Traffic Stream Models

Lecture Notes in Transportation Systems Engineering

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Contents

1 Overview

To figure out the exact relationship between the traffic parameters, a great deal of research has been done over the past several decades. The results of these researches yielded many mathematical models. Some important models among them will be discussed in this chapter.

2 Greenshield's macroscopic stream model

Macroscopic stream models represent how the behaviour of one parameter of traffic flow changes with respect to another. Most important among them is the relation between speed and density. The first and most simple relation between them is proposed by Greenshield. Greenshield assumed a linear speed-density relationship as illustrated in figure 1 to derive the model. The equation for this relationship is shown below.

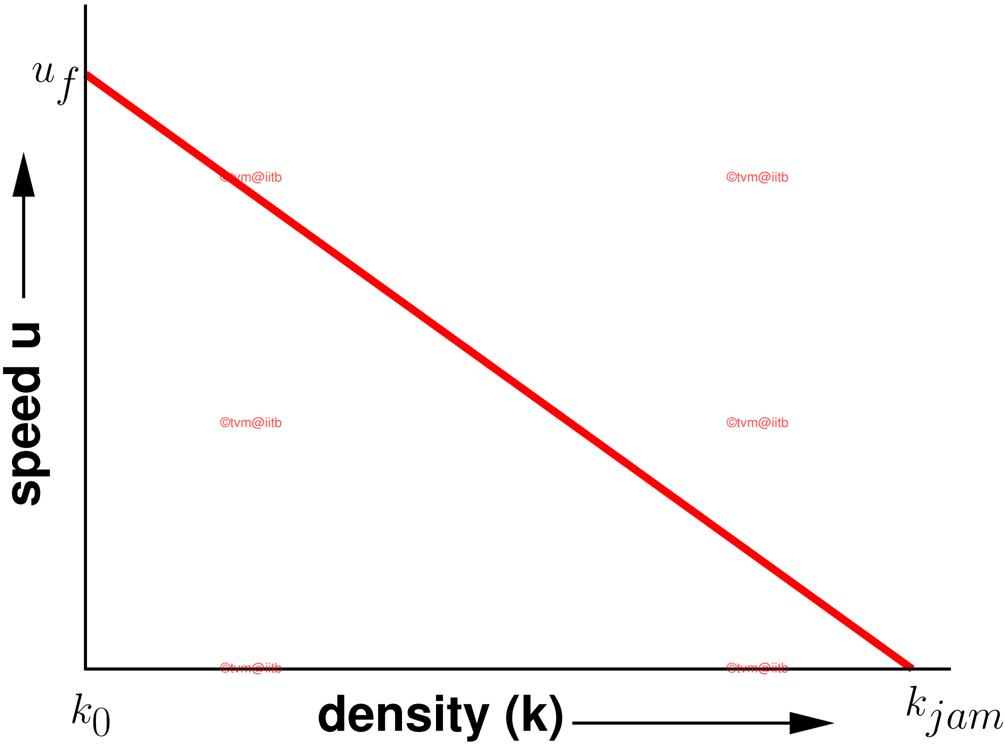

Figure 1: Relation between speed and density

$$v = v_f - \left[\frac{v_f}{k_j}\right].k \tag{1}$$

where v is the mean speed at density k, v_f is the free speed and k_j is the jam density. This equation (1) is often referred to as the Greenshield's model. It indicates that when density becomes zero, speed approaches free flow speed (ie. $v \rightarrow v_f$ when $k \rightarrow 0$).

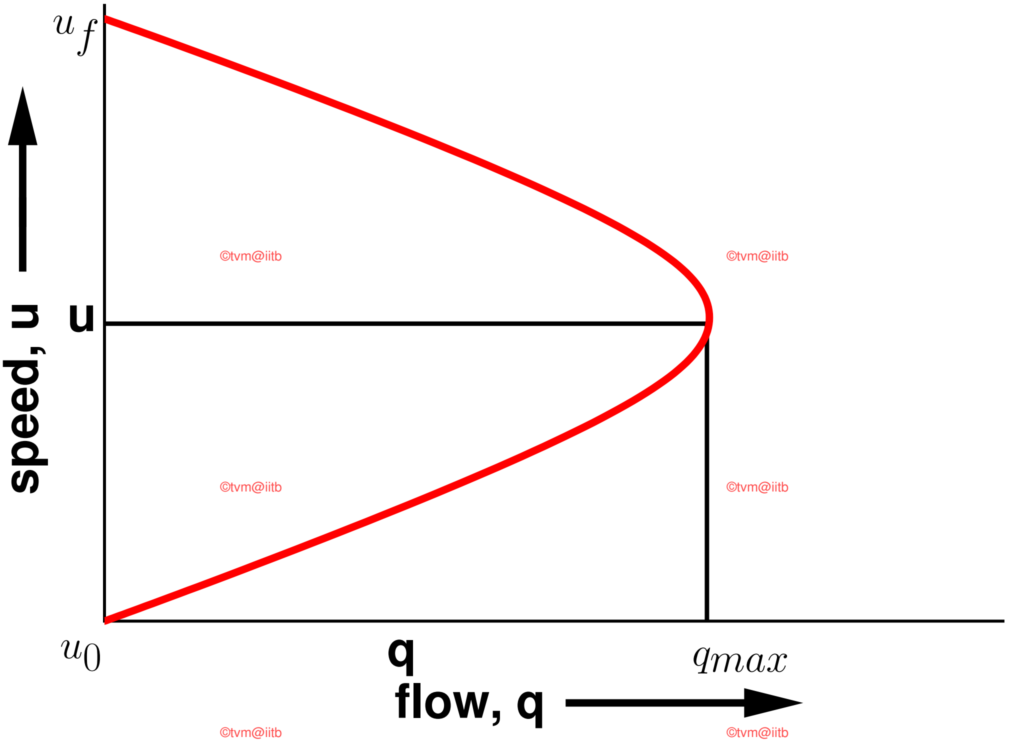

Figure 2: Relation between speed and flow

Once the relation between speed and flow is established, the relation with flow can be derived. This relation between flow and density is parabolic in shape and is shown in figure 3. Also, we know that

$$q = k.v \tag{2}$$

Now substituting equation 1 in equation 2, we get

$$q = v_f.k - \left[\frac{v_f}{k_j}\right] k^2 \tag{3}$$

Similarly we can find the relation between speed and flow. For this, put $k = \frac{q}{v}$ in equation 1 and solving, we get

$$q = k_j.v - \left[\frac{k_j}{v_f}\right] v^2 \tag{4}$$

This relationship is again parabolic and is shown in figure 2. Once the relationship between the fundamental variables of traffic flow is established, the boundary conditions

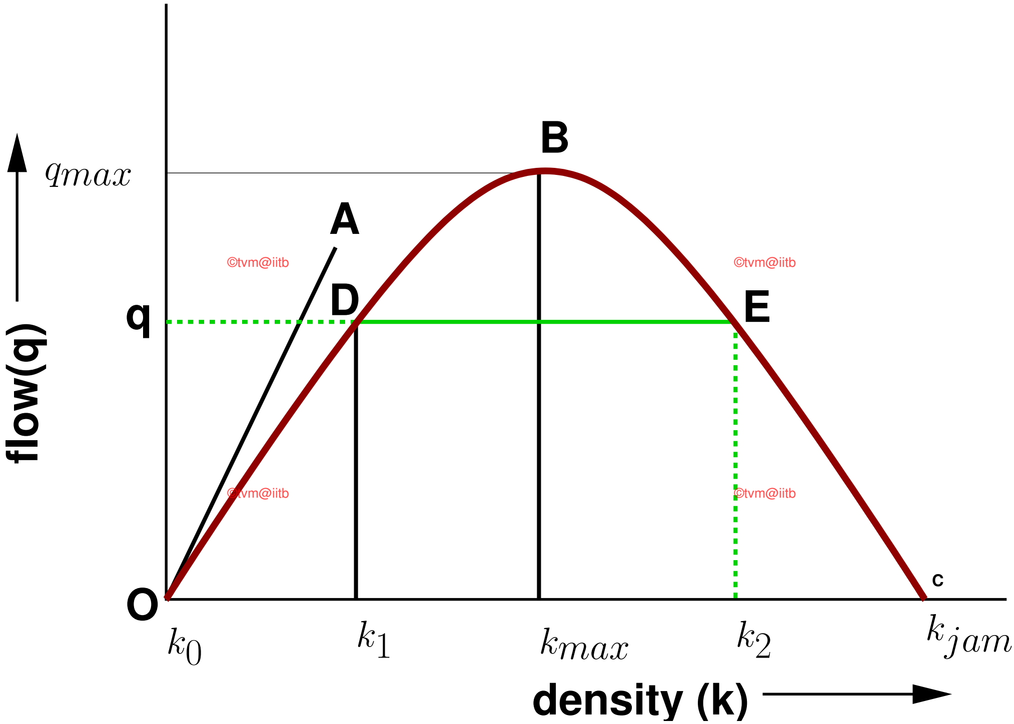

Figure 3: Relation between flow and density 1

can be derived. The boundary conditions that are of interest are jam density, free-flow speed, and maximum flow. To find density at maximum flow, differentiate equation 3 with respect to k and equate it to zero. ie.,

$$\begin{aligned} \frac{dq}{dk} &= 0 \\ v_f - \frac{v_f}{k_j}.2k &= 0 \\ k &= \frac{k_j}{2} \end{aligned}$$

Denoting the density corresponding to maximum flow as k_0,

$$k_0 = \frac{k_j}{2} \tag{5}$$

Therefore, density corresponding to maximum flow is half the jam density. Once we get k_0, we can derive for maximum flow, q_{max}. Substituting equation 5 in equation 3

$$\begin{aligned} q_{max} &= v_f.\frac{k_j}{2} - \frac{v_f}{k_j}.\left[\frac{k_j}{2}\right]^2 \\ &= v_f.\frac{k_j}{2} - v_f.\frac{k_j}{4} \\ &= \frac{v_f.k_j}{4} \end{aligned}$$

Thus the maximum flow is one fourth the product of free flow and jam density. Finally to get the speed at maximum flow, v_0, substitute equation 5 in equation 1 and solving we get,

$$v_0 = v_f - \frac{v_f}{k_j}.\frac{k_j}{2} = \frac{v_f}{2} \quad (6)$$

Therefore, speed at maximum flow is half of the free speed.

3 Calibration of Greenshield's model

In order to use this model for any traffic stream, one should get the boundary values, especially free flow speed (v_f) and jam density (k_j). This has to be obtained by field survey and this is called calibration process. Although it is difficult to determine exact free flow speed and jam density directly from the field, approximate values can be obtained from a number of speed and density observations and then fitting a linear equation between them. Let the linear equation be $y = a + bx$ such that y is density k and x denotes the speed v. Using linear regression method, coefficients a and b can be solved as,

$$b = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i.\sum_{i=1}^{n} y_i}{n.\sum_{i=1}^{n} {x_i}^2 - (\sum_{i=1}^{n} x_i)^2} \quad (7)$$

$$a = \bar{y} - b\bar{x} \quad (8)$$

Alternate method of solving for b is,

$$b = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \quad (9)$$

where x_i and y_i are the samples, n is the number of samples, and $\bar{x}$ and $\bar{y}$ are the mean of x_i and y_i respectively.

Table 1: Solution to numerical example

x(k)	y(v)	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x}^2)$
171	5	73.5	-16.3	-1198.1	5402.3
129	15	31.5	-6.3	-198.5	992.3
20	40	-77.5	18.7	-1449.3	6006.3
70	25	-27.5	3.7	-101.8	756.3
390	85			-2947.7	13157.2

3.0.1 Numerical example

For the following data on speed and density, determine the parameters of the Greenshield's model. Also find the maximum flow and density corresponding to a speed of 30 km/hr.

k	v
171	5
129	15
20	40
70	25

Solution Denoting y = v and x = k, solve for a and b using equation 8 and equation 9. The solution is tabulated as shown below. $\bar{x} = \frac{\Sigma x}{n} = \frac{390}{4}$ = 97.5, $\bar{y} = \frac{\Sigma y}{n} = \frac{85}{4}$ = 21.3. From equation 9, b = $\frac{-2947.7}{13157.2}$ = -0.2 $a = y - b\bar{x}$ = 21.3 + 0.2×97.5 = 40.8 So the linear regression equation will be,

$$v = 40.8 - 0.2k \quad (10)$$

Here v_f = 40.8 and $\frac{v_f}{k_j}$ = 0.2. This implies, $k_j = \frac{40.8}{0.2}$ = 204 veh/km. The basic parameters of Greenshield's model are free flow speed and jam density and they are obtained as 40.8 kmph and 204 veh/km respectively. To find maximum flow, use equation 6, i.e., $q_{max} = \frac{40.8 \times 204}{4}$ = 2080.8 veh/hr Density corresponding to the speed 30 km/hr can be found out by substituting $v = 30$ in equation 10. i.e, 30 = 40.8 - 0.2 × k Therefore, k = $\frac{40.8 - 30}{0.2}$ = 54 veh/km.

4 Other macroscopic stream models

In Greenshield's model, linear relationship between speed and density was assumed. But in field we can hardly find such a relationship between speed and density. Therefore, the validity of Greenshield's model was questioned and many other models came up. Prominent among them are Greenberg's logarithmic model, Underwood's exponential model, Pipe's generalized model, and multi-regime models. These are briefly discussed below.

4.1 Greenberg's logarithmic model

Greenberg assumed a logarithmic relation between speed and density. He proposed,

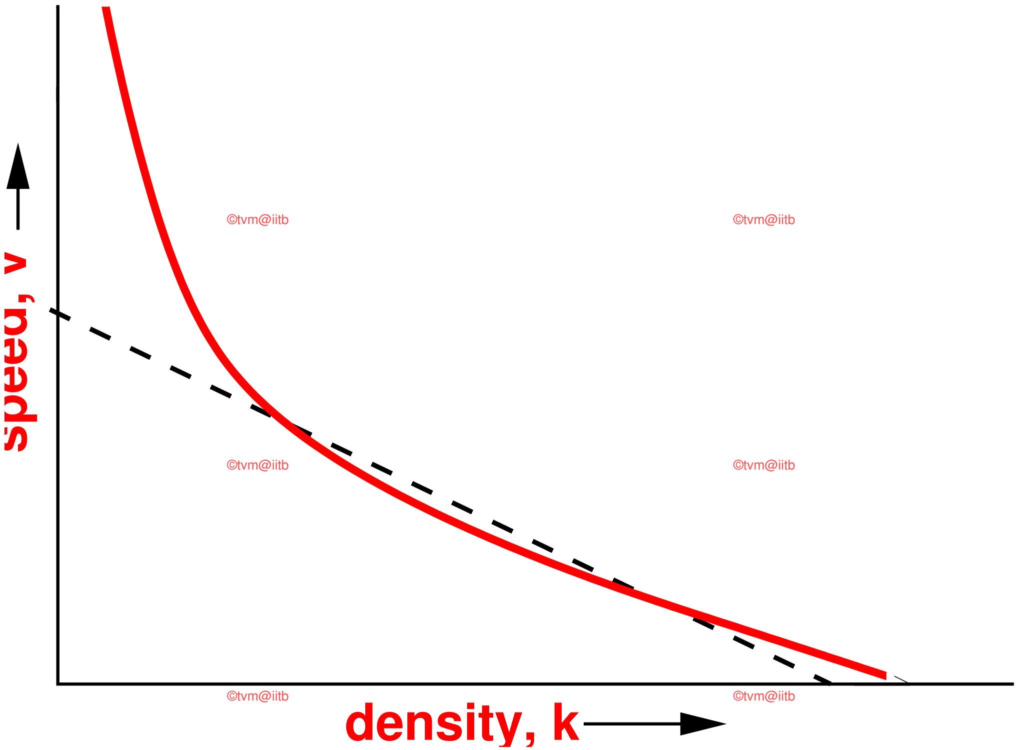

Figure 4: Greenbergs logarithmic model

$$v = v_0 \ln \frac{k_j}{k} \tag{11}$$

This model has gained very good popularity because this model can be derived analytically. (This derivation is beyond the scope of this notes). However, main drawbacks of this model is that as density tends to zero, speed tends to infinity. This shows the inability of the model to predict the speeds at lower densities.

4.2 Underwood's exponential model

Trying to overcome the limitation of Greenberg's model, Underwood put forward an exponential model as shown below.

$$v = v_f . e^{\frac{-k}{k_0}} \tag{12}$$

where v_f is the free flow speed and k_0 is the optimum density. The model can be graphically expressed as in figure 5. is the free flow speed and k_o is the optimum density, i.e. the density corresponding to the maximum flow. In this model, speed becomes zero only when density reaches infinity which is the drawback of this model. Hence this cannot be used for predicting speeds at high densities.

4.3 Pipes' generalized model

Further developments were made with the introduction of a new parameter (n) to provide for a more generalized modeling approach. Pipes proposed a model shown by the following equation.

$$v = v_f \left[1 - \left(\frac{k}{k_j} \right)^n \right] \tag{13}$$

When n is set to one, Pipe's model resembles Greenshield's model. Thus by varying the values of n, a family of models can be developed.

4.4 Multi-regime models

All the above models are based on the assumption that the same speed-density relation is valid for the entire range of densities seen in traffic streams. Therefore, these models are called single-regime models. However, human behaviour will be different at different densities. This is corroborated with field observations which shows different relations at

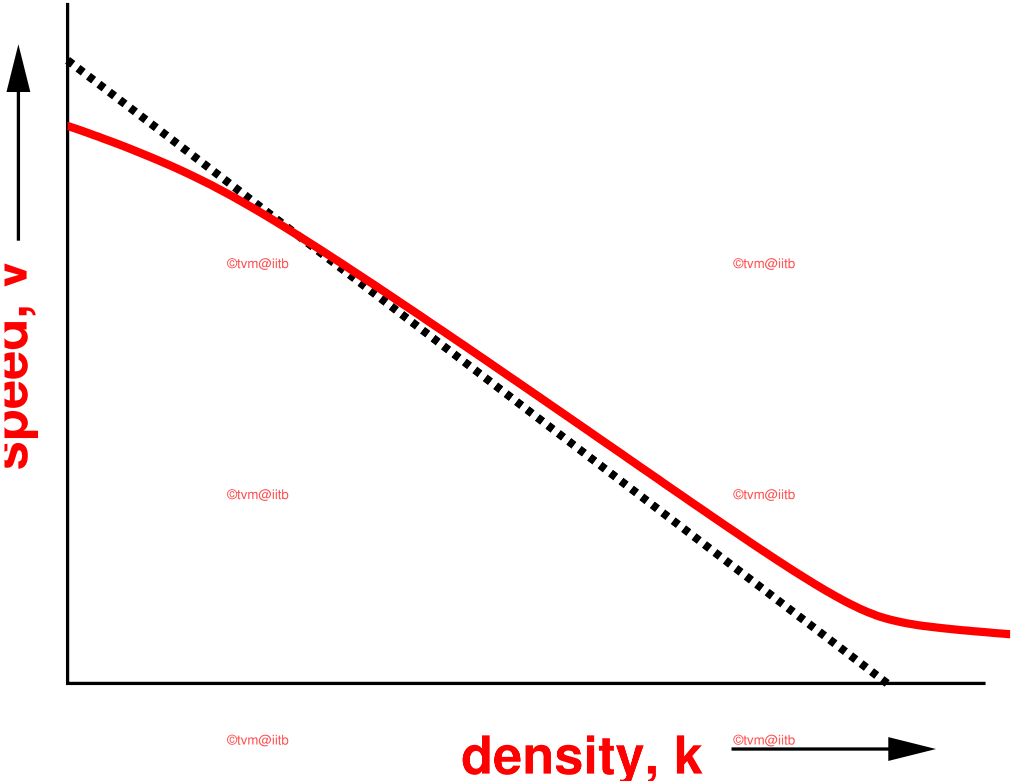

Figure 5: Underwood's exponential model

different range of densities. Therefore, the speed-density relation will also be different in different zones of densities. Based on this concept, many models were proposed generally called multi-regime models. The most simple one is called a two-regime model, where separate equations are used to represent the speed-density relation at congested and uncongested traffic.

5 Shock waves

The flow of traffic along a stream can be considered similar to a fluid flow. Consider a stream of traffic flowing with steady state conditions, i.e., all the vehicles in the stream are moving with a constant speed, density and flow. Let this be denoted as state A (refer figure 6. Suddenly due to some obstructions in the stream (like an accident or traffic block) the steady state characteristics changes and they acquire another state of flow, say state B. The speed, density and flow of state A is denoted as v_A, k_A, and q_A, and state B as v_B, k_B, and q_B respectively. The flow-density curve is shown in

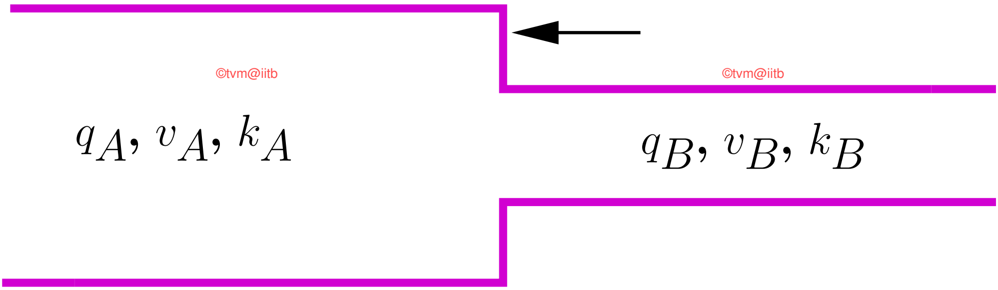

Figure 6: Shock wave: Stream characteristics

figure 7. The speed of the vehicles at state A is given by the line joining the origin and point A in the graph. The time-space diagram of the traffic stream is also plotted in figure 8. All the lines are having the same slope which implies that they are moving with constant speed. The sudden change in the characteristics of the stream leads to the formation of a shock wave. There will be a cascading effect of the vehicles in the upstream direction. Thus shock wave is basically the movement of the point that demarcates the two stream conditions. This is clearly marked in the figure 7. Thus the shock waves produced at state B are propagated in the backward direction. The speed of the vehicles at state B is the line joining the origin and point B of the flow-density curve. Slope of the line AB gives the speed of the shock wave (refer figure 7). If speed of the shock-wave is represented as ω_{AB}, then

$$\omega_{AB} = \frac{q_A - q_B}{k_A - k_B} \tag{14}$$

The above result can be analytically solved by equating the expressions for the number vehicles leaving the upstream and joining the downstream of the shock wave boundary (this assumption is true since the vehicles cannot be created or destroyed. Let N_A be the number of vehicles leaving the section A. Then, $N_A = q_B\ t$. The relative speed of these vehicles with respect to the shock wave will be $v_A - \omega_{AB}$. Hence,

$$N_A = k_A\left(v_A - \omega_{AB}\right) t \tag{15}$$

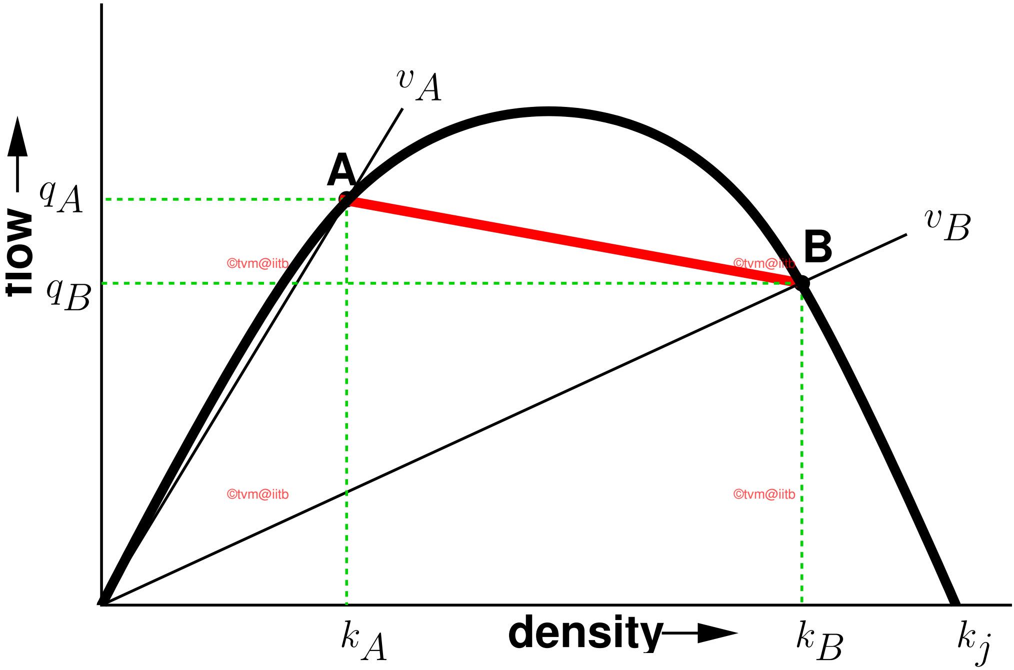

Figure 7: Shock wave: Flow-density curve

Similarly, the vehicles entering the state B is given as

$$N_B = k_A \left(v_B - \omega_{AB} \right) t \tag{16}$$

Equating equations 15 and 16, and solving for ω_{AB} as follows will yield to:

$$\begin{aligned}
N_A &= N_B \\
k_A \left(v_A - \omega_{AB} \right) t &= k_B \left(v_B - \omega_{AB} \right) t \\
k_A \, v_A \, t - k_A \, \omega_{AB} \, t &= k_B \, v_B \, t - k_B \omega_{AB} \, t \\
k_A \omega_{AB} \, t - k_B \omega_{AB} \, t &= k_A \, v_A - k_B \, v_B \\
\omega_{AB} \left(k_A - k_B \right) &= q_A - q_B
\end{aligned}$$

This will yield the following expression for the shock-wave speed.

$$\omega_{AB} = \frac{q_A - q_B}{k_A - k_B} \tag{17}$$

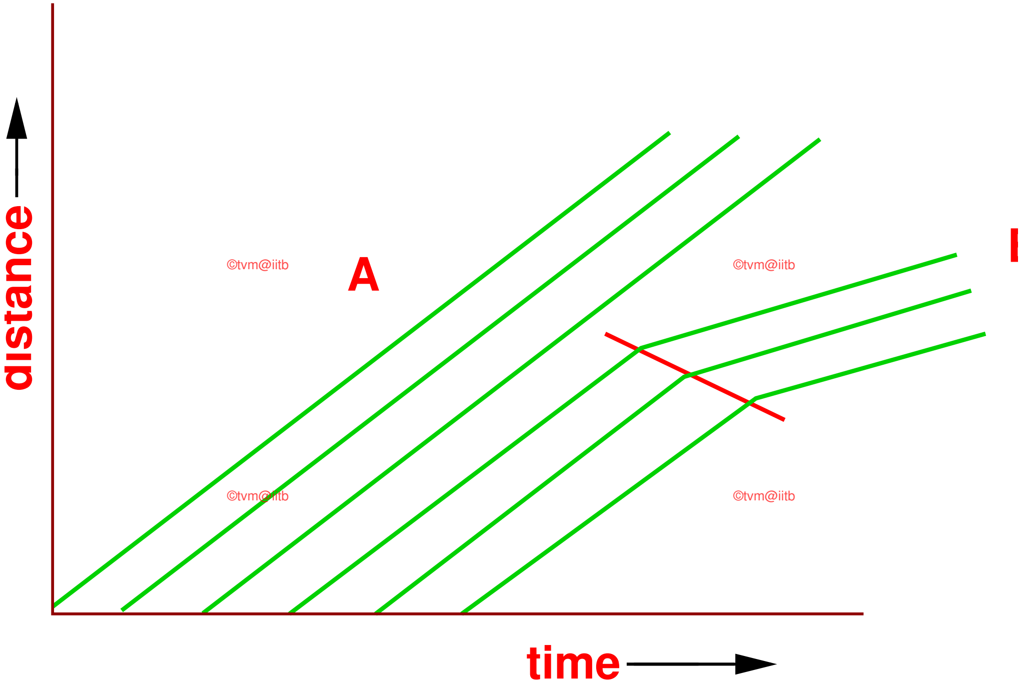

Figure 8: Shock wave : time-distance diagram

In this case, the shock wave move against the direction of traffic and is therefore called a backward moving shock wave. There are other possibilities of shock waves such as forward moving shock waves and stationary shock waves. The forward moving shock waves are formed when a stream with higher density and higher flow meets a stream with relatively lesser density and flow. For example, when the width of the road increases suddenly, there are chances for a forward moving shock wave. Stationary shock waves will occur when two streams having the same flow value but different densities meet. traffic parameters. These models were based on many assumptions, for instance, Greenshield's model assumed a linear speed-density relationship. Other models were also discussed in this chapter. The models are used for explaining several phenomena in connection with traffic flow like shock wave. The topics of further interest are multi-regime model (formulation of both two and three regime models) and three dimensional representation of these models.

Exercises

1. Illustrate neatly on a single graph the speed-density relation by Greenberg, Greenshield, Underwood, Pipe(n=0.5,2), two regime, and three regime models, along with typical field observations

2. Explain with neat sketch the need and examples of multi-regime stream models.

3. Sketch the three fundamental diagrams of traffic flow. Derive the relation between maximum flow (q_{max}), jam density (k_j), and free flow speed (u_f). Assume liner speed flow relation: $u = u_f \left(1 - \frac{k}{k_j}\right)$.

4. Plot typical speed-density field data points. Draw the shapes of various traffic stream models (5-7) including multi-regime models. Write the equations of these models as well.

5. Ilustrate a three regime speed-density relation considering low, medium, and high flow traffic states.

6. In a traffic study, the observed densities were 150, 120, 50, 70 and 20 veh/km and the corresponding speeds were 10, 25, 45, 40 and 32km/h. Find the jam density according to Greenberg's logarithmic traffic stream model. (Hint: Linearize the expression)

7. For the following data on speed and concentration, determine the parameters of Greenshields' model. Find the concentration corresponding to a speed of 40 kmph. Find also the maximum flow.

Concentration(veh/km)	Speed(kmph)
180	4
140	20
30	50
75	35

8. A study of flow at a particular location resulted in a calibrated speed-density relationship as follows. $v = 52.5\,(1 - 0.35\,k)$. For this relationship, determine free flow speed, jam density, maximum flow, and the relationship between fundamental parameters of traffic. (Illustrate with a sketch)

9. If the mean speeds in kmph observed from a road stretch at various time is given as: 10, 25, 45, 40, and 50, and the corresponding densities in veh/km are: 150, 120, 50, 70, and 20. What would be the maximum flow on this road stretch.

10. The speed density relationship of traffic on a section of highway was estimated to be $u_x = 18.2 \log \frac{220}{k}$. (i) What is the maximum flow, speed and density at this flow? (ii) What is the jam density?

11. Determine the parameters of Greenshields model for the following data. Find the maximum flow and density for a speed of 45 kmph.

Speed (kmph)	Density (veh/km)
5	150
20	120
30	100
40	70

12. A study of flow at a particular location resulted in a calibrated speed-density relationship as follows. $v = 47.5\ (1 - 0.32\ k)$ For this relationship, determine free flow speed, jam density, maximum flow, speed-flow relationship, and flow-density relationship. (Illustrate with a sketch)

13. In a traffic study experiment, density values are obtained as 160, 120, 40, and 72 veh/km corresponding to speed values of 3, 18, 55, 32 respectively. Determine the parameters of Greenshields' model. Find the density corresponding to a speed of 40 kmph. Find also the maximum flow.

14. The following speed and density is observed from a road section. If we assume the speed decreases linearly with respect to density, then: (a) what will be the density at a speed of 10 kmph, and (b) what will be the maximum flow across the section

Speed (kmph)	Density (veh/km)
5	120
20	90
30	40
40	10

15. The speed and density observed from a road is given below. What is the density and flow corresponding to a speed of 25 kmph. State the assumptions/model used in the computation.

Speed (kmph)	Density (veh/km)
10	200
20	170
30	120
40	100

References

1. Adolf D. May. *Fundamentals of Traffic Flow*. Prentice - Hall, Inc. Englewood Cliff New Jersey 07632, second edition, 1990.

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