Urban Freight Tour Models: State of the Art and Practice

José Holguín-Veras, Ellen Thorson, Qian Wang, Ning Xu, Carlos González-Calderón, Iván Sánchez-Díaz, John Mitchell

Center for Infrastructure, Transportation, and the Environment (CITE)
Introduction, Basic Concepts

Urban Freight Tours: Empirical Evidence

Urban Freight Tour Models
  - Simulation Based Models
  - Hybrid Models
  - Analytical Models

Analytical Tour Models

Conclusions
Introduction and Basic Concepts
Basic Concepts

- A supplier sending shipments from its home base (HB) to six receivers

The goal is to capture both the underlying economics of production and consumption, and realistic delivery tours.

Notation:
- Loaded vehicle-trip
- Empty vehicle-trip
- Commodity flow
- Consumer (receiver)

Diagram with nodes labeled S-1, S-2, S-3, R1, R2, R3, R4, R5, R6, and HB.
Empirical Evidence on Urban Freight Tours
Characterization of Urban Freight Tours (UFT)

- Number of stops per tour depends on: Country, city, type of truck, the number of trip chains, type of carrier, service time, and commodity transported.
Characterization of Urban Freight Tours (UFT)

- Denver, Colorado (Holguín-Veras and Patil, 2005):
  - By type of company:
    - Common carriers: 15.7 stops/tour
    - Private carriers: 7.1 stops/tour
  - By origin of tour:
    - New Jersey: 13.7 stops/tour
    - New York: 6.0 stops/tour

<table>
<thead>
<tr>
<th>Stops/Tour</th>
<th>Single Truck</th>
<th>Combination Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>6.5</td>
<td>7.0</td>
</tr>
<tr>
<td>1 tour/day</td>
<td>7.2</td>
<td>7.7</td>
</tr>
<tr>
<td>2 tours/day</td>
<td>4.5</td>
<td>3.7</td>
</tr>
<tr>
<td>3 tours/day</td>
<td>2.8</td>
<td>3.3</td>
</tr>
</tbody>
</table>
Characterization of Urban Freight Tours (UFT)

- NYC and NJ (Holguin-Veras et al. 2012):
  - Average: 8.0 stops/tour
  - 12.6%: 1 stop/tour
  - 54.9%: < 6 stops/tour
  - 8.7% do > 20 stops
  - Parcel deliveries: 50-100 stops/tour
Urban Freight Tour Models
Urban Freight Tour Models

- The UFT models could be subdivided into:
  - Simulation models
  - Hybrid models
  - Analytical models
Simulation
Models
Simulation Models

- Simulation models attempt to create the needed isomorphic relation between model and reality by imitating observed behaviors in a computer program.

- Examples include:
  - Tavasszy et al. (1998) (SMILE)
  - Boerkamps and van Binsbergen (1999) (GoodTrip)
Hybrid Models
Hybrid Models

- Hybrid models incorporate features of both simulation and analytical models (e.g., using a gravity model to estimate commodity flows, and a simulation model to estimate the UFTs)

- Examples include:
  - van Duin et al. (2007)
  - Wisetjindawat et al. (2007)
  - Donnelly (2007)
Analytical Models
Analytical Models

- Analytical models attend to achieve isomorphism using formal mathematic representations based on behavioral, economic, or statistical axioms
- Two main branches:
  - Spatial Price equilibrium models (disaggregate)
  - Entropy Maximization models (aggregate)
- Examples include:
  - Xu (2008), Xu and Holguín-Veras (2008)
  - Holguín-Veras et al. (2012)
  - Wang and Holguín-Veras (2009), Sanchez and Holguín-Veras (2012)
Entropy Maximization
Tour Flow Model
Entropy Maximization Tour Flow Models

- Based on entropy maximization theory (Wilson, 1969; Wilson, 1970; Wilson, 1970)
- Computes most likely solution given constraints
- Key concepts:
  - Tour sequence: An ordered listing of nodes visited
  - Tour flow: The flow of vehicle-trips that follow a sequence
- The problem is decomposed in two processes:
  - A tour choice generation process
  - A tour flow model
Entropy Maximization Tour Flow Model

- Tour choice: To estimate sensible node sequences
- Tour flow: To estimate the number of trips traveling along a particular node sequence
## Definition of states in the system:

<table>
<thead>
<tr>
<th>State</th>
<th>State Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micro state</td>
<td>Individual commercial vehicle journey starting and ending at a home base</td>
</tr>
<tr>
<td></td>
<td>(tour flow) by following a tour.</td>
</tr>
<tr>
<td>Meso state</td>
<td>The number of commercial vehicle journeys (tour flows) following a tour</td>
</tr>
<tr>
<td></td>
<td>sequence.</td>
</tr>
<tr>
<td>Macro state</td>
<td>Total number of trips produced by a node (production);</td>
</tr>
<tr>
<td></td>
<td>Total number of trips attracted to a node (attraction);</td>
</tr>
<tr>
<td></td>
<td>Formulation 1: ( C = ) Total time in the commercial network;</td>
</tr>
<tr>
<td></td>
<td>Formulation 2: ( C_T = ) Total travel time in the commercial network;</td>
</tr>
<tr>
<td></td>
<td>( C_H = ) Total handling time in the commercial network.</td>
</tr>
</tbody>
</table>
Static version of EM Tour Flow Model
Entropy Maximization Tour Flow Model

\[
\begin{align*}
\text{MIN} & \quad Z = \sum_{m=1}^{M} \left( t_m \ln t_m - t_m \right) \\
\text{Subject to:} & \quad \sum_{m=1}^{M} a_{im} t_m = O_i \quad (\lambda_i) \quad \text{Trip production constraints} \\
& \quad \sum_{m=1}^{M} c_{Tm} t_m = C_T \quad (\beta_1) \quad \text{Total travel time constraint} \\
& \quad \sum_{m=1}^{M} c_{Hm} t_m = C_H \quad (\beta_2) \quad \text{Total handling time constraint} \\
& \quad t_m \geq 0
\end{align*}
\]
First-order conditions (tour distribution models)

Formulation 1: $t_m^* = \exp\left(\sum_{i=1}^{N} \lambda_i^* a_{im} + \beta^* c_m\right) = \exp\left(\sum_{i=1}^{N} \lambda_i^* a_{im}\right) \exp(\beta^* c_m)$

Formulation 2: $t_m^* = \exp\left(\sum_{i=1}^{N} \lambda_i^* a_{im}\right) \exp(\beta_1^* c_{Tm} + \beta_2^* c_{Hm})$

Traditional gravity trip distribution model

$t_{ij}^* = A_i O_i B_j D_j \exp(\beta^* c_{ij})$

Formulation 1 and the traditional GM model have exactly the same number of parameters
The optimal tour flows are found under the objective of maximizing the entropy for the system.

The tour flows are a function of tour impedance and Lagrange multipliers associated with the trip productions and attractions along that tour.

Successfully tested with Denver, Colorado, data:
- The MAPE of the estimated tour flows is less than 6.7% given the observed tours are used.
- Much better than the traditional GM.
Case Study: Denver Metropolitan Area

- **Test network**
  - 919 TAZs among which 182 TAZs contain home bases of commercial vehicles
  - 613 tours, representing a total of 65,385 tour flows / day
  - Calibration done with 17,000 tours (from heuristics)

- **Estimation procedure**
  - Sorting input data: aggregate the observed tour flows to obtain trip productions and total impedance
  - Estimation: estimate the tour flows distributed on these tours using the entropy maximization formulations
  - Assessing performance: compare the estimated tour flows with the observed tour flows
Performance of the Models

\[ R^2 = 0.9992 \]
Time-Dependent Freight Tour Synthesis Model
**TD-FTS Model**

- **Bi-objective Program:**

  **PROGRAM TD-ODS 1**

  Minimize $z(x) = \sum_{d=1}^{K} \sum_{m=1}^{M} \left( x_m^d \ln(x_m^d) - x_m^d \right)$

  Minimize $e(x) = \sum_{a=1}^{A} \sum_{k=1}^{K} \frac{1}{2} \left( v_k^a - \sum_{d=1}^{K} \sum_{m=1}^{M} \delta_{am}^k x_m^d \right)^2$

  Subject to:

  \[ \sum_{d=1}^{K} \sum_{m=1}^{M} g_{im} x_m^d = O_i \quad \forall i \in N \]

  \[ \sum_{d=1}^{K} \sum_{m=1}^{M} \gamma_{jm} x_m^d = D_j \quad \forall j \in N \]

  \[ \sum_{d=1}^{K} \sum_{m=1}^{M} c_m x_m^d = C \]

  \[ x_m^d \geq 0, \quad \forall m \in M, d \in K \]
TD-FTS Model

Multi-attribute Value formulation:

 PROGRAM TD-FTS3

Minimize \(-U(e(x), z(x)) = -\alpha_1 v_1(e(x)) - \alpha_2 v_2(z(x))\)

Subject to:

\[
P_{\sum_{d=1}^{K} \sum_{m=1}^{M} Y_{jm} x_{m,y}^d = D_y^y} \quad \forall j \in N, y \in Y
\]

\[
P_{\sum_{d=1}^{K} \sum_{m=1}^{M} \sum_{y=1}^{Y} C_{m,y} x_{m,y}^d = C}
\]

\[
x_{m,y}^d \geq 0, \quad \forall m \in M, d \in K, y \in Y
\]
## Application to the Denver Region

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>MAPE</td>
<td>RMSE</td>
</tr>
<tr>
<td>S/TD-EM</td>
<td>39.8</td>
<td>31.2%</td>
<td>58.5</td>
</tr>
<tr>
<td>S/TD-FTS-A</td>
<td>17.2</td>
<td>16.6%</td>
<td>14.5</td>
</tr>
<tr>
<td>S/TD FTS-B</td>
<td>8.0</td>
<td>0.8%</td>
<td>9.5</td>
</tr>
<tr>
<td>S/TD-EM</td>
<td>39.8</td>
<td>31.2%</td>
<td>58.5</td>
</tr>
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<td>S/TD-FTS-A</td>
<td>17.2</td>
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<td>14.5</td>
</tr>
<tr>
<td>S/TD FTS-B</td>
<td>8.0</td>
<td>0.8%</td>
<td>9.5</td>
</tr>
<tr>
<td>DCGM</td>
<td>116.0</td>
<td>79.5%</td>
<td>39.6</td>
</tr>
<tr>
<td>S/TD-EM</td>
<td>32.1</td>
<td>21.0%</td>
<td>58.3</td>
</tr>
<tr>
<td>S/TD-FTS-A</td>
<td>13.8</td>
<td>11.3%</td>
<td>12.6</td>
</tr>
<tr>
<td>S/TD FTS-B</td>
<td>5.7</td>
<td>5.2%</td>
<td>7.5</td>
</tr>
</tbody>
</table>

- **TD-FTS MAPE’s**: 0.8%-76.1%
- **Static Entropy Maximization (S-EM)**: MAPE’s 31.2%-117%
- **Gravity Model (DCGM)**: MAPE 79.5%
- **Temporal aspect better captured using TD-FTS**
TD-FTS Performance per Time Interval

**TD-ODS AM OH**

- Estimated TD Tour Flows vs. Actual TD Tour Flows
- Linear regression: $y = 0.755x$
- Coefficient of determination: $R^2 = 0.913$

**TD-ODS AM PH**

- Estimated TD Tour Flows vs. Actual TD Tour Flows
- Linear regression: $y = 1.049x$
- Coefficient of determination: $R^2 = 0.996$

**TD-ODS PM PH**

- Estimated TD Tour Flows vs. Actual TD Tour Flows
- Linear regression: $y = 1.032x$
- Coefficient of determination: $R^2 = 0.998$

**TD-ODS PM OH**

- Estimated TD Tour Flows vs. Actual TD Tour Flows
- Linear regression: $y = 0.928x$
- Coefficient of determination: $R^2 = 0.983$
Multiclass Equilibrium Demand Synthesis
Multiclass traffic

- Multiclass: two or more classes of travelers with different behavioral or choice characteristics
- Vehicle classes are related under the same objective function
- Multiclass equilibrium demand synthesis (MEDS)
Travel time function depends on the vector of traffic flows for passenger cars and trucks.

The travel cost is affected by the value of time of each one of the classes.

The formula is a second order Taylor expansion of a general link-performance function:

\[ t_a(X_t, X_c) = \alpha_0 + \alpha_1 X_c + \alpha_2 X_t + \alpha_3 X_c^2 + \alpha_4 X_t^2 + \alpha_5 X_c X_t \]

Where:

- \( X_t \): Vector of truck traffic flow
- \( X_c \): Vector of passenger cars traffic flow
Multiclass Equilibrium

- In a multiclass equilibrium, the cost functions of the modes are asymmetric, traffic flows interact.
- The user optimal assignment cannot be written as an optimization problem.
- The User Equilibrium (UE) problem could be addressed using a Variational Inequality (VI) Problem.

\[
\sum_{m} C_m (t_m^*, T_{i,j}^*)^T \cdot (t_m - t_m^*) + \sum_{ij} C_{ij} (t_{m}^*, T_{i,j}^*)^T \cdot (T_{i,j} - T_{i,j}^*) \geq 0
\]

\[
\begin{cases}
0 \leq C_m - C_m^* \perp t_m \geq 0 \\
0 \leq C_{ij} - C_{ij}^* \perp T_{i,j} \geq 0
\end{cases}
\]
Although the vehicles share the transportation network and the travel time is the same (in equilibrium), the travel cost will be affected by the value of time of each one of the parties value of time

\[ c^t_a = \alpha_t \cdot t_a (x_t, x_c) \]
\[ c^c_a = \alpha_c \cdot t_a (x_t, x_c) \]

Considering paths, the cost functions for truck tours and passenger cars will be given by:

\[ C_m = \sum_a c^t_a \cdot \delta_{ma} \]
\[ C_{ij} = \sum_a c^c_a \cdot \delta_{ija} \]

In a multiclass equilibrium, the cost functions of modes are asymmetric, traffic flows interact. UE Variational Inequality

Where: \( \delta_{ma} \) : A binary variable indicating whether tour \( m \) uses link \( a \)
\( \delta_{ija} \) : A binary variable indicating whether trip from \( i \) to \( j \) uses link \( a \)
Multiclass EM Formulations

The multiclass EM formulations is given by:

$$\text{Max } W = \frac{T_t! \cdot T_c!}{\prod_{m,ij} (t_m! \cdot T_{ij}!)}$$

$$\text{Min } z = \sum_{m} \sum_{ij} \left( t_m \cdot \ln t_m - t_m + T_{ij} \cdot \ln T_{ij} - T_{ij} \right)$$

Where:

$W$: System entropy that represents the number of ways of distributing commercial vehicles tour flows and passenger cars flows

$T_t$: Total number of commercial vehicle tour flows in the network;

$T_c$: Total number of passenger cars flows in the network;

$t_m$: Number of commercial vehicle journeys (tour flows) following tour $m$;

$T_{ij}$: Number of car trips between $i$ and $j$
The objective is to find the most likely ways to distribute tours considering congestion.

VI problem to obtain a UE condition for cars and trucks.
Spatial Price Equilibrium
Tour Models
General principles

- The models estimate commodity flows and vehicle trips that arise under competitive market equilibrium

- Conceptual advantages:
  - Account for tours
  - Provide a coherent framework to jointly model the joint formation of commodity flows and vehicle trips

- Based on the seminal work of Samuelson (1952), as it seeks to maximize the economic welfare associated with the consumption and transportation of the cargo, taking into account the formation of UFTs
Two flavors

- Independent Shipper-Carrier Operations:
  - Carrier and Shipper are independent companies
  - Carrier travels empty from its base to pick up cargo at shipper’s location(s)
  - Carrier delivers cargo to shipper’s customers
  - Carrier travels empty back to its base

- Integrated Shipper-Carrier Operations:
  - Carrier and Shipper are part of the same company
  - Carrier is loaded at shipper’s location(s)
  - Carrier delivers cargo to shipper’s customers
  - Carrier travels empty back to its base
Five suppliers deploy tours from their bases (rhomboids) to distribute the cargo they produce to various consumer (demand) nodes (circles).

Legend:

- **Supplier**
- **Receiver**
- **Empty trips**
- **Loaded trips made by suppliers**

(Contested nodes are shown as shaded circles)
Samuelson’s model is reformulated to consider freight tours. A supplier $i$ sends a cargo ($e_{ip1}$, $e_{ip2}$, $e_{ip3}$, and $e_{ip4}$) to different customers ($p_1$, $p_2$, $p_3$, and $p_4$).

The cost of delivering to $p_3$ is not the (path) cost along $i-p_1-p_2-p_3$. It is the (incremental) cost from $p_2$ to $p_3$, plus part of the empty trip cost.
A Spatial Price Equilibrium UFT Model (P1)

\[
\text{MAX } NSP = \sum_{i=1}^{SN} S_i (E_i) - \sum_{u} \sum_{i=1}^{SN} \sum_{j=1}^{DN} C_{ij}^u (e_{ij}^u)
\]

(Social Welfare)

Subject to:
\[
S_i (E_i) = -\int_0^{E_i} s(x)dx
\]

(Area under excess supply function)

\[
E_i = \sum_u \sum_j e_{ij}^u
\]

(Excess supply)

\[
C_{i,p_i}^u (e_{i,p_i}^u) = c_D d_{p_i,1}^u + c_T t_{p_{i-1},p_i}^u + c_E p_i^u + c_H e_{i,p_i}^u
\]

(Delivery cost to demand node)

\[
we_{i,j}^u = Q \mathcal{G}_{ij}^u \leq 0
\]

(Linking flows to vehicle-trips)

\[
(t_0^L + t_{i,p_i}^u + \sum_{l=1}^{L^u-1} t_{p_i,p_{i+1}}^u + t_{L^u0}) < T
\]

(Tour length constraint)

\[
\begin{aligned}
\sum_{i=1}^{SN} \sum_{j=i+1}^{SN} we_{i,j}^u &\leq Q \\
\mathcal{G}_{ij}^u &\leq 1 \\
\mathcal{G}_{ij}^u &\in (0,1) \\
e_{ij}^u &\geq 0 \\
c_D, c_T, c_H &\geq 0 \\
d_{i,j}, t_{i,j} &\geq 0
\end{aligned}
\]

This term is equal to the summation of tour costs. Thus, it could be replaced by the summation of tour costs, which allows to eliminate the delivery cost constraint.
A Spatial Price Equilibrium UFT Model (P2)

\[ \text{MAX } NSP = \sum_{i=1}^{SN} S_i (E_i) - \sum_u C^u_v \]

(Net Social Payoff)

Subject to:

\[ S_i (E_i) = -\int_0^{E_i} s(x)dx \]

(Area under excess supply function)

\[ E_i = \sum_u \sum_j e_{ij}^u \]

(Excess supply)

\[ w e_{i,p_i^u=j}^u - Q \mathcal{G}_{ij}^u \leq 0 \]

(Linking flows to vehicle-trips)

\[ \left( t_{0i} + t_{i,p_i^u}^u + \sum_{l=1}^{L^u-1} t_{p_i^u,p_{l+1}^u} + t_{L^u}^u \right) \leq T \]

(Tour length constraint)

\[ \sum_i \sum_{j=i+1}^{L^u-1} \sum_{j=i+1}^{L^u} w e_{p_i^u,p_j^u}^u \leq Q \]

(Capacity constraint)

\[ \sum_{j,u} \mathcal{G}_{ij}^u \leq 1 \quad \forall j \]

(Conservation of flow)

\[ \mathcal{G}_{ij}^u \in (0,1) \quad \forall i, j, u \]

(Integrality)

\[ e_{ij}^u \geq 0 \quad c_D, c_T, c_H \geq 0 \quad d_{i,j}, t_{i,j} \geq 0 \]

(Non-negativity)
However...

- P2 is a nasty combinatorial and non-linear problem that is notoriously difficult to solve
- To solve it, frame it as:
  - A dispersed SPE problem
  - A problem of profit maximization subject to competition (which is equivalent to the NSP formulation produced by Samuelson)
  - A dynamic problem in which competitors adjust decisions based on the market competition results
- Use heuristics
Heuristic Solution Approach (Dispersed SPE)

**Initialization:** Assume prices

**First-stage:**
Production level, prices, profit margins, net

**Phase 1: Initialization of TS,** Generate initial solutions

**Phase 2: Initial Improvement:**
Perform neighborhood search procedure

**Phase 3: Second Improvement,** 2-Opt procedure and repeat 2

**Phase 4: Intensification,** Perform neighbor search on solutions from 3

**Convergence?**

**Yes**

**Market competition:** Compute purchases from suppliers

**Equilibrium?**

**No**

**Production dynamics:** Update production level

**STOP**

**Second-stage pricing:** Compute optimal prices, profit margins

**Equilibrium?**

**Yes**

**No**

**Production dynamics:** Update production level
Equilibrium Results

Two suppliers, four customers

\[ P_{ijt} = \frac{1}{\beta(1 - s_i)} - \frac{(1 + \alpha_{op})}{\beta D_j (s_i - s_i^2)} \sum_{j,u,h} (m_{Qijuht} + m_{Tijuht}) \]
Concluding Remarks
Conclusions

- There are reasons to be optimistic:
  - The community is cognizant of the need to model tours
  - Collecting data and developing tour models

- However:
  - The models developed are still in need of improvements
  - The data collected are small and not comprehensive

- Simulations and hybrid models require better behavioral foundations that are not always validated

- The most theoretically appealing models present significant computational challenges to be overcome

- Entropy Maximization models offer an interesting avenue, though disregarding commodity flows
References

References

Thanks! Questions?