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1 INTRODUCTION

The performance and accuracy of PLAXIS 3D FOUNDATION has been carefully tested by carrying out analyses of problems with known theoretical solutions. A selection of these benchmark analyses is described in Chapters 2 to 6. PLAXIS 3D FOUNDATION has also been used to carry out predictions and back-analysis calculations of the performance of full-scale structures as additional checks on performance and accuracy.

Soil model problems: A selection of soil model problems with known theoretical solutions is presented in Chapter 2.

Elastic benchmark problems: A large number of elasticity problems with known exact solutions is available for use as benchmark problems. A selection of elastic calculations is described in Chapter 3; these particular analyses have been selected because they resemble the calculations that PLAXIS might be used for in practice.

Plastic benchmark problems: A series of benchmark calculations involving plastic material behaviour is described in Chapter 4. This series includes the calculation of collapse loads for two different footings. As for the elastic benchmarks, only problems with known exact solutions are considered.

Structural element problems: In Chapter 5 the performance of structural elements has been verified with known theoretical solutions.

Practical foundation applications: PLAXIS has been used extensively for the prediction and back-analysis of full-scale projects. This type of calculations may be used as a further check on the performance of PLAXIS provided that good quality soil data and measurements of structural performance are available. Some such projects are published in the PLAXIS Bulletin and on the internet site: http://www.plaxis.nl. One validation example can be found in Chapter 6 of this manual.
SOIL MODEL PROBLEMS WITH KNOWN THEORETICAL SOLUTIONS

A series of calculations is described in this chapter. In each case the analytical solutions may be found in many of the various textbooks on elasticity solutions, for example Giroud (1973) and Poulos & Davis (1974).

2.1 BI-AXIAL TEST WITH LINEAR ELASTIC MODEL

**Input:** A bi-axial test is conducted on a volume of 1x1x1 m as shown in Figure 2.1. The bottom-left is fixed in all direction and the front, left and rear planes are fixed horizontally.

![Bi-axial test geometry](image)

The lateral pressure $\sigma_2$ is represented by a distributed load on the right plane. The axial load $\sigma_1$ is represented by a distributed load on the top and bottom plane. The density $\gamma$ is set to zero, the remaining properties of the soil are:

$$E = 1000 \text{ kN/m}^2 \quad \nu = 0.25$$

The sample is subjected to the following loading tests: lateral loading of $\sigma_2 = -1 \text{ kN/m}^2$, axial loading of $\sigma_1 = -1 \text{ kN/m}^2$ and bi-axial loading of $\sigma_1 = \sigma_2 = -1 \text{ kN/m}^2$. 
Output: The displacement of the upper right corner for the three loading tests is:

Phase 1: $u_x = 0.9375$ mm, $u_y = -0.3125$ mm, $u_z = 0$ mm

Phase 2: $u_x = 0.3125$ mm, $u_y = -0.9375$ mm, $u_z = 0$ mm

Phase 3: $u_x = u_y = -0.625$ mm, $u_z = 0$ mm

Since a block of unit length is considered, the values of these displacement components are equal to the strains in corresponding directions.

Verification: The theoretical solution of the amount of strain is:

$$
\varepsilon_{xx} = \frac{(\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz}))}{E}
$$

$$
\varepsilon_{yy} = \frac{(\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz}))}{E}
$$

$$
\varepsilon_{zz} = \frac{(\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy}))}{E} = 0 \rightarrow \sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})
$$

The theoretical strain is the following in each phase:

**Test 1:**
- $\sigma_{xx} = -1$ kN/m$^2$
- $\sigma_{yy} = 0$ kN/m$^2$
- $\sigma_{zz} = -0.25$ kN/m$^2$
- $\varepsilon_{xx} = -0.9375 \cdot 10^{-3}$
- $\varepsilon_{yy} = 0.3125 \cdot 10^{-3}$
- $\varepsilon_{zz} = 0$

**Test 2:**
- $\sigma_{xx} = 0$ kN/m$^2$
- $\sigma_{yy} = -1$ kN/m$^2$
- $\sigma_{zz} = -0.25$ kN/m$^2$
- $\varepsilon_{xx} = 0.3125 \cdot 10^{-3}$
- $\varepsilon_{yy} = -0.9375 \cdot 10^{-3}$
- $\varepsilon_{zz} = 0$

**Test 3:**
- $\sigma_{xx} = -1$ kN/m$^2$
- $\sigma_{yy} = -1$ kN/m$^2$
- $\sigma_{zz} = -0.5$ kN/m$^2$
- $\varepsilon_{xx} = -0.625 \cdot 10^{-3}$
- $\varepsilon_{yy} = 0.625 \cdot 10^{-3}$
- $\varepsilon_{zz} = 0$

Theoretical and calculated values are in agreement with each other.
2.2 BI-AXIAL SHEARING TEST WITH LINEAR ELASTIC MODEL

**Input:** A bi-axial shearing test is conducted on a volume with the same properties as given in Section 2.1. The sample is subjected to a shear loading of 1 kN/m² as shown in Figure 2.2. Additionally, the line (1, 0) – (1,1) in the $y = -1$ m plane is fixed in $y$- and $z$-directions.

![Bi-axial shearing test](image)

**Output:** The resulting deformations are shown in Figure 2.2, the shear strain is $2.5 \times 10^{-3}$. 

![Bi-axial shearing test result](image)
**Verification:** The shear modulus is equal to:

\[ G = \frac{E}{2(1 + \nu)} = \frac{1000}{2.5} = 400 \text{kN/m}^2 \]

and the shear strain is:

\[ \gamma_{xy} = \frac{\sigma_{xy}}{G} = \frac{1}{400} = 2.5 \cdot 10^{-3} \]

The computational results are in agreement with the theoretical solution.
2.3 BI-AXIAL TEST WITH MOHR-COULOMB MODEL

**Input:** A bi-axial test is conducted on a volume identical to the one presented in section 2.1. The material behaviour is now modelled by means of the Mohr-Coulomb model. The confining pressure $\sigma_2$ is represented by vertical distributed load on the right side plane. The axial load $\sigma_1$ is represented by distributed loads on top and bottom planes. The density $\gamma$ is set to zero, the remaining model parameters are:

$$E = 1000 \text{ kN/m}^2 \quad \nu = 0.25$$

$$c = 1 \text{ kN/m}^2 \quad \phi = 30^\circ$$

The sample is subjected to the following loading scheme: bi-axial loading of $\sigma_1 = \sigma_2 = -1$ kPa, axial loading of $\sigma_1 = -2$ kPa and further axial loading to $\sigma_1 = -10$ kPa. A tolerated error of 0.001 is used.

**Output:** The soil fails at a axial stress $\sigma_1 = -6.48 \text{ kN/m}^2$ as shown in Figure 2.3.

![Figure 2.3 Results of the Bi-axial loading test with the Mohr-Coulomb model, axial stress versus axial strain](image-url)
**Verification:** The theoretical solution to the failure of the sample is given by the Mohr–Coulomb criterion:

\[
f = \frac{\sigma_1 - \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cdot \sin \varphi - c \cdot \cos \varphi = 0
\]

Failure occurs in compression at:

\[
\sigma_1 = \sigma_2 \cdot \frac{1 + \sin \varphi}{1 - \sin \varphi} + 2c \cdot \frac{\cos \varphi}{1 - \sin \varphi} = -6.46 \text{ kN/m}^2
\]

The error in the numerical solution is therefore 0.3 %.
2.4 TRIAXIAL TEST WITH HARDENING SOIL MODEL

**Input:** A traxial test is conducted on a volume of 1x1x1 m as shown in Figure 2.3. The soil behaviour is modelled by means of the Hardening Soil model. The bottom-left is fixed in all directions and the left and rear planes are fixed horizontally. The pressure $\sigma_2$ is represented by a distributed load on the left plane and $\sigma_3$ is represented by a distributed load on the front plane. The axial load $\sigma_1$ is represented by a distributed load on the top and bottom planes. The density $\gamma$ and $\nu$ are set to zero, the remaining model parameters are:

\[
E_{50}^{\text{ref}} = 2.0 \cdot 10^4 \text{ kN/m}^2 \quad E_{\text{oed}} = 2.0 \cdot 10^4 \text{ kN/m}^2 \quad E_{\text{ur}}^{\text{ref}} = 6.0 \cdot 10^4 \text{ kN/m}^2
\]

\[
c_{\text{ref}}' = 1 \text{ kN/m}^2 \quad \phi' = 35^\circ \quad \psi' = 5^\circ
\]

The sample is subjected to the following loading: isotropic loading to 100 kN/m$^2$, (after which displacements are reset to zero), axial compression until failure and axial extension until failure.
**Output:** The triaxial sample fails at $\sigma_1 = -373.3 \text{ kN/m}^2$ in compression and at $\sigma_1 = -26.4 \text{ kN/m}^2$ in extension as can be seen in Figure 2.5.

![Figure 2.5](image-url)

**Verification:** The theoretical solution to the failure of the sample is given by the Mohr-Coulomb criterion:

$$f = \frac{|\sigma_1 - \sigma_3|}{2} + \frac{\sigma_1 + \sigma_3}{2} \cdot \sin \varphi - c \cdot \cos \varphi \leq 0$$

So that failure occurs in compression at:

$$\sigma_1 = \sigma_3 \cdot \frac{1 + \sin \varphi}{1 - \sin \varphi} + 2c \cdot \frac{\cos \varphi}{1 - \sin \varphi} = -372.9 \text{ kN/m}^2$$

And failure occurs in extension at:

$$\sigma_1 = \sigma_3 \cdot \frac{1 - \sin \varphi}{1 + \sin \varphi} - 2c \cdot \frac{\cos \varphi}{1 + \sin \varphi} = -26.1 \text{ kN/m}^2$$

The calculated and theoretical values are in good agreement with each other.
3 ELASTICITY PROBLEMS WITH KNOWN THEORETICAL SOLUTIONS

A series of elastic benchmark calculations is described in this Chapter. In each case the analytical solutions may be found in many of the various textbooks on elasticity solutions, for example Giroud (1972) and Poulos & Davis (1974).

3.1 STRIP FOOTING ON ELASTIC GIBSON SOIL

**Input:** Figure 3.1 shows the 3D mesh and the soil data for a ‘plane strain’ calculation of the settlement of a strip load on Gibson soil. (Gibson soil is an elastic layer in which the shear modulus increases linearly with depth). Using $z$ to denote depth, the shear modulus, $G$, used in the calculation is given by: $G = 100 \cdot z$. With a Poisson’s ratio of 0.495, the Young’s modulus varies by: $E = 299 \cdot z$. In order to prescribe this variation of Young’s modulus in the material properties window the reference value of Young’s modulus, $E_{\text{ref}}$, is taken very small and the Advanced option is selected from the reference level $y_{\text{ref}}$ is entered as 0.0 m, being the top of the geometry.

![Figure 3.1 Problem geometry](image)

**Output:** An exact solution to this problem is only available for the case of a Poisson’s ratio of 0.5; in the PLAXIS calculation a value of 0.495 is used for the Poisson’s ratio in order to approximate this incompressibility condition. The numerical results show an almost uniform settlement of the soil surface underneath the strip load as can be seen from the displacement shadings plot in Figure 3.2. Figure 3.3 shows the shadings of the total stresses. The computed settlement is 46.4 mm at the centre of the strip load.
Figure 3.2  Vertical displacement contours

Figure 3.3  Total stresses in soil
Verification: The analytic solution is exact only for an infinite half-space, whereas the PLAXIS solution is obtained for a layer of finite depth. However, the effect of a shear modulus that increases linearly with depth is to localise the deformations near the surface; it would therefore be expected that the finite thickness of the layer has only a small effect on the results. The exact solution for this particular problem, as given by Gibson (1967), gives a uniform settlement beneath the load of magnitude:

\[ \text{Settlement} = \frac{p}{2\alpha} \]

with \( \alpha = 100 \) for this case. The exact solution for this case gives a settlement of 50 mm. The numerical solution is 7% lower than the exact solution, which is partly due to the finite depth. If, for instance, the thickness of the soil layer is increased to 100 m, the settlement calculated by PLAXIS becomes 49 mm and the error is only 2%.
3.2 FLEXIBLE TANK FOUNDATION ON ELASTIC SATURATED SOIL

Problem: In this case a flexible tank on elastic saturated soil is tested. The test includes the verification of the settlement of the centre of the tank for the condition of homogeneous, isotropic soil of finite depth.

Input: The dimensions of the tank used in the test calculation are shown in Figure 3.4. The tank is founded on a homogeneous, isotropic soil of infinite depth. The tank will impose a pressure difference in the soil of $\Delta \rho_s = 263.3 \text{ kN/m}^2$. The remaining soil properties are:

$$E = 95.8 \text{ MN/m}^2 \quad \nu = 0.499$$

Output:

The vertical settlement of the surface at the centre of the tank, calculated by PLAXIS, is 73.6 mm. A coarse mesh has been used for this calculation. The vertical displacements and the deformed mesh are shown in the Figure 3.5 and Figure 3.6.
Figure 3.5  Vertical displacements

Figure 3.6  Deformed mesh
**Verification:** The settlement at the centre of the tank is given by:

\[ \rho = \frac{\Delta q \cdot R \cdot I_p}{E} \]

Where \( I_p \) is the influence coefficient, which can be determined with Figure 3.7.

![Figure 3.7 Influence coefficients for settlement under uniform load over circular area](image)

The settlement at the centre of the tank is therefore:

\[ \rho = \frac{263.3 \cdot 23.35 \cdot 1.15}{95.8 \cdot 1000} = 0.074 \text{ m} \]

This is in good correspondence with the numerical value from PLAXIS 3D FOUNDATION.
4 PLASTICITY PROBLEMS WITH THEORETICAL COLLAPSE LOADS

Two footing collapse problems involving plastic material behaviour are described in this chapter. The first involves a strip footing on a cohesive soil with strength increasing linearly with depth and the second involves a smooth square footing on a frictional soil.

4.1 BEARING CAPACITY OF STRIP FOOTING

Problem: In practice it is often found that clay type soils have a strength that increases with depth. This type of strength variation is particularly important for foundations with large physical dimensions. A series of plastic collapse solutions for rigid plane strain footings on soil with strength increasing linearly with depth, has been derived by Davis and Booker (1973). These solutions may be used to verify the performance of PLAXIS for this class of problems.

Input: The dimensions and material properties used in the test calculation are shown in Figure 4.1. In fact, only half of the symmetric problem is modelled. The cohesion at the soil surface, \( c_{\text{ref}} \), is taken to be 1 kN/m\(^2\) and the value of the cohesion gradient in the advanced settings, \( c_{\text{increment}} \), is 2 kN/m\(^2\)/m, using a reference level, \( y_{\text{ref}} = 0 \) m (= top of the layer). The stiffness at the top is given by \( E_{\text{ref}} = 299 \) kN/m\(^2\) and the increase of stiffness with the depth is defined by \( E_{\text{increment}} = 598 \) kN/m\(^2\)/m. Calculations are carried out for the case of a rough (x- and z-direction are fixed) and a smooth footing (x- and z-direction are free).

Output: The calculated maximum average vertical stress under the smooth footing is 7.82 kN/m\(^2\), giving a bearing capacity of 15.6 kN. For the rough footing this is 9.28 kN/m\(^2\), giving a bearing capacity of 18.6 kN. The computed load-displacement curves are shown in Figure 4.2. The deformed mesh for the smooth footing is shown in Figure 4.3.
Figure 4.2 Stress-displacement curves

Verification: The analytical solution derived by Davis & Booker (1973) for the mean ultimate vertical stress beneath the footing, $p_{\text{max}}$, is:

$$ p_{\text{max}} = \frac{F}{B} = \beta \left[ (2 + \pi) c_{\text{layer}} + \frac{B c_{\text{depth}}}{4} \right] $$

Where $B$ is the footing width and $\beta$ is a factor that depends on the footing roughness and the rate of increase of clay strength with depth. The appropriate values of $\beta$ in this case are 1.27 for the smooth footing and 1.48 for the rough footing. The analytical solution therefore gives average vertical stresses at collapse of 7.8 kN/m$^2$ for smooth footing and 9.1 kN/m$^2$ for the rough footing. These results indicate that the errors in the PLAXIS solution are 0.3% and 2% respectively.

Directional dependence: In addition the infinite long strip is modelled along the $x$-axis with the same parameters. The deformed mesh is shown in Figure 4.4. The results are exactly the same as obtained from the above calculation with the strip modelled along the $z$-axis. There is no directional dependency.
Figure 4.3  Deformed mesh (smooth)

Figure 4.4  Deformed mesh (rough)
4.2 BEARING CAPACITY OF A CIRCULAR FOOTING

**Input:** Figure 4.5 shows the mesh and material data for a smooth rigid circular footing with a radius of 1 m on a frictional soil. The thickness of the soil layer is taken to be 4 metres and the material behaviour is represented by the elasto-plastic Mohr-Coulomb model. The footing is represented by a distributed load on a plate with high flexural rigidity, but low normal stiffness. Around the footing an interface has been modelled, extending 0.5 metres below the footing. A virtual thickness of 0.3 metres has been assigned to this interface. During the ultimate level 3D plastic staged construction calculation the load is increased until failure.

**Output:** The load-displacement curve for the footing is shown in Figure 4.6. The final vertical load at failure is 227 kN/m². During the calculation a higher vertical load of 242 kN/m² is obtained and the final, lower, collapse load is only obtained if sufficient additional calculation steps are permitted. For this calculation a total of 1000 calculation steps have been used. Figure 4.7 shows the absolute displacement shadings at failure.
Verification: The exact solution for this collapse load problem for a circular footing is derived by Cox (1962). For $\gamma R/c = 10$ and $\phi = 30^\circ$. Cox presents the exact solution:

$$P_{max} = 141 \cdot c = 141 \cdot 1.6 = 225.6 \text{ kN/m}^2$$

The relative error of the end result calculated with PLAXIS is less than 1%.
5 CONSOLIDATION

In this Chapter, the results of a one-dimensional consolidation analysis in PLAXIS 3D FOUNDATION are compared to an analytical solution.

5.1 ONE-DIMENSIONAL CONSOLIDATION

**Input:** Figure 5.1 shows the finite element mesh for the one-dimensional consolidation problem. The thickness of the layer is 1.0 m. The layer surface (upper side) is allowed to drain while the other sides are kept undrained by imposing closed consolidation boundary condition. These are the standard boundary conditions in PLAXIS 3D FOUNDATION. An excess pore pressure, \( p_0 \), is generated by using undrained material behaviour and applying an external load \( p_0 \) in the first (plastic) calculation phase. In addition, ten consolidation analyses are performed to ultimate times of 0.1, 0.2, 0.5, 1.0, 2.0, 5.0, 10, 20, 50 and 100 days respectively.

\[ E = 1000 \text{ kN/m}^2 \]
\[ \nu = 0.0 \]
\[ k = 0.001 \text{ m/day} \]
\[ \gamma_w = 10 \text{ kN/m}^3 \]
\[ H = 1.0 \text{ m} \]

**Output:** Figure 5.2 shows the calculated relative excess pore pressure versus the relative vertical position as marked. Each of the above consolidation times is plotted. Figure 5.3 presents the development of the relative excess pore pressure at the (closed) bottom.
Figure 5.2 Development of excess pore pressure as a function of the sample height

\[ T = \frac{c_v t}{H^2} \]

\[ c_v = \frac{kE_{oed}}{\gamma_w} \]

\[ E_{oed} = \frac{(1-v)E}{(1+v)(1-2v)} \]

Figure 5.3  Development of excess pore pressure at the bottom of the sample as a function of time

\[ \text{Relative Excess Pore Pressure } \frac{p}{p_0} \]

\[ \text{Time } [\text{day}] \]
**Verification:** The problem of one-dimensional consolidation can be described by the following differential equation for the excess pore pressure $p$:

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

where:

$$c_v = \frac{kE_{oed}}{\gamma_w}, \quad E_{oed} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \quad z = H - y$$

The analytical solution of this equation, i.e. the relative excess pore pressure, $p / p_0$ as a function of time and position is presented by Verruijt (1983):

$$\frac{p}{p_0}(z,t) = \frac{4}{\pi} \sum_{j=1}^{\infty} \frac{(-1)^{j-1}}{2j-1} \cos\left((2j-1)\frac{\pi}{2} \frac{y}{H}\right) \exp\left(-\frac{(2j-1)^2 \pi^2}{4 \frac{c_v t}{H^2}}\right)$$

This solution is presented by the continuous lines in Figure 5.2. It can be seen that the numerical solution is close to the analytical solution, but has two distinct points of difference. First, the excess pore pressure initially calculated is $0.98 \ p_0$, instead of $1.0 \ p_0$. This is due to the fact that the pore water in PLAXIS is not completely incompressible. See *Undrained behaviour* in Section 3.5 of the Reference Manual for more information. Secondly, the consolidation rate is slightly lower than the theoretical consolidation rate. This is caused by the implicit time integration scheme used.
6 STRUCTURAL ELEMENT PROBLEMS

A series of structural element/elastic benchmark calculations is described in this chapter. In each case the analytical solutions may be found in many of the various textbooks on elasticity solutions, for example Giroud (1972) and Poulos & Davis (1974).

6.1 BENDING OF FLOOR ELEMENTS

Input: For the verification of a floor element two problems are considered. These problems involve a single line load and a uniformly distributed load on a plate respectively, as indicated in Figure 6.1. For these problems a plate of 1 m length and 1 m width has been selected. The properties, dimensions and the loads of the plate are:

\[
E = 1 \cdot 10^6 \text{kN/m}^2 \quad G = 5 \cdot 10^5 \text{kN/m}^2 \quad \nu = 0.0 \\
d = 0.1 \text{ m} \quad F = 100 \text{kN/m} \quad q = 200 \text{kN/m}^2
\]

Plates cannot be used individually. A single cluster may be used to create the geometry. The two plates are added to the top work plane with a spacing in between. Use line fixities on the end points of the plate. A coarse mesh is sufficient to model the situation. In the Initial conditions mode the soil cluster can be deactivated so that only the plates remain.

Output: The results of the two calculations are plotted in Figure 6.2, Figure 6.3 and Figure 6.4. For the extreme moments and displacements we find:

Line load: \( M_{\text{max}} = 25.22 \text{kNm/m} \quad u_{\text{max}} = 25.5 \text{ mm} \)

Distributed load: \( M_{\text{max}} = 25.58 \text{kNm/m} \quad u_{\text{max}} = 31.8 \text{ mm} \)
Verification: As a first verification, it is observed from Figure 6.2 that PLAXIS yields the correct distribution of moments. For further verification we consider the well-know formulas as listed below. These formulas give approximately the values as obtained from the PLAXIS analysis.

Point load: \[ M_{\text{max}} = \frac{1}{4}Fl = 25 \text{kNm} \quad u_{\text{max}} = \frac{Fl^3}{48EI} = 25 \text{mm} \]

Distributed load: \[ M_{\text{max}} = \frac{1}{8}ql^2 = 25 \text{kNm} \quad u_{\text{max}} = \frac{5}{384} \frac{ql^4}{EI} = 31.25 \text{mm} \]

The error of the results of PLAXIS is less than 2.5%.
6.2 BENDING OF WALL ELEMENTS

Input: For the verification of a wall element the same two problems are considered as in the last section. These problems involve a single line load and a uniformly distributed load on a plate respectively, as indicated in Figure 6.5. For these problems a plate of 1 m length and 1 m width has been selected. The properties, dimensions and the loads of the plate are:

\[
E = 1\cdot10^6 \text{ kN/m}^2 \quad G = 5\cdot10^5 \text{ kN/m}^2 \quad \nu = 0.0
\]
\[
d = 0.1 \text{ m} \quad F = 100 \text{ kN/m} \quad q = 200 \text{ kN/m}^2
\]

Plates cannot be used individually. A single cluster may be used to create the geometry. The two plates are added to the geometry, taking care that there is a gap between the plates and the boundaries of the problem. Use line fixities on the top and bottom sides of the plates. A coarse mesh is sufficient to model the situation. In the Initial conditions mode the soil cluster can be deactivated so that only the plates remain.

![Figure 6.5 Loading scheme for testing walls](image)

Output: The results of the two calculations are plotted in Figure 6.6, Figure 6.7 and Figure 6.8. For the extreme moments and displacements we find:

- Line load: \( M_{\max} = 25.00 \text{ kNm/m} \quad u_{\max} = 25.6 \text{ mm} \)
- Distributed load: \( M_{\max} = 25.46 \text{ kNm/m} \quad u_{\max} = 31.8 \text{ mm} \)

The error of the results of PLAXIS is less than 2.5 \%. 
Figure 6.6  Computed distribution of moments

Figure 6.7  Computed shear forces

Figure 6.8  Computed displacements
6.3 BENDING OF SHELL ELEMENTS

The wall of a circular pile can be modelled in PLAXIS using curved shell elements. By using this element, 3 types of deformations are taken into account: shear deformation, compression due to normal forces and obviously bending.

**Input:** A ring with a radius of $R = 1\,\text{m}$ and a width of $1\,\text{m}$ is considered. The Young's modulus and the Poisson's ratio of the material are taken respectively as $E = 1\cdot10^6\,\text{kN/m}^2$ and $\nu = 0$. For the thickness of the ring cross-section, $H$, several different values are taken so that we have rings ranging from very thin to very thick. To model such a ring one point of the ring is fixed with respect to translation. The other side is allowed to move freely and a load $F = 1.0\,\text{kN/m}$ is applied at that side. Geometric non-linearity is not taken into account.

**Output:** The calculated vertical deflections at the top point are presented in Figure 6.9. The deformed shape of the ring is shown in Figure 6.10. The calculated normal force at the belly of the ring is $0.50\,\text{kN}$ for all different values of ring thickness. The calculated bending moment at the belly is $0.182\,\text{kNm}$ for all different values of ring thickness. Typical graphs of the bending moment and normal force are shown in Figure 6.11.

![Figure 6.9](image)

*Figure 6.9  Calculated deflections compared with analytical solutions*
Verification: The analytical solution for the deflection of the ring is given by Blake (1959), and the analytical solution for the bending moment and the normal force can be found from Roark (1965). The vertical displacement at the top of the ring is given by the following formula:

$$\delta = \frac{F\lambda}{E} \left[ 1.788\lambda^2 + 3.091 - \frac{0.637}{1+12\lambda^2} \right]$$

with $\lambda = \frac{R}{H}$
The solid curve in Figure 6.11 is plotted according to this formula. It can be seen that the deflections calculated by PLAXIS fit the theoretical solutions very well. Only for a very thick ring some errors are observed, which is about 4 percent for $H/R = 0.5$. But for thin rings the error is nearly zero. The analytical solution for the bending moment and normal force at the belly is 0.181 kNm and 0.5 kN respectively. Thus even for very thick rings the error in the bending moment and normal force is almost zero.
6.4 PERFORMANCE OF SPRINGS

Springs are used to transport forces to the outside world. Springs are fully fixed on one side and are connected to the geometry on the other side. They only transport forces parallel to their direction and have no stiffness perpendicular to their direction. In the following example the performance of springs connected to floors and walls is verified.

**Input:** Two floors of 2 x 2 m are modelled (Figure 6.12). Each floor is loaded by a distributed load of 100 kN/m², acting downwards. One floor is directly supported by 4 vertical springs on the corners. The second floor is supported by two walls. The walls in turn are supported by vertical springs on their lower corner points. For stability two horizontal springs acting in x-direction are added at the bottom center of the walls, and two horizontal springs acting in z-direction are added to the floor.

All springs have a spring stiffness \( EA/L = 10^3 \) kN/m. All walls and floors have a Young’s modulus \( E = 10^8 \) kN/m², Poisson ratio \( \nu = 0 \) and a thickness \( d = 0.1 \) m.

**Verification:** The resulting force in all springs is equal to –100.00 kN. The vertical displacement of the corners of the floor directly supported by springs is –100.34 mm, which is a relative error of 0.3 %. For the second case, with the floors supported by walls, the vertical displacement of the bottom corner points of the walls is equal to -100.00 mm exactly.

![Figure 6.12  Geometry of floors and walls supported by springs](image-url)
7 SINGLE PILE AND PILE GROUP IN OVERCONSOLIDATED CLAY

(by Y. El-Mossallamy, Arcadis Germany)

In order to validate the program, a pile load test in Germany has been analysed. The load test investigated both the load-settlement behaviour of a single pile and that of a pile group. The behaviour of the single pile has been analysed using both PLAXIS 3D FOUNDATION as well as PLAXIS V8. Subsequently, the behaviour of the pile group has been analysed using PLAXIS 3D FOUNDATION.

7.1 INTRODUCTION

The load settlement behaviour of the piles in a pile group is totally different from the behaviour of the corresponding single pile. The group action represents the behaviour of the pile group compared to that of the single pile. Pile group action plays an important role for the behaviour of piled foundation both under vertical tension and compression loads and under horizontal loads. The group action of pile groups under vertical compression loads will be dealt with in this example.

As no possibility exists to take into account -in an adequate manner- the soil disturbance caused due to pile installation by theoretical means (El-Mossallamy 1999), pile load tests on single piles are frequently carried out to determine the load-settlement behaviour of a single pile. On the other hand it is costly and time consuming to carry out load tests on pile groups. Therefore, the pile group action is considered either by adapting simple correlations, or by comparing the pile group to simplified foundation shapes, or by applying advanced numerical analyses. The application of three dimensional finite element analyses to determine the pile group action will be demonstrated in this example.

7.2 NUMERICAL SIMULATION OF THE SINGLE PILE BEHAVIOUR (PILE LOAD TEST)

An extensive research program related to bored piles in overconsolidated clay was conducted by Sommer & Hambach (1974) to optimise the foundation design of a highway bridge in Germany. Load cells were installed at the pile base to measure the loads carried directly by pile base. Figure 7.1 gives the layout of the pile load test arrangement. The measured load-settlement curves and the distribution of loads between base resistance and skin friction are shown in Figure 7.2. The upper 4.5 m subsoil consist of silt (loam) followed by tertiary sediments down to great depths. These tertiary sediments are stiff plastic clay similar to the so-called Frankfurt clay, with a varying degree of overconsolidation. A pile load test is often used to verify the numerical modelling of pile behaviour in Frankfurt overconsolidated clay (El-Mossallamy 2004). The groundwater table is about 3.5 m below the ground surface. The considered pile has
a diameter of 1.3 m and a length of 9.5 m. It is located completely in the overconsolidated clay. The loading system consists of two hydraulic jacks working against a reaction beam. This reaction beam is supported by 16 anchors. These anchors were installed vertically at a depth between 15 and 20 m below the ground surface at a distance of about 4 m from the tested pile, in order to minimize the effect of the mutual interaction between the tested pile and the reaction system (Figure 7.1.a). Vertical and horizontal loading tests were carried out. The loads were applied in increments and maintained constant until the settlement rate was negligible. Both the applied loads and the corresponding displacements at the tested pile head were measured. Additionally the soil displacements near the pile at different depths were measured using deep settlement points (Figure 7.1b).

Figure 7.1 Layout of the pile load test and the measurement points

Figure 7.2 Measured load-settlement curves and distribution of loads between base resistance and skin friction
7.2.1 GEOMETRY OF THE MODEL

In order to analyse the behaviour of the single pile, at first a model has been made in PLAXIS V8 using an axisymmetric model for a completely homogeneous soil. A mesh of 15 m width and 16 m depth has been used. At the axis of symmetry the pile has been modelled with a length of 9.5 m and a diameter of 1.3 m. The soil is modelled as a single layer of overconsolidated stiff plastic clay, with properties as given in Table 7.1. The groundwater table is located at 3.5 m below the soil surface. Along the length of the pile an interface has been modelled. This interface extends to 0.5 m below the pile, in order to allow for sufficient flexibility around the pile tip. The resulting mesh composed of high order 15 node elements is shown in Figure 7.3.

Figure 7.3 The resulting 2D axisymmetric mesh

Figure 7.4 The dimensions of the 3D Foundation model
Secondly, a model has been made using PLAXIS 3D FOUNDATION. A working area 50 m x 50 m has been used. The pile is modelled as a solid pile using volume elements in the centre of the mesh. Interfaces are modelled along the pile. The soil consists of a single layer of overconsolidated stiff plastic clay, with properties as given in Table 7.1. The load is modelled as a distributed load at the pile top. 6 different meshes with different levels of refinement were applied to check the sensitivity of the mesh refinement on the results. Table 7.2 summarizes the main properties of the 6 tested meshes. This table also lists the number of elements used to model the pile in vertical direction. Figure 7.5 shows the different finite element meshes composed of 15 node volume elements.

![Figure 7.5 The finite element meshes used for the 3D analyses.](image-url)
7.2.2 MATERIAL PROPERTIES

The required soil parameters were determined based on the conducted laboratory and in-situ tests as well as on experience gained in similar soil conditions, see Table 7.1. The concrete pile is modelled as a non-porous linear elastic material with Young’s modulus $E = 3\times10^7$ kN/m$^2$, Poisson ratio $\nu = 0.2$ and unit weight $\gamma = 24$ kN/m$^3$. For the overconsolidated clay layer, two different material models have been considered.

Table 7.1 Model parameters for different soil data sets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Overcons. Clay 1</th>
<th>Overcons. Clay 2</th>
<th>Silt (Loam)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material model</td>
<td>Model</td>
<td>Mohr- Coulomb</td>
<td>Hardening Soil</td>
<td>Mohr- Coulomb</td>
<td>-</td>
</tr>
<tr>
<td>Type of material behaviour</td>
<td>Type</td>
<td>Drained</td>
<td>Drained</td>
<td>Drained</td>
<td>-</td>
</tr>
<tr>
<td>Soil weight above phr. level</td>
<td>$\gamma_{unsat}$</td>
<td>20</td>
<td>20</td>
<td>19</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>Soil weight below phr. level</td>
<td>$\gamma_{sat}$</td>
<td>20</td>
<td>20</td>
<td>19</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E$</td>
<td>$6\times10^4$</td>
<td>-</td>
<td>$1\times10^4$</td>
<td>kN/m$^2$</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>$\nu$</td>
<td>0.3</td>
<td>-</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>Secant stiffness</td>
<td>$E_{so}^{ref}$</td>
<td>-</td>
<td>$4.5\times10^4$</td>
<td>-</td>
<td>kN/m$^2$</td>
</tr>
<tr>
<td>Oedometer stiffness</td>
<td>$E_{oed}^{ref}$</td>
<td>-</td>
<td>$4.5\times10^4$</td>
<td>-</td>
<td>kN/m$^2$</td>
</tr>
<tr>
<td>Unloading-reloading stiffness</td>
<td>$E_{ur}^{ref}$</td>
<td>-</td>
<td>$9\times10^4$</td>
<td>-</td>
<td>kN/m$^2$</td>
</tr>
<tr>
<td>Power</td>
<td>$m$</td>
<td>-</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Unloading-reloading Poisson ratio</td>
<td>$\nu_{ur}$</td>
<td>-</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cohesion</td>
<td>$c$</td>
<td>20</td>
<td>20</td>
<td>5</td>
<td>kN/m$^2$</td>
</tr>
<tr>
<td>Friction angle</td>
<td>$\varphi$</td>
<td>22.5</td>
<td>22.5</td>
<td>27.5</td>
<td>°</td>
</tr>
<tr>
<td>Dilatancy angle</td>
<td>$\psi$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>°</td>
</tr>
<tr>
<td>Lateral earth pressure coeff.</td>
<td>$K_0$</td>
<td>0.8</td>
<td>0.8</td>
<td>0.5</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.2 Applied meshes for the three dimensional analyses (15 node wedge elements)

<table>
<thead>
<tr>
<th>Model name</th>
<th>No. of elements / nodes in top work plane</th>
<th>Total no. of elements / nodes for the whole 3D mesh</th>
<th>No. of layers in pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variety - 01</td>
<td>106 / 237</td>
<td>742 / 2238</td>
<td>4</td>
</tr>
<tr>
<td>Variety - 02</td>
<td>292 / 609</td>
<td>2044 / 5865</td>
<td>4</td>
</tr>
<tr>
<td>Variety - 03</td>
<td>350 / 741</td>
<td>2450 / 7060</td>
<td>4</td>
</tr>
<tr>
<td>Variety - 04</td>
<td>350 / 741</td>
<td>3150 / 8862</td>
<td>5</td>
</tr>
<tr>
<td>Variety - 05</td>
<td>350 / 741</td>
<td>3850 / 10664</td>
<td>7</td>
</tr>
<tr>
<td>Variety - 06</td>
<td>350 / 741</td>
<td>5250 / 14268</td>
<td>10</td>
</tr>
</tbody>
</table>
7.2.3 MODELLING THE SINGLE PILE

Initial stresses were generated using the $K_0$-procedure in the 2D axisymmetric case and using gravity loading in the 3D analyses. In both cases the initial $K_0$ value in the overconsolidated clay was taken 0.8. Pore pressures were generated based on a phreatic level. The actual load test was simulated by applying a distributed load at the top of the pile.

Figure 7.6 shows the load-settlement curves for the different 3D analyses. The vertical displacement of the top of the pile has been plotted. The results are similar up to 2000 kN, almost equal to the working load. At higher load levels, the results of meshes 3, 4, 5 and 6 show little differences. These results demonstrate the stability of the program. Nevertheless, it is recommended to check the sensitivity of the mesh refinement on the results for each individual case.

![Load-settlement curves](image)

**Figure 7.6** Results of different finite element meshes.

Figure 7.7 shows a comparison between the different numerical models. There is a good agreement between the results of different numerical models and those of the pile load test up to a working load of about 2000 kN. Nevertheless, the three dimensional analysis shows a relatively stiff behaviour at higher load level in comparison with the axisymmetric results for the same initial conditions. The effect of the initial stresses on the load settlement behaviour of single pile as well as on a pile group will be discussed in more details in Section 7.3.1.
Figure 7.7 Comparison between the results of different numerical models and measured results.

Figure 7.8 Deformation results using PLAXIS 3D FOUNDATION.
Figure 7.8 demonstrates some deformation results of PLAXIS 3D FOUNDATION for Variety 6 (see Table 7.2). At higher load levels, plastic deformation of the soil controls the settlement behaviour of the pile. These plastic deformations are concentrated in a narrow zone around the pile shaft. Outside this plastic narrow zone the soil behaviour remains mainly elastic. Therefore, the settlement trough under working loads (of 1500 kN (Figure 7.8a)) is wider than that under loads near the ultimate load level (of 4000 kN (Figure 7.8b)).

7.3 NUMERICAL SIMULATION OF THE PILE GROUP ACTION

From the pile load test of the single pile, it was determined that the ultimate skin friction was about 60 kN/m². Subsequently, an allowable skin friction of 30 kN/m² was selected for the foundation design, as at the corresponding load level, the settlement of the tested pile was measured to be in the order of 3 mm. A settlement of 3 mm was deemed to be acceptable for the bridge design. The bridge piers consists of 2 pillars, each founded on a separate pile group. The foundation piles have a diameter of 1.5 m and a length of 24.5 m with 6 piles under each pillar. The pile arrangement is shown in Figure 7.9a. The settlement of the entire foundation should be about 3 mm if there were no group action. The load-settlement behaviour of the whole foundation was monitored during and after the construction to obtain information on the group action. The load settlement relationship of one of the monitored pillars (Sommer/Hambach, 1974) is shown in Figure 7.9b.
The average measured settlement of the pillar was about 9.0 mm. The difference between the expected settlement and the measured value demonstrates the importance of considering the pile group action to predict a reliable settlement of the whole foundation. A three dimensional finite element analysis is applied to investigate its reliability determining the pile group action. The results of the boundary element method (El-Mossallamy 1999) will be used to compare with the results of the 3D finite element analyses.

The load settlement behaviour of a single foundation pile (pile length 24.5 m and pile diameter 1.5 m) was calculated using both the 3D-FEM as well as the BEM (El-Mossallamy 1999). In both cases the same soil parameters were used for the clay layers as in the verification analysis of the single pile in homogeneous soil conditions, see Table 7.1. In this analysis the top layer of silt is also taken into account. Figure 7.10 shows the 3D finite element mesh used to simulate the behaviour of a single foundation pile.

Figure 7.10 3D finite element mesh to simulate the behaviour of a single foundation pile.

Figure 7.11 shows a comparison between the different conducted analyses. The load settlement relationship up to a working load of about 3 MN is mainly linear. Furthermore, the different models behave very similar up to a load of about 7 MN (about twice the working load).

For the analysis of the pile group, three different mesh refinements were used, see Figure 7.12. Table 7.3 summarizes the main properties of the 3 different meshes.
Figure 7.11 Load settlement behaviour of a single foundation pile.

Figure 7.12 3D finite element meshes to simulate the foundation behaviour.
Table 7.3 Main properties of the 3 meshes used for the analysis of the pile group.

<table>
<thead>
<tr>
<th>Variety</th>
<th>Variety No. of elements / nodes in cross section</th>
<th>Total no. of elements / nodes for the whole 3D mesh</th>
<th>No. of pile subdivisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variety - 01</td>
<td>164 / 417</td>
<td>1804 / 5249</td>
<td>7</td>
</tr>
<tr>
<td>Variety - 02</td>
<td>161 / 412</td>
<td>2093 / 6038</td>
<td>8</td>
</tr>
<tr>
<td>Variety - 03</td>
<td>429 / 956</td>
<td>8151 / 22120</td>
<td>14</td>
</tr>
</tbody>
</table>

The calculated results of the load settlement behaviour of the whole pile group are shown in Figure 7.13. The different meshes give almost the same result up to 32 MN (twice the working load). Mesh variations 2 and 3 yield a good agreement at higher loads. The calculated settlement at the working load of 16 MN is about 10 mm and agrees well with the measurements.

Contour lines of equal settlement at the ground surface are shown in Figure 7.14a to demonstrate the 3D results. The settlement of the foundation alone is shown in Figure 7.14b. It can be recognized that the mutual interaction between the two pillars leads to some tilting of both pillars. The calculated tilting reaches about 1:3500. These results
show the ability of PLAXIS 3D FOUNDATION to predict the load settlement behaviour of pile groups under working conditions in order to check the serviceability requirements.

Figure 7.15 compares the behaviour of the single pile with the average behaviour of the pile group under the same average load. The calculated pile group action, resulting from the 3D finite element analyses as well as from the boundary element analyses (El-Mossallamy 1999) can be determined to be in the order of 3.0. This value agrees well with the results of the conducted measurements. These results demonstrate the ability of PLAXIS 3D FOUNDATION to predict the pile group action.

Figure 7.14 Deformation results of the bridge pillar using PLAXIS 3D FOUNDATION. a) Settlement at the ground surface. b) Settlement of the foundation plate

Figure 7.15 Pile group action
7.3.1 **EFFECT OF INITIAL STRESSES**

The previous analyses show that the load-settlement behaviour of a foundation pile can be accurately modelled under working load conditions. For instance Figure 7.13 shows that the correct settlement under working load conditions can be predicted for a pile group, and that this predicted settlement is not strongly influenced by the mesh refinements. On the other hand, the ultimate bearing capacity of the pile is strongly influenced by several factors, amongst which mesh refinements and the initial stress state. Figure 7.7, Figure 7.11 and Figure 7.15 show a comparison between results with different initial stresses. From each of these figures it can be seen that an accurate prediction of the settlement under working loads can be obtained. However, the ultimate bearing capacities obtained from these analyses depend strongly on the modelling scheme followed. Figure 7.15, for example, shows that the difference in ultimate bearing capacity of a single pile obtained using the boundary element method (El-Mossallamy, 1999) and PLAXIS 3D FOUNDATION amounts to approximately 3 MPa.

![Figure 7.16 Effect of initial stresses on the calculation results.](image-url)
Figure 7.16 summarizes the results of the comparison for the behaviour of pile load test, the single foundation pile and the whole pillar foundation, in order to demonstrate the effect of the initial stresses in more detail. Once again, this figure shows significant differences in predicted ultimate bearing capacity for different models, but also for different initial stress conditions. For example for the single pile, the deformation under working load conditions is hardly influenced by the initial value of $K_0$, but the ultimate bearing capacity may change as much as 3 MPa. The same trend is seen for the pile group.

7.4 CONCLUSIONS

The load settlement behaviour of the piles in overconsolidated clay is almost linear up to the working load. Therefore, the initial stresses have almost no effect on the results up to the working loads. On the other hand, the initial stresses have a dominant effect on the pile behaviour under higher load levels. The calculated ultimate bearing capacity depends strongly on the initial stresses. The results of Figure 7.16 show that the PLAXIS 3D FOUNDATION analyses have a good agreement with the results of the PLAXIS V8 (axisymmetric modelling with 15-node elements) under the working load. At higher load level, the PLAXIS 3D FOUNDATION analyses show stiffer behaviour than the axisymmetric analyses and predict a higher ultimate bearing capacity. Therefore, the ultimate bearing capacity should be checked using independent conventional methods. Nevertheless, it can be concluded that the calculated deformation under working conditions (serviceability limit analyses) can be adequately determined using PLAXIS 3D FOUNDATION.
8 REFERENCES


