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IMPRECISE PROBABILITY FOR MODELING PARTIAL IGNORANCE: APPLICATION TO WASTE LOAD ALLOCATION IN A RIVER SYSTEM

by

Sashank Musti[1] MISH, **Subimal Ghosh**[2] MISH, **and P. P. Mujumdar**[3] FISH,

ABSTRACT

Water resources systems problem are often characterized by uncertainty due to partial ignorance resulting from missing hydrologic data. Imprecise probability is a branch of advanced probability theory which models partial ignorance. The paper presents mathematical background of the theory of imprecise probability with its application to waste load allocation for river water quality management. The example problem of waste load allocation considers the fuzzy or imprecise goals of Pollution Control Agency (PCA) and dischargers discharging pollutants to river. The risk of low water quality due to the discharge of pollutants is computed with imprecise probability and subsequently minimized using fuzzy multiobjective optimization. Application of the model is illustrated with a hypothetical river system.

KEY WORDS : River Water Quality Management, Uncertainty Modeling, Imprecise Probability, Fuzzy Risk.

INTRODUCTION

Waste Load Allocation (WLA) in a river system refers to determining the required optimum fractional removal levels at a number of point source and non-point source locations, to attain a satisfactory water quality response at the check points in the river in an economically efficient manner. The stakeholders involved in a WLA are Pollution Control Agency (PCA) and dischargers. The goal of the PCA is to improve the water quality and those of dischargers are to minimize the fractional removal levels of the pollutants. These goals are in conflict with each other. Therefore a WLA problem can be modeled as a multiobjective optimization problem considering the conflicting goals. The imprecision lying in defining the goals has been modeled by Sasikumar and Mujumdar (1998), where the goals are treated as fuzzy goals with

1. Former Undergraduate Student, Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai – 400 076
2. Assistant Professor, Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai – 400 076
3. Professor, Department of Civil Engineering, Indian Institute of Science, Bangalore-560 012

appropriate fuzzy membership functions. To incorporate the uncertainty due to randomness along with uncertainty due to fuzziness, fuzzy risk approach has been proposed by Sasikumar and Mujumdar (2000). Considering minimization of risk as the fuzzy goal of PCA, a seasonal model for WLA has been developed by Mujumdar and Sasikumar (2002). However, imprecision lying in partial ignorance due to missing data of hydrologic variable is not considered in these models. Starting with the methodology proposed by Mujumdar and Sasikumar (2002), the present study uses the concept of imprecise or interval probability resulting imprecise fuzzy risk for determination of seasonal fractional removal levels for the dischargers.

Missing data is a very common problem for any hydrologic variable and thus only the available data may not fully compose the sample space, which is defined to contain all the possible events. The essence of imprecise probability is the assignment of ranges of probabilities to events which are characterized by such partial ignorance. Thus uncertainty about an event E, would be expressed by an interval $[\overline{P}(E), \underline{P}(E)]$, where, $0 \le \underline{P}(E) \le \overline{P}(E) \le 1$. $\overline{P}(E)$ and $\underline{P}(E)$, called respectively the upper and lower probabilities of E, are computed such that, given the present evidence, one can be reasonably sure that the probability if E is neither less than nor greater than (Tonn, 2005). To define imprecise probability, Walley (1991, 2000) states that it can accommodate all kinds of uncertainty and partial ignorance, including vague or qualitative judgments of uncertainty, models for complete ignorance or near ignorance, random sets and multivalued mappings, and partial information about an unknown probability measure. Fuzzy risk computed with imprecise probability is termed as imprecise fuzzy risk and is essentially an interval grey number. Simultaneous minimization of both the bounds of imprecise fuzzy risk is considered as the goal of pollution control agency and an optimization model is developed to compute the seasonal fractional removal levels. The following section presents the mathematical background of imprecise probability.

IMPRECISE PROBABILITY

Theory of imprecise probability is considered as the generalized version of probability theory for an event characterized by partial ignorance. It is also considered as the generalized form of other two uncertainty theories: (a) theory of possibility and necessity measures (Dubois and Prade, 1988), in which the model is characterized by qualitative knowledge and intuitions in the absence of precise measurement, and (b) theory of belief and plausibility functions following Dempster-Shafer structure (Dempster, 1967; Shafer, 1976; Caselton and Luo, 1992), when evidence is available in terms of sets or intervals and not as precise values. In the present work, uncertainty due to partial ignorance resulting from missing hydrologic data is modeled with imprecise probability. The following illustration is designed to show how imprecise probability can be more expressive in the highly uncertain hydrologic data modeling characterized with missing information. Let us assume that a hydrologic event has two states 1 and 2 and the data sample of that event has 100 data points. The probability of hydrologic event to be in state 1 and 2 are defined by P(1) and P(2), respectively. The following four cases are considered to explain the concept of imprecise probability:

1. If all the data points lie in state 1, then P(1)=1 and P(2)=0, and the probability model is considered to be a 'certain or deterministic model', which is quite uncommon in hydrology.

2. If 50 data points are in state 1 and rest are in state 2, then P(1)=0.5 and P(2)=0.5, and the probability model is considered to be a uncertain (due to randomness) model with 'complete knowledge'.

3. If 40 data points are in state 1, 40 data points are in state 2, and the rest are missing, then the two extreme possible cases are: (A) 60 data points are in state 1, and 40 data points are in state 2, i.e., P(1)=0.6 and P(2)=0.4; and (B) 40 data points are in state 1, and 60 data points are in state 2, i.e., P(1)=0.4 and P(2)=0.6. From these two possible cases it can be inferred that P(1) and P(2) can vary between 0.4 and 0.6, which results in imprecise probabilities, probabilities denoted by interval grey numbers. In such a case, the upper and lower bounds of P(1) will be 0.6 and 0.4. Similarly, the bounds of P(2) will also be 0.6 and 0.4. Such a probability model can be categorized as an uncertain model with 'partial ignorance'.

4. If all the data points are missing, then P(1) and P(2) can vary between 0 and 1 and the upper and lower bounds of both the probabilities will be 1 and 0. Such a model is termed as uncertain model with 'complete ignorance. Imprecise probability thus addresses uncertainty due to partial ignorance with interval grey numbers and the interval between the bounds of the probability reflects the incomplete nature of knowledge or partial ignorance. The properties of imprecise probability are as follows:

a. For universal sample set (Ω) and null set (Φ) both the bounds of imprecise probability follow axioms of probability.

$$\overline{P}(\Omega) = \underline{P}(\Omega) = 1 \tag{1}$$

$$\overline{P}(\Phi) = \underline{P}(\Phi) = 0 \tag{2}$$

b. Bounds of imprecise probability follow the following properties of complementarity:

$$\underline{P}(E) + \overline{P}(E') = 1 \qquad \forall E \subseteq \Omega \tag{3}$$

where, E' is the complement of E.

c. For two mutually exclusive sets E_3 and E_4, imprecise probability follows:

$$E_3 \cap E_4 = \Phi \;\Rightarrow\; \underline{P}(E_3 \cup E_4) \geq \underline{P}(E_3) + \underline{P}(E_4) \tag{4}$$

$$E_3 \cap E_4 = \Phi \;\Rightarrow\; \overline{P}(E_3 \cup E_4) \leq \overline{P}(E_3) + \overline{P}(E_4) \tag{5}$$

Replacing E_3 and E_4 by E and E', Eq (5) can be re-written as :

$$\overline{P}(E) + \overline{P}(E') \geq 1 \tag{6}$$

Therefore, the uncertainty due to partial ignorance (U), associated with event E is given by:

$$\overline{P}(E) + \overline{P}(E') - 1 = U \tag{7}$$

From Eq(3) and Eq. (7):

$$\overline{P}(E) - \underline{P}(E) = U \tag{8}$$

Therefore, the interval between probability bounds reflects the total uncertainty resulting from partial ignorance. It is true that, imprecise probability introduces an extra dimension or degree of freedom into the formal expression of uncertainty, i.e., probability models. This inevitably introduces some indeterminism into any subsequent decision analysis as the dimensionality of the problem is increased while dealing with less information. This indeterminism, rather than being a short-coming can be viewed as strength, more faithfully reflecting the reality of the situation (Caselton and Luo, 1992).

FUZZY RISK AND IMPRECISE FUZZY RISK

In a conventional risk analysis, risk can be defined as probability of failure. For a fuzzy event conventional crisp risk can be extended to fuzzy risk, defined by expected degree of failure. For WLA fuzzy risk is computed to evaluate the expected degree of low water quality at the checkpoints. In a river system for water quality indicator i (say, DO-deficit) at checkpoint l in season s, PCA sets a desirable level c_{il}^{Ds} and a maximum permissible level c_{il}^{Hs} ($c_{il}^{Hs} \geq c_{il}^{Ds}$). The set of concentration levels corresponding to the low water quality is defined as a fuzzy set W_{il}^{s}. Each concentration level in the fuzzy set W_{il}^{s} is assigned to a membership value lying in the closed interval [0,1]. Mathematically, the fuzzy set W_{il}^{s}, is expressed as follows (Subbarao et al., 2004):

$$W_{il}^{s} = c_{il}^{s} : 0 \leq \mu_{W_{il}^{s}}\left(c_{il}^{s}\right) \leq 1 \tag{9}$$

The membership value $\mu_{W_{il}^s}\left(c_{il}^s\right)$ of the fuzzy set W_{il}^s, indicates the degree of compatibility of the concentration level with the notion of low water quality. A typical membership function of low water quality for DO- deficit can be given by:

$$\mu_{W_{il}}\left(c_{il}\right)=\begin{cases} 0 & c_{il}<c_{il}^D \\ \dfrac{c_{il}-c_{il}^D}{c_{il}^H-c_{il}^D} & c_{il}^D \leq c_{il}<c_{il}^H \\ 1 & c_{il}>c_{il}^H \end{cases} \tag{10}$$

The fuzzy risk of low water quality is defined as the probability of occurrence of the fuzzy event of low water quality or in other words expected degree of low water quality and can be defined as:

$$r_{il}^s=\int_{o}^{c_{\max_{il}}^s} \mu_{W_{il}^s}\left(c_{il}^s\right) f\left(c_{il}^s\right) dc_{il}^s \tag{11}$$

where, $\mu_{W_{il}^s}\left(c_{il}^s\right)$ = membership function of the fuzzy set W_{il}^s of low water quality; $c_{\max_{il}}^s$ = maximum concentration level; and $f\left(c_{il}^s\right)$ = probability density function (pdf) of the concentration of water quality indicator i at the checkpoint l in season s. For discrete case the hydrologic variables in season s (Q_s) can be discretized into different states k (k=1,2,..K). If the hydrologic variable at a particular state k in season s will lead to the water quality indicator c_{il}^{ks} then, the fuzzy risk for season s can be computed as:

$$r_{il}^s=\sum_{k=1}^{K} \mu_{W_{il}^s}\left(c_{il}^{ks}\right) p_{ks} \tag{12}$$

where, p_k^s is the probability that the hydrologic variable is in the state k in season s. When the hydrologic variable is modeled with imprecise probability, Eq. (12) can be modified to:

$$r_{il}^{s\pm}=\sum_{k=1}^{K} \mu_{W_{il}^s}\left(c_{il}^{ks}\right) p_{ks}^{\pm} \tag{13}$$

where, $r_{il}^{s\pm}$ is the imprecise fuzzy risk at checkpoint l in season s and $p_{ks}^{\pm}$ is the imprecise probability with which the hydrologic variable (Q_s) is in state k in season s. Computation of the bounds of imprecise fuzzy risk is not as straight forward as precise fuzzy risk (Eq. 12). Following Eq. (5), it can be concluded that the sum of upper bounds of $p^{\pm}_{ks}$ for all k, is greater than 1

$$\sum_{k=1}^{K} \overline{p}_{ks} \geq \overline{P}\left(Q_s \in 1 \cup Q_s \in 2 \cup .... \cup Q_s \in K\right) = 1 \tag{14}$$

where, $\overline{p}_{ks}$ is the upper bound of probability with which, the hydrologic variable (Qs) is in state k in season s. Similarly, the sum of lower bounds of imprecise probability in a particular season s, for all k is less than 1.

$$\sum_{k=1}^{K} \underline{p}_{ks} \leq \underline{P}\left(Q_s \in 1 \cup Q_s \in 2 \cup .... \cup Q_s \in K\right) = 1 \tag{15}$$

where, $\underline{p}_{ks}$ is the lower bound of probability with which, the hydrologic variable (Qs) is in state k in season s. Therefore, the upper and lower bounds of imprecise probabilities can not directly be used in estimation of expected value expressed as interval grey number. Imprecise fuzzy risk is also the expected degree of failure in terms of interval grey number, and thus direct use of the bounds of probability is not possible in computation of the bounds of imprecise fuzzy risk. To compute them, the probabilities should be selected from their interval in such a way so that their sum will be 1 and they will lead to maximum/minimum values of risk resulting upper/ lower bounds of imprecise fuzzy risk. In the present study a linear optimization model is developed to compute the upper/lower bounds of imprecise fuzzy risk. The model for computing upper bound of risk is as follows:

$$MAX \qquad \overline{r}_{il}^{s} \tag{16}$$

$$\overline{r}_{il}^{s} = \sum_{k=1}^{K} \mu_{W_{il}^{s}}\left(c_{il}^{ks}\right) p'_{ks} \tag{17}$$

$$\sum_{k=1}^{K} p'_{ks} = 1 \tag{18}$$

$$\underline{p}_{ks} \leq p'_{ks} \leq \overline{p}_{ks} \qquad \forall k \tag{19}$$

The model will select p'_{ks} values from their range in such a way so that it will result in the upper bound of imprecise fuzzy risk. Similarly for the lower bound of imprecise fuzzy risk the following model is developed with the decision variables p''_{ks}:

$$MIN \qquad \underline{r}_{il}^{s} \tag{20}$$

$$\underline{r}_{il}^{s} = \sum_{k=1}^{K} \mu_{W_{il}^{s}}\left(c_{il}^{ks}\right) p_{ks}'' \tag{21}$$

$$\sum_{k=1}^{K} p_{ks}'' = 1 \tag{22}$$

$$\underline{p}_{ks} \leq p_{ks}'' \leq \overline{p}_{ks} \qquad \forall k \tag{23}$$

The imprecise risk thus computed at all the checkpoints for all the seasons are used in the fuzzy optimization model in defining the fuzzy goals of PCA. The next section presents the details of the optimization model for seasonal waste load allocation.

WASTE LOAD ALLOCATION : IMPRECISE FUZZY RISK APPROACH

A fuzzy optimization model based on fuzzy decision making (Bellman and Zadeh, 1970; Zimmermann, 1978) is developed for WLA incorporating the goal of dischargers to minimize the fractional removal levels of pollutants and the goals of PCA to minimize the upper and lower bounds of imprecise fuzzy risk. The model is as follows:

$$MAX \qquad \lambda \tag{24}$$

$$\frac{x_{imn}^{Ms} - x_{imn}^{s}}{x_{imn}^{Ms} - x_{imn}^{Ls}} \geq \lambda \qquad \forall i, m, n, s \tag{25}$$

$$\frac{\alpha + \beta}{2} \geq \lambda \tag{26}$$

$$\frac{r_{il}^{Ms} - \overline{r}_{il}^{s}}{r_{il}^{Ms} - r_{il}^{Ls}} \geq \alpha \qquad \forall i, l, s \tag{27}$$

$$\frac{r_{il}^{Ms} - \underline{r}_{il}^{s}}{r_{il}^{Ms} - r_{il}^{Ls}} \geq \beta \qquad \forall i, l, s \tag{28}$$

$$r_{il}^{Ls} \leq \underline{r}_{il}^{s} \leq \overline{r}_{il}^{s} \leq r_{il}^{Ms} \qquad \forall i, l, s \tag{29}$$

$$c_{il}^{Ds} \leq c_{il}^{ks} \leq c_{il}^{HS} \qquad \forall i,l,k,s \tag{30}$$

$$x_{imn}^{Ls} \leq x_{imn}^{s} \leq x_{imn}^{Ms} \qquad \forall i,m,n,s \tag{31}$$

$$0 \leq \alpha \leq 1 \tag{32}$$

$$0 \leq \beta \leq 1 \tag{33}$$

$$0 \leq \lambda \leq 1 \tag{34}$$

The LHS of constraint (25) in the optimization model presents the membership of fuzzy goals of the dischargers. x_{imn}^{Ms} x_{imn}^{Ls} and are the minimum permissible and maximum possible fractional removal levels for the discharges. x_{imn}^{s} is the fractional removal level of the pollutant n from the discharger m to control the water quality indicator i in the river system in season s and is a decision variable in the optimization model for each i, m, n and s. The LHS of constraint (27) and (28) denotes the membership of the goals of PCA in terms of upper and lower bounds of imprecise fuzzy risk. The quantities r_{il}^{Ls} and r_{il}^{Ms} are the minimum acceptable and maximum permissible risk levels. In constraint (27), α denotes the minimum goal fulfillment level of PCA, when the upper bounds of imprecise probability at all checkpoints in all the seasons are involved. Similarly, in constraint (28), β denotes the minimum goal fulfillment level of PCA, when the lower bounds of imprecise probability at all checkpoints in all the seasons are involved. The average of α and β is considered as the overall goal fulfillment level of PCA considering both the bounds of imprecise fuzzy at all the checkpoints in all the seasons. The concentration c_{il}^{ks} of water quality indicator i, at checkpoint l in season s for the hydrologic variable (here, streamflow) in state k, can be computed using a water quality simulation model. is the minimum satisfaction level of all the stakeholders (PCA and dischargers) and to be maximized in the optimization model. As the optimization model is non-linear, Probabilistic Global Search Laussane (PGSL), a global search algorithm for nonlinear optimization is used. Tests on benchmark problems having multi-parameter non-linear objective function revealed that PGSL performs better than Genetic Algorithm and advanced algorithms for Simulated Annealing (Raphael and Smith, 2003). The algorithm is based on the assumption that better sets of points are more likely to be found in the neighbourhood of good sets of points, therefore intensifying the search in the regions that contain good solutions. Details of algorithm may be found in Raphael and Smith (2003).

Probabilistic Global Search Laussane (PGSL) algorithm consists of four nested cycles: sampling cycle, probability updating cycle, focusing cycle and subdomain cycle. In the sampling cycle a number of points (say NSC) are generated randomly by generating a value for each variable according to the probability density function (pdf). Among them the best sample is selected. In a probability updating cycle the sampling cycle is invoked for a number of times (say NPUC). After each iteration, the pdf of each variable is modified. The interval containing the best solution is first selected and then the probability of that interval is multiplied by a factor greater than 1. The pdf thus generated, is then modified to make the area under the density function equal to unity. This ensures that the sampling frequencies in regions containing good points are increased. In a focusing cycle, probability updating cycle is repeated for NFC times. After each iteration, the search is increasingly focused on the interval containing the current best point. The interval containing the best point is divided into uniform subintervals. 50% probability is assigned to this interval. The remaining probability is then distributed to the region outside this interval in such a way so that the pdf decays exponentially from the best interval. In subdomain cycle, the focusing cycle is repeated NSDC times and at the end of each iteration, the current search space is modified. In the beginning the entire space is searched, but in subsequent iterations a subdomain is selected for search. The size of the subdomain decreases gradually and the solution converges to a point. PGSL is used in the present study with penalty function (Ghosh and Mujumdar, 2006) for constrained optimization. More details algorithm may be found in Raphael and Smith (2003). The algorithm of the proposed methodology is presented in Fig. 1.

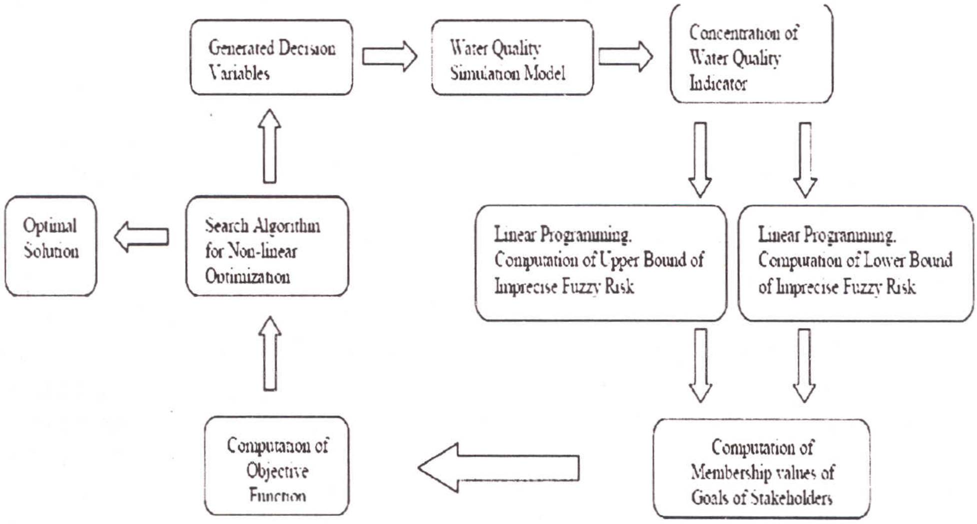

FIG. 1 ALGORITHM OF IMPRECISE FUZZY RISK APPROACH FOR WASTE LOAD ALLOCATION

APPLICATION TO A HYPOTHETICAL RIVER SYSTEM

The model is applied to a hypothetical river system, considered by Mujumdar and Sasikumar (2002), shown in Fig. 2. Details of the river reaches, effluent flow, saturated DO concentration, and position of checkpoints are taken from Mujumdar and Sasikumar (2002). Six seasons are considered in the case study, and for each of the season the streamflow is discretized into three states. The river flow corresponding to each state is taken from Mujumdar and Sasikumar (2002). The steady state imprecise probability associated with each of the state in a season is mentioned in Table 1. The constants appearing in constraints (25)-(28) are as follow:

$$c_{il}^{Ds} = 0.0,\ \ c_{il}^{Hs} = 5.0,\ \ x_{imn}^{Ms} = 0.95,\ \ x_{imn}^{Ls} = 0.30,\ \ r_{il}^{Ls} = 0.00,\ \ r_{il}^{Ms} = 0.60, \text{ for all s.}$$

Streeter-Phelp equation is used for the water quality simulation model. The minimum satisfaction level λ is obtained as 0.1535 by solving the optimization model (Eq. 24-34). The seasonal fractional removal levels are presented in Table 2. Since the river flows in seasons 4 and 5 are comparatively lower than those in the other seasons, higher fraction removal levels result in these seasons. The imprecise fuzzy risk obtained using the optimum fractional removal levels are presented in Table 3. Incorporation of imprecise fuzzy risk in the model enables minimization of both the bounds of risk in a single optimization framework.

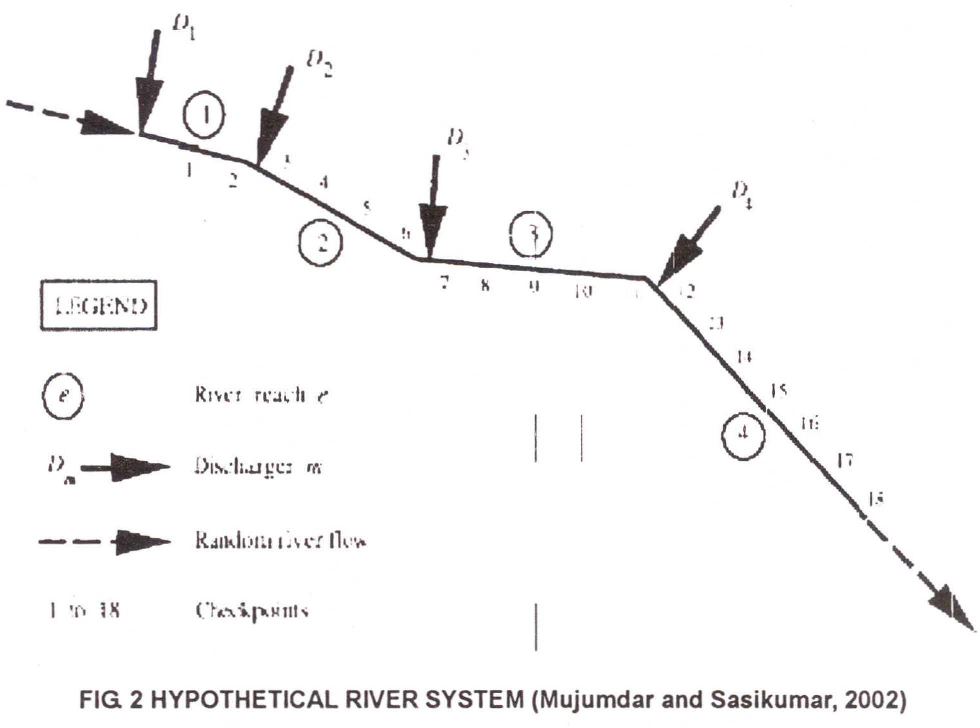

FIG. 2 HYPOTHETICAL RIVER SYSTEM (Mujumdar and Sasikumar, 2002)

TABLE-1
STEADY STATE IMPRECISE PROBABILITIES

	Probability											
	Season 1		Season 2		Season 3		Season 4		Season 5		Season 6	
States	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound
1	0.1	0.2	0.1	0.2	0.2	0.3	0.1	0.2	0.6	0.7	0.2	0.3
2	0.2	0.3	0.5	0.6	0.5	0.6	0.6	0.7	0.2	0.3	0.4	0.5
3	0.6	0.7	0.2	0.3	0.1	0.2	0.1	0.2	0.1	0.2	0.3	0.4

TABLE-2
OPTIMUM FRACTIONAL REMOVAL LEVELS

	Seasons					
Discharger						
1	2	3	4	5	6	
1	0.4678	0.6387	0.8502	0.7940	0.8122	0.7539
2	0.5898	0.4326	0.7599	0.8501	0.8450	0.8489
3	0.8499	0.6402	0.5706	0.7907	0.8502	0.5925
4	0.5497	0.5526	0.5571	0.8470	0.8502	0.6900

TABLE -3
IMPRECISE FUZZY RISK AT THE CHECKPOINTS WITH OPTIMUM FRACTIONAL REMOVAL LEVELS

Checkpoints	Season - 1 Imprecise Fuzzy Risk		Season - 2 Imprecise Fuzzy Risk		Season - 3 Imprecise Fuzzy Risk		Season - 4 Imprecise Fuzzy Risk		Season - 5 Imprecise Fuzzy Risk		Season - 6 Imprecise Fuzzy Risk	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound
1	0.0129	0.0136	0.0111	0.0116	0.0096	0.0100	0.0194	0.0210	0.0234	0.0249	0.0151	0.0161
2	0.0202	0.0214	0.0165	0.0173	0.0125	0.0131	0.0280	0.0305	0.0339	0.0362	0.0219	0.0235
3	0.0327	0.0342	0.0296	0.0308	0.0283	0.0295	0.0509	0.0546	0.0604	0.0639	0.0402	0.0425
4	0.0595	0.0627	0.0657	0.0691	0.0493	0.0518	0.0819	0.0888	0.1026	0.1093	0.0614	0.0654
5	0.0839	0.0887	0.0985	0.1038	0.0681	0.0718	0.1083	0.1178	0.1378	0.1472	0.0799	0.0854
6	0.1061	0.1122	0.1283	0.1352	0.0848	0.0896	0.1309	0.1423	0.1670	0.1784	0.0962	0.1028
7	0.1214	0.1279	0.1441	0.1514	0.1044	0.1097	0.1584	0.1710	0.1981	0.2106	0.1186	0.1261
8	0.1468	0.1547	0.1870	0.1966	0.1474	0.1554	0.2045	0.2216	0.2470	0.2631	0.1690	0.1805
9	0.1698	0.1790	0.2258	0.2374	0.1859	0.1962	0.2439	0.2643	0.2875	0.3062	0.2133	0.2280
10	0.1906	0.2009	0.2609	0.2742	0.2202	0.2323	0.2771	0.3001	0.3206	0.3413	0.2520	0.2694
11	0.2094	0.2206	0.2925	0.3072	0.2505	0.2643	0.3049	0.3297	0.3472	0.3693	0.2856	0.3052
12	0.2185	0.2299	0.3014	0.3163	0.2616	0.2756	0.3202	0.3455	0.3642	0.3868	0.2981	0.3179
13	0.2466	0.2595	0.3420	0.3588	0.3058	0.3222	0.3543	0.3820	0.3999	0.4246	0.3431	0.3658
14	0.2720	0.2861	0.3784	0.3969	0.3449	0.3634	0.3826	0.4120	0.4284	0.4545	0.3821	0.4072
15	0.2949	0.3100	0.4112	0.4310	0.3795	0.3996	0.4058	0.4363	0.4507	0.4777	0.4158	0.4427
16	0.3155	0.3314	0.4405	0.4614	0.4098	0.4313	0.4244	0.4555	0.4675	0.4950	0.4446	0.4729
17	0.3338	0.3505	0.4665	0.4883	0.4363	0.4588	0.4391	0.4703	0.4796	0.5072	0.4691	0.4983
18	0.3502	0.3674	0.4896	0.5121	0.4593	0.4825	0.4502	0.4812	0.4876	0.5150	0.4895	0.5193

CONCLUDING REMARKS

A seasonal fuzzy water quality management model is developed in the present study incorporating the partial ignorance due to missing hydrologic data along with uncertainty due to randomness and imprecision. Partial ignorance due to missing hydrologic data can be modeled with imprecise probability which leads to imprecise fuzzy risk. The developed model minimizes both the bounds of the imprecise fuzzy risk in a single optimization framework. The model does not limit its application to any particular pollutant or water quality parameter in the river system. Given appropriate models for spatial and temporal distribution of the pollutant in a water body, the methodology can be used to reduce the risk. In a general sense, it is adaptable to various environmental systems where a sustainable and efficient use of environment is of interest.

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