

# Risk Evaluation in Water Quality Management of a River System

V. R. Subbarao Vemula<sup>1</sup>; P. P. Mujumdar<sup>2</sup>; and Subimal Ghosh<sup>3</sup>

**Abstract:** A methodology for evaluation of risk for a river water quality management problem is presented. A fuzzy waste load allocation model is solved with a simulation–optimization approach for obtaining optimum fractional removal levels for the dischargers to the river system. With the optimal fractional removal levels, sensitivity analysis and first-order reliability analysis are applied to identify key random variables which influence the water quality simulation model output, and key checkpoints where the model output is more likely to be affected (i.e., has high variability). Frequency distributions of the output variable are obtained at the key checkpoints using Monte Carlo simulation with the key random variables as input variables. The event of low water quality at a checkpoint in a river system is considered as a fuzzy event, with appropriate membership functions defined for the fuzzy risk of low water quality. With the help of fuzzy membership functions and frequency distributions, fuzzy risk levels are computed at the key checkpoints. The proposed methodology is demonstrated through a case study of Tunga–Bhadra River in southern India.

**DOI:** 10.1061/(ASCE)0733-9496(2004)130:5(411)

**CE Database subject headings:** Water quality; River systems; Evolutionary computation; Fuzzy sets; Monte Carlo method; Optimization models; Risk analysis; Uncertainty analysis.

## Introduction

Water quality management problems are characterized by various types of uncertainties at different stages of the decision making process. Uncertainty in water quality management models arises primarily from: (1) Randomness associated with various input variables of the model; (2) uncertainty due to the water quality simulation model used; and (3) imprecision (or fuzziness) associated with the goals of dischargers and the pollution control agency (PCA).

Uncertainty due to randomness of variables and parameters of the river system has received due attention in the development of water quality management models. Major components of the river system that give rise to randomness are the quality and discharge characteristics of both headwater flow as well as effluent flows. These in turn render the water quality indicators (i.e., output variables of the water quality simulation model) random in nature. There are three widely adopted approaches for addressing randomness in water quality models (Takyi and Lence 1999). These are: (1) Chance-constrained optimization (e.g., Ellis 1987); (2) combined simulation–optimization (e.g., Takyi and Lence 1994);

and (3) the multiple realization approach (e.g., Burn and Lence 1992; Takyi and Lence 1999).

The second form of uncertainty, referred to as model uncertainty, arises because of simplifying assumptions used to derive mathematical relations between inputs and outputs in describing a complex process (Tyagi and Haan 2001). Cardwell and Ellis (1993) addressed model uncertainty by simultaneously considering multiple models, such as the Streeter–Phelps (Streeter and Phelps 1925) equations, QUAL2E (Brown and Barnwell 1987), and WASP4 (Ambrose et al. 1988) for a water quality management problem.

The third form of uncertainty, that due to imprecision, is associated with description of the goals and quantification of *desirable water quality*. Establishing water quality criteria for any particular use of a water body is an example of uncertainty due to imprecision. A second example is assignment of permissible risk levels for violation of water quality standards. A management model that accounts for uncertainties due to both randomness and fuzziness may be expected to provide a more realistic decision making tool for water quality management of river systems.

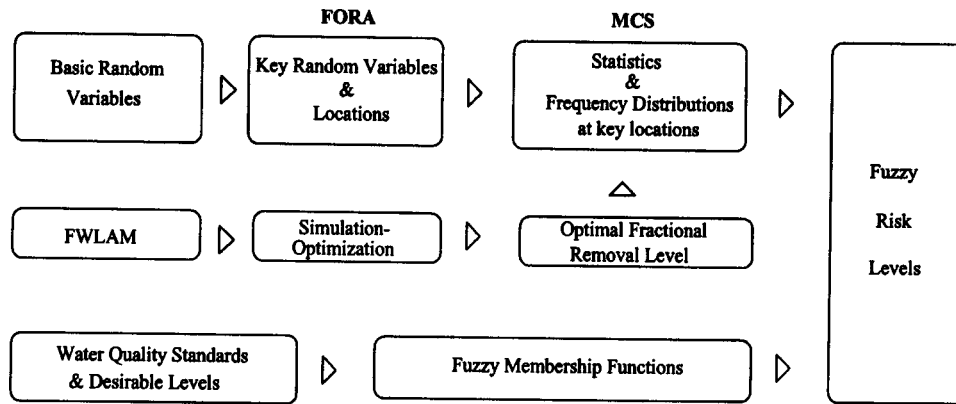
Efforts have recently been made for simultaneous treatment of randomness and fuzziness in water quality management of river systems. Sasikumar and Mujumdar (2000) presented a theoretical framework to include both randomness and fuzziness in river water quality management models. The concept of probability of a fuzzy event is used to link probability with fuzzy sets. Methods based on multiple scenarios and optimization (e.g., Burn 1989) and the multiple realization method (e.g., Takyi and Lence 1999) use Monte Carlo simulation (MCS) to generate several possible scenarios of hydrologic, hydraulic, and pollutant-loading conditions. The probability distribution estimated by MCS generally closely approximates the exact one, provided the number of realizations is sufficiently large (Maier et al. 2001). A major disadvantage of MCS, however, is its high computational requirements. To overcome this limitation to a certain extent, first-order reliability analysis (FORA) can be used. As introduced for water

<sup>1</sup>Scientist, Defense Research and Development Organization, 560 093 Bangalore, India. E-mail: vvrsubbarao@yahoo.com

<sup>2</sup>Associate Professor, Dept. of Civil Engineering, Indian Institute of Science, 560 012 Bangalore, India. E-mail: pradeep@civil.iisc.ernet.in

<sup>3</sup>Graduate Student, Dept. of Civil Engineering, Indian Institute of Science, 560 012 Bangalore, India. E-mail: subimal@civil.iisc.ernet.in

Note. Discussion open until February 1, 2005. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on August 2, 2002; approved on February 6, 2004. This paper is part of the *Journal of Water Resources Planning and Management*, Vol. 130, No. 5, September 1, 2004. ©ASCE, ISSN 0733-9496/2004/5-411–423/\$18.00.



**Fig. 1.** Evaluation of fuzzy risk (Note: FORA=first order reliability analysis; MCS=Monte Carlo simulation; and FWLAM=fuzzy waste load allocation model)

quality problems by Burges and Lettenmaier (1975), FORA helps in identifying the combinations of model input parameters and variables most likely contributing to failure of the system. It also aids in screening the key checkpoints [i.e., locations with high variability of the water quality indicator such as dissolved oxygen (DO) concentration] where risk due to uncertainty is likely to be high. In this study, FORA is used to identify the key variables and key checkpoints in the system, and MCS is applied to obtain the frequency distribution of water quality indicator levels at these checkpoints with respect to the key input variables. A procedure is demonstrated herein for evaluating fuzzy risk using FORA and MCS methods applied to the QUAL2E-UNCAS model (Brown and Barnwell 1987).

The methodology of computing fuzzy risk is illustrated in Fig. 1. The set of optimal fractional removal levels are determined using the fuzzy waste load allocation model (FWLAM). The simulation-optimization approach developed by Mujumdar and Subbarao (2003) is followed for implementing the FWLAM, with FORA used for identifying the key input variables and parameters and determining the key checkpoints in the river system. The frequency distribution of the water quality indicator levels at the key checkpoints with key variables and parameters treated as random, are obtained from MCS. Appropriate membership functions are assigned to the fuzzy set of *low water quality*. The frequency distribution of the water quality indicator level along with the fuzzy membership functions are then used to evaluate the fuzzy risk of low water quality at the key checkpoints.

### Fuzzy Waste Load Allocation Model

The FWLAM developed by Sasikumar and Mujumdar (1998) assumes a general river system consisting of a set of dischargers releasing pollutants into the river after removal of some fraction of the pollutants. Acceptable water quality conditions are ensured by monitoring water quality indicator levels (e.g., DO concentration) at a finite number of locations referred to as checkpoints. In a water quality management model, the concentration level of the water quality indicator is expressed as a function of the fractional removal levels for the pollutants released by the dischargers in the river system. An optimization problem is formulated with the set of fractional removal levels and the minimum satisfaction level forming the decision variables.

In the FWLAM, the following fuzzy optimization problem is formulated to take into account the fuzzy goals of the PCA and dischargers

$$\text{Maximize } \lambda \quad (1)$$

subject to

$$\mu_{E_{il}}(c_{il}) \geq \lambda \quad \forall i, l \quad (2)$$

$$\mu_{F_{imn}}(x_{imn}) \geq \lambda \quad \forall i, m, n \quad (3)$$

$$c_{il}^L \leq c_{il} \leq c_{il}^D \quad \forall i, l \quad (4)$$

$$\max[x_{imn}^L, x_{imn}^{\text{MIN}}] \leq x_{imn} \leq \min[x_{imn}^M, x_{imn}^{\text{MAX}}] \quad \forall i, m, n \quad (5)$$

$$0 \leq \lambda \leq 1 \quad (6)$$

where  $c_{il}$ =concentration level of water quality indicator  $i$  at checkpoint  $l$  of the river system. The PCA sets a desirable level  $c_{il}^D$  and a minimum permissible level  $c_{il}^L$  for water quality indicator  $i$  at checkpoint  $l$  ( $c_{il} \geq c_{il}^L$ ) which form the bounds on  $c_{il}$  as shown in crisp Constraint (4). Similarly,  $x_{imn}$  is the fractional removal level of pollutant  $n$  from discharger  $m$  to control water quality indicator  $i$  in the river system. The aspiration level and maximum fractional removal level acceptable to discharger  $m$  with respect to  $x_{imn}$  are represented as,  $x_{imn}^L$  and  $x_{imn}^M$ , respectively. The PCA imposes minimum fractional removal levels that are also expressed as lower bounds,  $x_{imn}^{\text{MIN}}$  in Constraint (5). The upper bound  $x_{imn}^{\text{MAX}}$  represents the technologically feasible maximum fractional removal level. Observing that the maximum acceptable level of pollutant treatment cannot exceed the technologically possible upper limit,  $x_{imn}^M$  is always considered the upper bound of Constraint (5). The fuzzy goal  $\mu_{E_{il}}(c_{il})$  in Constraint (2) is the goal of the PCA to maintain the concentration level,  $c_{il}$  of water quality indicator  $i$  at checkpoint  $l$  as close as possible to the desirable level. Similarly,  $\mu_{F_{imn}}(x_{imn})$  in Constraint (3) is the goal of the discharger to make the fractional removal level  $x_{imn}$ , as close as possible to the aspiration level,  $x_{imn}^L$  for all  $i, m$ , and  $n$ . The membership functions  $\mu_{E_{il}}$  and  $\mu_{F_{imn}}$  indicate variation of satisfaction levels of the PCA and dischargers with respect to the water quality indicator and fractional removal levels, respectively and Constraints (2) and (3) define the parameter  $\lambda$  as the minimum satisfaction level in the system. Crisp Constraints (5) and

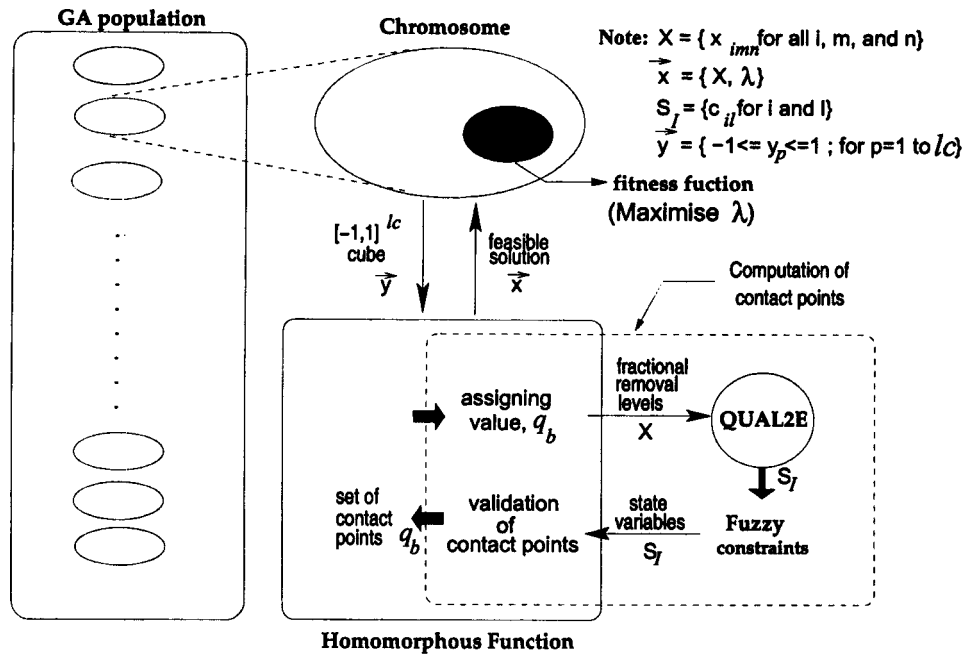


Fig. 2. Interaction between QUAL2E and genetic algorithm

(6) determine the space of alternatives. It may be noted that if the preferred maximum treatment level of the dischargers  $x_{imn}^M$  is less than the prescribed minimum  $x_{imn}^{\text{Min}}$  of the PCA, then a feasible solution exists. This leads to a case of complete conflict between the enforcing agency and dischargers.

Substituting expressions for the membership functions of the fuzzy goals  $\mu_{E_{il}}$  and  $\mu_{F_{imn}}$  (Sasikumar and Mujumdar 1998) in Constraints (2) and (3), respectively, the mathematical formulation of the fuzzy optimization is written as follows:

$$\text{maximize } \lambda \quad (7)$$

subject to

$$[(c_{il} - c_{il}^L)/(c_{il}^D - c_{il}^L)]^{\alpha_{il}} \geq \lambda \quad \forall i, l \quad (8)$$

$$[(x_{imn}^M - x_{imn})/(x_{imn}^M - x_{imn}^L)]^{\beta_{imn}} \geq \lambda \quad \forall i, m, n \quad (9)$$

$$c_{il}^L \leq c_{il} \leq c_{il}^D \quad \forall i, l \quad (10)$$

$$\max[x_{imn}^L, x_{imn}^{\text{Min}}] \leq x_{imn} \leq x_{imn}^M \quad \forall i, m, n \quad (11)$$

$$0 \leq \lambda \leq 1 \quad (12)$$

The exponents,  $\alpha_{il}$  and  $\beta_{imn}$ , appearing in Constraints (8) and (9), respectively, are nonzero positive real numbers. Assignment of numerical values to these exponents is subject to the desired shape of the membership functions and may be chosen appropriately by the decision maker. The concentration of water quality indicator  $c_{il}$  in Constraints (8) and (10) is determined using a water quality simulation model. In this study, the water quality simulation model QUAL2E, developed by the U.S. Environmental Protection Agency (Brown and Barnwell 1987) is used to estimate  $c_{il}$ . Inclusion of QUAL2E as the simulation model and the presence of the exponents,  $\alpha_{il}$  and  $\beta_{imn}$ , appearing in Constraints (8) and (9) render the optimization problem nonlinear. Genetic algorithms (GAs) are heuristic procedures for achieving global or near global solutions to complex problems. A GA is therefore selected

for solving the optimization problem using a simulation-optimization (SO) approach.

Since the GA is an unconstrained optimization technique, it is complemented with the homomorphous mapping (HM) (Koziel and Michalewicz 1999) method to handle the constraints. Interaction among GA, HM, and QUAL2E is shown in Fig. 2. Each chromosome of the GA is designed to represent the set of fractional removal levels  $x_{imn}$  and the satisfaction level  $\lambda$ , which are all decision variables of the optimization problem. The chromosome is coded in  $[-1, 1]^{lc}$  (where  $lc$  is the length of the chromosome) cube. It means that each element of a chromosome (which is called a *gene*) represents a real number between  $-1$  and  $1$ . Homomorphous mapping, after having multiple interactions with the simulation model QUAL2E, maps the  $[-1, 1]^{lc}$  cube to a feasible solution whose fitness function then is found. In the present case the decision variable  $\lambda$  acts as the fitness function for the chromosome. A similar procedure is followed for evaluating the fitness functions of all the chromosomes of the population. After evaluating the fitness functions, the GA applies the operators—reproduction, crossover, and mutation—to generate a new population with improved solutions. The procedure of fitness function evaluation of all the solutions in the new generation is repeated using QUAL2E. In a similar way, GA, HM, and QUAL2E are conjunctively used over the generations until the global solution criteria are met. The solution corresponding to the highest fitness function in the last generation is taken as the optimal solution with the objective function value equal to the fitness function. Again a fixed number of iterations are performed with a different set of GA parameters. In a similar way, different runs are made with different parameter sets to validate the optimal solution. The maximum of all such optimal solutions obtained after performing various runs is taken as the final solution for the purpose of risk evaluation.

A simulation run of QUAL2E with respect to the optimal solution yields the spatial distribution of the water quality indicator

level  $c_{il}$ . The checkpoints having the critical water quality indicator levels are chosen for evaluating the fuzzy risk of low water quality in the river system.

### Fuzzy Risk of Low Water Quality

The conventional water quality criteria at checkpoint  $l$  is such that any concentration of the water quality indicator less than a specified value, say,  $c_{il}^L$ , corresponds to a low water quality. This leads to a stringent definition of low water quality. To overcome this limitation and to account for imprecision in the description of low water quality, Sasikumar and Mujumdar (2000) and Mujumdar and Sasikumar (2002) introduced a fuzzy set based definition in place of the crisp set based definition of low water quality.

The set of concentration levels corresponding to the low water quality is defined as a fuzzy set  $W_{il}$ . Each concentration level in the fuzzy set  $W_{il}$  is assigned a membership value lying in the closed interval  $[0,1]$ . Mathematically, the fuzzy set  $W_{il}$ , is expressed as follows:

$$W_{il} = c_{il} : 0 \leq \mu_{W_{il}}(c_{il}) \leq 1 \quad (13)$$

The membership value  $\mu_{W_{il}}(c_{il})$  of the fuzzy set  $W_{il}$ , indicates the degree of compatibility of the concentration level with the notion of low water quality. The fuzzy risk of low water quality is defined as the probability of occurrence of the fuzzy event of low water quality. Mathematically, this can be stated as follows:

$$\text{fuzzy risk} = P(\text{fuzzy event of low water quality}) \quad (14)$$

$$= \tilde{P}(\text{low water quality}) \quad (15)$$

where  $\tilde{P}$  denotes the probability of a fuzzy event. The fuzzy risk is computed as

$$r_{il} = \int_0^{c_{\max,il}} \mu_{W_{il}}(c_{il}) f(c_{il}) dc_{il} \quad (16)$$

where  $\mu_{W_{il}}(c_{il})$  = membership function of the fuzzy set  $W_{il}$  of low water quality;  $c_{\max,il}$  = maximum concentration level; and  $f(c_{il})$  = probability density function (PDF) of the concentration of water quality indicator  $i$  at the checkpoint  $l$  in the river system. The fuzzy risk of low water quality at a checkpoint indicates the expected degree of low water quality and is a more general form of the crisp risk that indicates the probability of occurrence of a low water quality event.

### First-Order Reliability Analysis

Reliability-analysis methods such as FORA and MCS are based on multiple simulations and account for the combined effects of parameter sensitivity and parameter uncertainty in the identification of key input variables affecting the uncertainty of the model (Melching and Yoon 1996). An advantage of FORA over MCS is that, for suitable problems, it demands much less computational effort than MCS. Generally, however, the probability estimated by MCS approximates the exact value more closely as compared to other methods (e.g., Maier et al. 2001). Considering these aspects, FORA is used for identifying the key variables, whereas MCS is used to obtain the frequency distribution of  $c_{il}$ . However, MCS could be done without FORA for prediction of the uncertainty. First-order reliability analysis is used to provide quick evaluation of key parameters and locations.

First-order reliability analysis uses a first-order approximation of the relation between input and output variables for computing variances in multivariate situations. In FORA, a Taylor series expansion of the simulation model output is truncated after the first-order term (Melching and Yoon 1996)

$$y_v = G(X_{ue}) + \sum_{u=1}^{N_b} (x_u - x_{ue}) (\partial G / \partial x_u)_{X_{ue}} \quad (17)$$

where  $y_v$  = concentration of the constituent simulated in the selected water quality model; and  $G()$  = functional representation of the procedures simulating the constituent.  $G()$  may be mass balance equation which forms the basis for the QUAL2E water quality model;  $X_{ue}$  = vector of uncertain basic variables (e.g., model input variables, model parameters, etc.) representing the expansion point;  $X_u$  = vector of uncertain basic variables; and  $N_b$  = number of basic variables.

In FORA applications to water quality management problems, the expansion point is commonly assumed to be the mean value or some other convenient central value of the basic variables. For nonlinear systems, this assumption may lead to inaccurate estimation of mean and variance of the model output variable. Also, FORA is normally limited to problems where the random variables have relatively low variance [e.g., coefficient of variation (COV) of less than about 25%]. As FORA is applied in the present work only for identifying the key basic variables of the system, but not for quantifying the uncertainty of model output variables, this limitation of FORA may not be serious in this case. Thus, the expected value and variance of the model output, when the expansion point is considered at the mean value of the variables, are

$$E[y_v] \approx G(X_{uM}) \quad (18)$$

$$\text{var}(y_v) = \sigma_{y_v}^2 \approx \sum_{u_a=1}^{N_b} \sum_{u_b=1}^{N_b} (\partial G / \partial x_{u_a})_{X_{uM}} (\partial G / \partial x_{u_b})_{X_{uM}} E[(x_{u_a} - x_{u_aM})(x_{u_b} - x_{u_bM})]$$

where  $\sigma_{y_v}$  = standard deviation of  $y_v$ ; and  $X_{uM}$  = vector of mean values of the basic variables. In the above equation, subscripts  $u_a$  and  $u_b \in u$ . If the basic variables are statistically independent and derivatives are computed numerically, then

$$\text{var}(y_v) = \sigma_{y_v}^2 \approx \sum_{u=1}^{nb} [(\Delta G / \Delta x_u)_{X_{uM}}^2 \text{var}(x_u)] \quad (19)$$

The normalized sensitivity coefficient (NSC), which represents the percentage change in the output variable resulting from a unit percentage change in each input variable, is computed as follows (Brown and Barnwell 1987; Melching and Yoon 1996):

$$S_{uv} = (\Delta y_v / y_v) / (\Delta x_u / x_u) \quad (20)$$

where  $S_{uv}$  = normalized sensitivity coefficient for output  $y_v$  to input  $x_u$ ;  $x_u$  = base value of input variable [e.g., flow, DO, biological oxygen demand (BOD) values of headwater];  $\Delta x_u$  = magnitude of input perturbation;  $y_v$  = base value of output variables (e.g., DO of the river); and  $\Delta y_v$  = sensitivity of the output variable.

The ranking of NSC helps in identifying the key variables affecting the output variable of the river system.

First-order reliability analysis has been successfully applied to water quality models (Burgess and Lettenmaier 1975; Chadderton et al. 1982; Melching and Anmangandla 1992), despite conceptual problems such as the assumption of linearity in the functional

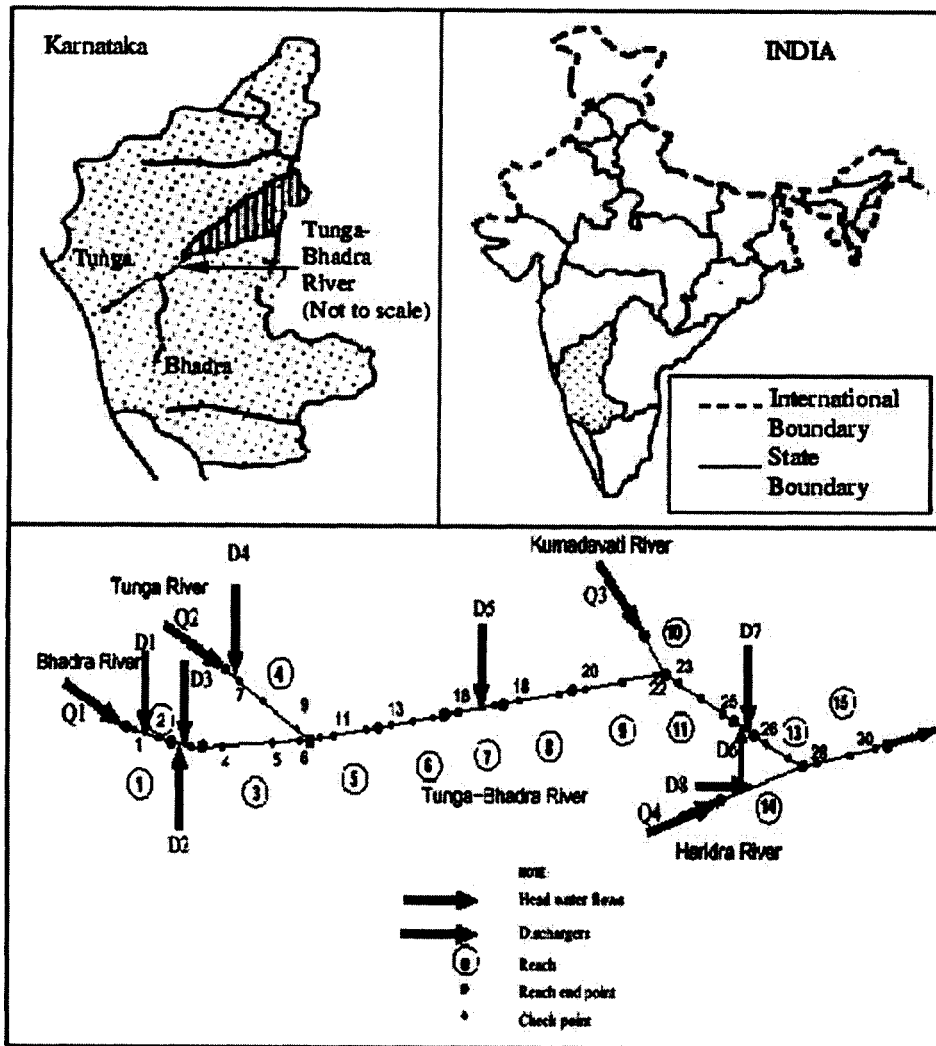


Fig. 3. Location map and schematic diagram of Tunga-Bhadra river system

approximation. First-order reliability analysis is performed initially at a few locations (or checkpoints), where the model output variables are likely to have a significant variability in magnitude. The key locations are identified based on the variance of the output variables at all the locations. Monte Carlo simulation is then applied at the key locations with key variables as the input random variables to obtain the frequency distribution and statistical parameters of the estimated output variable. The sources of water in the river system include distributed flow (or incremental flow) in addition to the headwater and point load flows. The distributed flow addition to the river may be due to runoff from predominantly either agricultural or forest areas and will accordingly affect water quality in the river. Here, fuzzy risk is computed for the case study with and without nonpoint source pollution through incremental flow and the contribution of incremental flow to the fuzzy risk is discussed.

### Model Application

Application of FORA to the fuzzy waste load allocation model is illustrated through a case study of the Tunga-Bhadra River system shown schematically in Fig. 3. The Tunga-Bhadra River is a perennial river formed by the confluence of Tunga and Bhadra

Rivers, both tributaries of the Krishna River, in southern India. The river has two other tributaries, the Kumadavati and Haridra Rivers. The river receives the waste loads from eight major effluent points which include five industrial effluents and three municipal effluents. The model is applied to a river stretch of 180 km that comprises the four headwaters (Tunga, Bhadra, Kumadavati, and Haridra) and eight point loads (five industrial and three municipal effluents). To maintain emphasis on simultaneous treatment of randomness and imprecision, the example is simplified by considering only one water quality indicator, the DO.

First-order reliability analysis and MCS are implemented using the uncertainty version of QUAL2E, viz., QUAL2E-UNCAS (Brown and Barnwell 1987). Melching and Yoon (1996) applied QUAL2E to determine the data required for reducing model-prediction uncertainty in a water quality model, and Han et al. (2001) modified QUAL2E-UNCAS for stochastic water quality analysis of downstream reaches of a river in Korea. Some limitations of the QUAL2E-UNCAS model are that it does not incorporate the effect of periphyton production on DO, and it does not consider nonpoint or diffuse sources of nutrients or oxygen demanding sources (Maier et al. 2001).

**Table 1.** Uncertainty Information of Basic Variables

Generic group	Basic variable	Coefficient of variation (%)	Source of data
Temperature coefficients	Biological oxygen demand decay	3.0	a
	Dissolved oxygen aeration	3.0	a
Hydraulic variables	Dispersion correction constant	50.0	a
	Manning's roughness	5.0	a
	Side slopes	5.0	a
	Bottom width	5.0	a
	Slope of channel	5.0	a
Reaction coefficients	Chemical biological oxygen demand oxidation rate	30.0	b
	Reaeration rate	50.0	b
Initial temperature	Initial temperature	10.0	c
Incremental flow	Discharge	27.0	c
	Temperature	8.0	c
	Dissolved oxygen	5.0	a
	Biological oxygen demand	10.0	a
Headwater flow	Discharge	27.0	c
	Temperature	8.0	c
	Dissolved oxygen	5.0	c
	Biological oxygen demand	10.0	c
Point load flow	Discharge	22.4	c
	Temperature	8.0	c
	Dissolved oxygen	5.0	a
	Biological oxygen demand	10.0	a

<sup>a</sup>QUAL2E-UNCAS manual.

<sup>b</sup>Melching and Yoon (1996).

<sup>c</sup>Historical data.

### Data Selection

The most important aspect of applying reliability-analysis methods, viz., FORA and MCS, for assessing the statistical parameters of DO concentration is to characterize uncertainty in the individual input variables required for QUAL2E-UNCAS. Uncertainty information is provided in two forms: (1) The coefficient of variation or relative standard deviation (COV), and (2) the specification of the PDF for each input variable. Table 1 presents the uncertainty information used in this present uncertainty analysis (UA). Table 1 includes a list of the basic variables (both natural variables and model parameters) considered in the analysis, with associated COV values and the source of the COV data. Historical data for the Tunga–Bhadra River system provides information on discharge and temperature characteristics of river and point load flows. Twenty-two years of mean annual flow discharge data obtained from daily flow records of a government agency (Water Resources Development Organization, Bangalore) are used in arriving at the COV of the headwater flow. The value of incremental flow is calculated based on gauge stations located in Bhadra (Reach 1), Tunga (Reach 4), and Tunga–Bhadra (Reach 7) Riv-

ers. Differences between flows at Tunga–Bhadra gage station and the sum of the flows at the Bhadra and Tunga gage stations is the flow incremented distributively. The ratio of this difference to the distance between the gage stations gives the distributed flow per unit distance, which is  $0.34 \text{ m}^3/\text{s}/\text{m}$  in the present case. This value is used as incremental flow throughout the river stretch, to account for the nonpoint source pollution due to runoff.

In Indian situations, agricultural sources form only a part of, but do not dominate, the contribution from diffuse pollution in the case of BOD (Agrawal 1999). The nonpoint BOD pollutant is primarily contributed by rural communities, animal husbandry, and on-stream activities in India. To conservatively account for uncertainty arising from lack of adequate data on nonpoint source pollution, a high value of  $30 \text{ mg/L}$  for BOD and a low value of  $4 \text{ mg/L}$  for DO are used for the incremental flow in the analysis. The COV of the incremental flow, BOD and DO, are assumed same as those of headwater flow. The COV values of the temperature coefficient, DO, BOD, and all hydraulic variable data except reaeration and deoxygenation rate coefficients are selected from the *typical* range (the other two ranges being *low* and *high*) for QUAL2E-UNCAS applications in Brown and Barnwell (1987, p. 86). The COV values of carbonaceous BOD and deoxygenation rate are obtained from Melching and Yoon (1996). The COVs of point, headwater and incremental inflow discharge and temperature were determined from historic data. Based on the literature (Melching and Yoon 1996; de Azevedo et al. 2000), all the input variables except headwater flow are assumed to follow a normal distribution for the purpose of analysis. A log-normal distribution is used for the headwater flow.

In the analysis, input variables are assumed to be uncorrelated because of lack of adequate data to estimate the correlation structure. If the variables are positively correlated, the assumption of statistical independence results in underprediction of the overall-model uncertainty. However, as described subsequently, the uncertainties in a few variables almost completely dominate the uncertainty of the simulated DO. These variables most likely have strong correlations between reaches. It may be noted that these variables are also the variables identified as key variables in FORA. In cases where the key variables identified by FORA contribute nearly all the output uncertainty, these results would be unlikely to change if variable correlations were considered (Melching and Yoon 1996).

The other important data necessary for performing the UA are the base values of all input variables of the model. For determining optimal waste load allocations for dischargers to the system, the values of discharge, DO of headwaters, point load flows, and temperature are selected with respect to adverse conditions (e.g., low flow, low DO, high temperature, etc.) prevailing in the river system. For example, a value of  $131.75 \text{ m}^3/\text{s}$  is taken for Tunga River headwater flow, while its mean value is  $166.89 \text{ m}^3/\text{s}$ . This yields conservative optimal waste load allocation to the dischargers. In the uncertainty analysis, however, the discharge, DO, and temperature are assigned values equal to their mean values given in Tables 2 and 3 to reflect general conditions of the river system. Thus, the mean value of simulated DO in FORA will equal the value obtained from the QUAL2E run corresponding to the base values of the input variables. For other variables, (e.g., hydraulic and reaction coefficients, etc.) base values assigned for the determination of optimal fractional removal levels are used in the UA. For solving the SO, BOD, and DO are taken as the pollutant and water quality indicator ( $i = n = 1$ ), respectively. Linear membership functions are considered ( $\alpha_{i1} = 1$  and  $\beta_{imn} = 1$ ) for the discharger and PCA goals in Constraints (8) and (9). The optimal

**Table 2.** Mean Values for Head Water Flow Conditions

River	River flow (m <sup>3</sup> /s)	Dissolved oxygen concentration (mg/L)	Biological oxygen demand concentration (mg/L)
Bhadra	17.80	6.5	1
Tunga	166.89	6.5	1
Kumadavati	14.94	6.5	1
Haridra	13.90	6.0	1

treatment levels and satisfaction level obtained from SO are given in Table 3. Both FORA and MCS are performed with respect to the optimal fractional removal levels computed from the SO approach (Table 3).

### Screening of Basic Variables

For carrying out FORA, 14 locations were selected based on the criteria of the lowest DO concentration in a reach and proximity of the locations (checkpoints) to the point loads and junctions. The DO concentrations are the result of a QUAL2E run with respect to the optimal fractional removal levels obtained from the SO. The significance of some locations chosen for the analysis is as follows: locations 1–3, (Reach 1, computational element 3), 2-3, 4-3, 7-10, and 13-1 are immediately downstream of point loads, where the lowest DO concentrations of the reach are observed in QUAL2E simulations. Locations 5-2, 11-2, and 15-2 are downstream of the river junctions of the Tunga–Bhadra, Kumadavati, and Haridra Rivers, respectively. There is no significance to location 7-1, which is merely chosen to learn the effect of the uncertainty in the middle portions of the river. Other locations are also similarly chosen based on judgment. QUAL2E-UNCAS requires the perturbation percentage to the input variable. The NSC and variances are computed for 5% perturbation. The application of a 5% increment in the parameter values was recommended by Brown and Barnwell (1987) for uncertainty calculation in QUAL2E-UNCAS (Melching and Yoon 1996).

Table 4 shows the NSC matrix for the output variable, DO, obtained from sensitivity analysis. The sensitivity analysis of DO was performed for all the basic variables listed in Table 1. The sensitivity analysis reveals that initial temperature has the highest sensitivity in all reaches except Reach 4. The reaeration coefficient, headwater flow, headwater DO, pointload flow, and point load BOD have a significant sensitivity (with NSC magnitude greater than 0.1) at only a few locations. An insignificant variable

**Table 3.** Effluent Flow Data and Optimal Fractional Removal Levels

Discharger	Biological oxygen demand concentration (mg/L)	Dissolved oxygen concentration (mg/L)	Effluent flow (m <sup>3</sup> /s)	Optimal fractional removal level <sup>a</sup> (%)
$D_1$	1000	2.0	1.167	74.6
$D_2$	440	2.0	0.539	74.6
$D_3$	300	2.0	0.032	66.5
$D_4$	900	2.0	0.763	35.0
$D_5$	222	2.0	0.042	35.0
$D_6$	600	2.0	0.225	35.0
$D_7$	450	2.0	1.672	35.0
$D_8$	900	2.0	1.515	45.0

<sup>a</sup>Obtained from fuzzy waste load allocation model; optimal satisfaction level,  $\lambda^*=0.28$ .

**Table 4.** Normalized Sensitivity Coefficient Matrix for Dissolved Oxygen Concentration without Incremental Flow

Basic variable	Reach 1		Reach 2		Reach 3		Reach 4		Reach 5		Reach 7		Reach 7		Reach 9		Reach 11		Reach 11		Reach 13		Reach 13		Reach 15		Reach 15	
	Element 3	Element 3	Element 3	Element 3	Element 2	Element 2	Element 3	Element 3	Element 2	Element 2	Element 1	Element 1	Element 10	Element 10	Element 19	Element 19	Element 2	Element 2	Element 16	Element 16	Element 12	Element 12	Element 2	Element 2	Element 19	Element 19		
Reaeration coefficient	0.10	0.15	0.04	0.03	0.05	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	
Initial temperature	-0.85	-0.93	-0.73	-0.18	-0.70	-0.81	-0.81	-0.18	-0.70	-0.81	-0.81	-0.81	-0.81	-0.79	-0.79	-0.79	-0.75	-0.75	-0.78	-0.78	-0.83	-0.83	-0.81	-0.81	-0.87	-0.87	-0.87	
Head water flow	0.08	0.13	0.00	0.00	0.00	<0.10	<0.10	0.00	0.00	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	
Head water dissolved oxygen	0.10	0.02	0.06	0.78	0.14	<0.10	<0.10	0.78	0.14	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	
Point load flow	-0.09	-0.15	-0.02	-0.01	-0.02	<0.10	<0.10	-0.01	-0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	
Point load biological oxygen demand	-0.09	-0.16	-0.02	-0.00	-0.02	<0.10	<0.10	-0.00	-0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	

**Table 5.** Components of Percentage of Variance Matrix for Dissolved Oxygen Concentrations without Incremental Flow

Basic variable	Reach 1	Reach 2	Reach 3	Reach 4	Reach 5	Reach 7	Reach 7	Reach 7	Reach 9	Reach 11	Reach 11	Reach 13	Reach 13	Reach 15	Reach 15
	Element 3	Element 3	Element 20	Element 3	Element 2	Element 1	Element 10	Element 19	Element 2	Element 16	Element 1	Element 12	Element 2	Element 19	Element 19
Biological oxygen demand decay	4.39	3.55	0.01	0.26	0.41	<1.00	<1.00	<1.00	<1.00	<1.00	0.02	0.43	0.98	2.27	2.27
Reaeration coefficient	23.41	30.41	7.64	8.31	11.73	1.83	1.25	0.92	0.92	1.27	1.02	2.05	3.80	12.10	12.10
Initial temperature	63.10	49.92	91.43	15.69	86.61	97.93	98.59	98.99	98.99	98.48	98.88	95.82	92.14	80.26	80.26
Head water flow	4.14	7.23	0.00	0.01	0.02	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	0.45	0.98	1.76	1.76
Head water dissolved oxygen	0.23	0.00	0.13	75.61	0.81	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00
Point load flow	3.78	6.64	0.45	0.08	0.29	<1.00	<1.00	<1.00	<1.00	<1.00	0.15	0.64	1.19	2.36	2.36
Point load biological oxygen demand	0.73	1.39	0.09	0.01	0.06	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00
SUM (%) <sup>a</sup>	99.78	99.15	99.75	99.97	99.92	99.76	99.85	99.90	99.90	99.75	99.90	99.40	99.09	98.75	98.75
Standard deviation	0.72	0.84	0.55	0.30	0.54	0.60	0.59	0.58	0.58	0.55	0.57	0.60	0.59	0.68	0.68

<sup>a</sup>Standard deviation of dissolved oxygen at the location.

has a sensitivity coefficient equal to zero and near to zero values at all 14 locations considered in the analysis. For this reason the NSC matrix of Table 4 shows sensitivity coefficients for DO concentration corresponding to only significant variables at the selected locations. QUAL2E-UNCAS performs sensitivity analysis only for five locations at a time (i.e., Locations 1-3, 2-3, 3-20, 4-3, 5-2; 7-1, 7-10, 9-19, 11-2, 11-16; and 13-1, 13-12, 15-2, 15-19). During the analysis, if the NSC of any basic variable (e.g., headwater DO in Table 4) is found to be <0.10 for all five locations, then the NSC is reported as <0.10 in the coefficient matrix. For this reason, some of the NSC values are reported as <0.10 in Table 4 at these five locations. The NSC shows the variables affecting DO concentration. Initial temperature is the predominant variable affecting DO as shown by its having the highest NSC value in all reaches except in Reach 4. The reaeration coefficient follows temperature in influencing DO with positive relationship although with a very low NSC. Other than these two variables, headwater DO also has an effect on DO, but only in Locations 1-3, 4-3, and 5-2. In Reach 4 (Tunga River) headwater flow is considerably higher (166.89 m<sup>3</sup>/s) than any other headwater flow and this affects the sensitivity of DO significantly. In middle reaches, DO is invariant to perturbation in variables other than initial temperature and reaeration coefficient. This is possibly due to the absence of point loads with high concentrations.

The variance analysis gives the magnitude of variance in the DO concentration due to the variance in an input variable. The contribution in percent of variance of each basic variable to the variance in the DO concentration estimated in QUAL2E-UNCAS with respect to the optimal allocation policy is given in Table 5. Similar to the sensitivity analysis, QUAL2E-UNCAS performs variance analysis only for five locations at a time. During the analysis, if the variance of any basic variable is less than 1% for all five locations, the value is reported as <1% at those locations in Table 5. The results show a similar but a somewhat modified pattern relative to the normalized sensitivity coefficients. As seen from the table, temperature and reaeration coefficient account for more than 95% of the variability in DO concentrations at most locations. Dissolved oxygen is more sensitive to temperature as is evident from both NSC matrix and percentage of variance matrix. Some marginal influence of other variables is observed in the last four locations (13-1, 13-12, 15-2, and 15-19). Headwater flow, headwater DO, and point load BOD have effect both at initial (1-3, 2-3, 3-20, 4-3 and 5-2) and end locations (13-1, 13-12, 15-2 and 15-19). The reasons discussed for NSC are valid here also for the dominance of headwater DO at Location 4-3. The BOD decay effect is slightly present in the first two locations and the last location.

Both NSC and percentage of variance are taken into consideration in screening the basic variables. Melching and Yoon (1996) have indicated that the NSC is not an appropriate way to determine the key parameters. The fraction of variance obtained from Eq. (19) is a far more powerful and useful tool. It is seen that only temperature and reaeration coefficient influence DO to a significant extent and headwater flow, headwater DO, and point load BOD to a lesser extent in the river system. Though headwater flow, headwater DO, and point load BOD have influence only at the beginning and end locations, they also have been considered as key random variables in the MCS analysis. Since DO is invariant to all variables except initial temperature and reaeration coefficient at the middle (7-1, 7-10, 9-19, 11-2, and 11-16) locations, these locations are neglected in the MCS analysis.

**Table 6.** Normalized Sensitivity Coefficient Matrix for Dissolved Oxygen Concentration with Incremental Flow

Basic variable	Reach 1 Element 3	Reach 2 Element 3	Reach 3 Element 20	Reach 4 Element 3	Reach 5 Element 2	Reach 7 Element 1	Reach 7 Element 1	Reach 7 Element 10	Reach 9 Element 19	Reach 11 Element 2	Reach 11 Element 16	Reach 13 Element 1	Reach 13 Element 12	Reach 15 Element 2	Reach 15 Element 19
Biological oxygen demand decay	-0.08	-0.11	-0.02	-0.01	-0.03	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Reaeration coefficient	0.12	0.18	0.09	0.03	0.08	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	0.06	0.07	0.08	0.11
Initial temperature	-0.86	-0.98	-0.75	-0.18	-0.72	-0.83	-0.83	-0.83	-0.81	-0.77	-0.80	-0.82	-0.85	-0.83	-0.89
Head water flow	0.09	0.16	0.04	0.00	0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Head water dissolved oxygen	0.10	0.02	0.06	0.78	0.13	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Point load flow	-0.09	-0.14	-0.02	-0.01	-0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10
Point load biological oxygen demand	-0.09	-0.15	-0.02	-0.00	-0.02	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10	<0.10

To examine the effect of nonpoint source pollution, FORA is carried out next by including the incremental flow. Tables 6 and 7 provide the results obtained from sensitivity analysis and FORA, respectively, for this case. Except for indicating the incremental flow effect, the FORA results show the same trend as in the case of noninclusion of incremental flows. Both the NSC and variance analyses indicate initial temperature and reaeration coefficient as major influencing variables at all the locations. Incremental flow, point load flow, headwater flow, and BOD are observed as other influencing variables. Incremental flow has the greatest influence after temperature and reaeration coefficient in all reaches except Reaches 1 and 2. Since the incremental flow is added uniformly, its cumulative magnitude is small in the initial section, Reaches 1 and 2. It may be observed that, at the locations away from point loads, the contribution from variance of incremental flow keeps building up on the downstream side. The point loads in Reaches 11 and 14 change the magnitude and the trend. Based on FORA and NSC results, reaeration coefficient, initial temperature, headwater flow, headwater DO, BOD decay, incremental flow, and point load BOD and flow are taken as basic variables. Since DO is invariant to all variables except initial temperature, reaeration coefficient, and incremental flow at the middle (7-1, 7-10, 9-19, 11-2, and 11-16) locations, these locations are neglected in the MCS analysis.

### Monte Carlo Simulation Analysis

Table 8 contains the summary statistics (base mean, simulated mean, bias, minimum, maximum, range, standard deviation, coefficient of variation, and skewness coefficient) for simulated DO (simulated with MCS) concentration at the key locations identified from FORA for analysis including the incremental flow. The results show similar trends for the case of noninclusion of incremental flows. The bias, shown in the table, is the difference between base value (resulting when the mean values of all parameters are used in the simulation) and simulated mean of DO concentration, whereas range is the difference between the minimum and maximum of all simulated DO concentrations. Initial analysis of MCS, varying the number of simulations, has shown that 2000 simulations are sufficient to achieve convergence of the statistics for the simulated DO variable. There is a good match between base and simulated means of DO concentration. Out of all the locations considered for MCS analysis, lowest values of simulated mean and minimum values and highest values of coefficient of variation are observed at Location 2-3. This is the most critical location being immediately downstream of two high BOD loads. The trend of the statistical parameters are the same at other locations, except for higher magnitudes in simulated mean, minimum, maximum values, and lower magnitudes of COV.

### Evaluation of Fuzzy Risk

The fuzzy risk of low water quality is computed with respect to the output variable DO concentration. Since only one variable is considered for the evaluation of the fuzzy risk, the suffix  $i$  is dropped. Denoting the fuzzy set of low water quality, DO concentration, and fuzzy risk of low water quality by  $W_l$ ,  $c_l$ , and  $r_l$ , respectively, the fuzzy risk is rewritten in discrete form as

$$r_l = \sum_{c_{\min_l}}^{\text{MIN}[c_{\max_l}, c_l^D]} \mu_{W_l}(c_l) p(c_l) \quad (21)$$

where  $c_{\min_l}$  and  $c_{\max_l}$  = minimum and maximum concentration lev-

**Table 7. Components of Percentage of Variance Matrix for Dissolved Oxygen Concentrations with Incremental Flow**

Basic variable	Reach 1 Element 3	Reach 2 Element 3	Reach 3 Element 20	Reach 4 Element 3	Reach 5 Element 2	Reach 7 Element 1	Reach 7 Element 10	Reach 9 Element 19	Reach 11 Element 2	Reach 11 Element 16	Reach 13 Element 1	Reach 13 Element 12	Reach 15 Element 2	Reach 15 Element 19
Biological oxygen demand decay	5.03	5.21	0.44	0.29	1.01	<1.00	<1.00	<1.00	<1.00	<1.00	0.74	1.62	2.23	3.37
Reaeration coefficient	26.75	36.64	26.59	8.52	20.99	10.16	9.67	11.44	12.12	11.58	11.18	13.67	16.01	26.27
Initial temperature	59.21	43.22	69.17	15.77	75.56	86.88	87.56	84.97	84.43	85.41	85.10	80.44	76.75	64.17
Incremental flow	0.12	0.57	1.73	0.02	0.80	1.05	1.09	1.68	1.61	1.46	1.45	1.39	1.20	1.04
Head water flow	4.66	8.22	1.15	0.00	0.60	0.87	0.79	1.00	0.87	0.73	1.17	1.91	2.37	2.89
Head water dissolved oxygen	0.21	0.00	0.12	75.28	0.60	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00	<1.00
Point load flow	3.16	4.40	0.26	0.08	0.23	<1.00	<1.00	<1.00	<1.00	<1.00	0.08	0.36	0.66	1.31
SUM (%) <sup>a</sup>	99.15	98.26	99.46	99.96	99.79	98.96	99.11	99.09	99.03	99.18	99.71	99.39	99.22	99.04
Standard deviation	0.74	0.91	0.61	0.30	0.57	0.62	0.62	0.61	0.58	0.60	0.61	0.64	0.63	0.73

<sup>a</sup>Standard deviation of dissolved oxygen at the location.

els of DO obtained from MCS at checkpoint  $l$ . Fig. 4 shows a typical membership function of low water quality  $\mu_{w_l}(c_l)$ , which is expressed as

$$\mu_{w_l}(c_l) = [(c_l^D - c_l) / (c_l^D - c_l^L)]^{\gamma_l} \quad (22)$$

where  $\gamma_l$  = nonzero positive real number defining the shape of the membership function at location  $l$ . The value of  $\gamma_l$  may be selected by the decision makers based on their perception of low water quality to a given value of DO.  $c_l^D$  is set to 95% of the saturated DO concentration, since an achievement of saturation DO is nearly impossible, even in natural conditions. The value of  $c_l^L$  is set to 4 mg/L for the locations in Reaches 1, 2, and 3, and 6 mg/L for the other locations. The 95% of saturation and 4 mg/L bounds of the membership function are for illustration purposes only, and a more realistic, ecologically based membership function should be developed in future research. The frequency distribution obtained from MCS is used to compute the probability distribution function  $p(c_l)$  in Eq. (21). The membership function  $\mu_{w_l}(c_l)$  of Eq. (22) and frequency density function  $p(c_l)$  at key location  $l$  are substituted in Eq. (21). The fuzzy risk is evaluated between  $c_{\min_l}$  and minimum of  $c_{\max_l}$  and desirable level  $c_l^D$ .

## Results and Discussion

Table 9 presents the results of the fuzzy risk levels as well as permissible and desirable DO concentration levels at the selected key locations of the river for three different values of  $\gamma$ . The fuzzy risk trends for the three  $\gamma$  are same.  $\gamma < 1$  (0.8) and  $\gamma > 1$  (1.2) give higher and lower values compared to linear membership ( $\gamma = 1$ ) based values and reflect, respectively, pessimistic and optimistic perceptions of the decision maker. The results are discussed with reference to the linear membership function.

Results for analysis without incremental flow show the highest fuzzy risk at Location 4-3. The higher fuzzy risk level at Location 4-3 compared to Locations 1-3 and 2-3 indicates the effect of minimum permissible and desirable levels.  $c_l^L$  at location 4-3 (6 mg/L) is more stringent than that of Locations 1-3 and 2-3 (4 mg/L). Setting of 6 mg/L at Locations 1-3 and 2-3 yields higher fuzzy risk level at those locations (43.51 and 66.45%) than at Location 4-3. Overall, the reason for higher fuzzy risk levels at 1-3, 2-3, and 4-3 is due to their location immediately downstream of high point loads. As the simulated mean DO values at many locations are greater than 6.5 mg/L with a very small variance, the number of simulated DO concentration levels that fall below  $c_l^L$  are nil and this results in zero fuzzy risk at those locations.

In Reach 15, the reaeration coefficient is low compared to that in all other reaches. Dissolved oxygen is more sensitive to reaeration in this reach, as reflected in the high variance values for the reaeration coefficient at Locations 15-2 and 15-19. This high variance resulted in low values of mean, minimum, and maximum DO values, and high standard deviation of DO at 15-2 and 15-19. These statistics when used in the MCS, result in a higher frequency of DO levels around minimum permissible level. Fuzzy membership values close to 1 near the minimum permissible value of DO resulted in a high fuzzy risk at these locations.

Inclusion of incremental flow in the analysis completely alters the trend and magnitudes of the fuzzy risk levels at all locations. In this case, the highest fuzzy risk levels are obtained at the last reach due to the obvious reason of the cumulative effect of incremental flow and nonpoint source of pollution resulting from it. The fuzzy risk considering the incremental flow may be seen as the sum of fuzzy risk with only point loads and fuzzy risk only

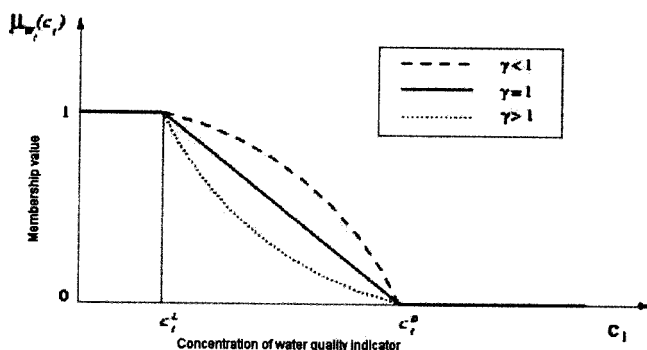
**Table 8.** Summary of Monte Carlo Simulation for Dissolved Oxygen (with Incremental Flow)

Reach	Element	Base mean	Simulated mean	Bias	Minimum	Maximum	Standard deviation	Coefficient of variation
1	3	6.61	6.56	-0.05	3.98	7.13	0.28	0.04
2	3	6.17	6.11	-0.06	3.56	6.99	0.40	0.07
3	20	6.85	6.83	0.02	6.08	7.08	0.09	0.01
4	3	6.62	6.62	0.00	5.79	7.47	0.26	0.04
5	2	6.92	6.91	-0.01	5.96	7.16	0.10	0.01
13	1	6.94	6.92	-0.02	6.23	7.09	0.09	0.01
13	12	6.75	6.73	-0.02	5.89	6.98	0.12	0.02
15	2	6.66	6.64	-0.02	5.67	6.93	0.14	0.02
15	19	6.63	6.61	-0.02	5.56	7.01	0.18	0.03

with incremental flow. There are no factors other than the incremental flow that change the trend and magnitude of the fuzzy risk levels resulting from only point loads.

Uncertainty analysis is also carried out with the low flow values used (e.g., 131.75 m<sup>3</sup>/s for Tunga headwater flow) in deriving the optimal fractional removal levels, with a view to determine the risk under design low flow conditions. The COV of headwater flows for this analysis was determined as 0.27 from the historic mean low flows. The key locations and key variables identified by FORA and sensitivity analysis were the same as those with mean flows. For both cases of neglecting and including incremental flow, the resulting fuzzy risk values at all locations were nearly the same as those obtained with mean flows, except at the last two locations, 15-2 and 15-19. At these two locations the fuzzy risk values are 17.64 and 22.58%, respectively, for the case of neglecting incremental flow (as against 11.31 and 13.29% with mean flows, shown in Table 9). The fuzzy risk at these two locations, with incremental flow are 46.99 and 52.29%, respectively (against, 39.62 and 42.77% with mean flows shown in Table 9).

The crisp risk, defined as  $P[c_i \leq c_i^L]$ , is also determined at the key locations. As the cumulative frequency of DO concentration level below 6 mg/L is near to zero in the case considering only point loads and low in the case considering incremental flow, the crisp risks are all negligible, being very low to zero. Since the fuzzy risk includes a wider range of DO concentration levels than the crisp risk, in general, the fuzzy risk values will be higher than the crisp risk. As seen from the membership function of low water quality, Fig. 4, the fuzzy risk and crisp risk will both be equal to 1 only in the unlikely event of all simulated values of  $c_i$  being less than  $c_i^L$ .

**Fig. 4.** Membership function for fuzzy set,  $W_l$ 

## Conclusions

A procedure is presented for risk evaluation in a river water quality control problem. The concept of fuzzy risk is used in the context of water quality-control problems. While the crisp risk denotes the probability of failure, the fuzzy risk indicates the expected degree of failure and, thus, provides a more general measure of risk. To account for uncertainty in the standards for determining failure, occurrence of failure itself is treated as a fuzzy event. The fuzzy definition of low water quality ensures that there is no single *threshold* value which defines a failed state. All discrete water quality concentrations are treated as *failures* of different degrees. The fuzzy set of low water quality maps all water quality levels to *low water quality* and its membership function denotes the degree to which the water quality is low. The membership functions of the fuzzy sets are subjective statements of the perceptions of the decision makers. For example, the membership function for the low water quality indicates the decision maker's perception of the degree of *low quality*, for a given level of water quality. The lower and upper bounds of the membership functions also are subjective, and in general depend on the particular problem being solved. Although not considered here, uncertainty in the lower and upper bounds of the membership functions may be treated as fuzzy and modeled using gray numbers (e.g., Chang et al. 1997). It may be noted that allowing the lower limits of the fuzzy membership functions to be less than normally used standard values such as 5 mg/L for DO, and shaping the membership functions with respect to biological information on DO requirements for aquatic life, would be a useful application of the methodology presented herein.

A fuzzy optimization model is first solved to obtain optimal fractional removal levels, and then, with these optimal fractional removal levels held fixed, the river system is simulated with the input variables treated as random. This implicit approach has the advantage of computational simplicity. Considering the random nature of input variables explicitly in QUAL2E and the optimization model would pose several computational difficulties associated with expressing the stochastic water quality simulation as a set of constraints in the fuzzy optimization model. With a number of variables treated randomly, it would be impossible to solve a stochastic water quality simulation integrated into the fuzzy optimization model. To overcome these difficulties, the implicit approach is applied, but with the limitation that fractional removal levels are determined independent of the uncertainty in the DO concentrations and, therefore, will not be truly optimal. Future research may be directed towards integrating the two models, viz., the optimal fractional removal model, and the risk analysis

**Table 9.** Fuzzy Risk at Key Locations Identified by First Order Reliability Analysis

Location no.	Reach no.	Element no.	Distance from $u/s$ (km)	Minimum permissible level ( $c_T^L$ ) (mg/L)	Desirable level ( $c_T^D$ ) (mg/L)	Without incremental flow			With incremental flow		
						Fuzzy risk ( $r_I$ ) (%)			Fuzzy risk ( $r_I$ ) (%)		
						$\gamma=0.8$	$\gamma=1.0$	$\gamma=1.2$	$\gamma=0.8$	$\gamma=1.0$	$\gamma=1.2$
1	1	3	3	4.00	7.13	22.35	15.74	11.17	24.88	17.92	13.02
2	2	3	7	4.00	7.15	33.04	25.56	19.91	40.74	32.98	26.84
3	3	20	27	4.00	7.14	0.07	0.03	0.01	15.56	9.85	6.26
4	4	3	3	6.00	7.09	47.51	41.13	35.90	49.22	42.81	37.53
5	5	2	29	6.00	7.09	1.00	0.51	0.27	20.40	14.19	9.96
6	13	1	130	6.00	6.97	0.00	0.00	0.00	0.04	0.02	0.01
7	13	12	141	6.00	6.97	0.89	0.53	0.32	32.86	25.41	19.81
8	15	2	143	6.00	7.06	16.47	11.31	7.88	47.33	39.62	33.31
9	15	19	160	6.00	7.06	18.21	13.29	9.87	50.11	42.77	36.70

model to derive policies that minimize risk of low water quality while maximizing goal satisfaction.

Optimal fractional removal levels have been obtained from the fuzzy optimization model using critical values of influencing variables (e.g., low stream flow, high temperature, etc.), whereas the entire range of possible values of the variables is used in evaluating the implications of the optimal fractional removal levels. It may be noted that for the purpose of risk evaluation, it is not really necessary to employ FORA. The MCS could be run with all parameters considered as uncertain at all possible output locations. Unfortunately, this necessitates modification of the QUAL2E code or requires multiple UNCAS runs since the number of output locations in QUAL2E is restricted. The key parameters could be determined by a simple regression analysis between the model output and each input parameter. First-order reliability analysis was used here mainly to provide a rapid evaluation of the key parameters and key locations.

## Acknowledgments

The writers sincerely thank the two anonymous reviewers for reviewing the manuscript and providing critical comments to improve the paper. The work reported in this paper was supported by the Department of Science and Technology, Government of India, Vide Project No. III.5(17)97-ET.

## References

- Agrawal, C. D. (1999). "Diffuse agricultural water pollution in India." *Water Sci. Technol.*, 39(3), 33–47.
- Ambrose, R. B., Wool, T. A., Connolly, J. P., and Robert, W. S. (1988). "WASP4—A hydrodynamic and water quality model—Model theory." *User's Manual, and Programmer's Guide, Rep. No. EPA/600/3-87/039*, Environmental Research Laboratory, U.S. Environmental Protection Agency, Athens, Ga.
- Brown, L. C., and Barnwell, T. O., Jr. (1987). "The enhanced stream water quality models QUAL2E and QUAL2E-UNCAS: Documentation and user manual." *Rep. No. EPA/600/3-87/007*, U.S. Environmental Protection Agency, Athens, Ga.
- Burges, S. J., and Lettenmaier, D. P. (1975). "Probabilistic methods in stream quality management." *Water Resour. Bull.*, 11(1), 115–130.
- Burn, D. H. (1989). "Water-quality management through combined simulation-optimization approach." *J. Environ. Eng.*, 115(5), 1011–1024.
- Burn, D. H., and Lence, B. J. (1992). "Comparison of optimization formulations for waste-load allocations." *J. Environ. Eng.*, 118(4), 597–612.
- Cardwell, H., and Ellis, H. (1993). "Stochastic dynamic programming models for water quality management." *Water Resour. Res.*, 29(4), 803–813.
- Chadderton, R. A., Miller, A. C., and McDonnell, A. J. (1982). "Uncertainty analysis of Dissolved Oxygen model." *J. Environ. Eng. Div. (Am. Soc. Civ. Eng.)*, 108(5), 1003–1013.
- Chang, N., Chen, H. W., Shaw, D. G., and Yang, C. H. (1997). "Water pollution control in river basin by interactive fuzzy interval multiobjective programming." *J. Environ. Eng.*, 123(12), 1208–1216.
- de Azevedo, L. G. T., Gates, T. K., Fontane, D. G., Labadie, J. W., and Porto, R. L. (2000). "Integration of water quantity and quality in strategic river basin planning." *J. Water Resour. Plan. Manage.*, 126(2), 85–97.
- Ellis, J. H. (1987). "Stochastic water quality optimization using embedded chance constraints." *Water Resour. Res.*, 23(12), 2227–2238.
- Han, K. Y., Kim, S. H., and Bae, D. H. (2001). "Stochastic water quality analysis using reliability method." *J. Am. Water Resour. Assoc.*, 37(3), 695–708.
- Koziel, S., and Michalewicz, Z. (1999). "Evolutionary algorithms, homomorphous mappings, and constrained parameter optimization." *Evol. Comput.*, 7(1), 19–44.
- Maier, H. R., Lence, B. J., Tolson, A., and Foschi, O. (2001). "First-order reliability method for estimating reliability, vulnerability, and resilience." *Water Resour. Res.*, 37(3), 779–790.
- Melching, C. S., and Anmangandla, S. (1992). "Improved first-order uncertainty method for water-quality modeling." *J. Environ. Eng.*, 118(5), 791–805.
- Melching, C. S., and Yoon, C. G. (1996). "Key sources of uncertainty in QUAL2E model of Passaic River." *J. Water Resour. Plan. Manage.*, 122(2), 105–113.
- Mujumdar, P. P., and Sasikumar, K. (2002). "A fuzzy risk approach for seasonal water quality management of a river system." *Water Resour. Res.*, 38(1), 5-1–5-9, doi: 10.1029/2000WR000126.
- Mujumdar, P. P., and Subbarao, V. V. R. (2003). "Fuzzy waste load allocation model for river systems: simulation—optimisation approach." *J. Comput. Civ. Eng.*, 18(2), 120–131.
- Sasikumar, K., and Mujumdar, P. P. (1998). "Fuzzy optimization model for water quality management of a river system." *J. Water Resour. Plan. Manage.*, 124(2), 79–84.
- Sasikumar, K., and Mujumdar, P. P. (2000). "Application of fuzzy probability in water quality management of a river system." *Int. J. Syst. Sci.*, 31(5), 575–591.
- Streeter, H. W., and Phelps, E. B. (1925). "A study of the pollution and natural purification of the Ohio River, III. Factors concerning the phe-

nomena of oxidation and reaeration." *Pub. Health Bull. No. 146*, U.S. Public Health Service, Washington, D.C.

Takyi, A. K., and Lence, B. J. (1994). "Incorporating input information uncertainty in a water quality management model using combined simulation and optimization." *Proc., Int. UNESCO Sym. on Water Resources Planning in a Changing World*, Karlsruhe, Germany.

Takyi, A. K., and Lence, B. J. (1999). "Surface water quality management using a multiple-realization chance constraint method." *Water Resour. Res.*, 35(5), 1657–1670.

Tyagi, A., and Haan, C. T. (2001). "Reliability, risk and uncertainty analysis using generic expectation functions." *J. Environ. Eng.*, 127(10), 938–945.