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Risk minimization in water quality control problems of a river system

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Abstract

Methodologies are presented for minimization of risk in a river water quality management problem. A risk minimization model is developed to minimize the risk of low water quality along a river in the face of conflict among various stake holders. The model consists of three parts: a water quality simulation model, a risk evaluation model with uncertainty analysis and an optimization model. Sensitivity analysis, First Order Reliability Analysis (FORA) and Monte–Carlo simulations are performed to evaluate the fuzzy risk of low water quality. Fuzzy multiobjective programming is used to formulate the multiobjective model. Probabilistic Global Search Laussane (PGSL), a global search algorithm developed recently, is used for solving the resulting non-linear optimization problem. The algorithm is based on the assumption that better sets of points are more likely to be found in the neighborhood of good sets of points, therefore intensifying the search in the regions that contain good solutions. Another model is developed for risk minimization, which deals with only the moments of the generated probability density functions of the water quality indicators. Suitable skewness values of water quality indicators, which lead to low fuzzy risk are identified. Results of the models are compared with the results of a deterministic fuzzy waste load allocation model (FWLAM), when methodologies are applied to the case study of Tunga–Bhadra river system in southern India, with a steady state BOD–DO model. The fractional removal levels resulting from the risk minimization model are slightly higher, but result in a significant reduction in risk of low water quality.

Keywords: Fuzzy sets; Optimization models; Uncertainty analysis; Waste management; Water quality

1. Introduction

Waste load allocation (WLA) in streams refers to the determination of required pollutant treatment levels at a set of point sources of pollution to ensure that an acceptable level of water quality is maintained throughout the stream. Waste load allocation problems are characterized by uncertainties due to both randomness and imprecision. Uncertainty due to randomness arises mainly due to the random nature of the input variables of water quality simulation model and that due to imprecision or fuzziness is associated with setting up the water quality standards and goals of Pollution Control Agencies (PCA), and the dischargers. Applications of fuzzy sets in addressing uncertainty due to imprecision in water resources problems may be found in Bender and Simonovic [2] and Teegavarapu and Simonovic [27].

Some early works related to the optimal Waste Load Allocation (WLA) problems are due to Liebman and Lynn [10] and Loucks et al. [12]. The WLA problem was formulated as a single objective optimization problem by them. Burn and McBean [5,6] presented a stochastic optimization model in which the concern

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was with the interaction between BOD and DO concentration in the river. Fujiwara et al. [8,9] developed a recursive procedure to express the dissolved oxygen concentration at a water quality checkpoint in a river system in terms of the fractional removal levels of the pollutants discharged to the river system. Takyi and Lence [24] proposed a Markov chain model for seasonal water quality management of a river system. Chebyshev criterion was used by Takyi and Lence [25] to develop direct regulation of a water quality management model that maximized excess water quality above the water quality goal at all checkpoints along the river. The Multiple Realization Model (MULTREA) proposed by Takyi and Lence [26], uses both simulation and optimization to address uncertainty in a WLA model.

Multiobjective problem solving technique was introduced by Monarchi et al. [14], which allowed the decision maker to trade off one objective versus another in an interactive manner. Tung and Hathhorn [28] reveal that fuzzy optimization is a valuable tool for solving the multiobjective water quality management problems. Wen and Lee [29] presented a multiobjective optimization approach based on neural networks for water quality management in a river basin. Sasikumar and Mujumdar [19] have formulated fuzzy waste load allocation model (FWLAM) to demonstrate the application of fuzzy decision making in a water quality management of a river system. Mujumdar and Subbarao [16] used genetic algorithm to solve FWLAM using the water quality simulation model QUAL2E, developed by the US Environmental Protection Agency.

A methodology for evaluating the fuzzy risk of low water quality is presented by Subbarao et al. [21]. First Order Reliability Analysis (FORA) and Monte–Carlo Simulations (MCS) were applied to compute the fuzzy risk. Starting with that methodology two models are developed in this paper to minimize the fuzzy risk of low water quality: (i) risk minimization model and (ii) Modified fuzzy waste load allocation model. The risk minimization model consists of three component models: (i) water quality simulation model; (ii) fuzzy risk evaluation model and (iii) optimization model to minimize the risk. The water quality simulation model is a simple one-dimensional BOD–DO model and is solved by the backward finite difference method. In the modified fuzzy waste load allocation, the FWLAM is modified accounting for skewness of the water quality indicator DO. Here base values are not considered in the optimization model; only the moments (i.e. mean, variance and skewness) of water quality indicator are used. Resulting non-linear optimization problems of both the models are solved by Probabilistic Global Search Lausanne (PGSL), a direct stochastic algorithm for global search developed by Raphael and Smith [18]. Tests on the benchmark problem having multi-parameter non-linear objective function have revealed that PGSL performs better than genetic algorithm and advanced algorithms for simulated annealing [17].

2. Risk minimization model

Uncertainties due to both randomness and imprecision in a water quality management problem are considered in developing the risk minimization model. Water quality at a location in a stream depends on the variability of a number of input parameters (e.g. temperature, streamflow, upstream water quality etc.). In FWLAM developed earlier, such variability is not considered, as it is a deterministic model. Risk minimization model, developed in this paper, has two parts: deterministic and stochastic. The deterministic part maximizes the minimum acceptability level of FWLAM for mean condition of the input parameters. The stochastic part minimizes the fuzzy risk of low water quality, incorporating both types of uncertainties in the optimization model. For ease in understanding the risk minimization model, a brief overview of FWLAM is first presented in the following Section 2.1.

2.1. FWLAM

The fuzzy waste load allocation model (FWLAM) developed by Sasikumar and Mujumdar [19] forms the basis for the optimization models developed in this paper. The FWLAM is described using a general river system. The river consists of a set of dischargers that are allowed to release pollutants into the river after removing some fraction of the pollutants. These fractional removal levels of the pollutants are necessary to maintain the acceptable water quality condition in the river as prescribed by the Pollution Control Agency (PCA). The acceptable water quality condition is ensured by checking the water quality in terms of water quality indicator levels (e.g., DO concentration) at a finite number of locations which are referred to as checkpoints. In a water quality management model, the concentration level of the water quality indicator at a checkpoint is expressed as a function of the fractional removal levels for the pollutants released by the dischargers upstream of that checkpoint in the river system. An optimization problem is formulated with the set of fractional removal levels and the minimum acceptability level forming the decision variables. The goal of the PCA is to improve the water quality and that of the dischargers is to minimize the fractional removal levels and they are in conflict with each other. These goals are treated as fuzzy goals and modeled using appropriate fuzzy membership functions. In the FWLAM, the following fuzzy optimization problem is formulated to take into account the fuzzy goals of the PCA and dischargers.

$$\text{Maximize} \quad \lambda \tag{1}$$

subject to

$$[(c_{il} - c_{il}^L)/(c_{il}^D - c_{il}^L)]^{\alpha_{il}} \geqslant \lambda \quad \forall i, l \tag{2}$$

$$[(x_{imn}^M - x_{imn})/(x_{imn}^M - x_{imn}^L)]^{\beta_{imn}} \geqslant \lambda \quad \forall i, m, n \tag{3}$$

$$c_{il}^L \leqslant c_{il} \leqslant c_{il}^D \quad \forall i, l \tag{4}$$

$$\max\,[x_{imn}^L, x_{imn}^{\mathrm{MIN}}] \leqslant x_{imn} \leqslant x_{imn}^M \quad \forall i, m, n \tag{5}$$

$$0 \leqslant \lambda \leqslant 1 \tag{6}$$

where, c_{il} is the concentration of water quality indicator i at checkpoint l. The model is a multiobjective formulation maximizing minimum satisfaction level (λ). In the fuzzy constraints (2) and (3) the goals of PCA and dischargers, respectively, are made greater than or equal to λ, to formulate this MAX–MIN model. The lower and upper bounds of water quality indicator i at the checkpoint l are fixed as permissible (c_{il}^L) and desirable level (c_{il}^D), respectively as set by PCA in constraint (4). The bounds of fractional removal level x_{imn} of the pollutant n from the discharger m to control the water quality indicator i in the river system, is given by constraint (5). The aspiration level and maximum fractional removal level acceptable to the discharger m with respect to x_{imn} are represented as, x_{imn}^L and x_{imn}^M, respectively. The PCA imposes minimum fractional removal levels that are also expressed as the lower bounds, x_{imn}^{MIN} in constraint (5).

The exponents, α_{il} and β_{imn}, appearing in constraints (2) and (3), respectively, are non-zero positive real numbers. Assignment of numerical values to these exponents is subject to the desired shape of the membership functions and may be chosen appropriately by the decision maker. The concentration of water quality indicator c_{il} in constraints (2) and (4) is determined using a water quality simulation model.

2.2. Model formulation

The conventional water quality criteria at checkpoint l are such that any concentration of the water quality indicator less than a specified value, say, c_{il}^L, corresponds to a low water quality. This leads to a very stringent definition of low water quality. To overcome this limitation and to account for imprecision in the description of low water quality, Sasikumar and Mujumdar [20] and Mujumdar and Sasikumar [15] have introduced a fuzzy set based definition in place of the crisp set based definition of low water quality. The fuzzy risk of low water quality is defined as the probability of occurrence of the fuzzy event of low water quality. Mathematically, this can be stated as follows:

$$\text{fuzzy risk} = P\ (\text{fuzzy event of low water quality}) \tag{7}$$

$$= \widetilde{P}\ (\text{low water quality}), \tag{8}$$

where $\widetilde{P}$ denotes the probability of a fuzzy event.

Denoting the fuzzy set of low water quality, DO concentration, and fuzzy risk of low water quality by W_l, c_l, and r_l, respectively, the fuzzy risk can be expressed in discrete form, as

$$r_l = \sum_{c_{\min_l}}^{\mathrm{MIN}[c_{\max_l}, c_l^D]} \mu_{W_l}(c_l) p(c_l) \tag{9}$$

where $c_{\min_l}$ and $c_{\max_l}$ are the minimum and maximum concentration levels of DO obtained from MCS at checkpoint l. The subscript i for the water quality indicator may be added in Eq. (9) in case of multiple water quality indicators.

Fig. 1 shows a typical membership function of low water quality, $\mu_{W_l}(c_l)$, which is expressed as,

$$\mu_{W_l}(c_l) = [(c_l^D - c_l)/(c_l^D - c_l^L)]^{\gamma_l} \tag{10}$$

where γ_l is the non-zero positive real number defining the shape of the membership function at location l. In the present study a linear membership function (i.e. $\gamma_l = 1$) is assumed. Details of the procedure for evaluating fuzzy risk may be found in Subbarao et al. [21]. In that work, Sensitivity analysis and First Order Reliability Analysis are performed for screening the basic variables, and Monte–Carlo simulation is used for deriving the probability density function of the water quality indicator.

Here, a non-linear multiobjective optimization model is developed to minimize the fuzzy risk of low water quality in river water quality management. The model consists of two objective functions, (i) to maximize the acceptability level resulting from fuzzy waste load allocation model (FWLAM) for base values of input variables and (ii) to minimize the sum of the risks of low water quality at all the checkpoints. The risk minimization model considers the probability density function of the output variable, as it is associated with fuzzy risk of low water quality. Mathematically the model is represented as

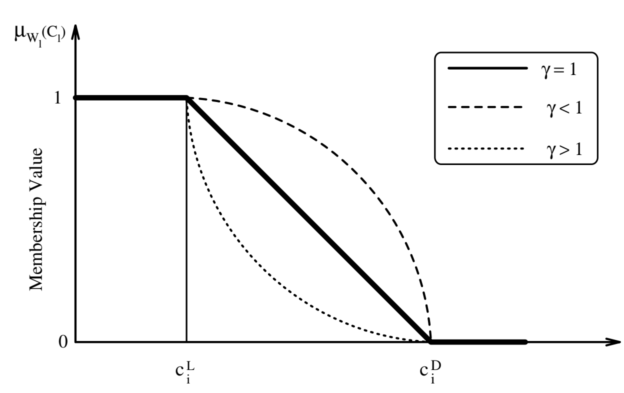

Fig. 1. Membership function for low water quality.

$$\text{Maximize} \quad \lambda \tag{11}$$
$$\text{Minimize} \quad \vartheta_s \tag{12}$$
subject to
$$[(c_{il} - c_{il}^L)/(c_{il}^D - c_{il}^L)]^{\alpha_{il}} \geqslant \lambda \quad \forall i, l \tag{13}$$
$$[(x_{imn}^M - x_{imn})/(x_{imn}^M - x_{imn}^L)]^{\beta_{imn}} \geqslant \lambda \quad \forall i, m, n \tag{14}$$
$$c_{il}^L \leqslant c_{il} \leqslant c_{il}^D \quad \forall i, l \tag{15}$$
$$\max\,[x_{imn}^L, x_{imn}^{\mathrm{MIN}}] \leqslant x_{imn} \leqslant x_{imn}^M \quad \forall i, m, n \tag{16}$$
$$\vartheta_s = f(X) \tag{17}$$
$$0 \leqslant \lambda \leqslant 1 \tag{18}$$

where $\vartheta_s =$ sum of fuzzy risk of low water quality at all the check points and $X =$ vector of fractional removal levels. $f(\cdot)$ is a function notation.

Fuzzy multiobjective programming [30] is used to solve the problem. The model is solved for each of the two objective functions at a time. The model is first solved with the objective to maximize the acceptability level, λ. The solution gives the best value of λ. With the fractional removal levels resulting from this first run, the sum of the risk ϑ_s is evaluated. This is the worst value of ϑ_s as it is not included in the objective function for the first run. In the second run, the model is solved with the objective to minimize ϑ_s. The ϑ_s value resulting from this run gives the best value of the sum of risk, and the corresponding λ value is the worst value of acceptability level. Using the best and worst values of λ and ϑ_s, appropriate membership functions are developed for each of the objective functions. For λ value a non-decreasing membership function is used since the objective of the model is to maximize λ. The membership function is expressed as follows:

$$\mu_\lambda = [(\lambda - \lambda^-)/(\lambda^+ - \lambda^-)]^\phi \tag{19}$$

where $\mu_\lambda =$ membership function for λ

$\lambda^+ =$ best value of λ;

$\lambda^- =$ worst value of λ;

Similarly, for the sum of fuzzy risk at all the checkpoints a non-increasing membership function is used because the objective is to minimize the fuzzy risk.

$$\mu_{\vartheta_s} = [(\vartheta_s^- - \vartheta_s)/(\vartheta_s^- - \vartheta_s^+)]^\eta; \tag{20}$$

where $\mu_{\vartheta_s} =$ membership function for sum of the risk (r_s)

$\vartheta_s^+ =$ best value of ϑ_s;

$\vartheta_s^- =$ worst value of ϑ_s

The exponents, ϕ and η, appearing in constraints (19) and (20) respectively, are non-zero positive real numbers. Assignment of numerical values to these exponents is subject to the desired shape of the membership functions and may be chosen appropriately by the decision maker. As with the increase in λ value the fractional removal level decreases with an increase in the value of sum of the risks at the checkpoints, the two objective functions are in conflict with each other. With the membership functions of the two objectives the following max–min multiobjective model is developed.

$$\text{Maximize} \quad \epsilon \tag{21}$$
subject to
$$[(\lambda - \lambda^-)/(\lambda^+ - \lambda^-)]^\phi \geqslant \epsilon \tag{22}$$
$$[(\vartheta_s^- - \vartheta_s)/(\vartheta_s^- - \vartheta_s^+)]^\eta \geqslant \epsilon \tag{23}$$
$$[(c_{il} - c_{il}^L)/(c_{il}^D - c_{il}^L)]^{\alpha_{il}} \geqslant \lambda \quad \forall i, l \tag{24}$$
$$[(x_{imn}^M - x_{imn})/(x_{imn}^M - x_{imn}^L)]^{\beta_{imn}} \geqslant \lambda \quad \forall i, m, n \tag{25}$$
$$c_{il}^L \leqslant c_{il} \leqslant c_{il}^D \quad \forall i, l \tag{26}$$
$$\max\,[x_{imn}^L, x_{imn}^{\mathrm{MIN}}] \leqslant x_{imn} \leqslant x_{imn}^M \quad \forall i, m, n \tag{27}$$
$$\vartheta_s = f(X) \tag{28}$$
$$0 \leqslant \lambda \leqslant 1 \tag{29}$$
$$0 \leqslant \epsilon \leqslant 1 \tag{30}$$

In the model a new variable, goal fulfillment level (ϵ) is introduced. The objective function (21) along with the minimum membership value constraints (22) and (23) ensure the maximization of minimum goal fulfillment level. The other constraints of the model are the same as those in FWLAM. The objective is to maximize the minimum goal fulfillment level, which results in a non-inferior solution, making both the objective values (λ and ϑ_s) as good as possible without violating the constraints.

As the model results in a non-linear optimization problem, Probabilistic Global Search Laussane (PGSL), a global search algorithm is used to solve the problem. Details of the algorithm are given in Section 4.

3. Modified fuzzy waste load allocation model

To minimize the fuzzy risk of low water quality with minimum computational effort, a Modified fuzzy waste load allocation model (MFWLAM) is developed. It is essentially a modified version of FWLAM, dealing with first, second and third order moments of the water quality indicator considering the random nature of input variables. The objective of this model is not only to determine the fractional removal levels of the effluents considering the aspirations and conflicting objectives of the pollution control agency and dischargers, but also to minimize the fuzzy risk of low water quality by incorporating the skewness of the probability density function of the water quality indicator.

Modified fuzzy waste load allocation model (MFWLAM) does not consider the base values of the input variables. It is a stochastic optimization model dealing with the moments (i.e. mean and variance) of the input variables. The model is based on fuzzy decision theory. To minimize the fuzzy risk of low water quality, the membership function of skewness of water quality indicator is also incorporated in the model. The nature of membership function is selected for the skewness depending on the water quality indicator. For example, for skewness of DO a non-increasing membership function should be assumed as negative skewness is preferred for lower value of risk. The fuzzy risk (Eq. (9)) is the integration (summation for discrete values) of the product of fuzzy membership function of low water quality and pdf (pmf for discrete values) of the water quality indicator. For DO this membership function is non-increasing, leading to a low value of risk for negative skewness. Similarly for skewness of BOD an non-decreasing membership function should be used as positive skewness is preferred.

The bounds of the water quality indicator are determined from Chebyshev's inequality. The proportion of observations lying k standard deviation outside the mean value is at most $1/k^2$, which can be mathematically stated as,

$$P(|Z - \bar{Z}| \geqslant k\sigma) \leqslant \frac{1}{k^2} \tag{31}$$

where Z = a random variable, $\bar{Z}$ = mean value of Z, σ = standard deviation of Z and $k \geqslant 0$. From the Chebyshev's inequality the constraint (4) is modified as follows:

$$c_{il}^L \leqslant (\overline{c_{il}} - k\sigma_{c_{il}}) \quad \forall i,l \tag{32}$$

$$c_{il}^D \geqslant (\overline{c_{il}} + k\sigma_{c_{il}}) \quad \forall i,l \tag{33}$$

where $\overline{c_{il}}$ = mean value of water quality indicator i at checkpoint l and $\sigma_{c_{il}}$ = standard deviation of water quality indicator i at checkpoint l. The condition (32) ensures that $P[c_{il} \leqslant c_{il}^L] \leqslant 1/k^2$.

It may not possible to use both the upper and lower bounds of Chebyshev's inequality for the water quality indicator, when there is a high variability in the output, but the difference between desirable and permissible values c_{il}^L and c_{il}^D is small leading to an infeasible solution. As a better water quality is desired, the upper bound for $\overline{c_{il}}$ (Eq. (33)) is modified by making the mean value $\overline{c_{il}}$ less than desirable level, c_{il}^D. Finally the MAX–MIN formulation of the model can be written as

$$\text{Maximize} \quad \lambda \tag{34}$$

subject to

$$[(\overline{c_{il}} - c_{il}^L)/(c_{il}^D - c_{il}^L)]^{\alpha_{il}} \geqslant \lambda \quad \forall i,l \tag{35}$$

$$[(x_{imn}^M - x_{imn})/(x_{imn}^M - x_{imn}^L)]^{\beta_{imn}} \geqslant \lambda \quad \forall i,m,n \tag{36}$$

$$\mu(s_{c_{il}}) \geqslant \lambda \quad \forall i,l \tag{37}$$

$$c_{il}^L \leqslant (\overline{c_{il}} - k\sigma_{c_{il}}) \quad \forall i,l \tag{38}$$

$$c_{il}^D \geqslant \overline{c_{il}} \quad \forall i,l \tag{39}$$

$$\max[x_{imn}^L, x_{imn}^{\text{MIN}}] \leqslant x_{imn} \leqslant x_{imn}^M \quad \forall i,m,n \tag{40}$$

$$0 \leqslant \lambda \leqslant 1 \tag{41}$$

where $\mu(s_{c_{il}})$ = membership function for the skewness of water quality indicator i at checkpoint l.

As the water quality simulation model used here is a non-linear model, PGSL is used to solve the optimization problem.

4. Probabilistic Global Search Lausanne

Probabilistic Global Search Lausanne (PGSL), a new search algorithm for non-linear optimization has been developed by Raphael and Smith [18]. Tests on benchmark problems having multi-parameter non-linear objective functions revealed that PGSL performs better than Genetic Algorithm and advanced algorithms for Simulated Annealing [17]. It is basically a direct search method which depends on the objective function only through ranking a countable set of function values. The methodology is similar to that of the Adaptive Cluster Covering (ACCO) algorithm which is widely used in solving water resources problems and that has been proved to be faster than Genetic Algorithms [22,23]. PGSL uses the assumption that better sets of points are more likely to be found in the neighborhood of good sets of points, therefore intensifying the search in regions that contain good solutions [7]. PGSL has already proved to be valuable for engineering tasks in areas of design, diagnosis and control [17].

PGSL algorithm consists of four nested cycles: sampling cycle, probability updating cycle, focusing cycle and subdomain cycle. In the sampling cycle a number of points (say NS) are generated randomly by generating a value for each variable according to the PDF of the variable. Among them the best sample is selected. In a probability updating cycle the sampling cycle is invoked for a number of times (say NPUC). After each iteration, the PDF of each variable is modified. The interval containing the best solution is first selected and then the probability of that interval is multiplied by a factor greater than 1. The PDF thus generated, is then modified to make the area under the density function equal to unity. This ensures that the sampling frequencies in regions containing good points are increased. In a focusing cycle, the probability updating cycle is repeated for NFC times. After each iteration, the search is increasingly focused on the interval containing the current best point. The interval containing the best point is divided into uniform subintervals. Fifty percent probability is assigned to this interval. The remaining probability is then distributed to the region outside this interval in such a way that the PDF decays exponentially from the best interval. In subdomain cycle, the focusing cycle is repeated NSDC times and at the end of each iteration, the current search space is modified. In the beginning the entire space is searched, but in subsequent iterations a subdomain is selected for search. The size of the subdomain decreases gradually and the solution converges to a point.

A feature that PGSL shares with other random search methods is the use of the probability density function (PDF). However, the differences between PGSL and other random search methods are as follows [7]:

1. Other random methods that make use of an explicitly defined PDF follow a creep procedure similar to simulated annealing. They aim for a point to point improvement by restricting the search to a region around the current point. The PDF is used to search within a small neighborhood. On the other hand, PGSL works by global sampling. There is no point to point movement.
2. The four nested cycles in PGSL are not similar to any features of other algorithms.
3. Representation of probabilities in PGSL is different. Other methods make use of a mathematical function with a single peak (e.g., Gaussian) for the PDF. PGSL uses a histogram, a discontinuous function with multiple peaks. This allows a fine control over probabilities in small regions by subdividing intervals.
4. Probabilities are updated differently in PGSL. The primary mechanism for updating probabilities in other methods is in changing the standard deviation. In PGSL, the entire shape and form of the PDF can be changed by subdividing as well as updating probabilities of the intervals.

5. Case study

Application of the models are illustrated through a case-study of Tunga–Bhadra river system shown schematically in Fig. 2. The Tunga–Bhadra river is a perennial river formed by the confluence of Tunga and Bhadra rivers, both tributaries of the Krishna river, in southern India. The river has two other tributaries, the Kumadavati and Haridra rivers. They join the Tunga–Bhadra river from the west and east at a distance of 84 km and 124 km, respectively, downstream of the confluence of Tunga and Bhadra rivers. The river network is discretized into 15 reaches depending on the river morphology and river environment. The river receives the waste loads from eight major effluent points which include five industrial effluents and three municipal effluents. The models are applied to a river stretch of 180 km that comprises the four headwaters (Tunga, Bhadra, Kumadavati, and Haridra) and eight point loads (five industrial and three municipal effluents). Tunga has the highest mean headwater flow of 166.89 m^3/s. Bhadra, Kumadavati and Haridra have the mean headwater flows 17.80 m^3/s, 14.94 m^3/s and 13.90 m^3/s respectively. The value of incremental flow is calculated based on the guage stations located in

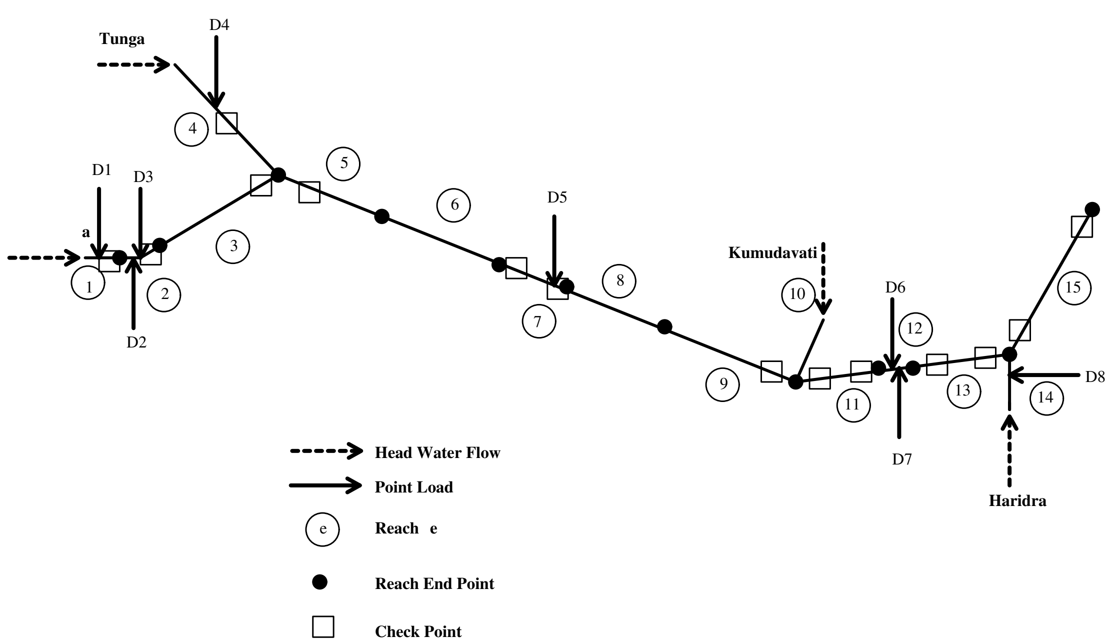

Fig. 2. Schematic diagram of the Tunga–Bhadra river system.

Bhadra (Reach 1), Tunga (Reach 4) and Tunga–Bhadra (Reach 7) rivers. Difference between sum of the flows at the Bhadra and Tunga guage stations and Tunga–Bhadra guage station is the flow incremented distributively. The ratio of this difference to the length between the guage stations gives the distributed load per unit distance, which is 0.34 m^3/s/km in the present case. This value is used as incremental flow throughout the river stretch, to account for the non-point source pollution due to runoff. To keep emphasis on simultaneous treatment of randomness and imprecision, the example is kept simple by considering only one water quality indicator, the DO. Table 1 presents the mean values of point load flow discharge, DO and BOD, calculated from the historical data. The non-point BOD pollutant is primarily contributed by rural communities, animal husbandry and on-stream activities in India. To conservatively account for uncertainty arising out of lack of adequate data on non-point source pollution in the present case, a high value of 30 mg/L for BOD and a low value of 4 mg/L for DO are used for the incremental flow in the analysis [21]. The uncertainty information of the basic variables are taken from Brown and Barnwell [3] and Subbarao et al. [21]. Based on the literature [1,4,9,13,21], all the input variables except headwater flow are assumed to follow a normal distribution for the purpose of analysis. A log-normal distribution is used for the headwater flow. A minimum fractional removal level of 35% and a maximum treatment level of 90% are assumed for the dischargers. Aspiration level and maximum permissible level of the dischargers are thus set to 35% and 90%, respectively. The maximum treatment level is imposed considering the technological constraints, whereas a minimum treatment level is imposed to ensure better quality conditions.

Table 1
Effluent flow data

Discharger	BOD conc. (mg/L)	DO conc. (mg/L)	Effluent flow (m^3/s)
D_1	1000	2.0	0.705
D_2	440	2.0	0.308
D_3	300	2.0	0.026
D_4	900	2.0	0.436
D_5	222	2.0	0.024
D_6	600	2.0	0.129
D_7	450	2.0	0.952
D_8	900	2.0	0.867

5.1. Model application

In the Tunga–Bhadra river system, the water quality is checked at 14 checkpoints. Details of the checkpoints including significance are given in Table 2. Two thousand runs of Monte–Carlo Simulations (MCS) are performed to derive the probability density function of the water quality indicator (DO). The methodology used for minimization of risk is shown in Fig. 3. PGSL method is used as an optimization tool to solve the non-linear problem. Two sampling cycles, 1 probability updating cycle, 80 focusing cycles and 30 subdomain cycles have been used in the present analysis. The model is solved for one objective function at a time. Maximization of λ gives the value of λ as 0.423, which is the best value of λ. With this solution the sum of the risk is found to be 5.035, which is the worst value of sum of the risk. Similarly, the model is solved for minimization of sum of risk of low water quality. The value of the sum of the risk is found to be 4.117, which is the best value of sum of the risk. The corresponding λ value is 0, which is the worst value of λ.

Appropriate membership functions have been assumed for the objective functions (Eqs. (19) and (20)). The problem is solved for three different values of ϕ and η; 0.8, 1 and 1.25 (Fig. 4). Taking all the combina-

Table 2
Details of the checkpoints

Loc.	Rch.	Ele.	Distance (km)	Permissible DO (mg/L)	Desirable DO (mg/L)	Significance
1	1	3	3	4.00	7.49	d/s of point load 1
2	2	3	7	4.00	7.49	d/s of point load 2 and 3
3	3	20	27	4.00	7.49	u/s of Tunga–Bhadra Junction
4	4	3	3	6.00	7.43	d/s of point load 4 (Tunga)
5	5	2	29	6.00	7.43	d/s of Tunga–Bhadra junction
6	7	1	68	6.00	7.43	An intermediate location
7	7	10	77	6.00	7.43	d/s of point load 5
8	9	19	110	6.00	7.43	u/s of Kumadavati junction
9	11	2	113	6.00	7.43	d/s of Kumadavati junction
10	11	16	127	6.00	7.43	u/s of point loads 6 and 7
11	13	1	130	6.00	7.34	d/s of point loads 6 and 7
12	13	12	141	6.00	7.34	u/s of Haridra junction
13	15	2	143	6.00	7.43	d/s of Haridra junction
14	15	19	160	6.00	7.43	most d/s location of the river

Note: u/s = upstream; d/s = downstream; Loc. = Location; Ele. = Element; Rch. = Reach.

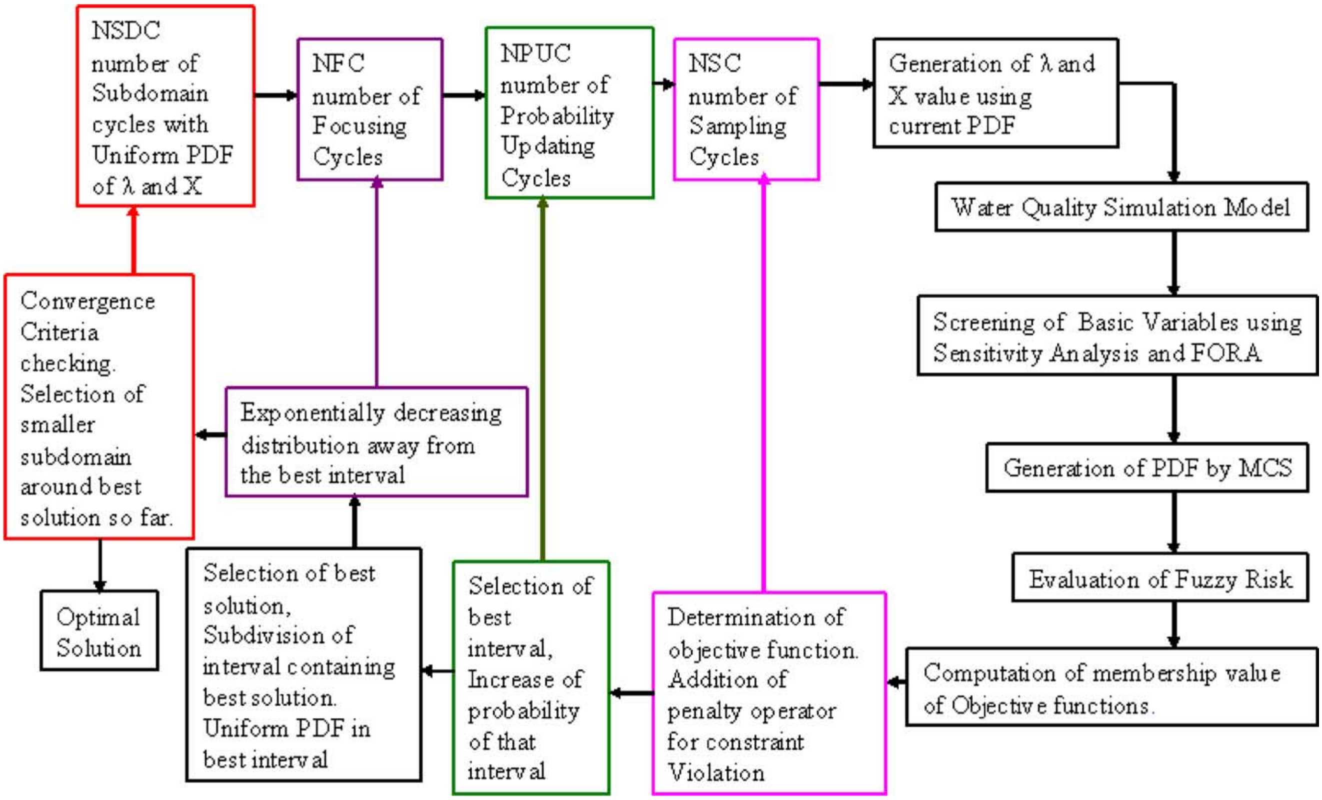

Fig. 3. Risk minimization model including PGSL algorithm.

tions, 9 sets of analyses have been performed in the present study. As both the membership functions are greater than or equal to ϵ, minimum of the membership function values of objective functions are taken as ϵ (goal fulfillment level).

For the modified fuzzy waste load allocation model, both the MAX–MIN and MAX-BIAS formulations are developed. As, the water quality indicator is DO, a non-increasing membership function is assumed for

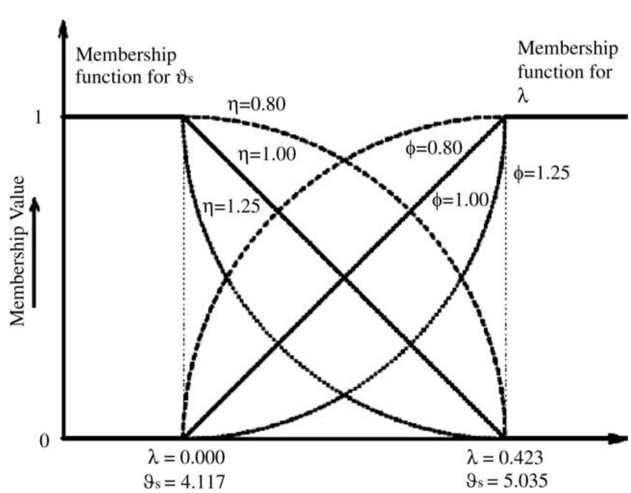

Fig. 4. Membership functions for objective functions.

skewness taking the best value as −5 and worst value as 5. In the MAX-BIAS formulation the deviations of satisfaction levels of PCA and discharger from minimum acceptability level (λ) are considered in the objective function [19].

$$d_{il} = \mu_{E_{il}}(c_{il}) - \lambda \quad \forall i, l \tag{42}$$

$$d_{imn} = \mu_{F_{imn}}(x_{imn}) - \lambda \quad \forall i, m, n \tag{43}$$

summing the deviations over all i, l, m and n

$$d_1 = \sum_{i=1}^{N_p} \sum_{l=1}^{N_q} \mu_{E_{il}}(c_{il}) - N_p N_q \lambda \tag{44}$$

$$d_2 = \sum_{n=1}^{N_r} \sum_{m=1}^{N_s} \sum_{i=1}^{N_p} \mu_{F_{imn}}(x_{imn}) - N_r N_s N_p \lambda \tag{45}$$

where, N_p = number of water quality indicators (1 in the present study), N_q = number of checkpoints (14 in the present study), N_r = number of pollutants (1 in the present study), N_s = number of dischargers (8 in the present study).

Minimization of d_1 and maximization of d_2 lead to a solution which is biased to the dischargers. Similarly maximization of d_1 and minimization of d_2 result in a solution that is biased to the PCA. So, the objective function Ω, which is to be maximized, for the model biased to the discharger can be given by,

$$\Omega = d_2 - d_1 \tag{46}$$

similarly, the objective function for the model biased to PCA can be given by,

$$\Omega = d_1 - d_2 \tag{47}$$

For the modified fuzzy waste load allocation model also, PGSL is used with the same values of parameters.

As both the problems are constrained optimization problems, they are converted into unconstrained problems by the penalty function method. In the present analysis, the bracket operator penalty term is used, which may be expressed as,

$$F = f(x) + \zeta \sum_{j=1}^{k} \delta_j v_j^2 \tag{48}$$

where

- F modified objective function value,
- $f(x)$ objective function value,
- k total number of constraints,
- ζ -1 (for maximization problem),
- δ_j penalty coefficient (a large value),
- v_j amount of violation.

Whenever there is a constraint violation the penalty function value is added to the objective function value to make the solution inferior. The results for risk minimization model and MFWLAM are discussed in the following section.

6. Results and discussion

Table 3 presents the fractional removal levels for dischargers as determined from the risk minimization model. Ten sets of results are presented including the result of FWLAM. Fig. 4 shows that optimal λ value should be the same for the sets of $\phi = 0.80$, $\eta = 0.80$; $\phi = 1.00$, $\eta = 1.00$; and $\phi = 1.25$, $\eta = 1.25$. The result shows that for all the three cases the optimal λ value is 0.230. $\phi = 0.80$, $\eta = 1.25$ and $\phi = 1.25$, $\eta = 0.80$ result in the two extreme solutions for the optimal values of λ, 0.194 and 0.261, respectively.

Table 4 gives the fuzzy risk of low water quality at the checkpoints resulting with optimal fractional removal levels, as determined from risk minimization model for all the 9 cases. The results are also compared with the fuzzy risk level resulting with the solutions of FWLAM. Convex membership function of λ gives lower value of λ and higher values of fractional removal levels. So the solutions for this membership function are pessimistic solutions. Similarly concave membership function of λ gives optimistic solutions. The reverse is true for the membership function of the sum of risk of low water quality. The concave value gives the pessimistic results, and the convex value gives the optimistic results. The selection of appropriate membership function depends on the decision maker.

The result shows that, applying the risk minimization model, it is possible to reduce the *fuzzy* risk of low water quality at some of the checkpoints by significant amount. For example, for the case of linear membership functions (i.e. $\phi = 1.00$, $\eta = 1.00$), at the first two checkpoints (locations 1–3 and 2–3) risks of low water quality have been reduced by 7.71% and 12.68%. In the last 3 reaches (location 13–12, 15–2 and 15–19), the risks are reduced by 4.48%, 5.55% and 6.83%, respectively, as compared to FWLAM. In the other locations also, the risks are reduced by 1–3%.

In the present case study the first two locations and the last three locations are the critical checkpoints. The first two locations are at the downstream of point loads 1 and 2 and the streamflow is significantly less compared to the main Tunga–Bhadra reach, making the checkpoints very much sensitive to the point loads. The last three locations are critical due to the cumulative effect of incremental flow. It is possible to reduce the

Table 3
Optimal fractional removal levels obtained from risk minimization model

	FWLAM MAX–MIN	$\phi = 0.80$ $\eta = 0.80$	$\phi = 0.80$ $\eta = 1.00$	$\phi = 0.80$ $\eta = 1.25$	$\phi = 1.00$ $\eta = 0.80$	$\phi = 1.00$ $\eta = 1.00$	$\phi = 1.00$ $\eta = 1.25$	$\phi = 1.25$ $\eta = 0.80$	$\phi = 1.25$ $\eta = 1.00$	$\phi = 1.25$ $\eta = 1.25$
ϵ	–	0.615	0.577	0.535	0.583	0.544	0.509	0.560	0.516	0.481
λ	0.423	0.230	0.213	0.194	0.247	0.230	0.215	0.261	0.244	0.230
x_1	0.667	0.773	0.783	0.792	0.764	0.773	0.782	0.755	0.764	0.773
x_2	0.665	0.772	0.781	0.791	0.759	0.773	0.781	0.753	0.759	0.772
x_3	0.624	0.730	0.746	0.742	0.737	0.654	0.746	0.739	0.737	0.643
x_4	0.555	0.773	0.782	0.790	0.763	0.773	0.782	0.753	0.763	0.773
x_5	0.437	0.753	0.742	0.676	0.669	0.749	0.742	0.750	0.669	0.712
x_6	0.567	0.772	0.781	0.780	0.756	0.769	0.781	0.709	0.756	0.759
x_7	0.448	0.773	0.782	0.792	0.763	0.773	0.782	0.751	0.763	0.773
x_8	0.603	0.771	0.782	0.790	0.762	0.773	0.782	0.750	0.762	0.773

Table 4
Fuzzy risk at different check points

Location	FWLAM MAX–MIN	Fuzzy Risks for different values of ϕ and η								
		$\phi = 0.80$ $\eta = 0.80$	$\phi = 0.80$ $\eta = 1.00$	$\phi = 0.80$ $\eta = 1.25$	$\phi = 1.00$ $\eta = 0.80$	$\phi = 1.00$ $\eta = 1.00$	$\phi = 1.00$ $\eta = 1.25$	$\phi = 1.25$ $\eta = 0.80$	$\phi = 1.25$ $\eta = 1.00$	$\phi = 1.25$ $\eta = 1.25$
1	0.3786	0.3016	0.2948	0.2878	0.3080	0.3015	0.2948	0.3146	0.3080	0.3014
2	0.5950	0.4678	0.4566	0.4452	0.4791	0.4682	0.4566	0.4892	0.4791	0.4683
3	0.1995	0.1893	0.1884	0.1875	0.1903	0.1894	0.1884	0.1911	0.1903	0.1894
4	0.5716	0.5630	0.5627	0.5623	0.5634	0.5630	0.5627	0.5638	0.5634	0.5630
5	0.3516	0.3250	0.3237	0.3226	0.3263	0.3250	0.3237	0.3277	0.3263	0.3250
6	0.2677	0.2516	0.2508	0.2501	0.2524	0.2516	0.2508	0.2532	0.2524	0.2515
7	0.2711	0.2563	0.2557	0.2550	0.2571	0.2563	0.2557	0.2578	0.2571	0.2563
8	0.2978	0.2865	0.2860	0.2857	0.2872	0.2865	0.2860	0.2876	0.2872	0.2866
9	0.3084	0.2985	0.2981	0.2977	0.2991	0.2985	0.2981	0.2995	0.2991	0.2985
10	0.2741	0.2663	0.2659	0.2656	0.2667	0.2663	0.2659	0.2670	0.2667	0.2663
11	0.2847	0.2622	0.2614	0.2607	0.2632	0.2622	0.2614	0.2644	0.2632	0.2623
12	0.3294	0.2846	0.2832	0.2819	0.2864	0.2846	0.2832	0.2888	0.2864	0.2849
13	0.4433	0.3879	0.3857	0.3838	0.3904	0.3878	0.3857	0.3936	0.3904	0.3880
14	0.4625	0.3944	0.3912	0.3887	0.3976	0.3942	0.3912	0.4018	0.3976	0.3943
ϑ_s	5.0353	4.5350	4.5042	4.4747	4.5672	4.5351	4.5042	4.6000	4.5672	4.5360

Note: ϑ_s = Sum of fuzzy risks at all locations.

Table 5
Optimal fractional removal levels obtained from modified fuzzy waste load allocation model (MFWLAM)

	FWLAM			MFWLAM		
	MAX–MIN	MAX-BIAS to discharger	MAX-BIAS to PCA	MAX–MIN	MAX-BIAS to discharger	MAX-BIAS to PCA
λ	0.423	0.375	0.000	0.219	0.137	0.000
x_1	0.667	0.684	0.900	0.779	0.774	0.900
x_2	0.665	0.354	0.900	0.778	0.671	0.900
x_3	0.624	0.353	0.900	0.758	0.625	0.900
x_4	0.555	0.351	0.900	0.773	0.653	0.900
x_5	0.437	0.350	0.900	0.745	0.438	0.900
x_6	0.567	0.356	0.900	0.736	0.586	0.900
x_7	0.448	0.353	0.900	0.778	0.815	0.900
x_8	0.603	0.351	0.900	0.767	0.822	0.900

fuzzy risk of low water quality significantly at these critical locations by using the model.

MAX–MIN formulation of MFWLAM gives similar results (Tables 5 and 6) as those of the risk minimization model. At locations 1–3 and 2–3 risks are reduced by 8.09% and 13.37%, respectively. In the last three reaches the risks are reduced by 4.47%, 5.48% and 6.71%. For comparison purpose the MAX–MIN, MAX-BIAS to dischargers and MAX-BIAS to PCA models resulting from FWLAM and MFWLAM have all been solved. As the number of checkpoints (i.e. 14) in this case study is greater than the number of dischargers (i.e. 8), acceptability level λ is having negative sign in the expression for objective function for the model biased to PCA (Eq. (47)), which leads to the solution having λ value 0. Thus the fractional removal levels are maximum i.e. 90%, which is not desirable. The solutions for the MAX-BIAS to PCA are therefore rejected and not used in further analysis. MAX–MIN formulation of MFWLAM gives lowest value of fuzzy risk at all checkpoints except the last two. At the last two locations the risks for MAX–MIN of MFWLAM, are higher than those for MAX-BIAS to discharger of MFWLAM, as the fractional removal levels for the last two dischargers resulting from MAX–MIN are lower. When the fuzzy risk of low water quality is taken into consideration, the MAX–MIN formulation of MFWLAM gives the best solution with little sacrifice in the λ value. For a better value of fuzzy risk, the fractional removal levels should be higher which in turn reduce the value of acceptability level.

In the analysis of all the models, the crisp risk is not considered; only the fuzzy risk is considered. The crisp risk indicates the probability of failure and the fuzzy risk indicates the expected degree of failure, and is, thus, a

Table 6
Statistics of DO concentration and fuzzy risks at checkpoints based on FWLAM and MFWLAM

Method		Rch 1 Ele 3	Rch 2 Ele 3	Rch 3 Ele 20	Rch 4 Ele 3	Rch 5 Ele 2	Rch 7 Ele 1	Rch 7 Ele 10	Rch 9 Ele 19	Rch 11 Ele 2	Rch 11 Ele 16	Rch 13 Ele 1	Rch 13 Ele 12	Rch 15 Ele 2	Rch 15 Ele 19
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
FWLAM MAX–MIN	Mean	6.176	5.420	6.754	6.610	6.923	7.047	7.041	6.998	6.981	7.031	6.952	6.889	6.782	6.750
	S.D.	0.320	0.400	0.866	0.245	0.336	0.368	0.371	0.393	0.374	0.396	0.401	0.418	0.403	0.440
	Skewness	2.175	4.021	−5.629	−1.026	−2.495	−3.006	−3.252	−4.205	−4.500	−4.502	−4.244	−4.519	−4.388	−4.740
	Risk	0.3783	0.5944	0.1995	0.5716	0.3516	0.2677	0.2711	0.2978	0.3084	0.2742	0.2847	0.3294	0.4432	0.4624
FWLAM MAX-BIAS to discharger	Mean	6.220	5.256	6.735	6.599	6.888	7.026	7.022	6.983	6.968	7.020	6.934	6.860	6.722	6.658
	S.D.	0.316	0.425	0.883	0.240	0.337	0.372	0.376	0.399	0.381	0.403	0.408	0.425	0.412	0.449
	Skewness	2.026	4.192	−5.478	−1.052	−2.396	−3.045	−3.309	−4.290	−4.572	−4.587	−4.289	−4.545	−4.203	−4.580
	Risk	0.3658	0.6413	0.2041	0.5796	0.3751	0.2820	0.2841	0.3075	0.3170	0.2809	0.2976	0.3499	0.4835	0.5240
MFWLAM MAX–MIN	Mean	6.460	5.888	6.780	6.623	6.963	7.072	7.064	7.016	6.997	7.044	6.984	6.952	6.863	6.850
	S.D.	0.296	0.357	0.812	0.250	0.330	0.360	0.363	0.382	0.363	0.385	0.388	0.405	0.389	0.425
	Skewness	1.075	2.780	−6.014	−0.999	−2.565	−2.893	−3.116	−4.028	−4.343	−4.330	−4.197	−4.534	−4.538	−4.808
	Risk	0.2974	0.4607	0.1888	0.5630	0.3248	0.2514	0.2562	0.2865	0.2985	0.2662	0.2623	0.2847	0.3884	0.3953
MFWLAM MAX-BIAS to discharger	Mean	6.448	5.794	6.788	6.616	6.943	7.059	7.053	7.007	6.989	7.038	6.977	6.946	6.865	6.860
	S.D.	0.297	0.367	0.824	0.247	0.331	0.362	0.366	0.385	0.367	0.389	0.392	0.409	0.393	0.430
	Skewness	1.125	3.101	−5.936	−1.014	−2.516	−2.928	−3.160	−4.091	−4.398	−4.392	−4.264	−4.606	−4.655	−4.911
	Risk	0.3007	0.4875	0.1913	0.5677	0.3386	0.2599	0.2640	0.2925	0.3038	0.2704	0.2670	0.2881	0.3866	0.3879

Note: Rch. = Reach; Loc. = Location; S.D. = Standard deviation.

more general indicator of the risk of low water quality in the river system.

7. Conclusion

Methodologies for minimization of risk in a river water quality control problem are presented in this paper. FWLAM, being a deterministic model, gives the fractional removal levels only for the base flow conditions and does not consider the random nature of the input variables of water quality simulation model. A major advantage of risk minimization model and MFWLAM presented in this paper is that they perform uncertainty analysis, considering all the input variables to be random. The risk minimization model performs uncertainty analysis for evaluation of fuzzy risk of low water quality at the checkpoints. The model is developed to derive policies that minimize the risk of low water quality while maximizing the acceptability level of FWLAM. Fuzzy multiobjective programming technique is used to solve the optimization model. MFWLAM considers only the basic statistics of the water quality indicator at the checkpoints, without considering the base flow conditions, and thus it is a more generalized waste load allocation model. As the risk minimization model and MFWLAM result in non-linear optimization problems, PGSL algorithm is used to solve the problems. The policies derived from the risk minimization model and MFWLAM are compared with those of FWLAM. The resulting fractional removal levels have been increased, but it is possible to reduce the risk of low water quality significantly by applying these models.

Risk minimization model and modified fuzzy waste load allocation model do not limit their application to any particular pollutant or water quality parameter in the river system. Given appropriate models for spatial and temporal distribution of the pollutant in a water body, the methodologies can be used to reduce the risk. In a general sense, they are adaptable to various environmental system where a sustainable and efficient use of environment is of interest.

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