Traffic stream models

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Traffic stream models

• **Macroscopic**
  – Expression of the average behavior of the vehicles at the specific location and time

• **Mesoscopic**
  – Small group of traffic entities with activities and interactions

• **Microscopic**
  – Space-time behavior of the systems’ entities (i.e. vehicle and drivers)
Traffic stream models
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Traffic stream models

• Macroscopic Stream Models
  – Greenshield's linear
  – Greenberg's logarithmic
  – Underwood's exponential
  – Pipe's generalized
  – Multi regime models
    • Two and Three regime
Greenshield's model

- Linear speed-density relationship

Relation between speed and density
Greenshield's model

• **Description**

\[ v = v_f - \left( \frac{v_f}{k_j} \right) k \]

– \( v \) = mean speed
– \( k \) = density
– \( v_f \) = free flow speed
– \( k_j \) = jam density

\( k = 0 \) when density approaches zero, speed approaches free flow speed.

\[ v_f = 80, \quad k_j = 200 \]
**Greenshield's model**

- **Relation between flow and density**

\[ v = v_f - \left( \frac{v_f}{k_j} \right) k \]

\[ q = k v \]

\[ q = k \left[ v_f - \left( \frac{v_f}{k_j} \right) k \right] \]

\[ q = v_f k - \left( \frac{v_f}{k_j} \right) k^2 \]
Greenshield's model

• **Boundary conditions**
  – Maximum flow $q_{\text{max}}$
  – Density corresponding to max. flow $k_o$
  – Speed corresponding to max. flow $v_o$

• **Model parameters**
  – Jam density $k_j$
  – Free flow speed $v_f$
Greenshield's model

- **Derivation of** $k_0$
  - We have \[ q = v_f k - \left( \frac{v_f}{k_j} \right) k^2 \]
  - Differentiating
    \[
    \frac{dq}{dk} = 0 \Rightarrow v_f - \frac{v_f}{k_j} \cdot 2k = 0 \\
    k = \frac{k_j}{2}
    \]
Greenshield's model

• Derivation of $q_{\text{max}}$

$$q_{\text{max}} = v_f \cdot \frac{k_j}{2} - \frac{v_f}{k_j} \cdot \left[ \frac{k_j}{2} \right]^2$$

$$= v_f \cdot \frac{k_j}{2} - v_f \cdot \frac{k_j}{4}$$

$$= \frac{v_f \cdot k_j}{4}$$

• Derivation of $v_o$

$$v_0 = v_f - \frac{v_f}{k_j} \cdot \frac{k_j}{2}$$

$$v_0 = \frac{v_f}{2}$$
Greenshield's model

- **Relation between speed and flow**

\[ v = v_f - \left( \frac{v_f}{k_j} \right) k \]

\[ k = k_j - \left( \frac{k_j}{v_f} \right) v \]

\[ q = k v \]

\[ q = k_j v - \left( \frac{k_j}{v_f} \right) v^2 \]
Greenshield's model

- **Calibration**
  - Determination of model parameters
  - Free flow speed (\(v_f\))
  - Jam density (\(k_j\))

\[
v = v_f - \left[ \frac{v_f}{k_j} \right] . k
\]

\[
y = a + bx
\]

Where \(x\) is density and \(y\) denotes speed.
Greenshield's model

• Calibration

– Using linear regression method

\[ b = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \]

OR

\[ a = \frac{n \sum_{i=1}^{n} xy - \sum_{i=1}^{n} x \sum_{i=1}^{n} y}{n \sum_{i=1}^{n} x^2 - (\sum_{i=1}^{n} x)^2} \]

\[ a \text{ is } \bar{y} - b\bar{x} \]
Greenshield's model

- **Example**
  - Calibrate Greenshields model using the data give in the table
  - Find the maximum flow
  - Find the density corresponding to a speed of 30 km/hr

<table>
<thead>
<tr>
<th>No</th>
<th>K</th>
<th>(\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>171</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>129</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>25</td>
</tr>
</tbody>
</table>
Solution

Greenshield’s model

Denoting \( y = \nu \) and \( x = k \), solve for \( a \) and \( b \) using equation 8 and equation 9. The solution is tabulated as shown below.

<table>
<thead>
<tr>
<th>( x(k) )</th>
<th>( y(\nu) )</th>
<th>( (x_i - \bar{x}) )</th>
<th>( (y_i - \bar{y}) )</th>
<th>( (x_i - \bar{x})(y_i - \bar{y}) )</th>
<th>( (x_i - \bar{x})^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>171</td>
<td>5</td>
<td>73.5</td>
<td>-16.3</td>
<td>-1198.1</td>
<td>5402.3</td>
</tr>
<tr>
<td>129</td>
<td>15</td>
<td>31.5</td>
<td>-6.3</td>
<td>-198.5</td>
<td>992.3</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>-77.5</td>
<td>18.7</td>
<td>-1449.3</td>
<td>6006.3</td>
</tr>
<tr>
<td>70</td>
<td>25</td>
<td>-27.5</td>
<td>3.7</td>
<td>-101.8</td>
<td>756.3</td>
</tr>
<tr>
<td>390</td>
<td>85</td>
<td></td>
<td>-2947.7</td>
<td>13157.2</td>
<td></td>
</tr>
</tbody>
</table>

\( \bar{x} = \frac{\sum x}{n} = \frac{390}{4} = 97.5 \), \( \bar{y} = \frac{\sum y}{n} = \frac{85}{4} = 21.3 \). From equation 9, \( b = \frac{-2947.7}{13157.2} = -0.2 \) \( a = y - b \bar{x} = 21.3 + 0.2 \times 97.5 = 40.8 \) So the linear regression equation will be,

\[
\nu = 40.8 - 0.2k \tag{10}
\]

Here \( \nu_f = 40.8 \) and \( \frac{\nu_f}{k_j} = 0.2 \) This implies, \( k_j = \frac{40.8}{0.2} = 204 \) veh/km The basic parameters of Greenshield’s model are free flow speed and jam density and they are obtained as 40.8 kmph and 204 veh/km respectively. To find maximum flow, use equation 6, i.e., \( q_{max} = \frac{40.8 \times 204}{4} = 2080.8 \) veh/hr Density corresponding to the speed 30 km/hr can be found out by substituting \( \nu = 30 \) in equation 10. i.e., \( 30 = 40.8 - 0.2 \times k \) Therefore, \( k = \frac{40.8 - 30}{0.2} = 54 \) veh/km
Model Comparison

speed vs density

- Empirical
- Emp mean
- Greenshields

speed, km/hr

density, veh/km

0 20 40 60 80 100 120 140 160 180

0 20 40 60 80 100 120

120 110 100 90 80 70 60 50 40 30 20 10 0

18
Model Comparison
Greenberg's model

- **Logarithmic relation**
  - Advantage
    - Analytical derivation
    - Good at congestion
  - Drawbacks
    - Infinite speed
    - Poor at low densities

\[ v = v_o \ln \left( \frac{k_j}{k} \right) \]
Underwood's model

- Exponential Model
  - Advantage
    - Good at low speed
  - Drawbacks
    - Speed is zero only at infinity density
    - Poor at high densities

\[ v = v_f e^{-\frac{k}{k_0}} \]
Pipes' model

• Generalized Model

\[ v = v_f \left[ 1 - \left( \frac{k}{k_j} \right)^n \right] \]

– When \( n \) is 1 Pipe’s model resembles Greenshield’s model
Comparison of Models

$k = 200, \ vf = 90, \ ko = 100, \ v0 = 45$

- Speed $V$ (kmph)
- Density $k$ (veh/km)

Liner $n = 1$
Comparison of Models

$k = 200, \ vf = 90, \ ko = 100, \ v_0 = 45$

- **Liner** $n = 1$
- **Log**
Comparison of Models

$k = 200, \ v_f = 90, \ k_0 = 100, \ v_0 = 45$

Liner $n = 1$

Log

Exp
Comparison of Models

$k = 200, \, v_f = 90, \, k_o = 100, \, v_0 = 45$

- Gen $n = 0.5$
- Liner $n = 1$
- Gen $n = 2$
- Log
- Exp
Model Comparison

![Graph showing speed vs density comparison between different models: Empirical, Greenshields, Greenberg, Underwood, Drake, Drew, and Pipes-Munjal. Each model is represented by a different line or marker on the graph.](image)
Model Comparison

![Flow vs Density Graph](image)
Model Comparison

• **Limitations**
  – Assumes uniform behaviour
  – Poor predictability
  – Generalization

• **Need for multi-regime models**
  – Traffic states divided into various regimes
  – Behaviour depends on the regime
  – Separate models for each regime
Multiregime model

• Eddie’s Two Regime Model
  – Based on field data (Chicago)

![Graph showing Speed vs. Density with two regimes: Exponential and Logarithmic.](attachment:graph.png)
Multiregime model

- **Eddie’s Two Regime Model**
  - Based on field data (Chicago)
Multiregime model

- Eddie’s Two Regime Model
Multiregime model

- Three Regime Model
  - Free flow
  - Normal
  - Congested
Multiregime model
	hree-regime model developed by Drake et al.

\[ u = \begin{cases} 
50 - 0.098k & \text{for } k \leq 40 \\
81.4 - 0.913k & \text{for } 40 \leq k \leq 65 \\
40 - 0.265k & \text{for } k \geq 65 
\end{cases} \]
Multiregime model

three-regime model developed by Drake et al.

\[ u = \begin{cases} 
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40 - 0.265k & \text{for } k \geq 65 
\end{cases} \]
Multi-regime models
Speed –Flow: Effect of location
Conclusion

• **Concerns**
  
  – The current status of mathematical models for speed-flow concentration relationships is in a state of flux
  – The models that dominated for nearly 30 years are incompatible with the data currently being obtained
  – but no replacement models have yet been developed


US DOT, Federal Highway Administration

http://www.tfhrc.gov/pubrds/janfeb99/traffic.htm
Conclusion

• Trends
  – Despite those words of caution, it is important to note that there have been significant advances in understanding traffic stream behavior since 1980’s leading to a better understanding of traffic operation
  – Efforts to implement ITS will provide challenges for applying this improvement
  – Equally important, ITS will likely provide the opportunity for acquiring more and better data to advance understanding of traffic operations