A Fuzzy Waste Load Allocation Model Integrating Skewness of Distributions

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Abstract

The Fuzzy Waste Load Allocation Model (FWLAM) developed earlier, is based on fuzzy decision making, and derives optimal fractional levels for the base flow conditions considering conflicting goals of Pollution Control Agency (PCA) and dischargers. The Modified Fuzzy Waste Load Allocation Model (MFWLAM), described in this paper additionally considers the moments (mean, variance and skewness) of frequency distribution of the water quality indicator, thus incorporating the uncertainty due to randomness of input variables of the water quality simulation model along with the uncertainty due to imprecision or fuzziness. The main goal of the model is to minimise the risk of low water quality. The Probabilistic Global Search Laussane (PGSL) is used as a nonlinear optimization tool, to solve the resulting model. The methodology is applied to a case study of Tunga-Bhadra river system in south India.

Introduction

Waste load allocation (WLA) in streams refers to the determination of required pollutant treatment level at a set of point sources of pollution to ensure that water quality standards are maintained throughout the stream. Water quality management problems are characterized by various types of uncertainties at different stages of decision making process. The two major types of uncertainties that influence decision making process are uncertainty due to randomness and uncertainty due to imprecision. Uncertainty due to randomness arises mainly due to the random nature of the input variables used in the water quality simulation model. Uncertainty due to imprecision is associated with the goals related to water quality and pollutant abatement. In the present study an effort has been made to address both kinds of uncertainties in waste load allocation models in deriving the optimal fractional removal levels of the point loads coming to the river.

River water quality management problems have been addressed as multiobjective optimization problems in a number of earlier studies [1, 2]. Fuzzy Waste Load Allocation Model (FWLAM) was developed by Sasikumar and Mujumdar [3] to incorporate the uncertainty due to imprecision associated with describing the goal of Pollution Control Agency (PCA) and the dischargers. FWLAM, being a deterministic model does not consider the variability of the input variables used in water quality simulation model. Efforts have recently been made for simultaneous treatment of randomness and fuzziness in term of fuzzy risk of low water quality in water quality management of river systems by Sasikumar and Mujumdar [4], Mujumdar and Sasikumar [5] and Subbarao et. al.[6]. To incorporate the uncertainty due to randomness in the optimization model Risk

Minimization Model [7] was developed. But Risk Minimization Model requires multiple runs of the optimization model with high computational effort. In the present study, a Modified Fuzzy Waste Load Allocation Model (MFWLAM) is developed which is capable of considering the uncertainty due to randomness in the optimization model by using first, second and third order moments (mean, variance and skewness) of the water quality indicator. The objective of this model is not only to determine the fractional removal levels of the effluents considering the aspirations and conflicting objectives of the pollution control agency and dischargers, but also to minimize the fuzzy risk of low water quality by incorporating the skewness of the probability density function of water quality indicator.

FWLAM

The fuzzy waste load allocation model (FWLAM) developed by Sasikumar and Mujumdar [3] forms the basis for the optimization models developed in this paper. The FWLAM is described using a general river system. The river consists of a set of dischargers who are allowed to release pollutants into the river after removing some fraction of the pollutants. The goal of the Pollution Control Agency (PCA) is to improve the water quality and those of dischargers are to minimize the fractional removal levels and they are in conflict with each other. These goals are treated as fuzzy events and modelled using appropriate fuzzy membership functions. In the FWLAM, the following fuzzy optimization problem is formulated to take into account the fuzzy goals of the PCA and dischargers.

$$Maximize \quad \lambda \qquad (1)$$

$$subject\ to$$

$$\left[(c_{il} - c_{il}^{L})/(c_{il}^{L} - c_{il}^{D})\right]^{\alpha_{il}} \geq \lambda \qquad \forall\ i, l \qquad (2)$$

$$\left[(x_{imn}^{M} - x_{imn})/(x_{imn}^{M} - x_{imn}^{L})\right]^{\beta_{imn}} \geq \lambda \qquad \forall\ i,\ m, n \qquad (3)$$

$$c_{il}^{L} \leq c_{il} \leq c_{il}^{D} \qquad \forall\ i, l \qquad (4)$$

$$max\ \left[x_{imn}^{L}, x_{imn}^{MIN}\right] \leq x_{imn} \leq x_{imn}^{MAX} \quad \forall\ i,\ m, n \qquad (5)$$

$$0 \leq \lambda \leq 1 \qquad (6)$$

The model is a multiobjective formulation maximizing minimum satisfaction level (λ). In the fuzzy constraints (2) and (3) the goals of PCA and dischargers respectively are made greater than or equal to λ, to formulate this MAX-MIN model. The lower and upper bounds of water quality indicator i at the checkpoint l are fixed as permissible (c_{il}^{L}) and desirable level (c_{il}^{D}), respectively as set by PCA in constraint (4). The bounds of fractional removal level x_{imn} of the pollutant n from the discharger m to control the water quality indicator i in the river system, is given by constraint (5). The aspiration level and maximum fractional removal level acceptable to the discharger m with respect to x_{imn} are represented as, x_{imn}^{L} and x_{imn}^{M}, respectively. The PCA imposes minimum fractional removal levels that are also expressed as the lower bounds, x_{imn}^{MIN} in constraint (5). The exponents, α_{il} and β_{imn}, appearing in constraints (2) and (3) respectively, are nonzero positive real numbers.

Modified Fuzzy Waste Load Allocation Model

The model, developed in this section is a modified version of FWLAM, dealing with the moments (mean, variance and skewness) of the water quality indicator considering the random nature of all the input variables. The goal of this model is not only to determine the fractional removal levels of the effluents considering the aspirations and conflicting objectives of the pollution control agency and dischargers, but also to improve the water quality by incorporating the skewness of the probability density function of water quality indicator.

MFWLAM does not consider the base values of the input variables. It is a stochastic optimization model that includes the moments of the distribution. The model is based on fuzzy decision theory as does FWLAM. To improve the water quality, the membership function of skewness of water quality indicator is also incorporated in the model. The concept, "higher the skewness the better" or "higher the skewness the worse" is modeled through fuzzy logic by choosing appropriate membership functions for the skewness resulting from optimization. The nature of membership function for skewness is selected depending on the water quality indicator. In the higher range of Dissolved Oxygen (DO) concentration, for example, a high frequency is desired and thus negative skewness is preferred for DO (Fig. 1). A non-increasing membership function is thus assumed for the skewness of DO. Similarly for BOD, high frequency is desired in the lower range of BOD values and thus positive skewness is preferred. A non-decreasing membership function is used for BOD.

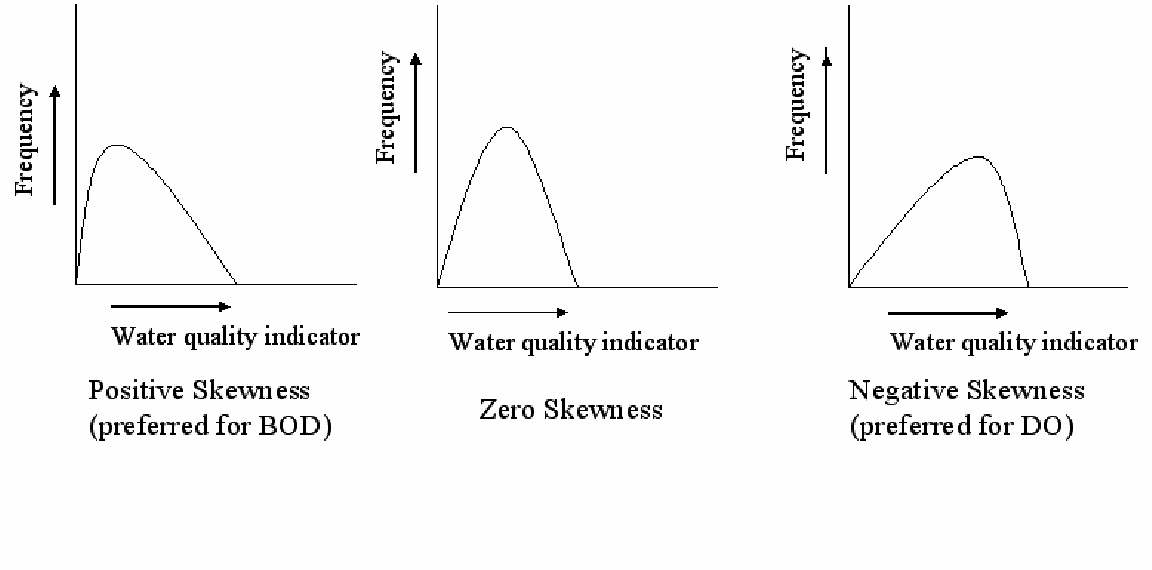

Fig. 1: Skewness of Distribution of a Water Quality Indicator

The bounds of the water quality indicator are determined from Chebyshev's inequality. According to Chebyshev's inequality, the proportion of observations lying k standard deviation outside the mean value is at most $1/k^2$, which can be mathematically stated by:

$$P\left(\left|Z - \bar{Z}\right| \geq k\sigma\right) \leq \frac{1}{k^2} \tag{7}$$

where, Z = a random variable; $\bar{Z}$ = mean value of Z; σ = standard deviation and $k \geq 0$.
From Chebyshev's inequality,

$$P\left(Z \leq \bar{Z} - k\sigma\right) + P\left(Z \geq \bar{Z} + k\sigma\right) \leq \frac{1}{k^2} \tag{8}$$

$$P\left(Z \leq \bar{Z} - k\sigma\right) \leq \frac{1}{k^2} \tag{9}$$

Replacing Z by water quality indicator c_{il},

$$P\left(c_{il} \leq \bar{c}_{il} - k\sigma_{c_{il}}\right) \leq \frac{1}{k^2} \quad \forall i, l \tag{10}$$

In the present model the lower bound of the water quality indicator is modified as follows:

$$c_{il}^{L} \leq \left(\bar{c}_{il} - k\sigma_{c_{il}}\right) \quad \forall i, l \tag{11}$$

This ensures that the probability of water quality indicator level less than the acceptable level set by PCA is at most $1/k^2$.

$$P\left(c_{il} \leq c_{il}^{L}\right) \leq \frac{1}{k^2} \quad \forall i, l \tag{12}$$

Finally the MAX-MIN formulation of the model can be given by:

$$Maximize \quad \lambda \tag{13}$$

subject to

$$\left[(\bar{c}_{il} - c_{il}^{L})/(c_{il}^{L} - c_{il}^{D})\right]^{\alpha_{il}} \geq \lambda \quad \forall\ i, l \tag{14}$$

$$\left[(x_{imn}^{M} - x_{imn})/(x_{imn}^{M} - x_{imn}^{L})\right]^{\beta_{imn}} \geq \lambda \quad \forall\ i,\ m, n \tag{15}$$

$$\mu\left(s_{c_{il}}\right) \geq \lambda \quad \forall i, l \tag{16}$$

$$c_{il}^{L} \leq \left(\bar{c}_{il} - k\sigma_{c_{il}}\right) \quad \forall i, l \tag{17}$$

$$\bar{c}_{il} \leq c_{il}^{D} \quad \forall\ i, l \tag{18}$$

$$max\ \left\lfloor x_{imn}^{L}, x_{imn}^{MIN} \right\rfloor \leq x_{imn} \leq x_{imn}^{MAX} \quad \forall\ i,\ m, n \tag{19}$$

$$0 \leq \lambda \leq 1 \tag{20}$$

where $\mu\left(s_{c_{il}}\right)$= membership function for the skewness of water quality indicator i at checkpoint l.

For the solution of the water quality simulation model a backward finite difference method is used. Probability density function of water quality indicator is derived from Monte-Carlo simulations. For non linear optimization Probabilistic Global Search Laussane (PGSL), a global search algorithm [8] is applied. Tests on benchmark problems having multi-parameter non-linear objective function revealed that PGSL performs better than Genetic Algorithm and advanced algorithms for Simulated

Annealing [9]. The algorithm is based on the assumption that better sets of points are more likely to be found in the neighbourhood of good sets of points, therefore intensifying the search in the regions that contain good solutions. Details of algorithm may be found in Raphael and Smith [9].

Model Application

Application of the model is illustrated through a case-study of Tunga-Bhadra River system shown schematically in Fig. 2. The Tunga-Bhadra River is a perennial river formed by the confluence of Tunga and Bhadra rivers, both tributaries of the Krishna River, in southern India. The river has two other tributaries, the Kumadavati and Haridra rivers. The river network is discretized into 15 reaches depending on the river morphology and river environment. The river receives the waste loads from eight major effluent points. Non-point source of pollution is also taken into account in the present study. Details of the data and the uncertainty information of the basic variables are taken from Central Water Commission (CWC), Karnataka State Water Resources Development Organisation (KSWRDO), Karnataka State Pollution Control Board (KSPCB) and Subbarao et al. [6]. A minimum fraction removal level of 35% and a maximum treatment level of 90% are assumed for the dischargers. 14 checkpoints are selected in the river reach depending on the positions of dischargers and the confluence of tributaries.

Dissolved Oxygen is considered as the water quality indicator of the stream. For deriving the PDF of water quality indicator 2000 number of Monte–Carlo simulations have been performed. PGSL is used with bracket operator penalty function for constrained non-linear optimization.

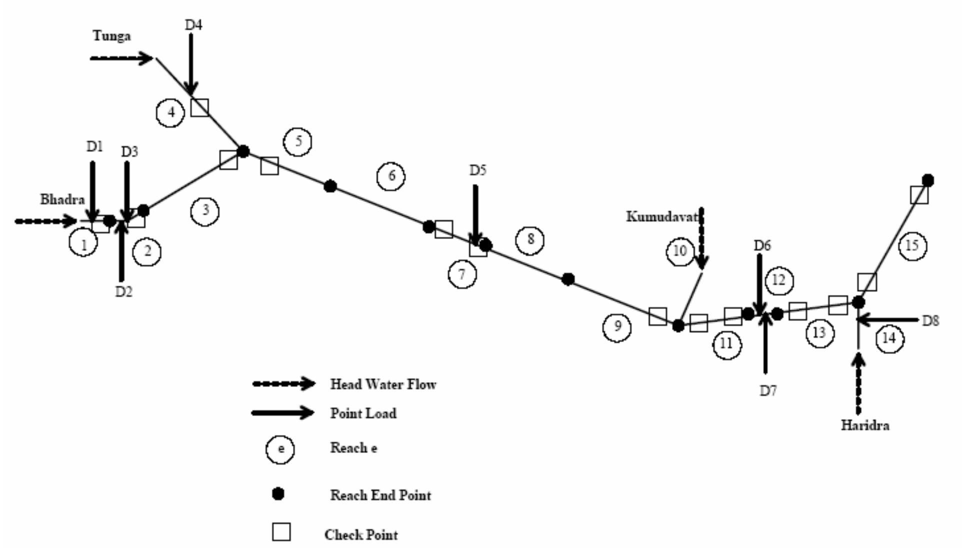

Fig. 2 Schematic Diagram of Tunga-Bhadra River System

Table 1 shows the fractional removal level derived from FWLAM and MFWLAM. Inclusion of new constraints for considering randomness results in increase in the fractional removal levels with a decrease in λ value. Here for comparison purpose fuzzy risk of low water quality [6] is taken as a measure of the performance of the model.

Fuzzy risk is defined as the probability of fuzzy event of low water quality. Denoting the fuzzy set of low water quality, DO concentration, and fuzzy risk of low water quality by W_l, c_l, and r_l, respectively, the fuzzy risk is written in discrete form as:

$$r_l \quad = \quad \sum_{c_{\min_l}}^{MAX[c_{man_l}, c_l^D]} \mu_{W_l}(c_l) p(c_l) \tag{21}$$

Where, $c_{\min_l}$ and $c_{\max_l}$ are the minimum and maximum concentration levels of DO obtained from MCS at checkpoint l. A typical membership function of low water quality, $\mu_{W_l}(c_l)$, may be expressed as,

$$\mu_{W_l}(c_l) \quad = \quad \left[\left(c_l^D - c_l\right) / \left(c_l^D - c_l^L\right)\right] \tag{22}$$

Table 1: Optimal λ and Fractional Removal Levels (%) Obtained from FWLAM and MFWLAM

	FWLAM	MFWLAM
λ	0.42	0.219
x_1	66.7	77.9
x_2	66.5	77.8
x_3	62.4	75.8
x_4	55.5	77.3
x_5	43.7	74.5
x_6	56.7	73.6
x_7	44.8	77.8
x_8	60.3	76.7

It is possible to reduce the fuzzy risk significantly by using MFWLAM. At locations 1 and 2 fuzzy risks are reduced by 8.09% and 13.37% respectively (Table 2). In the last three reaches the risks are reduced by 4.47%, 5.48% and 6.71%. At other locations also, the risks are reduced by 1-3%. In the present case study the first two locations and the last three locations are critical checkpoints, from water quality point of view. The first two locations are immediately downstream of point loads 1 and 2 and the streamflow is less as compared to the main Tunga- Bhadra reach, making the checkpoints very much sensitive to the point loads. The last three locations are critical due to the cumulative effect of incremental flow. At the critical checkpoints the fuzzy risk of low water quality is reduced by a significant amount by using the model.

Conclusions

A methodology for incorporating uncertainty due to randomness in a fuzzy optimization model of river water quality control problem is presented. The policy derived from MFWLAM is compared with that of FWLAM. The resulting fractional removal levels from the MFWLAM are higher but it is possible to reduce the risk of low water quality significantly by applying this model. As a measure of performance, in place of crisp risk, the fuzzy risk is considered. While crisp risk indicates the probability of failure, the fuzzy risk indicates the expected degree of failure, and is, thus, a more general indicator of the risk of low water quality in the river system. MFWLAM does not limit its application to any particular pollutant or water quality parameter in the river system. Given appropriate models for spatial and temporal distribution of the pollutant in a water body, the methodology can be used to reduce the risk. In a general sense, it is adaptable to various environmental system where a sustainable and efficient use of environment is of interest.

Table 2: DO Concentration with Summary Statistics and Fuzzy Risks at the Checkpoints

	FWLAM				MFWLAM			
Check points	Mean	Standard Deviation.	Skew-ness	Fuzzy Risk (%)	Mean	Standard Deviation.	Skew-ness	Fuzzy Risk (%)
1	6.18	0.32	2.17	37.83	6.46	0.30	1.07	29.74
2	5.42	0.40	4.02	59.44	5.89	0.36	2.78	46.07
3	6.75	0.87	-5.63	19.95	6.78	0.81	-6.01	18.88
4	6.61	0.24	-1.03	57.16	6.62	0.25	-0.99	56.30
5	6.92	0.34	-2.50	35.16	6.96	0.33	-2.56	32.48
6	7.05	0.37	-3.01	26.77	7.07	0.36	-2.89	25.14
7	7.04	0.37	-3.25	27.11	7.06	0.36	-3.12	25.62
8	7.00	0.39	-4.20	29.78	7.02	0.38	-4.03	28.65
9	6.98	0.37	-4.50	30.84	7.00	0.36	-4.34	29.85
10	7.03	0.40	-4.50	27.42	7.04	0.38	-4.33	26.62
11	6.95	0.40	-4.24	28.47	6.98	0.39	-4.20	26.23
12	6.89	0.42	-4.52	32.94	6.95	0.40	-4.53	28.47
13	6.78	0.40	-4.39	44.32	6.86	0.39	-4.54	38.84
14	6.75	0.44	-4.74	46.24	6.85	0.42	-4.81	39.53

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