Trip Assignment - (contd...)

Lecture Notes in Transportation Systems Engineering

Prof. Tom V. Mathew

Contents

---

1 Introduction

This chapter covers a numerical method to solve traffic assignmenr by user equilibrium or system optimum method. The method is known as Frank-Wolfe algorithm or convex combination algorithm.

2 All-OR-Nothing Assignment

Numerical illustartion: under preparation.

3 Frank-Wolfe Algorithm for UE

Step 0: ***Initilize***

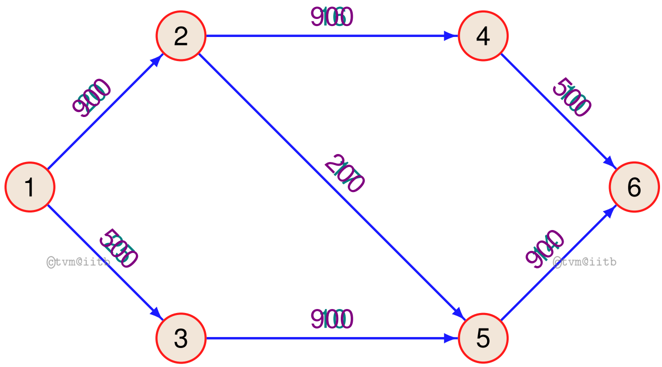

Figure 1: Network Flow Conservation Problem

Set $n = 1$, $t_a = t_a(0) \quad \forall a$

Do $AON \rightarrow \{x_a^n\}$

Step 1: ***Update***

Set $t_a^n = t_a(x_a^n)$

Step 2: ***Direction finding***

Do AON based on $t_a^n \rightarrow \{y_a^n\}$

where $\{y_a^n\}$ is the auxiliary flow

Step 3: ***Line search***

Find α_n^* that solves

$$\min_{0 \le \alpha_n \le 1} \sum_a \int_o^{x_a^n + \alpha_n (y_a^n - x_a^n)} t_a(x) dx$$

Step 4: ***Move***

Set $x_a^{n+1} = x_a^n + \alpha_n^* (y_a^n - x_a^n)$

Step 5: ***Convergence***

If $|x_a^{n+1} - x_a^n| \le \epsilon \quad \forall a$ then STOP

else set $n = n + 1$, go to step 1.

3.1 Numerical Example

To demonstrate the Frank-Wolfe algorithm, consider an example network give in figure 2. This network has two nodes having two paths as links. The travel time is a function of link flow for both the links and is given as: $t_1 = 10 + 3x_1$ and $t_2 = 15 + 2x_2$, and total flows from 1 to 2 is given as $q_{12} = 12$. Do an UE assignment using Frank-Wolfe algorithm and show two full iterations.

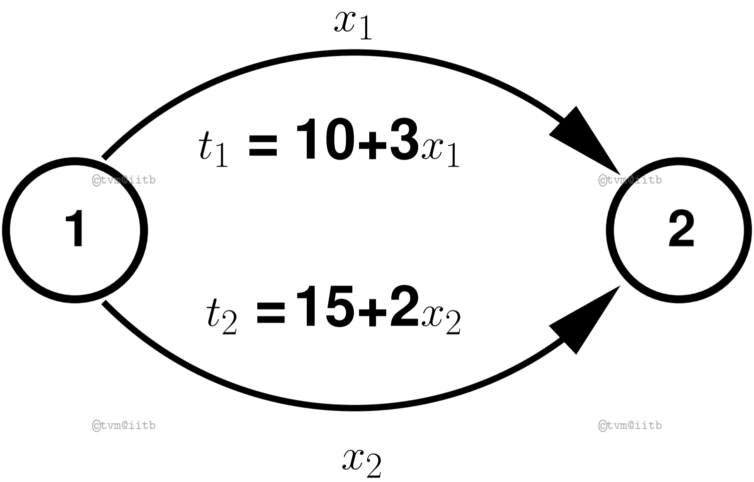

Figure 2: Two link problem for solving UE assignment by Franl-Wolfe algorithm

Solution

Step 0: *Initilize*

Set $n = 1$, $t_1^1 = t_1^1(0) = 10$, $t_2^1 = t_2^1(0) = 15$

Doing $AON \implies x_1^1 = 12,\ x_2^1 = 0$

Step 1: *Update*

Setting $t_1^1 = t_1(x_1^1) = t_1(15) = 10 + 3 \times 15 = 46$

and $t_2^1 = t_2(x_2^1) = t_2(0) = 15 + 2 \times 0 = 15$

Step 2: *Direction finding*

Doing AON based on t_1^1 and t_2^1 $\implies$ $y_1^1 = 0,\ y_2^1 = 12$.

Step 3: *Line search*

Setting $u_1 = x_1^1 + \alpha_1(y_1^1 - x_1^1) = 12 + \alpha_1(0 - 12) = 12 - 12\alpha_1$,

Setting $u_2 = x_2^1 + \alpha_1(y_2^1 - x_2^1) = 0 + \alpha_1(12 - 0) = 12\alpha_1$,

Writing the objective function in terms of α_1^*

$$\begin{aligned}
\min_{0 \le \alpha_1 \le 1} Z &= \sum_a \int_0^{u_1} (10 + 3 \times x)\, dx + \int_0^{u_2} (15 + 2 \times x)\, dx \\
&= 10x_1 + \left.\frac{3x_1^2}{2}\right|_0^{u_1} + 15x_1 + \left.\frac{2x_2^2}{2}\right|_0^{u_2} \\
&= 10(12 - 12\alpha) + \frac{3}{2}(12 - 12\alpha)^2 + 15(12\alpha) + (12\alpha)^2
\end{aligned}$$

Solving by taking $\frac{dz}{d\alpha} = 0 \implies \alpha^* = 0.517$

Step 4: *Move*

Get $x_1 = 12 - 12\alpha = 12 - 12 \times 0.517 = 5.796$

Get $x_2 = 12\alpha = 12 \times 0.517 = 6.204$

Step 5: *Convergence*

If $|x_a^{n+1} - x_a^n| \le \epsilon \quad \forall a$ then STOP

else set $n = n + 1$, go to step 1.

Exercises

1. Not Available

References

1. Yosef Sheffi. *Urban transportation networks: Equilibrium analysis with mathematical programming methods*. New Jersey, 1984.

2. R Thomas. *Traffic Assignment Techniques*. Avebury Technical publication,England, 1991.

Acknowledgments

I wish to thank several of my students and staff of NPTEL for their contribution in this lecture. I also appreciate your constructive feedback which may be sent to **tvm@civil.iitb.ac.in**

Prof. Tom V. Mathew
Department of Civil Engineering
Indian Institute of Technology Bombay, India