

Rigid pavement design

Lecture Notes in Transportation Systems Engineering

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1 Overview

As the name implies, rigid pavements are rigid i.e, they do not flex much under loading like flexible pavements. They are constructed using cement concrete. In this case, the load carrying capacity is mainly due to the rigidity and high modulus of elasticity of the slab (slab action). H. M. Westergaard is considered the pioneer in providing the rational treatment of the rigid pavement analysis.

1.1 Modulus of sub-grade reaction

Westergaard considered the rigid pavement slab as a thin elastic plate resting on soil sub-grade, which is assumed as a dense liquid. The upward reaction is assumed to be proportional to the deflection. Based on this assumption, Westergaard defined a *modulus of sub-grade reaction* K in kg/cm^3 given by $K = \frac{p}{\Delta}$ where Δ is the displacement level taken as 0.125 cm and p is the pressure sustained by the rigid plate of 75 cm diameter at a deflection of 0.125 cm.

1.2 Relative stiffness of slab to sub-grade

A certain degree of resistance to slab deflection is offered by the sub-grade. The sub-grade deformation is same as the slab deflection. Hence the slab deflection is direct measurement of the magnitude of the sub-grade pressure. This pressure deformation characteristics of rigid pavement lead Westergaard to define the term *radius of relative stiffness* l in cm is given by the equation 1.

$$l = \sqrt[4]{\frac{Eh^3}{12K(1 - \mu^2)}} \quad (1)$$

where E is the modulus of elasticity of cement concrete in kg/cm^2 (3.0×10^5), μ is the Poisson's ratio of concrete (0.15), h is the slab thickness in cm and K is the modulus of sub-grade reaction.

1.3 Critical load positions

Since the pavement slab has finite length and width, either the character or the intensity of maximum stress induced by the application of a given traffic load is dependent on the

location of the load on the pavement surface. There are three typical locations namely the *interior*, *edge* and *corner*, where differing conditions of slab continuity exist. These locations are termed as critical load positions.

1.4 Equivalent radius of resisting section

When the interior point is loaded, only a small area of the pavement is resisting the bending moment of the plate. Westergaard's gives a relation for equivalent radius of the resisting section in cm in the equation 2.

$$b = \begin{cases} \sqrt{1.6a^2 + h^2} - 0.675 h & \text{if } a < 1.724 h \\ a & \text{otherwise} \end{cases} \quad (2)$$

where a is the radius of the wheel load distribution in cm and h is the slab thickness in cm.

2 Wheel load stresses - Westergaard's stress equation

The cement concrete slab is assumed to be homogeneous and to have uniform elastic properties with vertical sub-grade reaction being proportional to the deflection. Westergaard developed relationships for the stress at interior, edge and corner regions, denoted as σ_i , σ_e , σ_c in kg/cm² respectively and given by the equation 3-5.

$$\sigma_i = \frac{0.316 P}{h^2} \left[4 \log_{10} \left(\frac{l}{b} \right) + 1.069 \right] \quad (3)$$

$$\sigma_e = \frac{0.572 P}{h^2} \left[4 \log_{10} \left(\frac{l}{b} \right) + 0.359 \right] \quad (4)$$

$$\sigma_c = \frac{3 P}{h^2} \left[1 - \left(\frac{a\sqrt{2}}{l} \right)^{0.6} \right] \quad (5)$$

where h is the slab thickness in cm, P is the wheel load in kg, a is the radius of the wheel load distribution in cm, l the radius of the relative stiffness in cm and b is the radius of the resisting section in cm

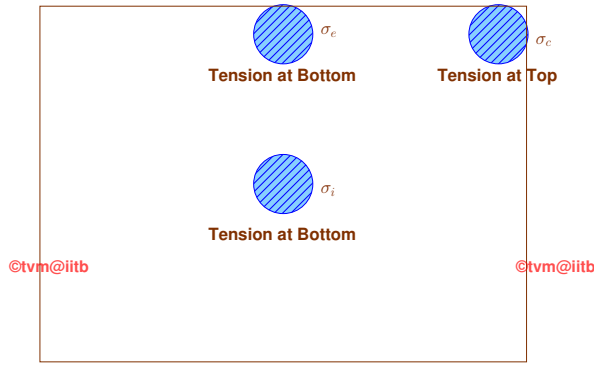


Figure 1: Critical stress locations

3 Temperature stresses

Temperature stresses are developed in cement concrete pavement due to variation in slab temperature. This is caused by (i) *daily variation* resulting in a temperature gradient across the thickness of the slab and (ii) *seasonal variation* resulting in overall change in the slab temperature. The former results in **warping** stresses and the later in **frictional** stresses.

3.1 Warping stress

The warping stress at the interior, edge and corner regions, denoted as σ_{ti} , σ_{te} , σ_{tc} in kg/cm² respectively and given by the equation 6-8.

$$\sigma_{ti} = \frac{E\epsilon t}{2} \left(\frac{C_x + \mu C_y}{1 - \mu^2} \right) \quad (6)$$

$$\sigma_{te} = \text{Max} \left(\frac{C_x E \epsilon t}{2}, \frac{C_y E \epsilon t}{2} \right) \quad (7)$$

$$\sigma_{tc} = \frac{E\epsilon t}{3(1 - \mu)} \sqrt{\frac{a}{l}} \quad (8)$$

where E is the modulus of elasticity of concrete in kg/cm² (3×10^5), ϵ is the thermal coefficient of concrete per °C (1×10^{-7}) t is the temperature difference between the top and bottom of the slab, C_x and C_y are the coefficient based on L_x/l in the desired direction and L_y/l right angle to the desired direction, μ is the Poisson's ration (0.15), a is the radius of the contact area and l is the radius of the relative stiffness.

3.2 Frictional stresses

The frictional stress σ_f in kg/cm^2 is given by the equation

$$\sigma_f = \frac{WLf}{2 \times 10^4} \quad (9)$$

where W is the unit weight of concrete in kg/cm^2 (2400), f is the coefficient of sub grade friction (1.5) and L is the length of the slab in meters.

4 Combination of stresses

The cumulative effect of the different stress give rise to the following three critical cases

- *Summer, mid-day*: The critical stress is for edge region given by $\sigma_{critical} = \sigma_e + \sigma_{t_e} - \sigma_f$
- *Winter, mid-day*: The critical combination of stress is for the edge region given by $\sigma_{critical} = \sigma_e + \sigma_{t_e} + \sigma_f$
- *Mid-nights*: The critical combination of stress is for the corner region given by $\sigma_{critical} = \sigma_c + \sigma_{t_c}$

5 Design of joints

5.1 Expansion joints

The purpose of the expansion joint is to allow the expansion of the pavement due to rise in temperature with respect to construction temperature. The design consideration are:

- Provided along the longitudinal direction,
- design involves finding the joint spacing for a given expansion joint thickness (say 2.5 cm specified by IRC) subjected to some maximum spacing (say 140 as per IRC)

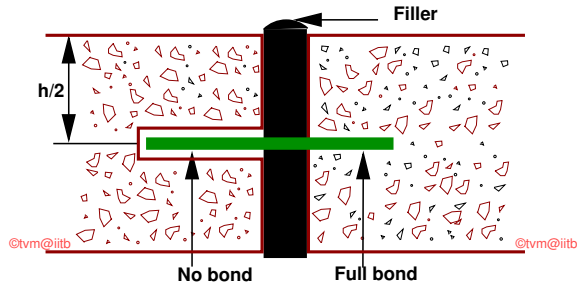


Figure 2: Expansion joint

5.2 Contraction joints

The purpose of the contraction joint is to allow the contraction of the slab due to fall in slab temperature below the construction temperature. The design considerations are:

- The movement is restricted by the sub-grade friction
- Design involves the length of the slab given by:

$$L_c = \frac{2 \times 10^4 S_c}{W.f} \quad (10)$$

where, S_c is the allowable stress in tension in cement concrete and is taken as 0.8 kg/cm², W is the unit weight of the concrete which can be taken as 2400 kg/cm³ and f is the coefficient of sub-grade friction which can be taken as 1.5.

- Steel reinforcements can be use, however with a maximum spacing of 4.5 m as per IRC.

5.3 Dowel bars

The purpose of the dowel bar is to effectively transfer the load between two concrete slabs and to keep the two slabs in same height. The dowel bars are provided in the direction of the traffic (longitudinal). The design considerations are:

- Mild steel rounded bars,
- bonded on one side and free on other side

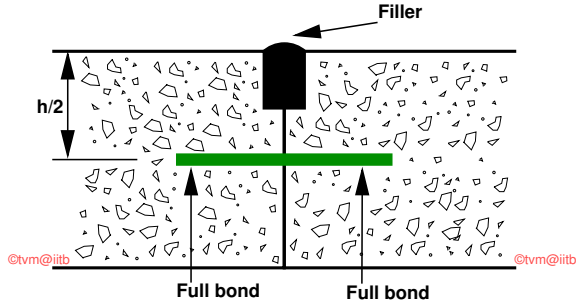


Figure 3: Contraction joint

5.3.1 Bradbury's analysis

Bradbury's analysis gives load transfer capacity of single dowel bar in shear, bending and bearing as follows:

$$P_s = 0.785 d^2 F_s \quad (11)$$

$$P_f = \frac{2 d^3 F_f}{L_d + 8.8\delta} \quad (12)$$

$$P_b = \frac{F_b L_d^2 d}{12.5 (L_d + 1.5\delta)} \quad (13)$$

where, P is the load transfer capacity of a single dowel bar in shear s , bending f and bearing b , d is the diameter of the bar in cm, L_d is the length of the embedment of dowel bar in cm, δ is the joint width in cm, F_s , F_f , F_b are the permissible stress in shear, bending and bearing for the dowel bar in kg/cm².

5.3.2 Design procedure

Step 1 Find the length of the dowel bar embedded in slab L_d by equating Eq. 12=Eq. 13, i.e.

$$L_d = 5d \sqrt{\frac{F_f (L_d + 1.5\delta)}{F_b (L_d + 8.8\delta)}} \quad (14)$$

Step 2 Find the load transfer capacities P_s , P_f , and P_b of single dowel bar with the L_d

Step 3 Assume load capacity of dowel bar is 40 percent wheel load, find the load

capacity factor f as

$$\max \left\{ \frac{0.4P}{P_s}, \frac{0.4P}{P_f}, \frac{0.4P}{P_b} \right\} \quad (15)$$

Step 4 Spacing of the dowel bars.

- Effective distance upto which effective load transfer take place is given by $1.8 l$, where l is the radius of relative stiffness.
- Assume a linear variation of capacity factor of 1.0 under load to 0 at $1.8 l$.
- Assume a dowel spacing and find the capacity factor of the above spacing.
- Actual capacity factor should be greater than the required capacity factor.
- If not, do one more iteration with new spacing.

5.3.3 Example

Design size and spacing of dowel bars at an expansion joint of concrete pavement of thickness 25 cm. Given the radius of relative stiffness of 80 cm. design wheel load 5000 kg. Load capacity of the dowel system is 40 percent of design wheel load. Joint width is 2.0 cm and the permissible stress in shear, bending and bearing stress in dowel bars are 1000, 1400 and 100 kg/cm^2 respectively.

Solution: Given, $P = 5000 \text{ kg}$, $l = 80 \text{ cm}$, $h = 25 \text{ cm}$, $\delta = 2 \text{ cm}$, $F_s = 1000 \text{ kg/cm}^2$, $F_f = 1400 \text{ kg/cm}^2$ and $F_b = 100 \text{ kg/cm}^2$; and assume $d = 2.5 \text{ cm}$ diameter.

Step-1: length of the dowel bar L_d

$$\begin{aligned} L_d &= 5 \times 2.5 \sqrt{\frac{1400 (L_d + 1.5 \times 2)}{100 (L_d + 8.8 \times 2)}} \\ &= 12.5 \times \sqrt{14 \frac{(L_d + 3)}{(L_d + 17.6)}} \end{aligned}$$

Solve for L_d by trial and error:

put $L_d = 45.00 \Rightarrow L_d = 40.95$

put $L_d = 45.95 \Rightarrow L_d = 40.50$

put $L_d = 45.50 \Rightarrow L_d = 40.50$

Minimum length of the dowel bar is $L_d + \delta = 40.5 + 2.0 = 42.5 \text{ cm}$, So, provide 45 cm long and 2.5 cm ϕ . Therefore $L_d = 45 - 2 = 43 \text{ cm}$.

Step 2: Find the load transfer capacity of single dowel bar

$$P_s = 0.785 \times 2.5^2 \times 1000 = 4906 \text{ kg}$$

$$P_f = \frac{2 \times 2.5^3 \times 1400}{43.0 + 8.8 \times 2} = 722 \text{ kg}$$

$$P_b = \frac{100 \times 2.5 \times 43.0^2}{12.5 (43.0 + 1.5 \times 2)} = 804 \text{ kg}$$

Therefore, the required load transfer capacity

$$\max \left\{ \frac{0.4 \times 5000}{4906}, \frac{0.4 \times 5000}{722}, \frac{0.4 \times 5000}{804} \right\}$$

$$\max \{0.41, 2.77, 2.487\} = 2.77$$

Step-3 : Find the required spacing: Effective distance of load transfer = $1.8 l = 1.8 \times 80 = 144 \text{ cm}$. Assuming 35 cm spacing,

Actual capacity is

$$1 + \frac{144 - 35}{144} + \frac{144 - 70}{144} + \frac{144 - 105}{144} + \frac{144 - 140}{144}$$

$$= 2.57 < 2.77 \text{ (the required capacity)}$$

Therefore assume 30 cm spacing and now the actual capacity is

$$1 + \frac{144 - 30}{144} + \frac{144 - 60}{144} + \frac{144 - 90}{144} + \frac{144 - 120}{144}$$

$$= 2.92 > 2.77 \text{ (the required capacity)}$$

Therefore provide 2.5 cm ϕ mild steel dowel bars of length 45 cm @ 30 cm center to center.

5.4 Tie bars

In contrast to dowel bars, tie bars are not load transfer devices, but serve as a means to tie two slabs. Hence tie bars must be deformed or hooked and must be firmly anchored into the concrete to function properly. They are smaller than dowel bars and placed at large intervals. They are provided across longitudinal joints.

Step 1 Diameter and spacing: The diameter and the spacing is first found out by equating the total sub-grade friction to the total tensile stress for a unit length (one

meter). Hence the area of steel per one meter in cm^2 is given by:

$$A_s \times S_s = b \times h \times W \times f$$

$$A_s = \frac{bhWf}{100S_s} \quad (16)$$

where, b is the width of the pavement panel in m , h is the depth of the pavement in cm , W is the unit weight of the concrete (assume 2400 kg/cm^2), f is the coefficient of friction (assume 1.5), and S_s is the allowable working tensile stress in steel (assume 1750 kg/cm^2). Assume 0.8 to 1.5 cm ϕ bars for the design.

Step 2 Length of the tie bar: Length of the tie bar is twice the length needed to develop bond stress equal to the working tensile stress and is given by:

$$L_t = \frac{d S_s}{2 S_b} \quad (17)$$

where, d is the diameter of the bar, S_s is the allowable tensile stress in kg/cm^2 , and S_b is the allowable bond stress and can be assumed for plain and deformed bars respectively as 17.5 and 24.6 kg/cm^2 .

5.4.1 Example

A cement concrete pavement of thickness 18 cm, has two lanes of 7.2 m with a joint. Design the tie bars.

(Solution:)

Given $h=18 \text{ cm}$, $b=7.2/2=3.6\text{m}$, $S_s = 1700 \text{ kg/cm}^2$ $W = 2400 \text{ kg/cm}^2$ $f = 1.5$ $S_b = 24.6 \text{ kg/cm}^2$.

Step 1: diameter and spacing: Get A_s from

$$A_s = \frac{3.6 \times 18 \times 2400 \times 1.5}{100 \times 1750} = 1.33 \text{ cm}^2/m$$

Assume $\phi = 1 \text{ cm}$, $\Rightarrow A = 0.785 \text{ cm}^2$. Therefore spacing is $\frac{100 \times 0.785}{1.33} = 59 \text{ cm}$, say 55 cm

Step 2: Length of the bar: Get L_t from

$$L_t = \frac{1 \times 1750}{2 \times 246} = 36.0 \text{ cm}$$

[Ans] Use 1 ϕ tie bars of length of 36 cm @ 55 cm c/c

6 Summary

Design of rigid pavements is based on Westergaard's analysis, where modulus of sub-grade reaction, radius of relative stiffness, radius of wheel load distribution are used. For critical design, a combination of load stress, frictional stress and warping stress is considered. Different types of joints are required like expansion and contraction joints. Their design is also dealt with.

7 Problems

1. Design size and spacing of dowel bars at an expansion joint of concrete pavement of thickness 20 cm. Given the radius of relative stiffness of 90 cm. design wheel load 4000 kg. Load capacity of the dowel system is 40 percent of design wheel load. Joint width is 3.0 cm and the permissible stress in shear, bending and bearing stress in dowel bars are 1000, 1500 and 100 kg/cm^2 respectively.
2. Design the length and spacing of tie bars given that the pavement thickness is 20cm and width of the road is 7m with one longitudinal joint. The unit weight of concrete is $2400 \text{ kg}/\text{m}^3$, the coefficient of friction is 1.5, allowable working tensile stress in steel is $1750 \text{ kg}/\text{cm}^2$, and bond stress of deformed bars is $24.6 \text{ kg}/\text{cm}^2$.

8 Solutions

1. Given, $P = 4000 \text{ kg}$, $l = 90 \text{ cm}$, $h = 20 \text{ cm}$, $\delta = 3 \text{ cm}$, $F_s = 1000 \text{ kg}/\text{cm}^2$, $F_f = 1500 \text{ kg}/\text{cm}^2$ and $F_b = 100 \text{ kg}/\text{cm}^2$; and assume $d = 2.5 \text{ cm}$ diameter.

Step-1: length of the dowel bar L_d ,

$$\begin{aligned} L_d &= 5 \times 2.5 \sqrt{\frac{1500 (L_d + 1.5 \times 3)}{100 (L_d + 8.8 \times 3)}} \\ &= 12.5 \times \sqrt{15 \frac{(L_d + 4.5)}{(L_d + 26.4)}} \end{aligned}$$

Solving for L_d by trial and error, it is =39.5cm Minimum length of the dowel bar is $L_d + \delta = 39.5 + 3.0 = 42.5 \text{ cm}$, So, provide 45 cm long and 2.5 cm ϕ . Therefore $L_d = 45 - 3 = 42 \text{ cm}$.

Step 2: Find the load transfer capacity of single dowel bar

$$\begin{aligned} P_s &= 0.785 \times 2.5^2 \times 1000 = 4906.25 \text{ kg} \\ P_f &= \frac{2 \times 2.5^3 \times 1500}{42.0 + 8.8 \times 3} = 685.307 \text{ kg} \\ P_b &= \frac{100 \times 2.5 \times 42.0^2}{12.5 (42.0 + 1.5 \times 3)} = 758.71 \text{ kg} \end{aligned}$$

Therefore, the required load transfer capacity (refer equation)

$$\max \left\{ \frac{0.4 \times 4000}{4906.25}, \frac{0.4 \times 4000}{685.307}, \frac{0.4 \times 4000}{758.71} \right\} \\ \max \{0.326, 2.335, 2.10\} = 2.335$$

Step-3 : Find the required spacing: Effective distance of load transfer = $1.8 \times l = 1.8 \times 90 = 162 \text{ cm}$. Assuming 35 cm spacing,

Actual capacity is

$$1 + \frac{162 - 35}{162} + \frac{162 - 70}{162} + \frac{162 - 105}{162} + \frac{162 - 140}{162} = 2.83$$

Assuming 40cm spacing, capacity is,

$$1 + \frac{162 - 40}{162} + \frac{162 - 80}{162} + \frac{162 - 120}{162} + \frac{162 - 160}{162} = 2.52$$

So we should consider 2.52 & 2.335 as it is greater and more near to other value. Therefore provide 2.5 cm ϕ mild steel dowel bars of length 45 cm @ 40 cm center to center.

2. 2. Given $h=20 \text{ cm}$, $b=7/2=3.5\text{m}$, $S_s = 1750 \text{ kg/cm}^2$ $W = 2400 \text{ kg/cm}^2$ $f = 1.5$ $S_b = 24.6 \text{ kg/cm}^2$.

Step 1: diameter and spacing:

$$A_s = \frac{3.5 \times 20 \times 2400 \times 1.5}{100 \times 1750} = 1.44 \text{ cm}^2/\text{m}$$

Assume $\phi = 1 \text{ cm}$, $\Rightarrow A = 0.785 \text{ cm}^2$. Therefore spacing is $\frac{100 \times 0.785}{1.44} = 54.57 \text{ cm}$, say 55 cm

Step 2: Length of the bar:

$$L_t = \frac{1 \times 1750}{2 \times 24.6} = 36.0 \text{ cm}$$

[Ans] Use 1 ϕ tie bars of length of 36 cm @ 55 cm c/c

References

1. IRC. *Rigid pavement design*. Number 58. New Delhi, 1988.

Acknowledgments

I wish to thank several of my students and staff of NPTEL for their contribution in this lecture. I also appreciate your constructive feedback which may be sent to tvm@civil.iitb.ac.in

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