1. In a case study, the average travel time for a particular stretch was found out to be 22.8 seconds, standard deviation is 5.951 and model time step duration is 10 sec. Find out the Robertson's model parameters and also the flow at downstream at different time steps where the upstream flows are as given below
$q_{10} = 20$, $q_{20} = 10$, $q_{30} = 15$, $q_{40} = 18$, $q_{50} = 14$

***Solution:***

- Calculation of Robertson's parameters

$$\beta_n = \frac{2T_a + n - \sqrt{n^2 + 4\sigma^2}}{2T_a} = \frac{2\text{x } 22.8 + 10 - \sqrt{(10^2 + 4 \text{ x } 5.951^2)}}{2\text{x } 22.8} = 0.878$$

$$\alpha_n = \frac{1 - \beta_n}{\beta_n} = \frac{1 - 0.878}{0.878} = 0.139$$

$$F_n = n\frac{\sqrt{n^2 + 4\sigma^2} - n}{2\sigma^2} = \frac{10 \text{ x } \sqrt{(10^2 + 4 \text{ x } 5.951^2)} - 10}{2 \text{ x } 5.951^2} = 0.783$$

$T = \beta\, T_a = 0.878 \text{ x } 22.8 = 20$ sec

$$q_t^d = \sum_{i=T}^{\infty} (F_n (1 - F_n)^{i-T} \times q_{(t-i)})$$

- $q^d_{10} =$ --
- $q^d_{20} = F \text{ x } (1-F)^{20-20} \text{ x } q_{10}$

  $= F \text{ x } q_{10}$

  $= 0.783 \text{ x } 20 = 15.66 \approx$ **16** veh

- $q^d_{30} = F \text{ x } (1-F)^{20-20} \text{ x } q_{20} + F \text{ x } (1-F)^{21-20} \text{ x } q_{10}$

  $= F \text{ x } q_{20} + F \text{ x } (1-F)^1 \text{ x } q_{10}$

  $= 0.783 \text{ x } 10 + 0.783 \text{ x } (1-0.783)^1 \text{ x } 20$

  $= 7.83 + 3.39 = 11.22 \approx$ **11** veh

- $q^d_{40} = F \text{ x } (1-F)^{20-20} \text{ x } q_{30} + F \text{ x } (1-F)^{21-20} \text{ x } q_{20} + F \text{ x } (1-F)^{22-20} \text{ x } q_{10}$

  $= F \text{ x } q_{30} + F \text{ x } (1-F)^1 \text{ x } q_{20} + F \text{ x } (1-F)^2 \text{ x } q_{10}$

  $= 0.783 \text{ x } 15 + 0.783 \text{ x } (1-0.783)^1 \text{ x } 10 + 0.783 \text{ x } (1-0.783)^2 \text{ x } 20$

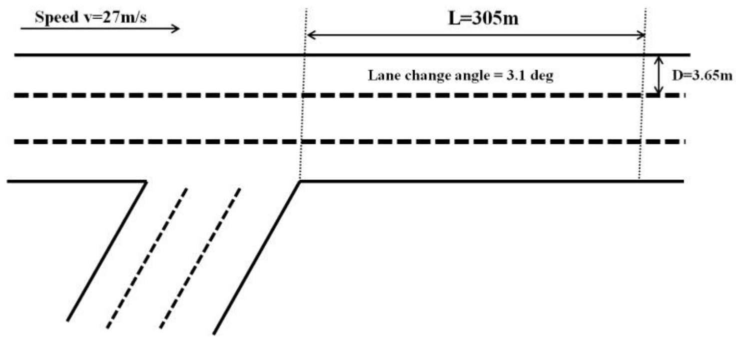
  $= 11.75 + 1.69 + 0.737 = 14.18 \approx$ **14** veh

Calculating on similar lines, we get

- $q^d_{50} =$ **17** veh

- $q^{d}_{60}$ = **15** veh

2. (a) Derive an expression for lane-changing intensity factor of a kinematic wave model of traffic. (b) Consider a uniform, three-lane lane-changing region, which is downstream to a merging junction connecting two three-lane roads as shown in Figure. The length of the lane-changing area is L = 305m, the width of a lane D = 3.65m, and both branches have the same density of ρ/2; assume that all lanes in the lane-changing region are fully balanced. The vehicle speed is 27m/s and lane changing angle is $3.1^{0}$.Compute the lane changing intensity factor.




***Solution:***

***(a) Derivation of lane-changing intensity factor***

- At the macroscopic level, the lane-changing intensity variable ϵ is related to the number of lane changes and lane changing duration.
- For a lane-changing section of length L, we have the following quantities:
  1. $\rho_{LC}$ is the density of lane-changing traffic,
  2. $\rho_{NLC}$ the density of non-lane-changing traffic, $\rho = \rho_{LC} + \rho_{NLC}$ the total density,
  3. $v$ the speed of both lane-changing and non-lane-changing vehicles,
  4. $q_{LC} = \rho_{LC} v$ is the flow-rate of lane-changing traffic, and
  5. $q_{NLC} = \rho_{NLC} v$ is the flow-rate of non-lane -changing traffic.
  6. The time for all vehicles to traverse the lane-changing region is $T = Lv$.
  7. total number of lane-changes in the lane-changing area during a period of $T$ is given by
  $$N_{LC} = \alpha q_{LC} T = \alpha \rho_{LC} L \qquad (1)$$
  where the coefficient $\alpha$ is the average number of lane-changes of each lane-changing vehicle
  8. If the width of a lane is D, and the average lane-changing angle θ, we then have the lane changing duration
  $$t_{LC} = \frac{D}{v \tan \theta} \qquad (2)$$
  9. The intensity factor ϵ is given by
  $$\epsilon = \frac{\text{Number of lane changing vehicles}}{\text{Total number of vehicles}} \qquad (3)$$
  $$= \frac{N_{LC} t_{LC}}{NT}$$
  $$= \frac{\alpha \rho_{LC} L t_{LC}}{\rho L T}$$
  $$\epsilon = \frac{\alpha \rho_{LC} t_{LC}}{\rho T} \qquad (4)$$

***(b) Computation of lane changing intensity factor***

- Traffic density on each lane is $\rho/6$ before merging.
- All lanes in the lane-changing region are fully balanced; i.e., traffic density on each lane in this region is twice as that before merging, $\rho/3$. Therefore, one third of vehicles from the merging branch, i.e., $\rho/6$, have to stay on the rightmost lane in the lane-changing region with no lane-change, one third will change one lane to the middle lane, and one third will change two lanes to the leftmost lane.
- The density of lane-changing traffic is that from the merging branch; i.e., $\rho_{LC}$ = $\rho/3$.
- The average number of lane-changes of a merging vehicle

$$\alpha = \frac{\rho/6.0+\rho/6.1+\rho/6.2}{\rho/3}=1.5$$

- If vehicle speed v =27 m/s,
  lane-change duration $t_{LC} = \frac{D}{v \tan \theta}=\frac{D3.65}{27 \tan 3.1}$=2.5 sec.
  the time for a vehicle to traverse the lane changing region is T = L/v=11.3 sec.

$$\epsilon = \frac{\alpha\rho_{LC} t_{LC}}{\rho T} = \frac{1.5.\left(\frac{\rho}{3}\right).(2.5)}{\rho.(11.3)} = 0.11$$

3. Find the jam density from the logarithmic traffic flow stream model for the following set of speed and density observations.

| k(veh/km) | 150 | 120 | 50 | 70 | 20 |
|---|---|---|---|---|---|
| v(km/h) | 10 | 25 | 45 | 40 | 50 |

***Solution:***

- The logarithmic traffic flow stream model is given by

$$u = u_0 \ln\left(\frac{k_j}{k}\right)$$

  Expanding the above equation we have:

$$u = u_0 ln(k_j) - u_0 \ln(k)$$

  The above expression is linear equation in $u$ and $\ln(k)$
- Regressing between $u$ and $\ln(k)$ we have

$$u_0=18.479 \text{ km.h}$$
$$u_0 ln(k_j) = 111.442$$
$$k_j = 416.064 \text{ veh/km}$$

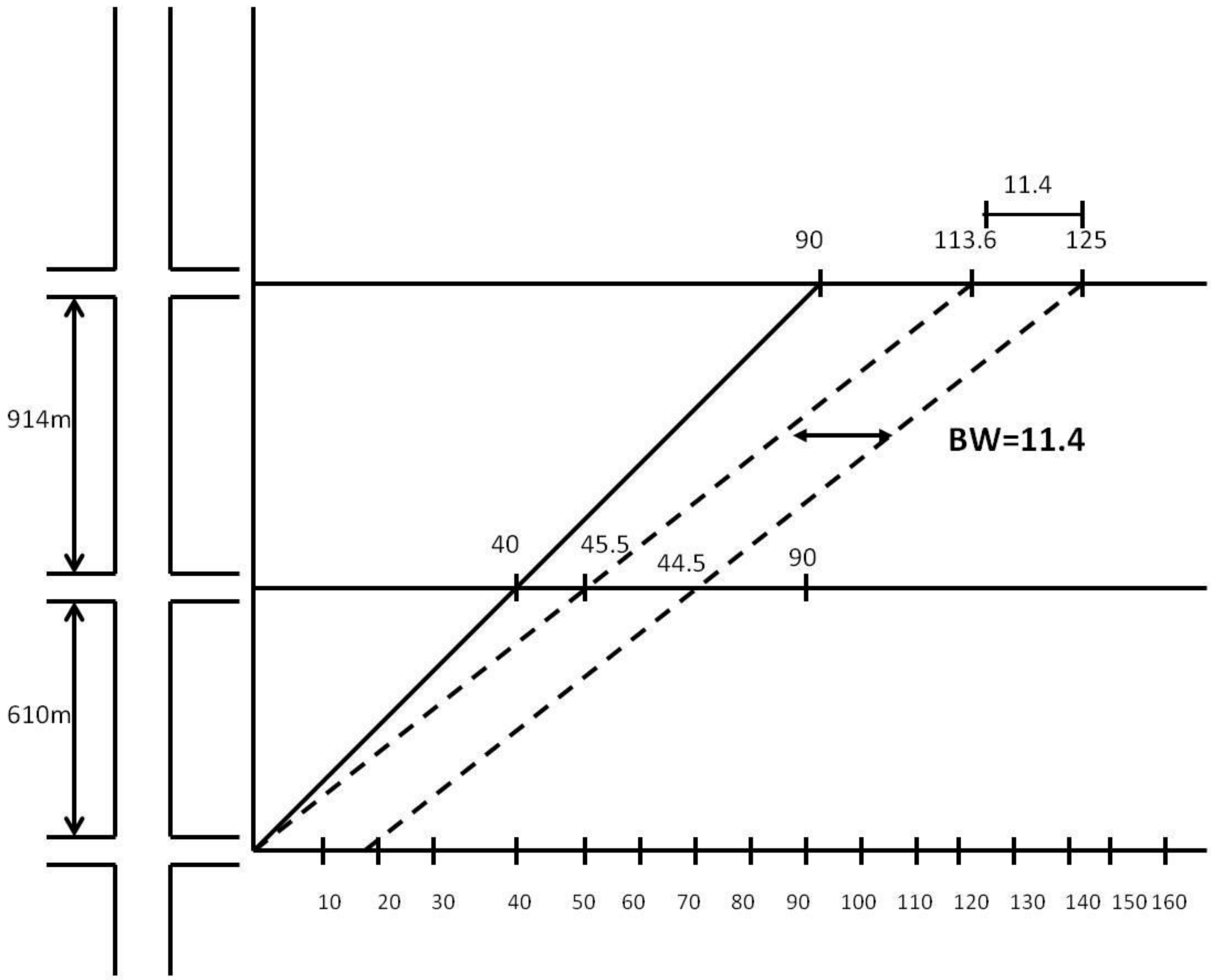
- The required calculations are shown in the table below:

| ln(k)=x | u=y | x-xmean | y-ymean | (x-xm)*(y-ym) | (x-xmean)^2 | | | |
|---|---|---|---|---|---|---|---|---|
| 5.011 | 10.000 | 0.820 | -24.000 | -19.674 | 0.672 | | b | -18.479 |
| 4.787 | 25.000 | 0.597 | -9.000 | -5.370 | 0.356 | | a | 111.442 |
| 3.912 | 45.000 | -0.279 | 11.000 | -3.067 | 0.078 | | | |
| 4.248 | 40.000 | 0.058 | 6.000 | 0.346 | 0.003 | | $u_0$ | 18.479 |
| 2.996 | 50.000 | -1.195 | 16.000 | -19.122 | 1.428 | | $u_0 ln(k_j)$ | 111.442 |
| 4.191 | 34.000 | | Sum | -46.888 | 2.537 | | $k_j$ | 416.064 |

4. The signals at an intersection of the one-way street have been pretimed and coordinated as presented in the following table. Given a design speed of 48.3km/h, determine the width of the resulting through band.

| Intersection | Green | Yellow | Red | Offset | Distance from A |
|---|---|---|---|---|---|
| A | 40 | 5 | 35 | 0 | - |
| B | 50 | 5 | 25 | 40 | 610 |
| C | 35 | 5 | 40 | 10 | 1524 |

***Solution:***




Minimum Band Width=11.4s

5. Compute the stopped delay and intersection control delay from the following field observations.

No. of lanes = 2; Free flow speed = 65kmph; Total vehicles arriving = 520; Stopped vehicles count = 207. Consider the survey count interval as 15 sec.

| Clock time | Cycle Number | Number of vehicles in queue | | | | | |
|---|---|---|---|---|---|---|---|
| | | Count interval | | | | | |
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| 4.34 | 1 | 2 | 7 | 10 | 9 | 3 | 0 |
| | 2 | 5 | 11 | 14 | 7 | 2 | 0 |
| | 3 | 6 | 8 | 12 | 5 | 0 | 0 |
| | 4 | 7 | 10 | 13 | 7 | 4 | 0 |
| 4.42 | 5 | 3 | 5 | 7 | 7 | 1 | 0 |
| | 6 | 7 | 9 | 11 | 11 | 4 | 0 |
| | 7 | 8 | 10 | 11 | 9 | 3 | 0 |
| | 8 | 7 | 9 | 10 | 4 | 0 | 0 |

***Solution:***

Total vehicles in queue $\sum V_{iq} = 278$

Time-in-queue per vehicle / Stopped delay $d_{vq} = I_s \times \sum V_{iq} \times 0.9/V_{tot} = 7.217$

No. of vehicles stopping per lane each cycle = $V_{stop}/(N_c \times N) = 12.9375$

Number of cycles surveyed, $N_c = 8$

Accel/Decln correction factor CF (Ex. A16-2) = 4

Fraction of vehicles stopping FVS = $V_{stop}/V_{tot} = 0.398$

Accel/Decln correction delay $d_{ad}$ = FVS x CF = 1.592

Control delay/vehicle d = $d_{vq} + d_{ad} = 8.81$

6. Consider a 1km homogeneous road with speed, $v$ = 60kmph, jam density, $k_j$ = 180veh/km and maximum flow, $q_{max}$ = 3600veh/hr. Initially traffic is flowing undisturbed at 75% of capacity. The capacity is 3600veh/hr. Then, a partial lane blockage lasting 1 min occurs on $l/3^{rd}$ of the distance from the end of the road. The blockage effectively restricts flow to 20% of the maximum. Clearly, a queue is going to build and dissipate behind the restriction. After 1 minute, the flow in cell 3 is maximum possible flow. Predict the evolution of the traffic for 3 clock ticks. Take one clock tick as 20 seconds.

***Solution:***

Cell length = Distance travelled by vehicle in one clock tick = $v \times t$ = 60 x (1/180) = 1/3

No. of cells = 1/(1/3) = 3 cells

Maximum number of vehicles that can be at time $t$ in cell $I$, $N$ = cell length x jam density

= 180 x 1/3

= 60 vehicles

Maximum number of vehicles that can flow into cell $I$ from time $t$ to $t$+1,

Q = 3600 x 1/180

= 20 vehicles

For 20% of the maximum = 3600 x 0.2 x (1/180) = 4 vehicles
For 75% of the maximum = 3600 x 0.75 x (1/180) = 15 vehicles

Now, by applying cell transmission rules the following table can be generated.

| Time | Source(00) | Gate(0) | cell 1 | cell 2 | cell 3 | sink (4) | Q3 |
|---|---|---|---|---|---|---|---|
| Q | - | 15 | 20 | 20 | - | 20 | |
| N | - | 999 | 60 | 60 | 60 | 999 | |
| 1 | 999 | 15 | 15 | 15 | 15 | - | 4 |
| 2 | 999 | 15 | 15 | 26 | 4 | - | 4 |
| 3 | 999 | 15 | 15 | 37 | 4 | - | 4 |

7. a) Derive the hyperbolic (explicit) form of LWR model using the following parameters in Greenshield's equation.
   Free flow speed =80kmph; Jam density = 320veh/hr.
   b) Find the values of density at points ($t$ = 1/4hr, $x$ = 15km) and ($t$ = 1hr, $x$ = 70km) using the method of characteristics with the following initial condition:

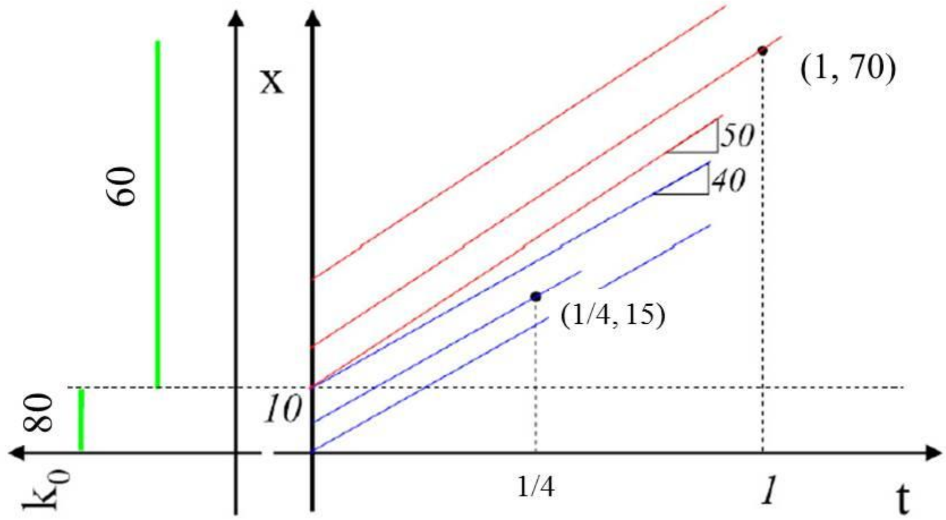
$$k(x,0) = k_0(x)\begin{cases} 80veh/km & if\ 0 < x \leq 10km \\ 60veh/km & if\ 0 > 10km \end{cases}$$

***Solution:***

Free flow speed, $v_f$ = 80km/hr
Jam density, $k_j$=320
Derivation result: $k_t$+(80-k/2)$k_x$=0
Between 0<x<10, slope of the characteristics, c = 80-80/2=40
Between x>10, slope of the characteristics, c = 80-60/2=50

From the figure or by analytical method using the equation for line, it can be verified that point (1/4,15) is within the blue line area.

**Analytical method for checking:**
y=mx+c
y=40x1/4+10=20. Point (1/4, 15) is below this. Therefore it is within the blue line area.




To find out the y-intercept,
15=40x1/4+c
c=5
Therefore, k(1/4,15)=k(0,5)=80
In a similar way, it can be verified that point (1/4, 15) is within the red line area.
k(1,70)=k(0,20)=60