result in a sample size of 200. The monthly data along with anomaly (deviations from individual means) and scatter plot are presented in Figure 2. It shows significant rainfall correlation of two meteorological subdivisions. Also, the anomaly shows that high rainfall in one subdivision for a particular month is associated with high rainfall in other subdivision. Individual modelling may not preserve such correlation and hence copula is proposed in this study. The mean and standard deviation of monthly rainfall for AM are 377·6 and 118·2 mm/month, respectively, and those of NMMT are 319·2 and 95·1 mm/month, respectively. The first

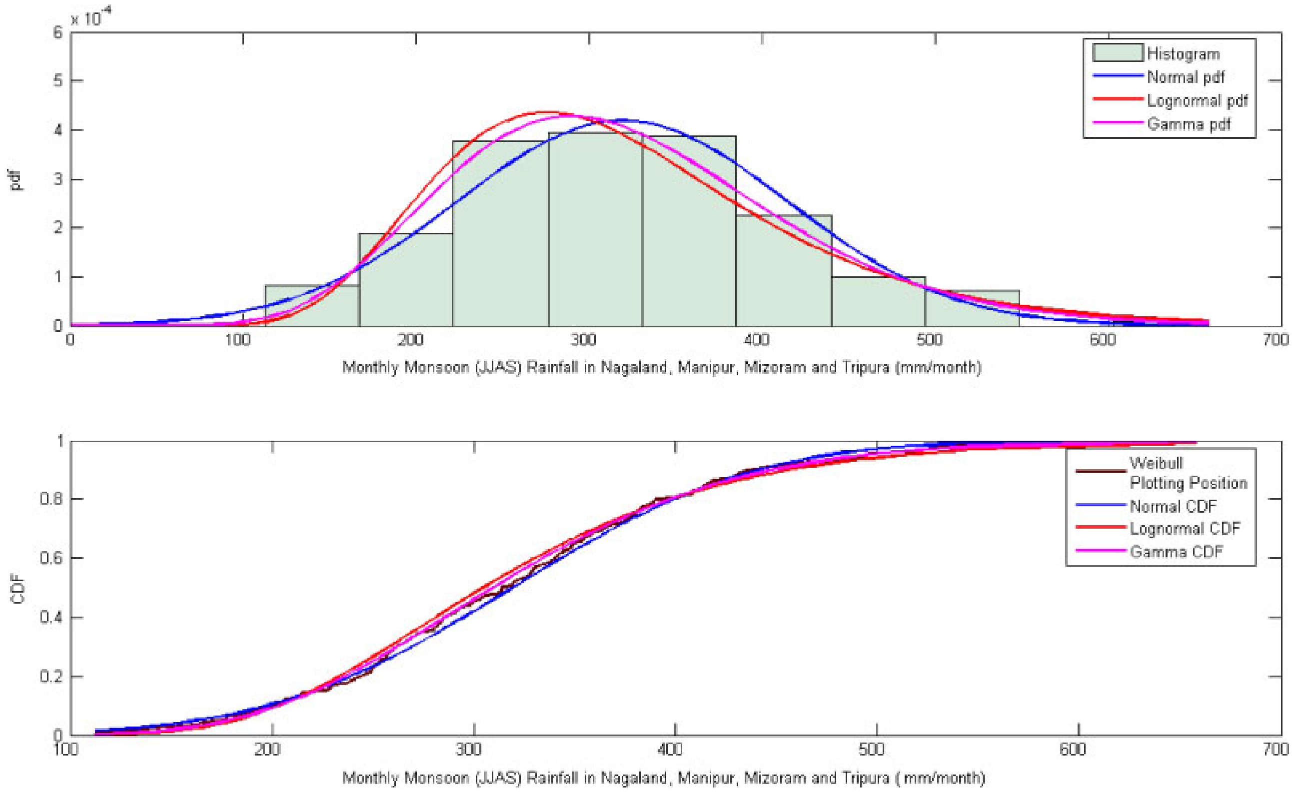

Figure 4. Selection of marginal distribution for NMMT

Table I. Selection of marginal distribution

Selection criteria	Assam and Meghalaya			Nagaland, Manipur, Mizoram, Tripura		
	Normal	Lognormal	Gamma	Normal	Lognormal	Gamma
AIC	−326·66	−650·64	−731·39	$-1{\cdot}6 \times 10^3$	$-1{\cdot}5 \times 10^3$	$-1{\cdot}7 \times 10^3$
BIC	−320·02	−644·00	−724·75	−538·09	−405·48	−637·05

step in modelling bivariate distribution using copula is to obtain the marginal distributions. For fitting marginal distribution for monsoon rainfall for AM and NMMT, normal, lognormal and gamma distribution have been tried. First the distributions are fitted to the dataset with the parameters obtained from method of moments. The probability density functions (PDFs) and cumulative distribution functions (CDFs) of the distributions along with the histogram of data set for both AM and NMMT are presented in Figures 3 and 4, respectively. For validation, rank-based CDFs are also obtained from Weibull plotting position formula. The plots for CDF for AM show that gamma and lognormal distributions fit better than normal distribution; however, it is difficult to select the best from the plots. For NMMT, the performance of all the distribution seems to be similar and hence decision regarding selection of distributions from the plots is difficult. Therefore, Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) are obtained with mean square error (MSE) for selection of best marginal distributions. AIC and BIC obtained from all the three distributions mentioned are shown in Table I. It shows, for both the cases, gamma distribution fits the best and hence Gamma distribution is used for modelling marginal distribution. After the selection of marginal distribution, Archimedean copulas are fitted to derive bivariate distribution. The next section presents the mathematical details of Archimedean copula.

ARCHIMEDEAN COPULA

There are different families (types) of copula, among which, Archimedean copula is more popular for hydrologic applications. It can be easily constructed and a large variety of copula families belong to this family. Archimedean copula can be applied for both positive and negative correlation between the multiple variables (Zhang and Singh, 2006). In order to express Archimedean copula for two random variables, X and Y, with their CDFs, respectively, as $F_x(x)$ and $F_Y(y)$, let $U = F_x(x)$ and $V = F_Y(y)$, then U and V are uniformly distributed random variables with their values u and v. If $\phi(\bullet)$ be the copula generator, a convex decreasing function, then the one parameter Archimedean copula,

Table II. Archimedean copulas

Copula	Equation	Generating function $\phi(t)$	Relationship with τ
Gumbel–Hoggard	$C_\theta(u,v)=\exp\{-[(-\ln u)^\theta+(-\ln v)^\theta]^{1/\theta}\}$	$\phi(t)=(-\ln t)^\theta$	$\tau=1-\theta^{-1}$
Ali–Mikhail–Haq	$C_\theta(u,v)=\frac{uv}{1-\theta(1-u)(1-v)}$	$\phi(t)=\ln\frac{1-\theta(1-t)}{t}$	$\tau=\left(\frac{3\theta-2}{\theta}\right)-\frac{2}{3}\left(1-\frac{1}{\theta}\right)^2\ln(1-\theta)$
Frank	$C_\theta(u,v)=\frac{1}{\theta}\ln\left[1+\frac{[\exp(\theta u)-1][\exp(\theta v)-1]}{\exp(\theta)-1}\right]$	$\phi(t)=\ln\left[\frac{\exp(\theta t)-1}{\exp(\theta)-1}\right]$	$\tau=1-\frac{4}{\theta}[D_1(-\theta)-1]$
Clayton	$C_\theta(u,v)=[u^{-\theta}+v^{-\theta}-1]^{-\frac{1}{\theta}}$	$\phi(t)=t^{-\theta}-1$	$\tau=\frac{\theta}{\theta+2}$

D_1 = First order Debye function (Zhang and Singh, 2006).

denoted by C_θ, can be expressed as (Nelsen, 1999; Zhang and Singh, 2006):

$$C_\theta(u,v)=\phi^{-1}\{\phi(u)+\phi(v)\};\quad 0<u,v<1 \tag{3}$$

where θ is the parameter of the copula generating function. Nelsen (1999) provided some important single-parameter families of Archimedean copulas, along with their generators, the range of the parameter and some special and limiting cases. The mathematical expressions of single-parameter bivariate Archimedean copulas and their fundamental properties applied in this study are listed in Table II. In these copula functions, the parameter θ synthesizes the dependence strength among the dependent random variables. For each bivariate Archimedean copula, the value of θ can be obtained by considering the mathematical relationship (Nelsen, 1999) between Kendall's coefficient of correlationτ, and generating function $\phi(t)$, which is given by (Karmakar and Simonovic, 2009):

$$\tau=1+4\int_0^1\frac{\phi(t)}{\phi'(t)}\mathrm{d}t \tag{4}$$

where t is either u or v. Kendall's coefficient of correlation (τ) is a well-known nonparametric measure of dependence or association in multivariate statistics. Kendall's τ from the observation is determined (estimated) from:

$$\tau_n=\binom{n}{2}^{-1}\sum_{i<j}\text{sign}[(x_i-x_j)(y_i-y_j)] \tag{5}$$

where sign $=1$ if $[(x_i-x_j)(y_i-y_j)]>0$, sign $=-1$ if $[(x_i-x_j)(y_i-y_j)]<0$; $i, j=1, 2, \ldots, n$. X and Y denote the two random variables denoting the monthly monsoon rainfall of AM and NMMT. Kendall's coefficient of correlation for the present case is obtained as 0·49.

Following Zhang and Singh (2006), it should be noted that Gumbel–Hoggard can be applied when the dependence structure between the random variables is positive. The Ali–Mikhail–Haq copula can be applied to both positive and negative correlation; however, may not be suitable for very high positive correlation or very low negative correlation. Frank copula does not have this limitation. Cook–Johnson copula is suitable only for positive correlations. With these four copulas, the multivariate distribution functions are derived for the monsoon rainfall of AM and NMMT (Figure 5). The following section presents the method for the selection of best copula.

SELECTION OF COPULA

Following Genest and Rivest (1993) and Zhang and Singh (2006), the steps involved in identification of copula is presented:

1. Define an intermediate random variable $Z=Z(x, y)$ which has a distribution function $K(z)=P(Z\le z)$, where z is value of Z. The distribution function is related to the generating function by $K(z)=z-\phi(z)/\phi'(z)$.
2. Construct a nonparametric estimate of K as follows:

- Obtain that $z_i=\{$number of (x_j, y_j) such that $x_j<x_i$ and $y_j<y_i\}/(N-1)$ for $i=1,\ldots,N$.
- Construct the estimate of K as $K_N(z)=$ the proportion of z_i' s $\le$ z.

3. Construct a parametric estimation of K using step 1, with z obtained from step 2.
4. Plot nonparametrically obtained $K_N(z)$ with parametrically obtained K for each copula. The plot, which is in best agreement with 45° line, the corresponding copula will be considered as the best fitted.

The plots for the four copulas, Ali–Mikhail–Haq, Frank, Cook–Johnson and Gumbel–Hoggard are presented in Figure 6, which shows the Gumbel–Hoggard copula is the best fitted copulas. For further validation, AIC and BIC are computed with respect to empirical joint distribution formula, given by (Zhang and Singh, 2006):

$$H(x_i,y_i)=P(X\le x_i, Y\le y_i)=\frac{\text{No. of }(x_j\le x_i\text{ and }y_j\le y_i)-0{\cdot}44}{N+0{\cdot}12} \tag{6}$$

where N is the sample size and $i, j=1, 2,\ldots, N$.

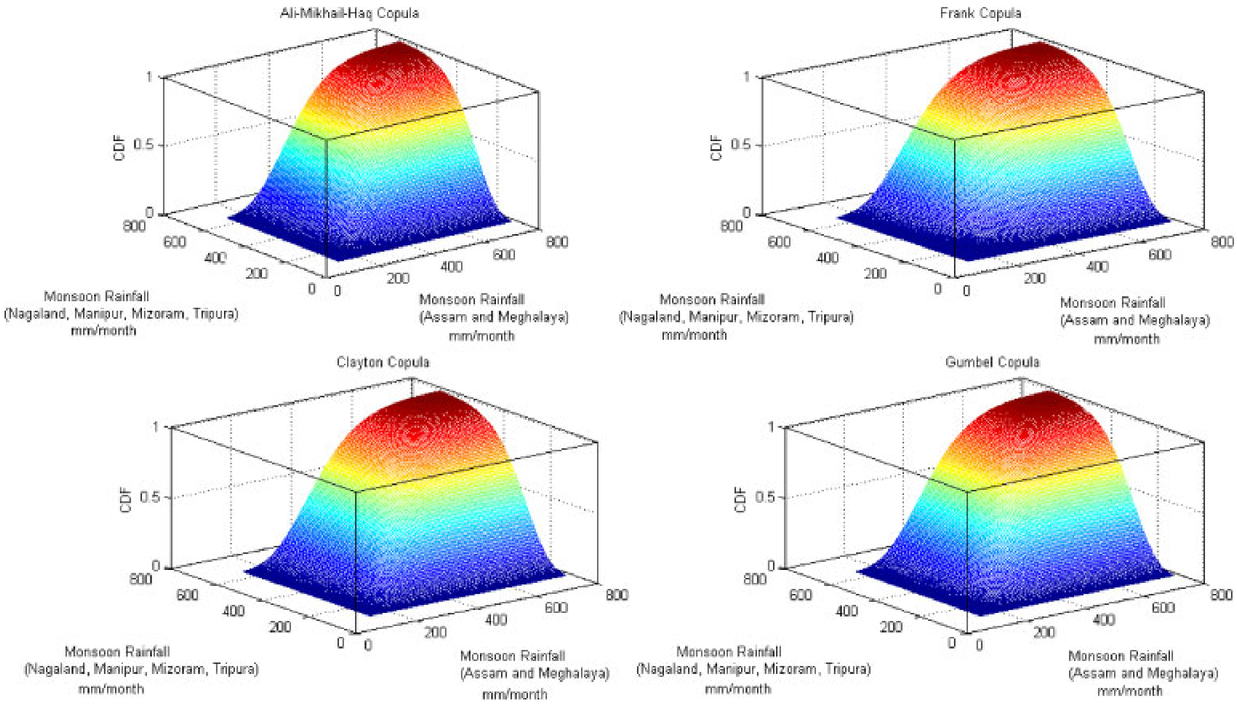

Figure 5. Bivariate distributions with Archimedean copulas

The results are presented in Table III which shows Gumbel–Hoggard copula is the best fitted copula and is in agreement with the nonparametric–parametric plot (Figure 6). As the correlation between the two variables of interest is positive, the assumption of positive dependence for Gumbel–Hoggard copula is also satisfied. Therefore, the Gumbel–Hoggard copula is selected for the present case study.

Table III. MSE, AIC and BIC for joint distributions

Copula	MSE	AIC	BIC
Gumbel–Hoggard	0·0099	−940·5247	−937·2066
Ali–Mikhail–Haq	0·0171	−828·5176	−825·1994
Frank	0·0267	−737·1128	−733·7947
Clayton	0·0101	−934·5519	−931·2338

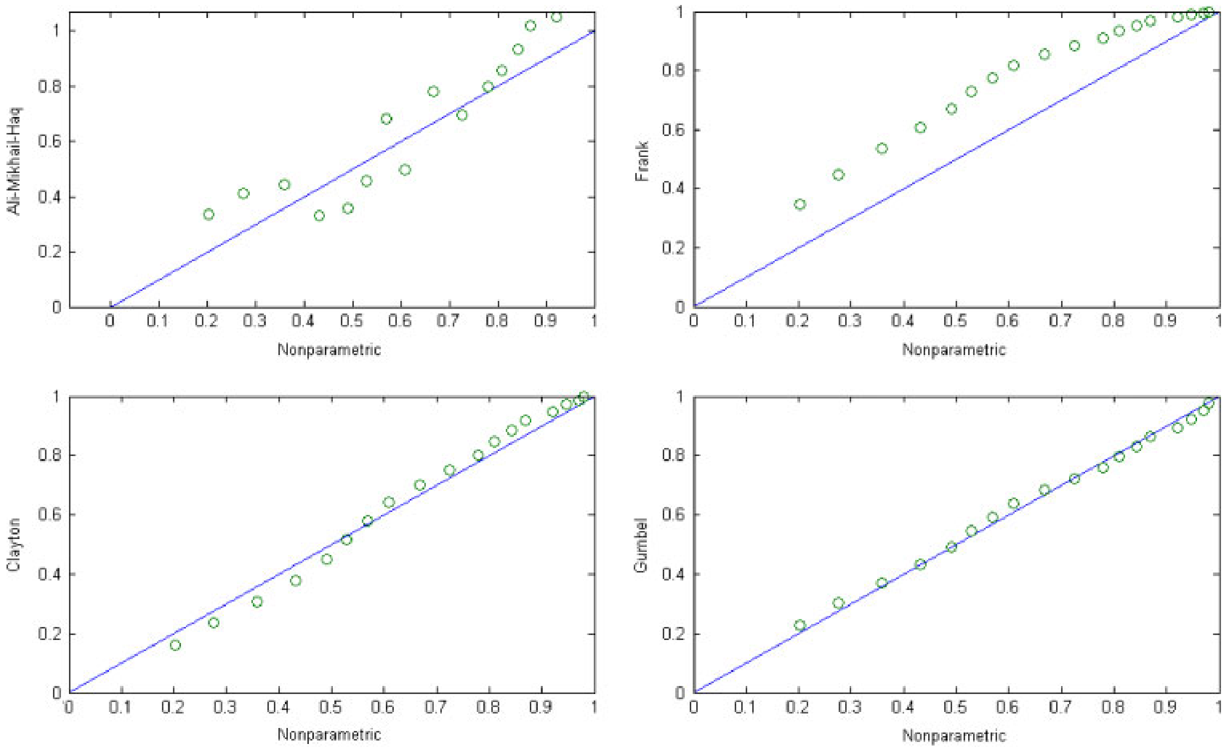

Figure 6. Selection of best fitted copula

TAIL DEPENDENCE

The notion of tail dependence relates to the amount of dependence in the upper-right quadrant tail or lower-left quadrant tail of a bivariate distribution (Poulin *et al.*, 2007). It is extremely important for the study of dependence between extreme values. The concept was introduced by Sibuya (1960) and the formal definition has been given by Joe (1997). Following the definition, the upper tail dependence coefficient is given by:

$$\lambda_U = \lim_{t \to 1^-} P\{F_X(x) > t | F_Y(y) > t\} \tag{7}$$

Similarly, the lower tail dependence coefficient is given by:

$$\lambda_L = \lim_{t \to 0^+} P\{F_X(x) < t | F_Y(y) < t\} \tag{8}$$

As this study deals with monsoon rainfall of North-East India, which is normally characterized by extreme rainfalls, and not really by low rainfall, the upper tail dependence is important for the present case study. Parametric estimators of upper tail dependence coefficient for Archimedean copula are presented in Table IV with their values.

For computing the upper tail dependence coefficient from a given sample, nonparametric estimators are used. They include LOG (Coles *et al.*, 1999), SEC (secant) (Joe, 1997) and CFG (Caperaa, Fougeres, Genest) estimators (Capéraà *et al.*, 1997). LOG and SEC estimators require a threshold, whereas the CFG estimator does not need a threshold and is used in the present study. As per CFG estimator, the nonparametric upper tail dependence coefficient is given by:

$$\hat{\lambda}_U^{CFG} = 2 - 2\exp\left\{\frac{1}{N}\sum_{i=1}^{N}\log\left[\frac{\sqrt{\log\frac{1}{u_i}\log\frac{1}{v_i}}}{\log\frac{1}{\max(u_i, v_i)^2}}\right]\right\} \tag{9}$$

where $u_i = F_X(x_i)$ and $v_i = F_Y(y_i)$. Although this estimator relies on the hypothesis that underlying empirical copula approximates an extreme value (EV) copula, Frahm *et al.* (2005) have shown that this assumption is not strong and the estimator performs well even, when the copula does not belong to EV class (Serinaldi, 2008). The upper tail dependence coefficient for the present case study using this estimator is obtained as 0·53, which in comparison with Table IV, shows Gumbel–Hoggard to be the best fitted copula (in agreement with section ‘Selection of Copula’). Therefore, the Gumbel–Hoggard copula is selected for further analysis, namely, bivariate data generation.

Table IV. Upper tail dependence coefficient for different copulas

Copula	λ_U	Value
Gumbel–Hoggard	$2 - 2^{1/\theta}$	0·57
Ali–Mikhail–Haq	0	0
Frank	0	0
Clayton	0	0

BIVARIATE DATA GENERATION

One of the primary applications of copulas is in simulation and Monte–Carlo studies. Following Nelsen (1999), the methodology used to generate bivariate random variable is given below:

1. Generate two independent uniform (0, 1) variates u and t.
2. Set $v = c_u^{(-1)}(t)$, where $c_u(v) = \frac{\partial}{\partial u}C(u, v)$ and $c_u^{(-1)}$ denotes a quasi-inverse of c_u.
3. The desired pair of CDFs is (u, v).
4. The desired variates are $x = F_X^{-1}(u)$ and $y = F_Y^{-1}(v)$

Bivariate random numbers are generated for the present case study with sample size 100, 1000 and 10 000. They are shown in Figure 7 with the observed data. All of them show a good match, which concludes that Gumbel–Hoggard copula well simulates the monsoon rainfall of AM and NMMT. As desired, as the sample size increase, i.e. it converges to population, the sample estimates converges to the population estimates, both in terms of Kendall's τ and upper tail dependence coefficient.

CONDITIONAL DISTRIBUTION AND RETURN PERIOD

The condition distribution based on copula may be given by:

$$F_{X|Y \le y} = C_{U|V \le v} = \frac{C(u, v)}{v} \tag{10}$$

Likewise, an equivalent formula for the conditional distribution function for V given $U \le u$ can be obtained. This is used to derive the conditional rainfall distribution of monsoon rainfall of each of the subdivisions, given the rainfall in other subdivision is equal to 25, 50, 75 and 99 percentile. The results are presented in Figure 8. Shifting of conditional CDF of monsoon rainfall towards the higher values in one subdivision with the increase of percentile value of other subdivision shows the positive nature of correlation. Similar analysis is performed with the total monsoon rainfall for both the subdivisions and it is observed that Gumbel–Hoggard copula fits best also for total monsoon rainfall. This is plotted in Figure 9 with the joint return period. The joint return period ($T(x, y)$) of a bivariate random variable (X, Y) corresponding to a value (x, y) is given by:

$$T(x, y) = \left(\frac{1}{1 - F_{XY}(x, y)}\right) \tag{11}$$

The information about joint return period has importance in designing hydraulic structure. For these two

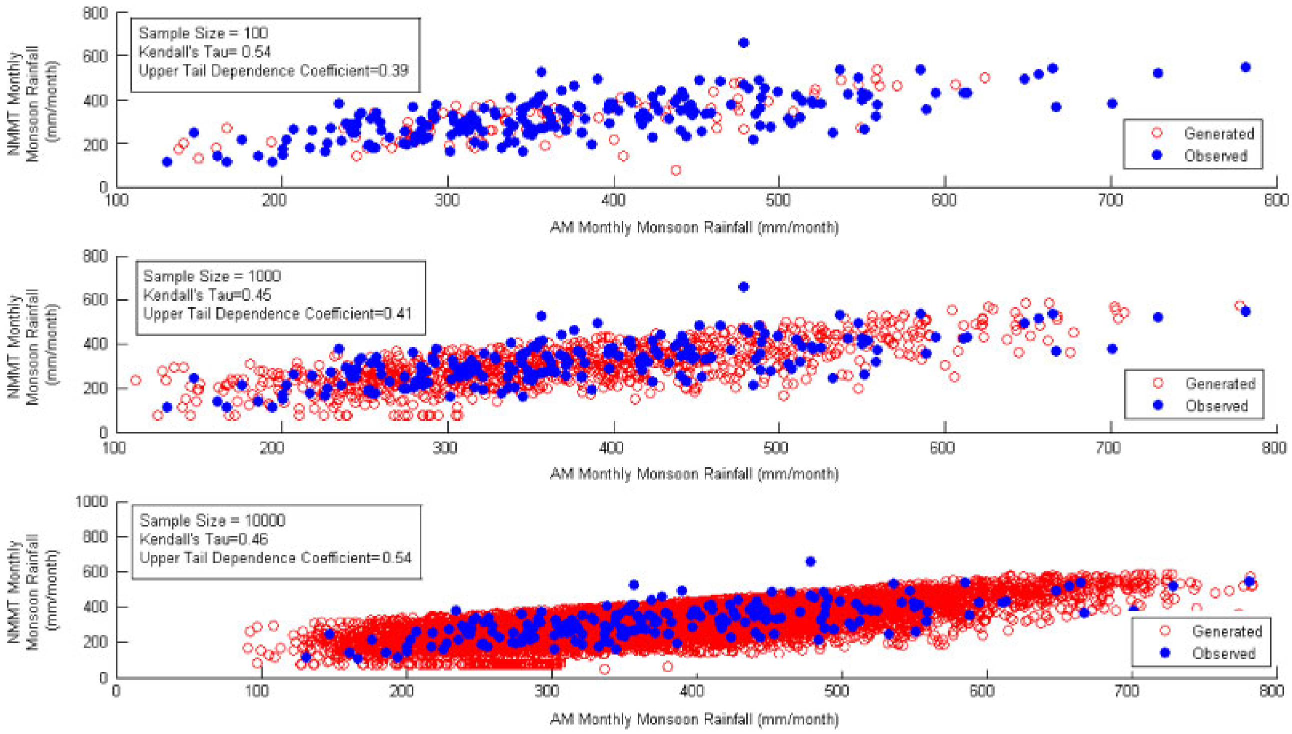

Figure 7. Generation of bivariate data using copula

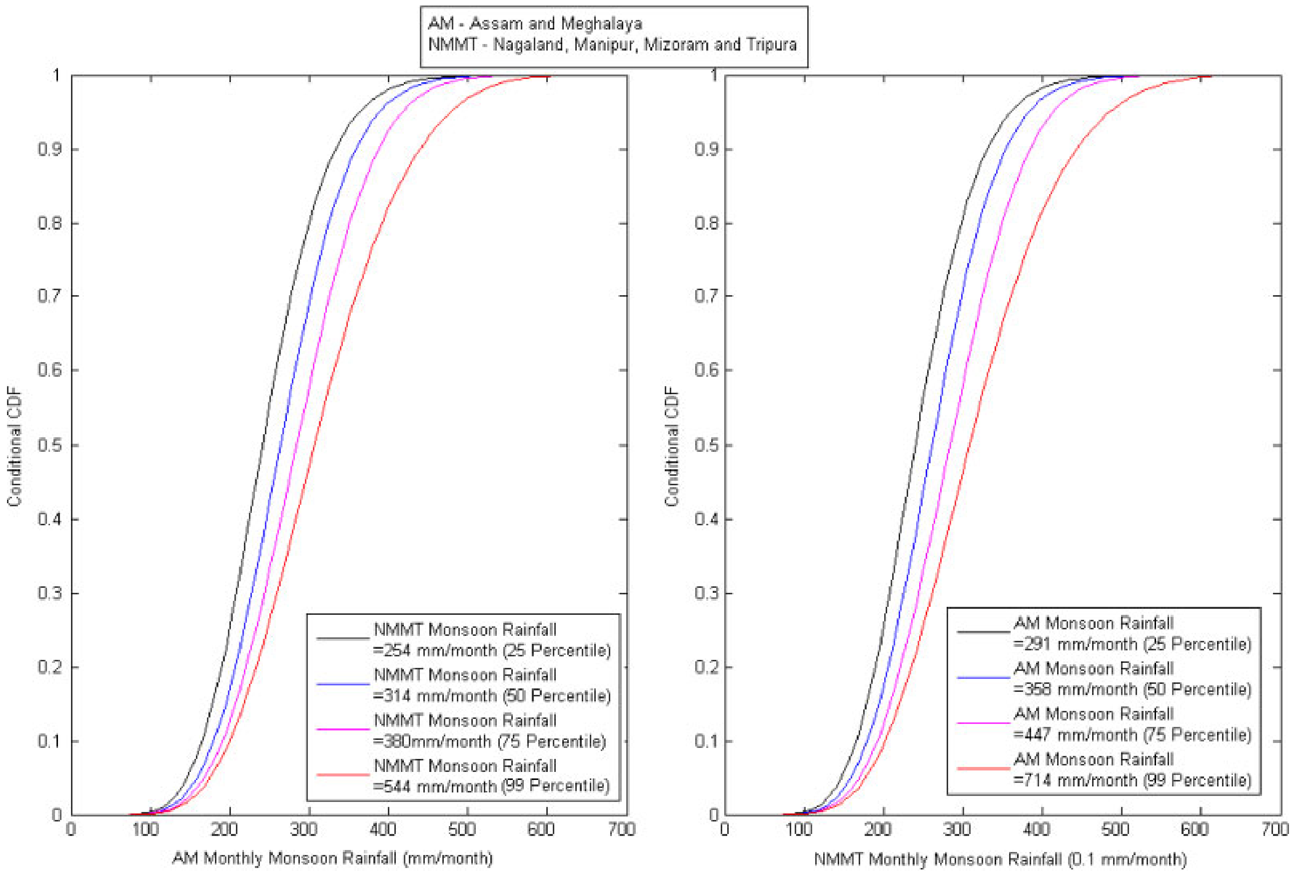

Figure 8. Conditional distributions

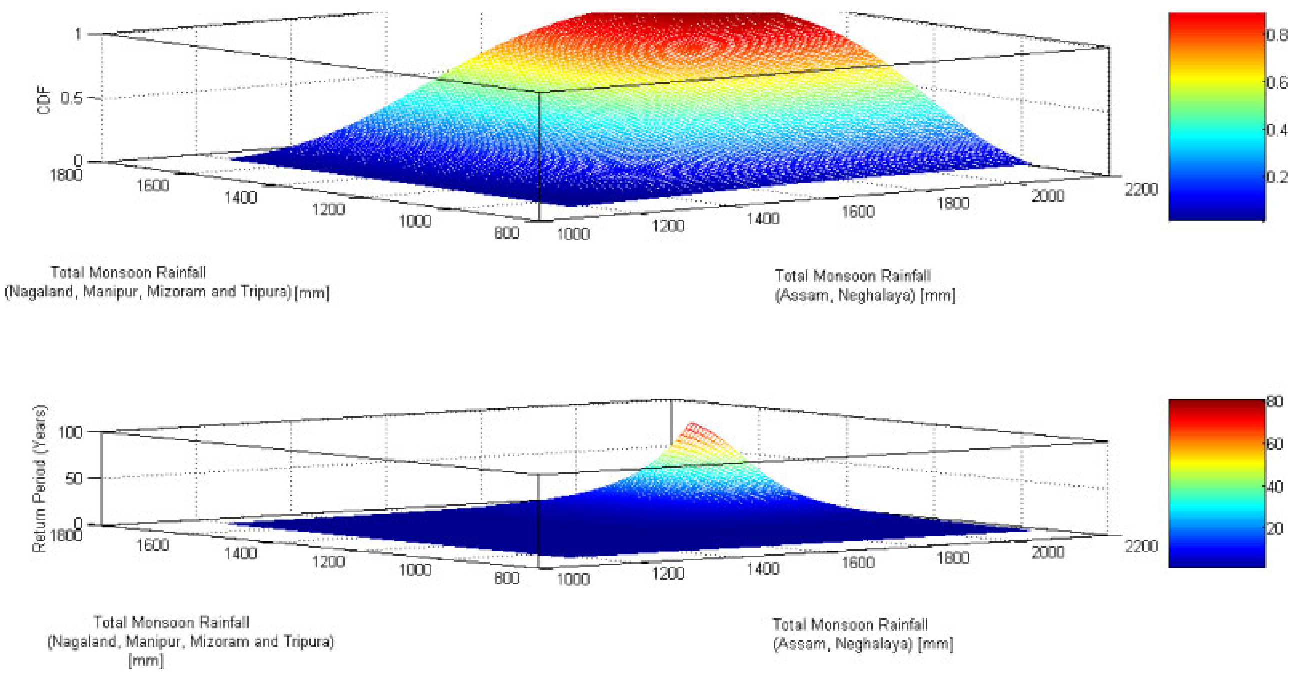

Figure 9. Joint distribution and return period for total monsoon rainfall in AM and NMMT

meteorological subdivisions, which belong to Brahmaputra river basin, Figure 9 will be helpful in computing the return periods of streamflows (computed with rainfall from these two subdivisions), which, in turn, will be helpful in designing river training works, water resources planning management and agricultural planning with monsoon rainfall.

CONCLUDING REMARKS

The concept of copula is used for derivation of joint and conditional distribution and simulation of monsoon rainfall in neighbouring meteorological subdivisions. Gumbel–Hoggard copula is found to be best fitted copula among Archimedean family in terms of nonparametric test, AIC, BIC and tail dependence. The data generation method with copula shows the convergence of sample estimate to population estimate with the increase of sample size. It should be noted that only Archimedean copulas are used in the present analysis. Use of elliptical copulas may result in a more robust model; however, the methodology and algorithms for selection of copula should remain the same. It should be noted that with multivariate nonparametric technique such as kernel distributions it is possible to generate multivariate PDFs/CDFs. However, major challenge in kernel density estimation is the selection of bandwidth and it has serious implications on final output. Generation of bivariate data is also quite straightforward using copula, which may not be for kernel density estimation method. Future scope of the work lies in use of copula generated correlated multi-station rainfall data in generating runoff and inclusion of copula for multi-site statistical downscaling to capture rainfall correlation between neighbouring stations for climate change impacts studies. The limitation of the present bivariate model is that it is unable to capture the autocorrelation. In this study, the objective of the work is to capture cross-correlation and is achieved by using copula. Capturing autocorrelation is important for time series analysis and generation of time series. Generation of multi-site time series considering both cross- and autocorrelation can be considered as the future scope of this study.

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