MINIMIZATION OF CONSTRAINT VIOLATION IN FUZZY MULTIOBJECTIVE PROGRAMMING

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Fuzzy multiobjective programming for a deterministic case involves maximizing the minimum goal satisfaction level among conflicting goals of different stakeholders using Max-min approach. Uncertainty due to randomness in a fuzzy multiobjective programming may be addressed by modifying the constraints using probabilistic inequality (e.g., Chebyshev's inequality) or by addition of new constraints using statistical moments (e.g., skewness). Such modifications may result in the reduction of the optimal value of the system performance. In the present study, a methodology is developed to allow some violation in the newly added and modified constraints, and then minimizing the violation of those constraints with the objective of maximizing the minimum goal satisfaction level. Fuzzy goal programming is used to solve the multiobjective model. The proposed methodology is demonstrated with an application in the field of Waste Load Allocation (WLA) in a river system.

INTRODUCTION

Water resources management problems are characterized by uncertainty due to both randomness and fuzziness. Uncertainty due to randomness is associated with the random behavior of different hydrologic variables. Uncertainty due to fuzziness arises from imprecisely defined goals of stakeholders and different standards for different purposes of water use. Imprecisely defined multiple goals in a management problem can be addressed by fuzzy multiobjective programming for a deterministic case. Incorporation of randomness in such models requires some modifications and additional constraints associated with different statistical moments of the random variables. Such an inclusion of constraints in the optimization model may lead to a low system performance, in terms of the goal fulfillment. The present paper introduces a new methodology for handling such problem, by allowing some violation in the newly added and modified constraints, and then minimizing the violation of those constraints with the objective of maximizing the minimum goal satisfaction level. Proposed methodology is demonstrated with an application to a waste load allocation problem in streams.

Waste load allocation (WLA) in streams refers to the determination of required pollutant treatment levels at a set of point sources of pollution to ensure that water quality standards are maintained throughout the stream. It is characterized by uncertainty due to randomness and imprecision. Starting with the Fuzzy Waste Load Allocation Model (FWLAM) [1] which considers conflicting imprecise goals of Pollution Control Agency (PCA) and dischargers, uncertainty due to randomness is incorporated in Modified Fuzzy Waste Load Allocation Model (MFWLAM) [2] by using membership function of skewness and Chebyshev's inequality. Incorporation of the new constraints, however leads to a low value of system performance measure, λ. To improve the value of λ, a multiobjective model is developed in this paper, allowing some violation in the new constraints, considering objectives of minimization of violations and maximization of satisfaction level, λ. Fuzzy goal programming is used to solve the optimization problem. The next sections provide a brief overview of FWLAM and MFWLAM, based on which the current work is developed.

FUZZY WASTE LOAD ALLOCATION MODEL

The fuzzy waste load allocation model (FWLAM) developed by Sasikumar and Mujumdar [1] forms the basis for the optimization model developed in this paper. The FWLAM is described using a general river system. The river consists of a set of dischargers who are allowed to release pollutants into the river after removing some fraction of the pollutants. The goal of the Pollution Control Agency (PCA) is to improve the water quality and those of dischargers are to minimize the fractional removal levels. These goals are in conflict with each other. The goals are treated as fuzzy goals and modelled using appropriate fuzzy membership functions. In the FWLAM, the following fuzzy optimization problem is formulated to take into account the fuzzy goals of the PCA and dischargers.

$$\text{maximize} \quad \lambda \tag{1}$$

subject to

$$\left[(c_{il} - c_{il}^{L})/(c_{il}^{L} - c_{il}^{D})\right]^{\alpha_{il}} \geq \lambda \qquad \forall\ i, l \tag{2}$$

$$\left[(x_{imn}^{M} - x_{imn})/(x_{imn}^{M} - x_{imn}^{L})\right]^{\beta_{imn}} \geq \lambda \qquad \forall\ i,\ m, n \tag{3}$$

$$c_{il}^{L} \leq c_{il} \leq c_{il}^{D} \qquad \forall\ i, l \tag{4}$$

$$max\ \left[x_{imn}^{L}, x_{imn}^{MIN}\right] \leq x_{imn} \leq x_{imn}^{MAX} \qquad \forall\ i,\ m, n \tag{5}$$

$$0 \leq \lambda \leq 1 \tag{6}$$

The model is a multiobjective formulation maximizing minimum satisfaction level (λ). In the fuzzy constraints (2) and (3), the goals of PCA and dischargers respectively are made greater than or equal to λ, to formulate this MAX-MIN model. The lower and upper bounds of water quality indicator i at the checkpoint l are fixed as permissible (c_{il}^{L}) and desirable level (c_{il}^{D}), respectively as set by PCA in constraint (4). The bounds of fractional removal level x_{imn} of the pollutant n from the discharger m to control the water quality indicator i in the river system, is given by constraint (5). The aspiration level and maximum fractional removal level acceptable to the discharger m with respect to x_{imn} are represented as, x_{imn}^{L} and x_{imn}^{MAX}, respectively. The PCA imposes minimum fractional removal levels that are also expressed as the lower bounds, x_{imn}^{MIN} in constraint (5). The exponents, α_{il} and β_{imn}, appearing in constraints (2) and (3) respectively, are nonzero positive real numbers, that define the shape of the membership functions.

MODIFIED FUZZY WASTE LOAD ALLOCATION MODEL

The Modified Fuzzy Waste Load Allocation Model (MFWLAM) [2] incorporates randomness of input variables by introducing mean, variance and skewness of the water quality indicator. The objective of the model is to determine the fractional removal levels of the effluents considering the conflicting objectives of the pollution control agency and dischargers, and to improve the water quality by incorporating the skewness of the probability density function of water quality indicator. The lower bound of the water quality indicator is fixed using Chebyshev's inequality. Details of the model may be found in Ghosh and Mujumdar [2]. The model is written as follows:

$$\text{maximize} \quad \lambda \tag{7}$$

subject to

$$\left[(\bar{c}_{il} - c_{il}^{L})/(c_{il}^{L} - c_{il}^{D})\right]^{\alpha_{il}} \geq \lambda \qquad \forall\ i, l \tag{8}$$

$$\left[(x_{imn}^{M} - x_{imn})/(x_{imn}^{M} - x_{imn}^{L})\right]^{\beta_{imn}} \geq \lambda \qquad \forall\ i,\ m,\ n \tag{9}$$

$$\mu\left(s_{c_{il}}\right) \geq \lambda \quad \forall i,l \tag{10}$$

$$c_{il}^{L} \leq \left(\bar{c}_{il} - k\sigma_{c_{il}}\right) \quad \forall i,l \tag{11}$$

$$\bar{c}_{il} \leq c_{il}^{D} \qquad \forall\ i,\ l \tag{12}$$

$$max\ \left[x_{imn}^{L}, x_{imn}^{MIN}\right] \leq x_{imn} \leq x_{imn}^{MAX} \qquad \forall\ i,\ m,\ n \tag{13}$$

$$0 \leq \lambda \leq 1 \tag{14}$$

where, $\bar{c}_{il}$, $\sigma_{c_{il}}$, $s_{c_{il}}$ and $\mu\left(s_{c_{il}}\right)$ are mean, standard deviation, skewness and membership function for the skewness, of water quality indicator i at checkpoint l, respectively.

Constraints (10) and (11) are newly introduced to incorporate the uncertainty due to randomness. The concept, "higher the skewness the better" or "higher the skewness the worse" is modelled through fuzzy logic by choosing appropriate membership functions for the skewness [2] and used in constraint (10). Lower bound of thw water quality indicator is fixed by constraint (11) using Chebyshev's inequality. The resulting nonlinear model is solved by using Probabilistic Global Search Lausanne (PGSL) [3, 4].

COMBINATION OF TWO MODELS: A MULTIOBJECTIVE APPROACH

In MFWLAM, some of the constraints of FWLAM are modified and new constraints are included. Inclusion of additional constraints for membership function of skewness (constraint 10) and Chebyshev's inequality (constraint 11) leads to a low value of satisfaction level. A multiobjective programming technique is developed here to improve the λ value, allowing some violations in the above mentioned two constraints, with an additional objective of minimization of violations of the new constraints. The model has two sets of objectives: (i) maximization of the minimum satisfaction level and (ii) minimization of the violation of the two constraints (10) and (11). The model is written as:

$$\text{maximize} \quad \lambda \tag{15}$$

$$\text{Minimize} \quad v_1 \tag{16}$$

$$\text{Minimize} \quad v_2 \tag{17}$$

subject to

$$\left[(\overline{c}_{il} - c_{il}^{L})/(c_{il}^{L} - c_{il}^{D})\right]^{\alpha_{il}} \geq \lambda \qquad \forall\ i, l \tag{18}$$

$$\left[(x_{imn}^{M} - x_{imn})/(x_{imn}^{M} - x_{imn}^{L})\right]^{\beta_{imn}} \geq \lambda \qquad \forall\ i,\ m, n \tag{19}$$

$$\overline{c}_{il} \leq c_{il}^{D} \qquad \forall\ i, l \tag{20}$$

$$max\ \left[x_{imn}^{L}, x_{imn}^{MIN}\right] \leq x_{imn} \leq x_{imn}^{MAX} \qquad \forall\ i,\ m, n \tag{21}$$

$$0 \leq \lambda \leq 1 \tag{22}$$

where, v_1 is the vioaltion of constraint (10) and is given by:

$$v_1 = \begin{cases} \lambda - \mu\left(s_{c_{il}}\right) & \text{if positive} \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

similarly, v_2 is the vioaltion of constraint (11) and is given by:

$$v_2 = \begin{cases} c_{il}^{L} - (\overline{c}_{il} - k\sigma_{c_{il}}) & \text{if positive} \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

As the newly introduced constraints (10) and (11) are relaxed in the model, (15)-(24), the model will lead to a higher value of minimum satisfaction level as compared to MFWLAM. Fuzzy goal programming [5] is used in the present study, to solve the model. Fuzzy goal programming is a combination of fuzzy multiobjective programming and goal programming, used to solve multiobjective optimization models. The concept is to assign suitable membership functions to each of the objectives setting their target as 1 and then to minimize the sum of the weighted deviations from their target. In the present study, the best and the worst values of each objective are used for determination of its membership function.

The constraints (10) and (11), whose violations are to be minimized, and which are responsible for reducing value of λ, are not considered in FWLAM, and therefore it gives the best value of λ. The corresponding violation value for FWLAM will be the worst (maximum) value of violations. The solution of MFWLAM gives the worst value of λ. The violations values are 0 for MFWLAM as the solution considers the new constraints. These are the best values of v_1 and v_2. The best and worst values, thus derived are used to get appropriate membership functions of violations v_1 and v_2. For λ value a non-decreasing membership function is used as the model maximizes λ.

$$\mu_\lambda = \left[\left(\lambda - \lambda^-\right)/\left(\lambda^+ - \lambda^-\right)\right] \tag{25}$$

where, μ_λ= membership function for λ;
λ^+ = best value of λ;
λ^- = worst value of λ;

Similarly, for the violations at non-increasing membership functions are used because the objective is to minimize the violations of constraints.

$$\mu_{v_k} = \left[\left(v_k^- - v_k\right)/\left(v_k^- - v_k^+\right)\right] \qquad k = 1,2 \tag{26}$$

where, μ_{v_k} = membership function for v_k;
v_k^+ = best value of v_k;
v_k^- = worst value of v_k;
Finally the following model is developed

$$Minimize \qquad \left(u_1 \times d_1 + u_2 \times d_2 + u_3 \times d_3\right) \tag{27}$$

subject to

$$\mu_\lambda + d_1 = 1 \tag{28}$$

$$\mu_{v_1} + d_2 = 1 \tag{29}$$

$$\mu_{v_2} + d_3 = 1 \tag{30}$$

$$\left[(\bar{c}_{il} - c_{il}^{L})/(c_{il}^{L} - c_{il}^{D})\right]^{\alpha_{il}} \geq \lambda \quad \forall\ i, l \tag{31}$$

$$\left[(x_{imn}^{M} - x_{imn})/(x_{imn}^{M} - x_{imn}^{L})\right]^{\beta_{imn}} \geq \lambda \quad \forall\ i,\ m, n \tag{32}$$

$$\bar{c}_{il} \leq c_{il}^{D} \qquad \forall\ i, l \tag{33}$$

$$max \left\lfloor x_{imn}{}^{L}, x_{imn}{}^{MIN} \right\rfloor \leq x_{imn} \leq x_{imn}{}^{MAX} \quad \forall \; i, \; m, n \tag{34}$$

$$0 \leq \lambda \leq 1 \tag{35}$$

d_z (z=1,2,3) is the deviation of the memberships of objective from the target value 1and u_z is the corresponding weight. The wight u_z can be given by [6]:

$$u_z = \frac{1}{|b_z - w_z|} \qquad \forall z \tag{36}$$

where b_z and w_z are the best and worst values of z^{th} objective, respectively. A major feature of fuzzy goal programming is that it is a goal programming which considers the fuzzy membership functions, and the weights to the goals are predetermined.

CASE-STUDY

Application of the model is illustrated through a case-study of Tunga-Bhadra River system shown schematically in Fig. 1. The Tunga-Bhadra River is a perennial river formed by the confluence of Tunga and Bhadra rivers having two other tributaries, Kumadavati and Haridra rivers. The river receives the waste loads from eight major effluent points. Non-point source of pollution is also taken into account in the present study [7]. Details of the data and the uncertainty information of the basic variables are taken from Subbarao et al. [7]. 14 checkpoints are selected in the river reach depending on the positions of dischargers and the confluence of tributaries. Dissolved Oxygen is considered as the water quality indicator of the stream. For deriving the PDF of water quality indicator 2000 number of Monte–Carlo simulations have been performed. PGSL is used with bracket operator penalty function for constrained non-linear optimization.

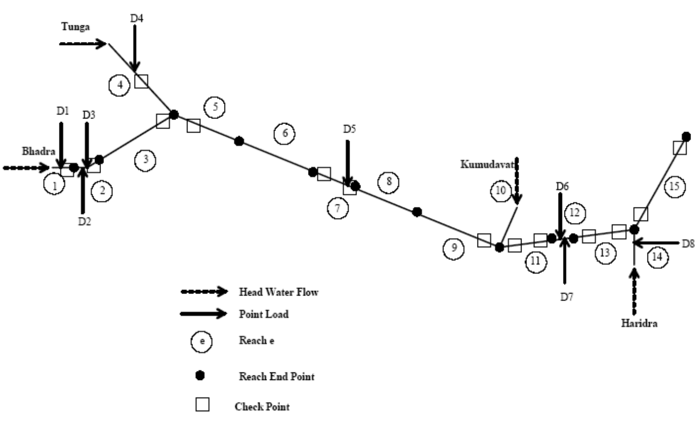

Figure 1. Schematic Diagram of Tunga-Bhadra River System

To compare FWLAM, MFWLAM and the combined model, fuzzy risk of low water quality [7] is taken as a measure of the performance of the model. Fuzzy risk is defined as the probability of fuzzy event of low water quality. Denoting the fuzzy sets of low water quality, DO concentration, and fuzzy risk of low water quality by W_l, c_l, and r_l, respectively, the fuzzy risk is written in discrete form as:

$$r_l = \sum_{c_{\min_l}}^{MAX[c_{man_l}, c_l^D]} \mu_{W_l}(c_l)p(c_l) \qquad (37)$$

where, $c_{\min_l}$ and $c_{\max_l}$ are the minimum and maximum concentration levels of DO obtained from MCS at checkpoint l.

Table 1. Results Obtained from FWLAM, MFWLAM and Combined Model

	FWLAM	MFWLAM	Combined Model
λ	0.42	0.219	0.289
x_1	66.7	77.9	74.2
x_2	66.5	77.8	74.2
x_3	62.4	75.8	72.3
x_4	55.5	77.3	74.2
x_5	43.7	74.5	70.0
x_6	56.7	73.6	72.6
x_7	44.8	77.8	74.2
x_8	60.3	76.7	74.2
Location	Risk for FWLAM	Risk for MFWLAM	Risk for Combined Model
1	37.83	29.74	32.40
2	59.44	46.07	50.44
3	19.95	18.88	19.23
4	57.16	56.30	56.42
5	35.16	32.48	32.92
6	26.77	25.14	25.41
7	27.11	25.62	25.87
8	29.78	28.65	28.83
9	30.84	29.85	30.01
10	27.42	26.62	26.75
11	28.47	26.23	26.51
12	32.94	28.47	28.99
13	44.32	38.84	39.52
14	46.24	39.53	40.40

A typical membership function of low water quality, $\mu_{W_l}(c_l)$, may be expressed as,

$$\mu_{W_l}(c_l) \quad = \quad \left[\left(c_l^D - c_l\right) / \left(c_l^D - c_l^L\right)\right] \tag{38}$$

Table 1 shows the results obtained from FWLAM, MFWLAM and the combined method. Application of MFWLAM results in a very low value of λ as 0.219, whereas deterministic FWLAM gives λ as 0.42. The combined methodology gives λ as 0.289, better than that of MFWLAM. By using the combined model, at the first two reaches the risk is reduced by 5.43% and 9.00% and at the last three reaches the risk is reduced by 3.95%, 4.80% and 5.84% as compared to FWLAM. Using this model, therefore it is possible to get a higher value of satisfaction level, with a satisfactory value of risk.

CONCLUSION

A methodology for improvement of system performance in a fuzzy multiobjective programming of waste load allocation problem with random inputs is presented. A compromise solution of higher value of system performance with a satisfactory value of risk is obtained by using the proposed model. The model does not limit its application to any particular pollutant or water quality parameter in the river system. Given appropriate transfer function for spatial and temporal distribution of the pollutant in a water body, the methodology can be used to derive the optimal fractional removal levels.

REFERENCES

[1] Sasikumar K. and Mujumdar P. P., “Fuzzy optimization model for water quality management of a river system” , Journal of Water Resources Planning and Management, ASCE, Vol. 124, No. 2, (1998), pp 79-88.

[2] Ghosh, S. and Mujumdar, P. P., “Risk minimization in water quality control problems of a river system”, Adv. in Water Resour, Vol. 29, No. 3, (2006) pp 458-470.

[3] Raphael, B. and Smith, B., “A probabilistic search algorithm for finding optimally directed solutions”, Proc., Construction Information Technology, Icelandic Building Research Institute, Reykjavik, Iceland, (2000), pp 708-721.

[4] Raphael, B. and Smith, B., “A direct stochastic algorithm for global search”, Applied Mathematics and Computation, Vol. 146, No. 3, (2003), pp 729-758.

[5] Pal, B.B., Moitra, B.N. and Maulik, U., “A goal programming procedure for fuzzy multiobjective linear fractional programming problem”, Fuzzy Sets and Systems, Vol. 139, (2003), pp 395-405.

[6] Mohamed, R. H., “The Relationship between Goal Programming and Fuzzy Programming”, Fuzzy Sets and Systems, Vol. 89, (1997) pp 215-222.

[7] Subbarao, V. V. R., Mujumdar, P. P. and Ghosh, S., “Risk evaluation in water quality management of a river system”, Journal of Water Resour. Plng and Mgmt, ASCE, Vol. 130, No. 5, (2004), pp 411-423.