

Car Following Models

Lecture Notes in Transportation Systems Engineering

@tvm@iitb

Prof. Tom V. Mathew

@tvm@iitb

Contents

1 Overview-1	1
2 Car following models	1
2.1 Notation	1
2.2 Pipe's model	2
2.3 Forbes' model	2
2.4 General Motors' model	3
2.5 Optimal velocity model	3
3 General motor's car following model	4
3.1 Basic Philosophy	4
3.2 Follow-the-leader model	4
4 Derivation of Greenberg's formula	9
5 Summary	11
Exercises	11
References	15
Acknowledgments	15

1 Overview-1

Longitudinal spacing of vehicles are of particular importance from the points of view of safety, capacity and level of service. The longitudinal space occupied by a vehicle depend on the physical dimensions of the vehicles as well as the gaps between vehicles. For measuring this longitudinal space, two microscopic measures are used- distance

headway and distance gap. Distance headway is defined as the distance from a selected point (usually front bumper) on the lead vehicle to the corresponding point on the following vehicles. Hence, it includes the length of the lead vehicle and the gap length between the lead and the following vehicles.

2 Car following models

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Car following theories describe how one vehicle follows another vehicle in an uninterrupted flow. Various models were formulated to represent how a driver reacts to the changes in the relative positions of the vehicle ahead. Models like Pipes, Forbes, General Motors and Optimal velocity model are worth discussing.

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2.1 Notation

Before going in to the details, various notations used in car-following models are discussed here with the help of figure 1. The leader vehicle is denoted as n and the following vehicle as $(n + 1)$. Two characteristics at an instant t are of importance; location and speed. Location and speed of the lead vehicle at time instant t are represented by x_n^t and v_n^t respectively. Similarly, the location and speed of the follower are denoted by x_{n+1}^t and v_{n+1}^t respectively. The following vehicle is assumed to accelerate at time $t + \Delta T$ and not at t , where ΔT is the interval of time required for a driver to react to a changing situation. The gap between the leader and the follower vehicle is therefore $x_n^t - x_{n+1}^t$.

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2.2 Pipe’s model

The basic assumption of this model is “A good rule for following another vehicle at a safe distance is to allow yourself at least the length of a car between your vehicle and the vehicle ahead for every ten miles per hour of speed at which you are traveling” According to Pipe’s car-following model, the minimum safe distance headway increases linearly with speed. A disadvantage of this model is that at low speeds, the minimum

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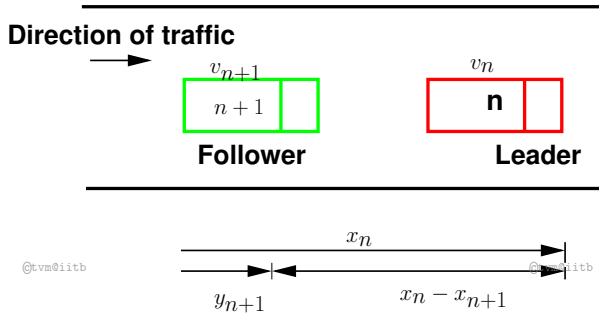


Figure 1: Notation for car following model

headways proposed by the theory are considerably less than the corresponding field measurements.

2.3 Forbes' model

In this model, the reaction time needed for the following vehicle to perceive the need to decelerate and apply the brakes is considered. That is, the time gap between the rear of the leader and the front of the follower should always be equal to or greater than the reaction time. Therefore, the minimum time headway is equal to the reaction time (minimum time gap) and the time required for the lead vehicle to traverse a distance equivalent to its length. A disadvantage of this model is that, similar to Pipe's model, there is a wide difference in the minimum distance headway at low and high speeds.

2.4 General Motors' model

The General Motors' model is the most popular of the car-following theories because of the following reasons:

1. Agreement with field data; the simulation models developed based on General motors' car following models shows good correlation to the field data.
2. Mathematical relation to macroscopic model; Greenberg's logarithmic model for speed-density relationship can be derived from General motors car following model.

In car following models, the motion of individual vehicle is governed by an equation, which is analogous to the Newton's Laws of motion. In Newtonian mechanics, accel-

eration can be regarded as the response of the particle to *stimulus* it receives in the form of force which includes both the external force as well as those arising from the interaction with all other particles in the system. This model is the widely used and will be discussed in detail later.

2.5 Optimal velocity model

The concept of this model is that each driver tries to achieve an optimal velocity based on the distance to the preceding vehicle and the speed difference between the vehicles. This was an alternative possibility explored recently in car-following models. The formulation is based on the assumption that the desired speed $v_{n_{desired}}$ depends on the distance headway of the n th vehicle. i.e. $v_{n_{desired}}^t = v^{opt}(\Delta x_n^t)$ where v^{opt} is the optimal velocity function which is a function of the instantaneous distance headway Δx_n^t . Therefore a_n^t is given by

$$a_n^t = [1/\tau][V^{opt}(\Delta x_n^t) - v_n^t] \tag{1}$$

where $\frac{1}{\tau}$ is called as sensitivity coefficient. In short, the driving strategy of n^{th} vehicle is that, it tries to maintain a safe speed which in turn depends on the relative position, rather than relative speed.

3 General motor's car following model

3.1 Basic Philosophy

The basic philosophy of car following model is from Newtonian mechanics, where the acceleration may be regarded as the response of a matter to the stimulus it receives in the form of the force it receives from the interaction with other particles in the system. Hence, the basic philosophy of car-following theories can be summarized by the following equation

$$[Response]_n \propto [Stimulus]_n \tag{2}$$

for the n th vehicle ($n=1, 2, \dots$). Each driver can respond to the surrounding traffic conditions only by accelerating or decelerating the vehicle. As mentioned earlier, different theories on car-following have arisen because of the difference in views regarding the

nature of the stimulus. The stimulus may be composed of the speed of the vehicle, relative speeds, distance headway etc, and hence, it is not a single variable, but a function and can be represented as,

$$a_n^t = f_{sti}(v_n, \Delta x_n, \Delta v_n) \quad (3)$$

where f_{sti} is the stimulus function that depends on the speed of the current vehicle, relative position and speed with the front vehicle.

3.2 Follow-the-leader model

The car following model proposed by General motors is based on follow-the leader concept. This is based on two assumptions; (a) higher the speed of the vehicle, higher will be the spacing between the vehicles and (b) to avoid collision, driver must maintain a safe distance with the vehicle ahead.

Let Δx_{n+1}^t is the gap available for $(n + 1)^{th}$ vehicle, and let Δx_{safe} is the safe distance, v_{n+1}^t and v_n^t are the velocities, the gap required is given by,

$$\Delta x_{n+1}^t = \Delta x_{safe} + \tau v_{n+1}^t \quad (4)$$

where τ is a sensitivity coefficient. The above equation can be written as

$$x_n - x_{n+1}^t = \Delta x_{safe} + \tau v_{n+1}^t \quad (5)$$

Differentiating the above equation with respect to time, we get

$$\begin{aligned} v_n^t - v_{n+1}^t &= \tau a_{n+1}^t \\ a_{n+1}^t &= \frac{1}{\tau} [v_n^t - v_{n+1}^t] \end{aligned}$$

General Motors has proposed various forms of sensitivity coefficient term resulting in five generations of models. The most general model has the form,

$$a_{n+1}^t = \left[\frac{\alpha_{l,m} (v_{n+1}^t)^m}{(x_n^t - x_{n+1}^t)^l} \right] [v_n^t - v_{n+1}^t] \quad (6)$$

where l is a distance headway exponent and can take values from +4 to -1, m is a speed exponent and can take values from -2 to +2, and α is a sensitivity coefficient. These parameters are to be calibrated using field data. This equation is the core of traffic simulation models.

In computer, implementation of the simulation models, three things need to be remembered:

1. A driver will react to the change in speed of the front vehicle after a time gap called the reaction time during which the follower perceives the change in speed and react to it.
2. The vehicle position, speed and acceleration will be updated at certain time intervals depending on the accuracy required. Lower the time interval, higher the accuracy.
3. Vehicle position and speed is governed by Newton's laws of motion, and the acceleration is governed by the car following model.

Therefore, the governing equations of a traffic flow can be developed as below. Let ΔT is the reaction time, and Δt is the updation time, the governing equations can be written as,

$$v_n^t = v_n^{t-\delta t} + a_n^{t-\delta t} \times \delta t \quad (7)$$

$$x_n^t = x_n^{t-\delta t} + v_n^{t-\delta t} \times \delta t + \frac{1}{2} a_n^{t-\delta t} \delta t^2 \quad (8)$$

$$a_{n+1}^t = \left[\frac{\alpha_{l,m} (v_{n+1}^{t-\delta t})^m}{(x_n^{t-\Delta T} - x_{n+1}^{t-\Delta T})^l} \right] (v_n^{t-\Delta T} - v_{n+1}^{t-\Delta T}) \quad (9)$$

The equation 7 is a simulation version of the Newton's simple law of motion $v = u + at$ and equation 8 is the simulation version of the Newton's another equation $s = ut + \frac{1}{2}at^2$. The acceleration of the follower vehicle depends upon the relative velocity of the leader and the follower vehicle, sensitivity coefficient and the gap between the vehicles.

3.2.1 Numerical Example

Let a leader vehicle is moving with zero acceleration for two seconds from time zero. Then he accelerates by 1 m/s^2 for 2 seconds, then decelerates by 1 m/s^2 for 2 seconds. The initial speed is 16 m/s and initial location is 28 m from datum. A vehicle is following this vehicle with initial speed 16 m/s, and position zero. Simulate the behavior of the following vehicle using General Motors' Car following model (acceleration, speed and position) for 7.5 seconds. Assume the parameters $l=1$, $m=0$, sensitivity coefficient $(\alpha_{l,m}) = 13$, reaction time as 1 second and scan interval as 0.5 seconds.

Solution The first column shows the time in seconds. Column 2, 3, and 4 shows the acceleration, velocity and distance of the leader vehicle. Column 5,6, and 7 shows the acceleration, velocity and distance of the follower vehicle. Column 8 gives the difference in velocities between the leader and follower vehicle denoted as dv . Column 9 gives the difference in displacement between the leader and follower vehicle denoted as dx . Note that the values are assumed to be the state at the beginning of that time interval. At time $t=0$, leader vehicle has a velocity of 16 m/s and located at a distance of 28 m from a datum. The follower vehicle is also having the same velocity of 16 m/s and located at the datum. Since the velocity is same for both, $dv = 0$. At time $t = 0$, the leader vehicle is having acceleration zero, and hence has the same speed. The location of the leader vehicle can be found out from equation as, $x = 28 + 16 \times 0.5 = 36$ m. Similarly, the follower vehicle is not accelerating and is maintaining the same speed. The location of the follower vehicle is, $x = 0 + 16 \times 0.5 = 8$ m. Therefore, $dx = 36 - 8 = 28$ m. These steps are repeated till $t = 1.5$ seconds. At time $t = 2$ seconds, leader vehicle accelerates at the rate of 1 m/s^2 and continues to accelerate for 2 seconds. After that it decelerates for a period of two seconds. At $t = 2.5$ seconds, velocity of leader vehicle changes to 16.5 m/s. Thus dv becomes 0.5 m/s at 2.5 seconds. dx also changes since the position of leader changes. Since the reaction time is 1 second, the follower will react to the leader's change in acceleration at 2.0 seconds only after 3 seconds. Therefore, at $t=3.5$ seconds, the follower responds to the leader's change in acceleration given by equation i.e., $a = \frac{13 \times 0.5}{28.23} = 0.23 \text{ m/s}^2$. That is the current acceleration of the follower vehicle depends on dv and reaction time Δ of 1 second. The follower will change the speed at the next time interval. i.e., at time $t = 4$ seconds. The speed of the follower vehicle at $t = 4$ seconds is given by equation as $v = 16 + 0.231 \times 0.5 = 16.12$ The location of the follower vehicle at $t = 4$ seconds is given by equation as $x = 56 + 16 \times 0.5 + \frac{1}{2} \times 0.231 \times 0.5^2 = 64.03$ These steps are followed for all the cells of the table.

The earliest car-following models considered the difference in speeds between the leader and the follower as the stimulus. It was assumed that every driver tends to move with the same speed as that of the corresponding leading vehicle so that

$$a_n^t = \frac{1}{\tau} (v_n^{t+1} - v_{n+1}^t) \quad (10)$$

where τ is a parameter that sets the time scale of the model and $\frac{1}{\tau}$ can be considered

Table 1: Car-following example

t (1)	$a(t)$ (2)	$v(t)$ (3)	$x(t)$ (4)	$a(t)$ (5)	$v(t)$ (6)	$x(t)$ (7)	dv (8)	dx (9)
t	$a(t)$	$v(t)$	$x(t)$	$a(t)$	$v(t)$	$x(t)$	dv	dx
0.00	0.00	16.00	28.00	0.00	16.00	0.00	0.00	28.00
0.50	0.00	16.00	36.00	0.00	16.00	8.00	0.00	28.00
1.00	0.00	16.00	44.00	0.00	16.00	16.00	0.00	28.00
1.50	0.00	16.00	52.00	0.00	16.00	24.00	0.00	28.00
2.00	1.00	16.00	60.00	0.00	16.00	32.00	0.00	28.00
2.50	1.00	16.50	68.13	0.00	16.00	40.00	0.50	28.13
3.00	1.00	17.00	76.50	0.00	16.00	48.00	1.00	28.50
3.50	1.00	17.50	85.13	0.23	16.00	56.00	1.50	29.13
4.00	-1.00	18.00	94.00	0.46	16.12	64.03	1.88	29.97
4.50	-1.00	17.50	102.88	0.67	16.34	72.14	1.16	30.73
5.00	-1.00	17.00	111.50	0.82	16.68	80.40	0.32	31.10
5.50	-1.00	16.50	119.88	0.49	17.09	88.84	-0.59	31.03
6.00	0.00	16.00	128.00	0.13	17.33	97.45	-1.33	30.55
6.50	0.00	16.00	136.00	-0.25	17.40	106.13	-1.40	29.87
7.00	0.00	16.00	144.00	-0.57	17.28	114.80	-1.28	29.20
7.50	0.00	16.00	152.00	-0.61	16.99	123.36	-0.99	28.64
8.00	0.00	16.00	160.00	-0.57	16.69	131.78	-0.69	28.22
8.50	0.00	16.00	168.00	-0.45	16.40	140.06	-0.40	27.94
9.00	0.00	16.00	176.00	-0.32	16.18	148.20	-0.18	27.80
9.50	0.00	16.00	184.00	-0.19	16.02	156.25	-0.02	27.75
10.00	0.00	16.00	192.00	-0.08	15.93	164.24	0.07	27.76
10.50	0.00	16.00	200.00	-0.01	15.88	172.19	0.12	27.81
11.00	0.00	16.00	208.00	0.03	15.88	180.13	0.12	27.87
11.50	0.00	16.00	216.00	0.05	15.90	188.08	0.10	27.92
12.00	0.00	16.00	224.00	0.06	15.92	196.03	0.08	27.97
12.50	0.00	16.00	232.00	0.05	15.95	204.00	0.05	28.00
13.00	0.00	16.00	240.00	0.04	15.98	211.98	0.02	28.02
13.50	0.00	16.00	248.00	0.02	15.99	219.98	0.01	28.02
14.00	0.00	16.00	256.00	0.01	16.00	227.98	0.00	28.02
14.50	0.00	16.00	264.00	0.00	16.01	235.98	-0.01	28.02
15.00	0.00	16.00	272.00	0.00	16.01	243.98	-0.01	28.02
15.50	0.00	16.00	280.00	0.00	16.01	251.99	-0.01	28.01

16.00	0.00	16.00	288.00	0.00	16.01	260.00	-0.01	28.00
16.50	0.00	16.00	296.00	0.00	16.01	268.00	-0.01	28.00

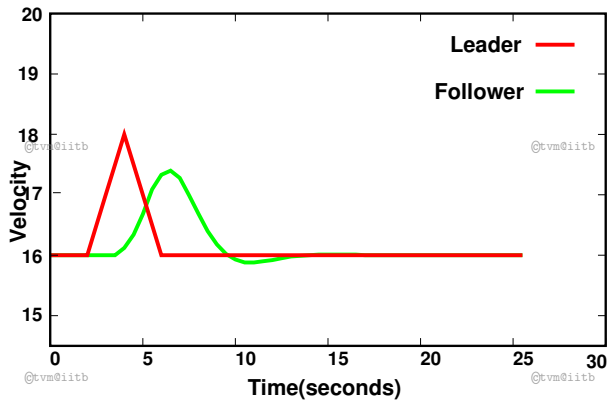


Figure 2: Velocity v_z Time

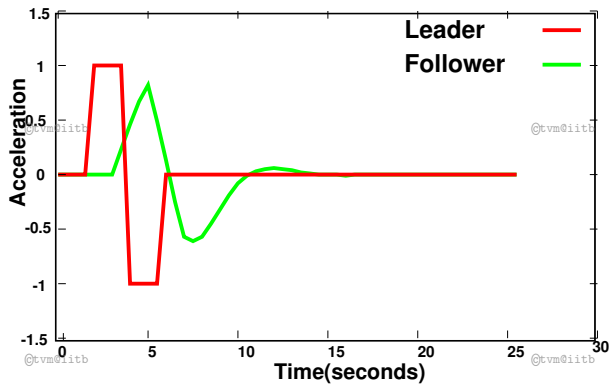


Figure 3: Acceleration v_z Time

as a measure of the sensitivity of the driver. According to such models, the driving strategy is to follow the leader and, therefore, such car-following models are collectively referred to as the follow the leader model. Efforts to develop this stimulus function led to five generations of car-following models, and the most general model is expressed mathematically as follows.

$$a_{n+1}^t = \frac{\alpha_{l,m} [v_{n+1}^{t-\delta t}]^m}{[x_n^{t-\Delta T} - x_{n+1}^{t-\Delta T}]^l} (v_n^{t-\Delta T} - v_{n+1}^{t-\Delta T}) \quad (11)$$

where l is a distance headway exponent and can take values from +4 to -1, m is a speed exponent and can take values from -2 to +2, and α is a sensitivity coefficient. These parameters are to be calibrated using field data.

4 Derivation of Greenberg's formula

The Greenberg's stream model can be derived from General Motors car following model. The GM model is given as:

$$a_n^t = \frac{\alpha_{l,m} [v_{n+1}^{t-\delta t}]^m}{[x_n^{t-\Delta T} - x_{n+1}^{t-\Delta T}]^l} (v_n^{t-\Delta T} - v_{n+1}^{t-\Delta T}) \quad (12)$$

Putting $l = 1$ and $m = 0$ the model becomes:

$$a_n^t = \frac{\alpha_{l,m} (v_n^{t-\Delta T} - v_{n+1}^{t-\Delta T})}{(x_n^{t-\Delta T} - x_{n+1}^{t-\Delta T})} \quad (13)$$

At steady state condition, the equation becomes:

$$a_n = \frac{\alpha (v_{n-1} - v_n)}{(x_{n-1} - x_n)} \quad (14)$$

Now integrating both sides with respect to t we get:

$$\int a_n dt = \int \frac{\alpha (v_{n-1} - v_n)}{(x_{n-1} - x_n)} dt \quad (15)$$

Now, put $u = x_{n-1} - x_n$, then

$$\frac{du}{dt} = v_{n-1} - v_n, \text{ or } du = (v_{n-1} - v_n) dt. \quad (16)$$

Therefore,

$$\therefore \int a_n dt = \alpha \int \frac{du}{u}. \quad (17)$$

$$\therefore v_n = \alpha \log u + c \quad (18)$$

$$= \alpha \log (x_{n-1} - x_n) + c \quad (19)$$

$$= \alpha \log s_n + c. \quad (20)$$

where, s_n is the spacing. Now, under steady state conditions, there is no identity for vehicles. Hence, the above expression will be as follows:

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$$v = \alpha \log s + c \quad (21)$$

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$$= \alpha \log \frac{1}{k} + c. \quad (22)$$

To find the constant c , apply the boundary condition, namely, at jam density, the speed is zero. That is:

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$$0 = \alpha \log \frac{1}{k_j} + c$$

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$$\therefore c = -\alpha \log \frac{1}{k_j}$$

$$\therefore v = \alpha \log \frac{1}{k} - \alpha \log \frac{1}{k_j}$$

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$$= \alpha \log \frac{1}{k} + \alpha \log k_j$$

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$$v = \alpha \log \frac{k_j}{k}. \quad (23)$$

Now, to find α we do the following:

$$q = kv = \alpha k \log \frac{k_j}{k} \quad (24)$$

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Differentiating with respect to density and equating to zero:

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$$\frac{dq}{dk} = \alpha k \left[\frac{d}{dk} \log \frac{k_j}{k} \right] + \alpha \left(\log \frac{k_j}{k} \right)$$

$$= \alpha k \left[\frac{d}{dk} \log k_j - \frac{d}{dk} \log k \right] + \alpha \left(\log \frac{k_j}{k} \right)$$

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$$= \alpha k \left[-\frac{1}{k} \right] + \alpha \left(\log \frac{k_j}{k} \right)$$

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$$= \alpha \log \frac{k_j}{k} - \alpha = 0.$$

$$\therefore \log \frac{k_j}{k} = 1,$$

$$\text{or } \frac{k_j}{k} = e, \text{ or, } k_0 = \frac{k_j}{e}. \quad (25)$$

Noted that we denote k corresponding to the maximum flow as k_0 . Now, substituting the above result, we get the value of alpha as

$$v_0 = \alpha \log e = \alpha. \quad (26)$$

Now the Greenberg's stream model is:

$$v = v_0 \log \frac{k_j}{k} \quad (27)$$

5 Summary

Microscopic traffic flow modeling focuses on the minute aspects of traffic stream like vehicle to vehicle interaction and individual vehicle behavior. They help to analyze very small changes in the traffic stream over time and space. Car following model is one such model where in the stimulus-response concept is employed. Optimal models and simulation models were briefly discussed.

Exercises

1. Discuss the concepts and model formulations of Generalised GM model, Gipps' model, and Wiedemann 74 car-following models.
2. Discuss the evolution of the five generations of General Motors car following model highlighting the drawback and advantages of each generation of the model.
3. A line of vehicles are in car following mode and all vehicles are travelling at 18 m/s with distance headway of 20 m. After 1.2 seconds, the lead vehicle suddenly decelerates at a rate of 1.2 m/s^2 until it stops completely. simulate the behaviour of first following vehicle using the GM fifth car following model for the first 2.5 seconds. Tabulate the results. Assume headway exponent 1.2, speed exponent 1.6, sensitivity coefficient 0.8, reaction time 0.6 seconds, and scan interval 0.3 seconds.
4. Simulate the following vehicle behaviour for the following data using Wiedemann 74 model. (a) For the case of stand still distance 3.5m, additive part of safety distance 1.5, and multiplicative part of safety distance 0.8. (b) For the case of stand still distance 3.5m, additive part of safety distance 1.5, and multiplicative part of safety distance 0.8. Comment on the following vehicle behaviour for the above two cases.

5. A car is travelling with a speed of 16 m/sec at time $t=0$. Another car follows the first at a distance of 28 m with same velocity. If the first car accelerated by 1 m/sec^2 from $t=1$ to 2 and decelerate by 1 m/sec^2 from $t=2$ to 3, find the speed, acceleration and spacing of the follower at time $t=3.0$ sec. Assume the reaction time is 1 sec, vehicle dynamics are updated every 0.5 seconds, and the car following model is given by Eq. 28. (Use of a tabular form is encouraged).

$$a_{n+1}(t) = 15 \times \left[\frac{u_n(t-1) - u_{n+1}(t-1)}{x_n(t-1) - x_{n+1}(t-1)} \right] \quad (28)$$

6. In a simulation experiment on a single lane road, one vehicle is travelling at 18 m/s. After 1.5 seconds, the vehicle suddenly accelerates at a rate of 1.5 m/s^2 for the next 1.8 seconds. Simulate the behaviour of subsequent vehicle with an initial speed of 16 m/s using GM fifth car following model for the first 3 seconds if the initial distance headway is 20 m. Tabulate the results. Assume headway exponent 1.2, speed exponent 1.5, sensitivity coefficient 0.8, reaction time 0.6 seconds, and update interval of 0.3 seconds.
7. A line of vehicles are in car following mode and all vehicles are travelling at 15 m/s with distance headway of 20 m. After 1.2 seconds, the lead vehicle suddenly decelerates at a rate of 1.2 m/s^2 until it stops completely. simulate the behaviour of first following vehicle using the GM fifth car following model for the first 2.5 seconds. Tabulate the results. Assume headway exponent 1.2, speed exponent 1.6, sensitivity coefficient 0.6, reaction time 0.6 seconds, and scan interval 0.3 seconds.
8. In a simulation experiment on a single lane road, one vehicle is travelling at 16 m/s. After 0.6 seconds, the vehicle suddenly accelerates at a rate of 1.2 m/s^2 for the next 0.9 seconds. Simulate the behaviour of subsequent vehicle with an initial speed of 16 m/s using GM fifth car following model for the first 2.1 seconds if the initial distance headway is 25 m. Tabulate the results. Assume headway exponent 1.2, speed exponent 1.4, sensitivity coefficient 0.6, reaction time 0.6 seconds, and update interval of 0.3 seconds.
9. A line of vehicles are in car following mode and all vehicles are travelling at 15 m/s with distance headway of 25 m. After 1 second, the lead vehicle suddenly decelerates at a rate of 1.2 m/s^2 until it stops completely. Simulate the behaviour of first following vehicle using the GM fifth car following model for the first 3 seconds. Tabulate the results. Assume headway exponent 1.0, speed exponent 1.5, sensitivity coefficient 0.5, reaction time 0.5 seconds, and scan interval 0.25 seconds.

Program Codes

GMV CF Model

Instructions

Modify the input file and use. To compile in linux use `gcc prog.c -lm -o prog.exe` and to run `.\prog.exe`.

Input File

```
GMV_Sensitivity_Coefficient_a 26.0
GMV_Distance_Exponent_____l 2.0
GMV_Speed_Exponent_____m 1.0
Reaction_Time_____DT 1.0
Update_Interval_____dt 0.5
Initial_Speed_Leader_____vl 16.0
Initial_Position_Leader____xl 28.0
Initial_Speed_Follower_____vf 16.0
Initial_Position_Follower__xf 0.0
No_of_Update_Intervals_____n 50
Leader_Accelration_Values_for_each_n_update_intervals_below
0.0 0.0 0.0 0.0 1.0 1.0 1.0 1.0 -1.0 -1.0
-1.0 -1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
```

C-code

```
#include<math.h>
#include<stdio.h>
#include<stdlib.h>
#define MAX_DUR 1000
int main()
{
    char text[10][100];
    int curTime=0; /* current time in seconds */
    int maxTime=0; /* maximum simulation time in seconds */
    int DelT=1; /* reaction time in no of update intervals = DT/dt */
    float delX,delV; /* relative speed and relative distance */
    float DT=0.0; /* reaction time in seconds */
    float dt=0.0; /* updated interval in seconds */
    float gm5_a=0.0;
    float gm5_l=0.0;
    float gm5_m=0.0;
    float vFol[MAX_DUR];
    float xFol[MAX_DUR];
    float aFol[MAX_DUR];
```

```

float vLdr[MAX_DUR];
float xLdr[MAX_DUR];
float aLdr[MAX_DUR];
float clockTime[MAX_DUR]={0.0}; /* simulaion clock time in seconds */
FILE* finp=fopen("input.dat","r");

/* Read all input data from the file */
fscanf(finp,"%s %f",text[0],&gm5_a);
fscanf(finp,"%s %f",text[1],&gm5_l);
fscanf(finp,"%s %f",text[2],&gm5_m);
fscanf(finp,"%s %f",text[3],&DT);
fscanf(finp,"%s %f",text[4],&dt);
fscanf(finp,"%s %f",text[5],&vLdr[0]);
fscanf(finp,"%s %f",text[6],&xLdr[0]);
fscanf(finp,"%s %f",text[7],&vFol[0]);
fscanf(finp,"%s %f",text[8],&xFol[0]);
fscanf(finp,"%s %d",text[9],&maxTime);
fscanf(finp,"%s",text[11]);
for(curTime=0;curTime<maxTime;curTime++)
    fscanf(finp,"%f",&aLdr[curTime]);

/* Compuation of the states of leader and follower */
for(curTime=1;curTime<maxTime;curTime++)
{
    clockTime[curTime]=clockTime[curTime-1]+dt;
    vLdr[curTime]=vLdr[curTime-1]+aLdr[curTime-1]*dt;
    xLdr[curTime]=xLdr[curTime-1]+vLdr[curTime-1]*dt+0.5*aLdr[curTime-1]*dt*dt;
    if(clockTime[curTime]>DT) DelT=(int)(DT/dt);
    delX=xLdr[curTime-DelT]-xFol[curTime-DelT];
    delV=vLdr[curTime-DelT]-vFol[curTime-DelT];
    aFol[curTime]=gm5_a*pow(vFol[curTime-1],gm5_m)/pow(delX,gm5_l)*delV;
    vFol[curTime]=vFol[curTime-1]+aFol[curTime-1]*dt;
    xFol[curTime]=xFol[curTime-1]+vFol[curTime-1]*dt+0.5*aFol[curTime-1]*dt*dt;
}

/* Printing all results */
printf("%s\t%.1f",text[0],gm5_a);
printf("\n%s\t%.1f",text[1],gm5_l);
printf("\n%s\t%.1f",text[2],gm5_m);
printf("\n%s\t%.1f",text[3],DT);
printf("\n%s\t%.2f",text[4],dt);
printf("\n%s\t%.2f",text[5],vFol[0]);
printf("\n%s\t%.2f",text[6],xFol[0]);
printf("\n%s\t%.2f",text[7],vLdr[0]);
printf("\n%s\t%.2f",text[8],xLdr[0]);

```

```

printf("\n%s\t%d",text[9],maxTime);
printf("\nNo\tTime\taLdr\tvLdr\txLdr\taFol\tvFol\txFol");
for(curTime=0;curTime<maxTime;curTime++)
{
    printf("\n%d\t%.2f\t%f\t%f\t%f\t%f\t%f\t%f",\
        curTime+1,clockTime[curTime],aLdr[curTime],vLdr[curTime],\
        xLdr[curTime],aFol[curTime],vFol[curTime],xFol[curTime]);
}
return 0;
}
/* End of File */

```

References

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Prof. Tom V. Mathew

Department of Civil Engineering
Indian Institute of Technology Bombay, India