

Transport Network Design

Lecture Notes in Transportation Systems Engineering

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1 Introduction

This document discusses the aspects of network design. The reader assumed to be familiar with the basics of various types of traffic assignment techniques especailly user equilibrium and system optimum including the mathematical formulation and solution

approaches. Then the concept of bilevel programming and few examples will be presented. Finally one such example, namely the network capacity expansion will be formulated as a bilevel optimization problem and will be illustrated using a numerical example.

Transportation network design in a broad sense deals with the configuration of network to achieve specified objectives. There are two variations to the problem, the continuous network design and the discrete network design. Examples of the form include

- a The determination of road width.
- b The calculation of signal timings.
- c The setting of road user charges.

Although this document covers the continuous network design in detailed, basic underlying principles are some from the discrete case. Conventional network design has been concerned with minimization of total system cost. However, this may be unrealistic in the sense that how the user will respond to the proposed changes is not considered. Therefore, currently the network design is thought of as supply demand problem or leader-follower game. The system designer leads, taking into account how the user follows. The core of all network design problems is how a user chooses his route of travel. The class of traffic assignment problem tries to model these behaviours.

2 Bilevel

The bilevel programming (BLP) problem is a special case of multilevel programming problems with a two level structure. The problem can be expressed as follows: the transport planner, wishes to determine an optimal policy as a function of his or control variables (y) and the users response to these controls, where users response generally takes the form of a network flow (x). The transport planner then seeks to minimise a function of both y and x , where some constraints may be imposed upon y as well as the fact that x should be a user equilibrium flow, parameterised by the control vector, y . There exists many problems in transportation that can be formulated as bilevel

programming problem. They include network capacity expansion, network level signal setting and optimum toll pricing. They are discussed briefly here:

3 Examples of Bilevel

3.1 Network Capacity Expansion

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The network capacity expansion problem is to determine capacity enhancements of existing facilities of a transportation network which are, in some sense, optimal. Network design models concerned with adding indivisible facilities (modeled as integer variables) are said to be *discrete*, whereas those dealing with divisible capacity enhancements (modeled as continuous variables) are said to be *continuous*. Thus network expansion problem is a continuous network design problem, which determines the set of link capacity expansions and the corresponding equilibrium flows for which measures of performance index for network is optimal. A bilevel programming technique can be used to formulate this equilibrium network design problem. At the upper level problem, the system performance index is defined as the sum of total travel times and investment costs of link capacity expansions. At the lower level problem, the user equilibrium flow is determined by Wardrop's first principle and can be formulated as an equivalent minimization problem. The most well-studied equilibrium network design problem is user equilibrium network design with fixed transportation demand.

3.2 Combined traffic assignment and signal control

For a road network with flow responsive signal control and fixed origin-destination travel demands. Combined traffic assignment and signal control problem tries to allocate the demand matrix to the network subject to user equilibrium assumption and computes the optimal signal control parameter from the generated link flows. Consider $\mathbf{f}$ and $\mathbf{g}$, which denote respectively, a vector of link flows and a vector of signal settings for the network; assuming that the signal plan structure is given (specified by number, type, and sequence of phases), signal settings may consist of cycle length's, green splits, and offsets. Traffic equilibrium, $\mathbf{x}^*$ is a set of link flows satisfying Wardrop's first principle.

The equilibrium traffic signal setting is a pair $(\mathbf{x}^*, \mathbf{s}^*)$ such that $\mathbf{x}^*$ is a traffic equilibrium when signals are set at $\mathbf{s}^*$.

$$\mathbf{x}^* = \mathbf{f}^e(\mathbf{s}^*) \quad (1)$$

where $\mathbf{s}^*$ is the signal settings corresponding to $\mathbf{x}^*$ under specified control policy P ;

$$\mathbf{s}^* = \mathbf{g}^P(\mathbf{x}^*) \quad (2)$$

If there exists a pair $(\mathbf{x}^*, \mathbf{s}^*)$, then link flows and signal settings are

$$\mathbf{x}^* = \mathbf{f}^e[\mathbf{g}^P(\mathbf{x}^*)] \quad \text{or} \quad \mathbf{g}^* = \mathbf{g}^P[\mathbf{f}^e(\mathbf{s}^*)] \quad (3)$$

3.3 Optimising Toll

In general, traffic flow and queue size on a road network depend on road toll pattern and also traffic control. An efficient pricing scheme should therefore take into account the effects of the altered network flow pattern and queueing due to road pricing to achieve a global optimal solution. This requires development of an efficient procedure for calculating optimal toll patterns in general road networks while anticipating driver response in terms of route choice. The procedure should be able to estimate queueing delay and queue length, both of which are critical in queue management in congested urban road networks. Optimum toll pricing problem can be formulated as a bilevel programming in general road networks. The users route choice behaviour under condition of queing and congestion in a road network for any given toll pattern can be represented by the mathematical programming model. Global evaluation of likely effects of road pricing thus becomes possible. The model can be formulated to find an optimal set of link tolls such that a particular system performance criterion is optimized. A meaningful objective is to optimize is to minimize the total network cost or to maximize total revenue raised from toll charges. In this kind of problem, it is assumed that for any given toll pattern, $\mathbf{u}$, there is a unique equilibrium flow distribution, $\mathbf{x}$, obtained from the lower-level problem. $\mathbf{x}$ is also called the response or reaction function. An efficient toll pattern, $\mathbf{u}$, will greatly depend on how to evaluate the reaction function $\mathbf{x}$, or in other words, how to predict route changes of users in response to alternative toll charges. This interaction game can be represented as the following bi-level programming problem:

Lower Level:

$$\mathbf{x}^* = \mathbf{f}^e(\mathbf{u}^*) \quad (4)$$

where u^* is the signal settingd corresponding to x^* under specified control policy P;

$$u^* = g^e(x^*) \quad (5)$$

If there exists such a pair (x^*, u^*) , then link flows and signal settings are

$$x^* = f^e[g^p(x^*)] \quad or \quad g^* = g^p[f^e(s^*)] \quad (6)$$

There can be three types of formulation on upper level, one is total network travel cost F_1 , the sum of travel times and queueing delays experienced by all vehicles:

$$F_1 = \sum_{a \in A} v_a t_a(v_a) + v_a d_a \quad (7)$$

The total revenue, denoted as F_2 , arising from toll charges can be expressed as:

$$F_2 = \sum_{a \in A^*} u_a v_a \quad (8)$$

A third objective function can be to maximize the ratio, denoted as F_3 , of the total revenue to total cost :

$$F_3 = \frac{\sum_{a \in A^*} v_a u_a}{\sum_{a \in A} v_a t_a(v_a)} \quad (9)$$

where u_a is travel cost v_a is exit flows and d_a are the queueing delay.

3.4 Formal Notation

Consider a road network and suppose that origin destination travel demand is fixed and known. Let x and y denote respectively vector of link flows and a vector of network expansion policy. A Budget control policy, denoted by B is in general any rule or procedure that can be used to determine the components of 'y' when 'x' is known. The network design problem consists of finding a pair (x^*, y^*) , such that x^* is at traffic equilibrium when capacity is y^* .

$$x^* = f^e(y^*) \quad (10)$$

where y^* is the capacity improvement corresponding to a x^* under specified Budget B and f^e is the function that gives the vector of link flows.

$$y^* = g^p(x^*) \quad (11)$$

where g^P is a function that given optimal capacity expansion vector for a given x^* . If there exists such a pair (x^*, y^*) then link flows and capacity improvement are mutually consistent or in equilibrium, in the sense that users choice when controls are at y^* yield link flows equal to those from which y^* arises under budget constraint B. In other words

$$x^* = x^e[y^P(x^*)] \mid constant y \tag{12}$$

or equivalently

$$y^* = y^P[x^e(y^*)] \mid constant x \tag{13}$$

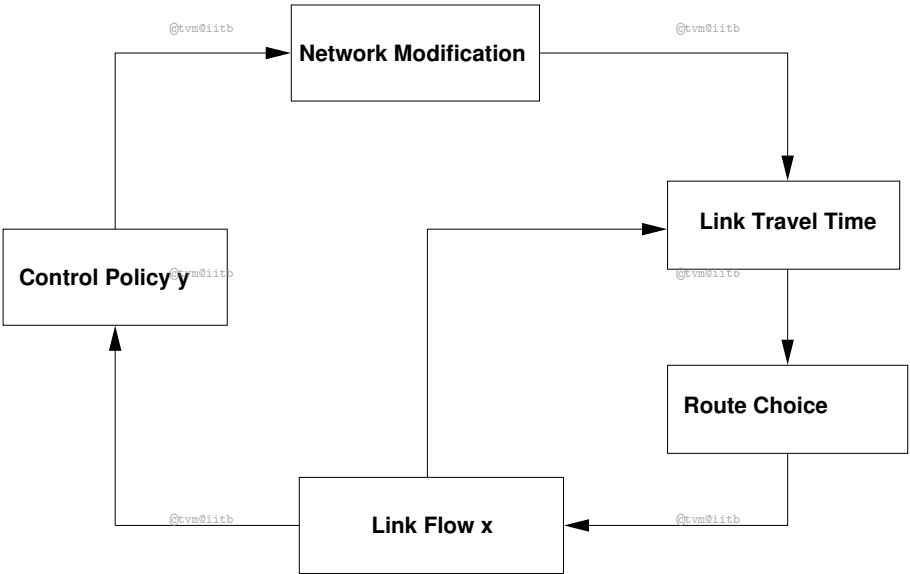


Figure 1: Bilevel

4 Formulation of capacity expansion problem

The following notation has been used for CNDP formulation:
 Let A be the set of links in the network, Ω the set of OD pairs, q the vector of fixed OD pair demands, $q = q_{rs}$, K the set of paths between OD pair ω , f the vector of path flows

between OD pair r,s on path k which means $f = [f_k^{rs}]$, x the vector of link flows, $x = x_a$, y the vector of link capacity expansion, $y = y_a$, B the allocated budget for expansion, t_a travel time on link a , γ_a is the coefficient of link expansion vector y , f_k^{rs} flow on path k connecting O-D pair $r-s$, q_{rs} trip rate between r and s .

Upper Level

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$$\min Z(x) = \sum_{\forall a} x_a t_a(x_a, y_a) \quad (14)$$

subject to

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$$\sum_{\forall a} \gamma_a y_a \leq B \quad (15)$$

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$$y_a \geq 0 : \forall a \in A \quad (16)$$

Lower Level

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$$\min Z(x) = \sum_{\forall a} \int_0^{x_a} t_a(x_a) ds \quad (17)$$

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subject to

$$\sum_{\forall k} f_k^{rs} = q_{rs} : k \in K; r, s \in \Omega \quad (18)$$

$$x_a = \sum_r \sum_s \sum_k \delta_a, k^{rs} f_k^{rs} : a \in A; k \in K \quad (19)$$

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$$f_k^{rs} \geq 0 : r, s \in \Omega; k \in K \quad (20)$$

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$$x_a \geq 0 : a \in A \quad (21)$$

x_a equilibrium flows in link a , t_a travel time on link a , y_a link capacity expansions in link a , $\gamma_a y_a$, f_k^{rs} flow on path k connecting O-D pair $r-s$, q_{rs} trip rate between r and s . To illustrate how the bilevel problem of network capacity expansion works an example network was considered. This network had four nodes and five links. Two links were considered for improvement. The figure shows the network.

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5 Numerical Example

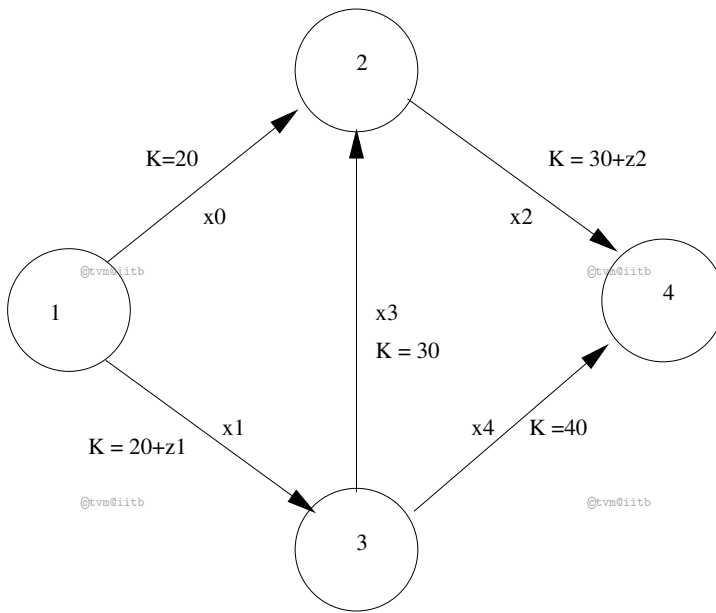


Figure 2: Bilevel example problem

5.1 Input

$$q_{23} = 50 \quad (22)$$

$$q_{03} = 80 \quad (23)$$

$$t = t_o \left(1 + \alpha \left(\frac{x}{K} \right)^\beta \right) \quad (24)$$

$$B = 10 \quad (25)$$

From	To	α	β	length(Km)	Capacity(K)	Speed(Km/hr)
1	2	0.15	4.0	1.0	20.0	60.0
1	3	0.15	4.0	1.0	20.0	60.0
2	4	0.15	4.0	1.0	30.0	60.0
3	2	0.15	4.0	1.0	30.0	60.0
3	4	0.15	4.0	1.0	40.0	60.0

5.2 Output

no.	x0*	x1*	x2*	x3*	x4*	UE	TSTT	z1*	z2*	SO
1	42.5	38.71	52.87	10.37	79.09	317.39	692.8	5.36	4.64	609.41
2	38.78	42.27	56.05	17.27	75.65	296.85	564.23	5.79	4.21	564
3	38.61	42.44	55.67	17.06	76.02	296.73	564.48	5.95	4.05	564.45
4	38.45	42.56	55.58	17.13	76.07	296.53	563.5	6.02	3.98	563.5
5	38.51	42.53	55.48	16.97	76.21	296.68	564.61	6.04	3.96	564.61
6	38.5	42.53	55.46	16.96	76.22	296.66	564.6	6.04	3.96	564.6

5.3 Discussion

The initial link expansion vector is taken as 0. User equilibrium is performed to get the required link flows. Now these flows are input to upper level from where we get a new set of link expansion vectors which minimizes the system travel time. This iteration is repeated until the total system travel time from lower level and upper level converges.

Exercises

1. Not Available

References

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