CE-307 :DESIGN OF STRUCTURES – I

Concrete

SLOT- 2

MONDAY 9.30 am

TUESDAY 11.30 am

WEDNESDAY 8.30 am

www.civil.iitb.ac.in/~abhijit/ce307.htm

CE-315: DESIGN OF STRUCTURES – I LAB

MONDAY 2.00 pm

www.civil.iitb.ac.in/~abhijit/ce315.htm

Contents

- Introduction
- Concrete
- Design by working stress method
- Design by Limit state method

Structural Analysis

- To determine the response of the structure under the action of loads.
- Response may be displacement, internal forces like axial force, bending moment, shear force etc.
- Structure geometry and material properties are known.

Methods of Analysis

1. Manual computation methods

   Slope – Deflection Method

   Strain Energy Method

   Moment Distribution Method

   Kani's Method

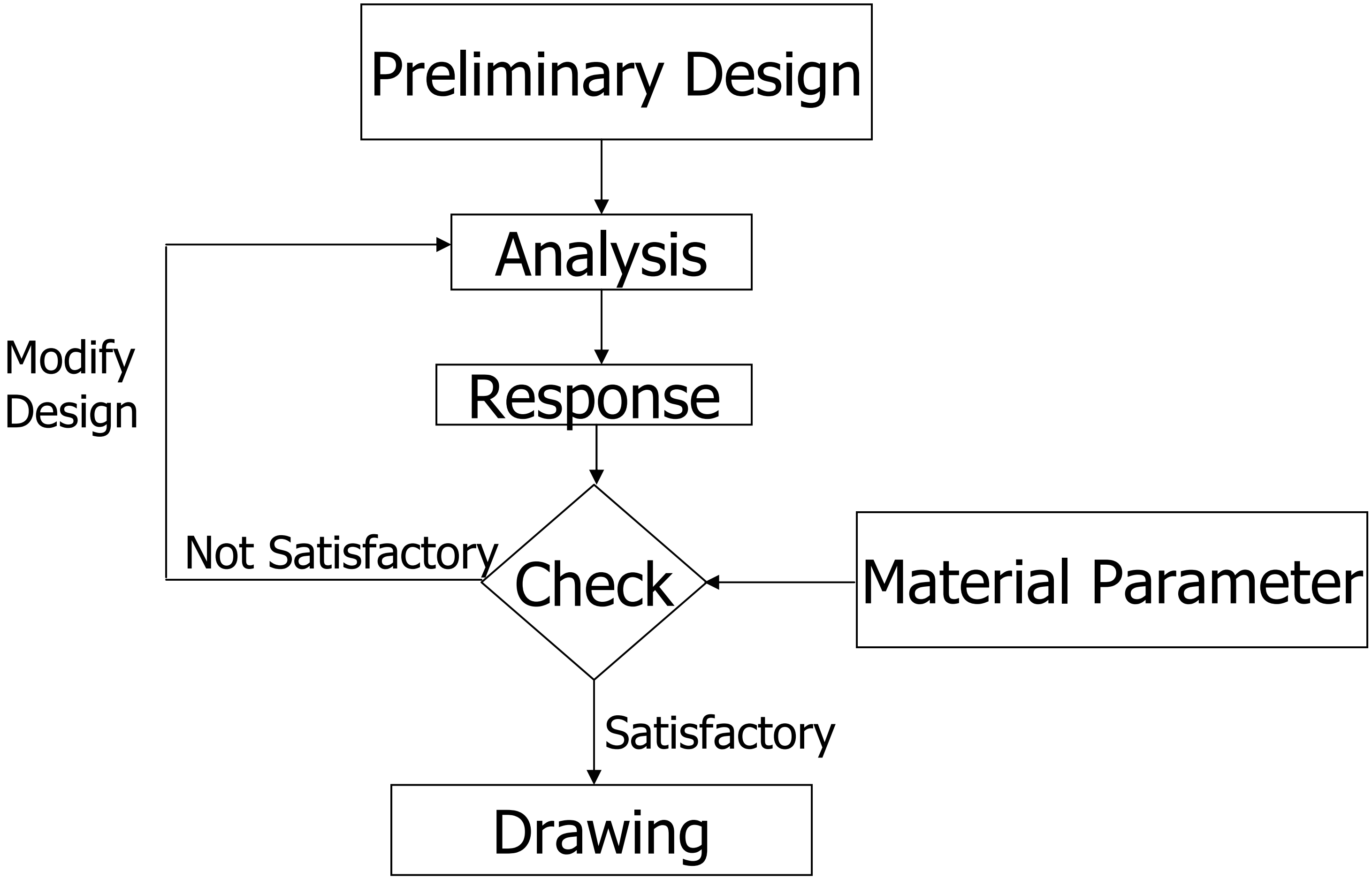
2. Computer Methods

   Matrix Methods

   Finite Element Method

   Finite Difference Method

Design Process

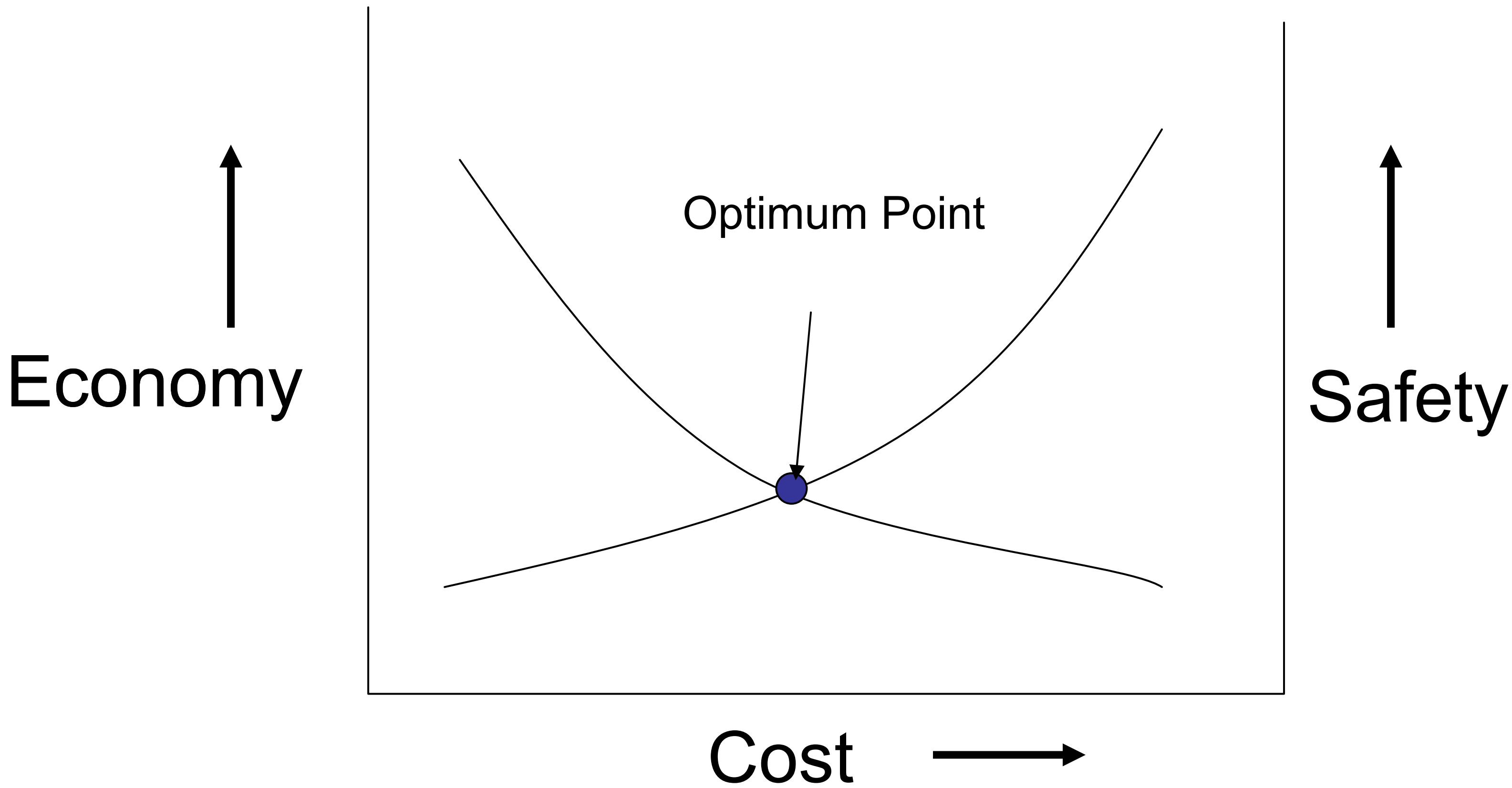


Structural Design

- Structural design is an art and science of creation, with economy and elegance, a safe, servicable and durable structure.

- Besides knowledge of structural engineering it requires knowledge of practical aspects, such as relevant codes and bye laws backed up by ample experience, intuition and judgment.

Optimum Design



Concrete

binder

+

concrete = coarse aggregates

+

fine aggregates

+

water

air

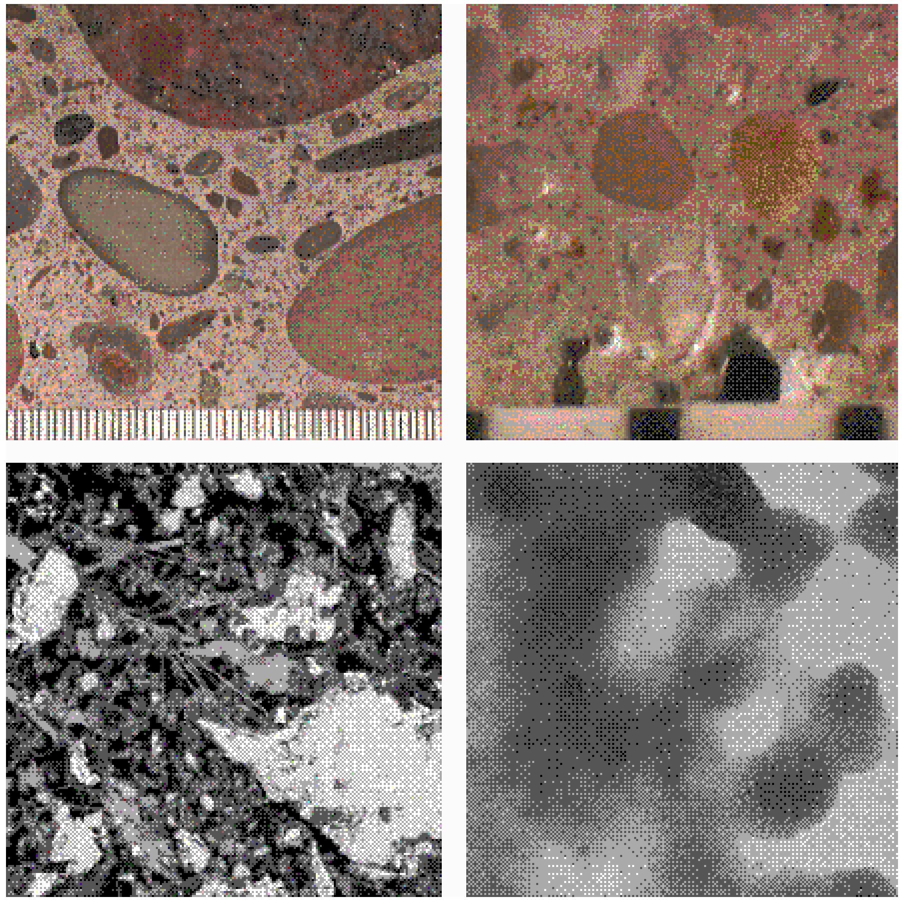
+

admixtures

A closer look

Centimeter

Milimeter



Micro meter

Nanometer

Coliseum of Rome

History of concrete

3000 BC	The Egyptians began to use mud mixed with straw to bind dried bricks. They also used gypsum mortars and mortars of lime in the building of the pyramids
800 BC	The Greeks used lime mortars that were much harder than later Roman mortars. This material was also in evidence in Crete and Cyprus at this time.
300 BC	The Babylonians and Assyrians used bitumen to bind stones and bricks together
299 BC – 476 AD	The Romans used pozzolana cement from Pozzuoli, Italy near Mt. Vesuvius to build the Roman Baths of Caracalla, the Basilica of Maxentius, the Coliseum and Pantheon in Rome. They used broken brick aggregate embedded in a mixture of lime putty with brick dust or volcanic ash by the Romans.

History of concrete contd…

1200-1500	The quality of cementing materials deteriorated and even the use of concrete died out during The Middle Ages as the art of using burning lime and pozzolan (admixture) was lost, but it was later reintroduced in the 1300s
1414	Fra Giocondo used pozzolanic mortar in the pier of the Pont de Notre Dame in Paris. It is the first acknowledged use of concrete in modern times
1744	John Smeaton discovered that combining quicklime with other materials created an extremely hard material that could be used to bind together other materials.
1793	John Smeaton found that the calcination of limestone containing clay produced a lime that hardened under water (hydraulic lime). He used hydraulic lime to rebuild Eddystone Lighthouse in Cornwall, England.

Eddystone Lighthouse

History of concrete contd…

1813 - 1813	Louis Vicat of France prepared artificial hydraulic lime by calcining synthetic mixtures of limestone and clay.
1816	The world's first unreinforced concrete bridge was built at Souillac, France.
1824	Joseph Aspdin, a British bricklayer, produced and patented the first Portland cement, made by burning finely pulverized lime and clay at high temperatures in kilns. The sintered product was then ground and he called it Portland cement since it looked like the high quality building stones quarried at Portland, England
1828	I. K. Brunel is credited with the first engineering application of Portland cement, which was used to fill a breach in the Thames Tunnel

History of concrete contd…

1887	Henri le Chatelier of France established oxide ratios to prepare the proper amount of lime to produce Portland cement
1894	Anatole de Baudot designs and builds the Church of St. Jean de Montmarte with slender concrete columns and vaults and enclosed by thin reinforced concrete walls
1900	Basic cement tests were standardized.
1903	The first concrete high rise was built in Cincinnati, Ohio.
1916	The Portland Cement Association was formed in Chicago.

Hoover dam (first concrete dam)

History of concrete contd…

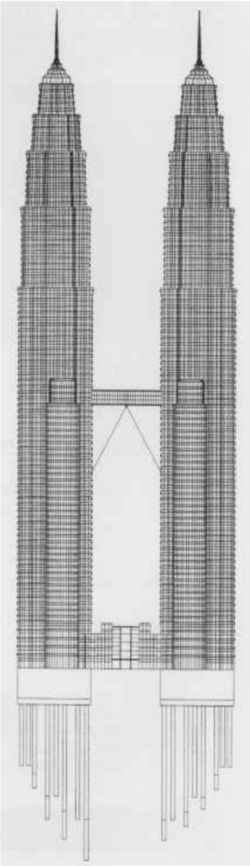
1917	The National Bureau of Standards (now the National Bureau of standards and Technology) and the American Society for Testing Materials established a standard formula for Portland cement.
1936	The first major concrete dams, Hoover Dam and Grand Coulee Dam, were built
1948	Pre-stressed concrete was introduced and first used in airport pavements.
1970	Fiber reinforcement in concrete was introduced.
1973	The Opera House in Sydney, Australia was opened. Its distinctive concrete peaks quickly became a symbol for the city.

Opera house (Sydney)

History of concrete contd…

1980	Superplasticizers were introduced as admixtures
1985	Silica fume was introduced as a pozzolanic additive.
1992	The tallest reinforced concrete building in the world was constructed at 311 South Wacker Drive in Chicago, Illinois. This was later surpassed by the Petronas Tower, Kualalumpur.
1993	The J. F. K. Museum in Boston, Massachusetts was completed. The dramatic concrete and glass structure was designed by renowned architect I. M. Pei.

Petronas Tower

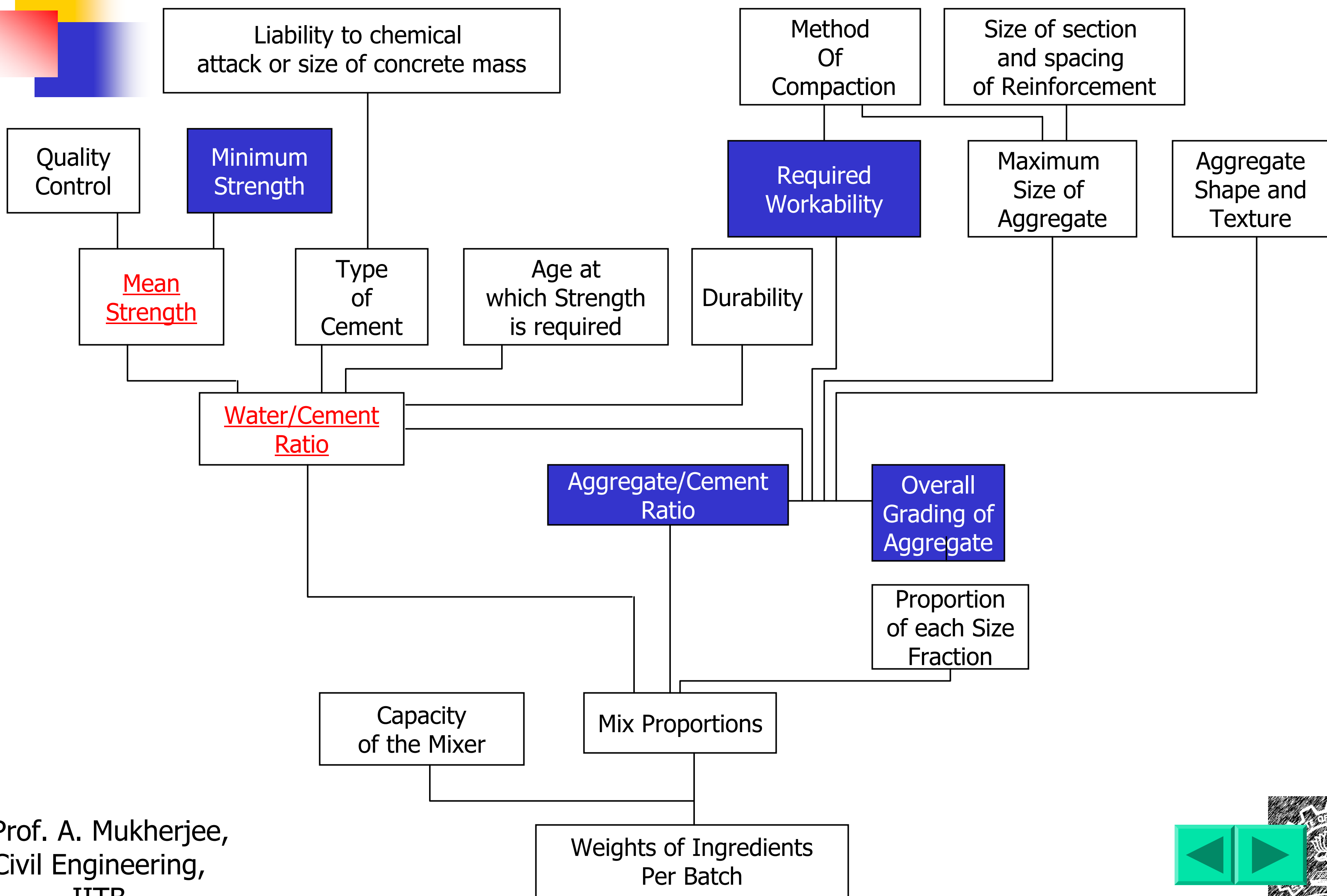


Concrete
(various strength up to grade 0)
160,000 cu m in the superstructure

Concrete Mix Design

The process of selecting suitable ingredients of concrete and determining their relative quantities with the object of producing as economically as possible concrete of certain minimum properties, notably consistence, strength, and durability.

Basic factors in the process of Mix Design



Basic definitions

- **Mean strength:** This is the average strength obtained by dividing the sum of strength of all the cubes by the number of cubes.

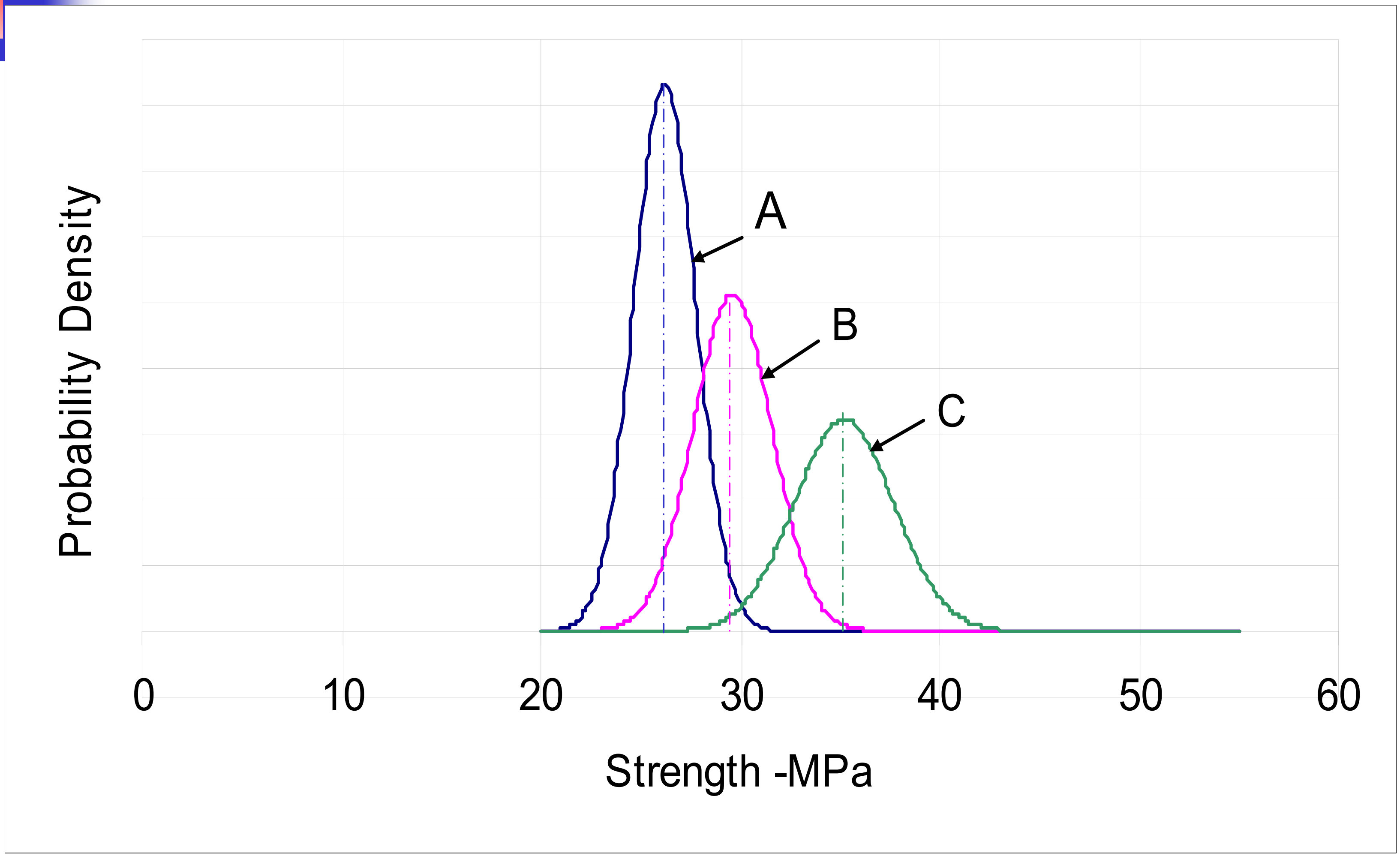
$$\bar{x} = \frac{\sum x}{n}$$

where $\bar{x}$ = mean strength

x = sum of strengths of cubes

n = number of cubes

Gaussian distribution curves for concretes with a minimum strength of 20.6 Mpa

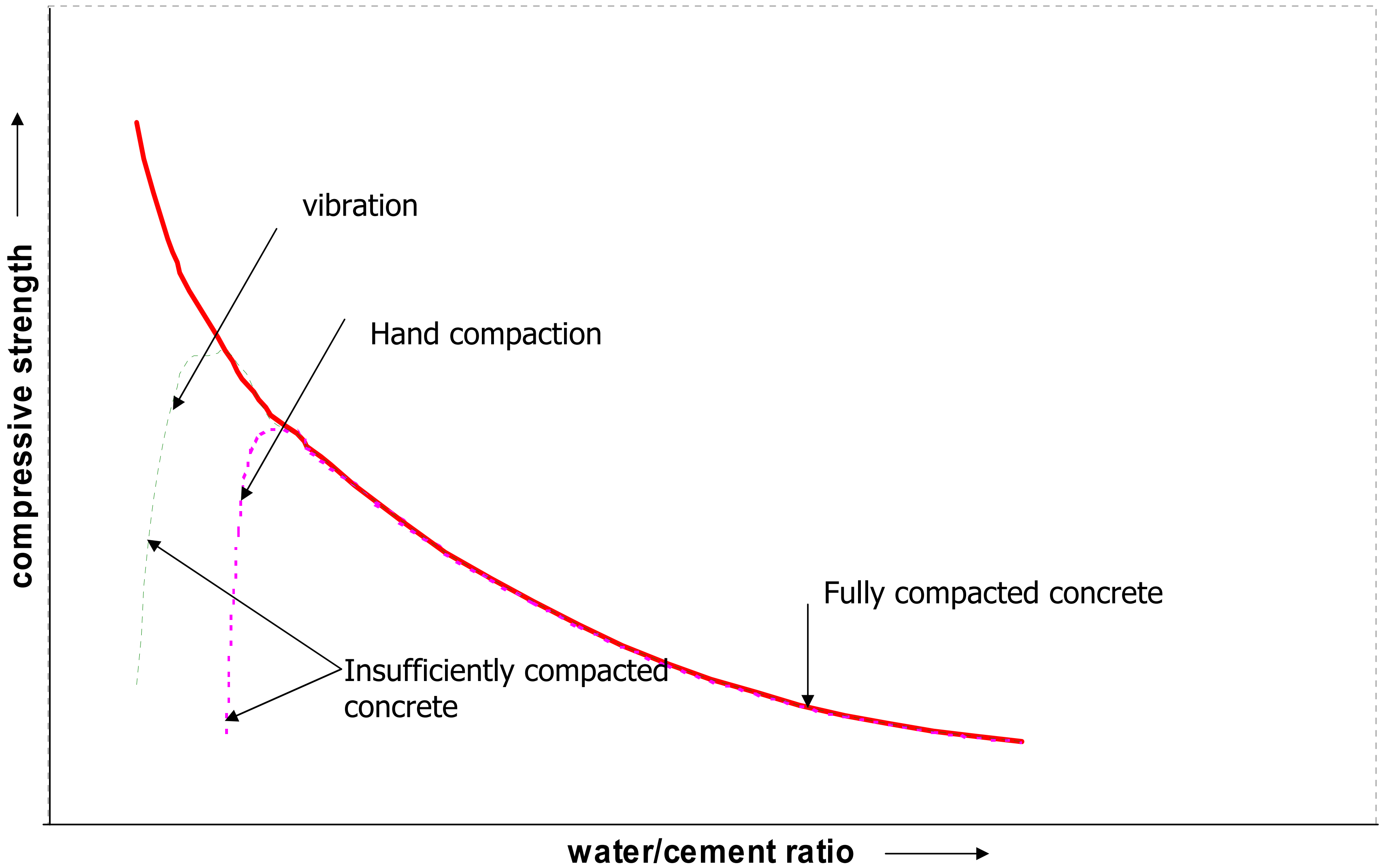


Percentage of Specimens having a strength lower than (Mean – k x Standard deviation)

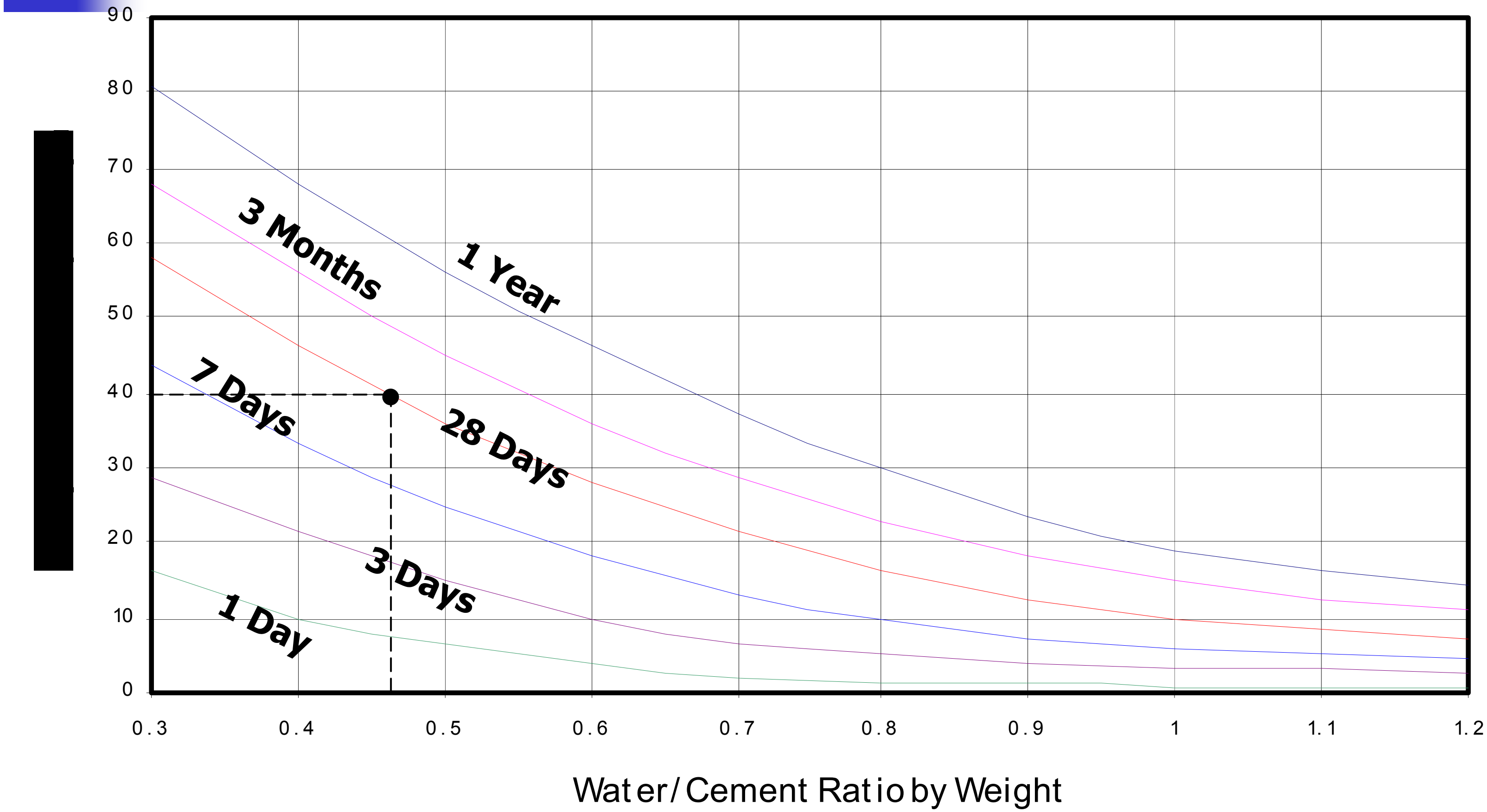
Degree of control

k	Percentage of specimen having a strength below than ($\bar{x} - k\sigma$)
1.00	15.9
1.50	6.7
1.96	2.5
2.33	1.0
2.50	0.6
3.09	0.1

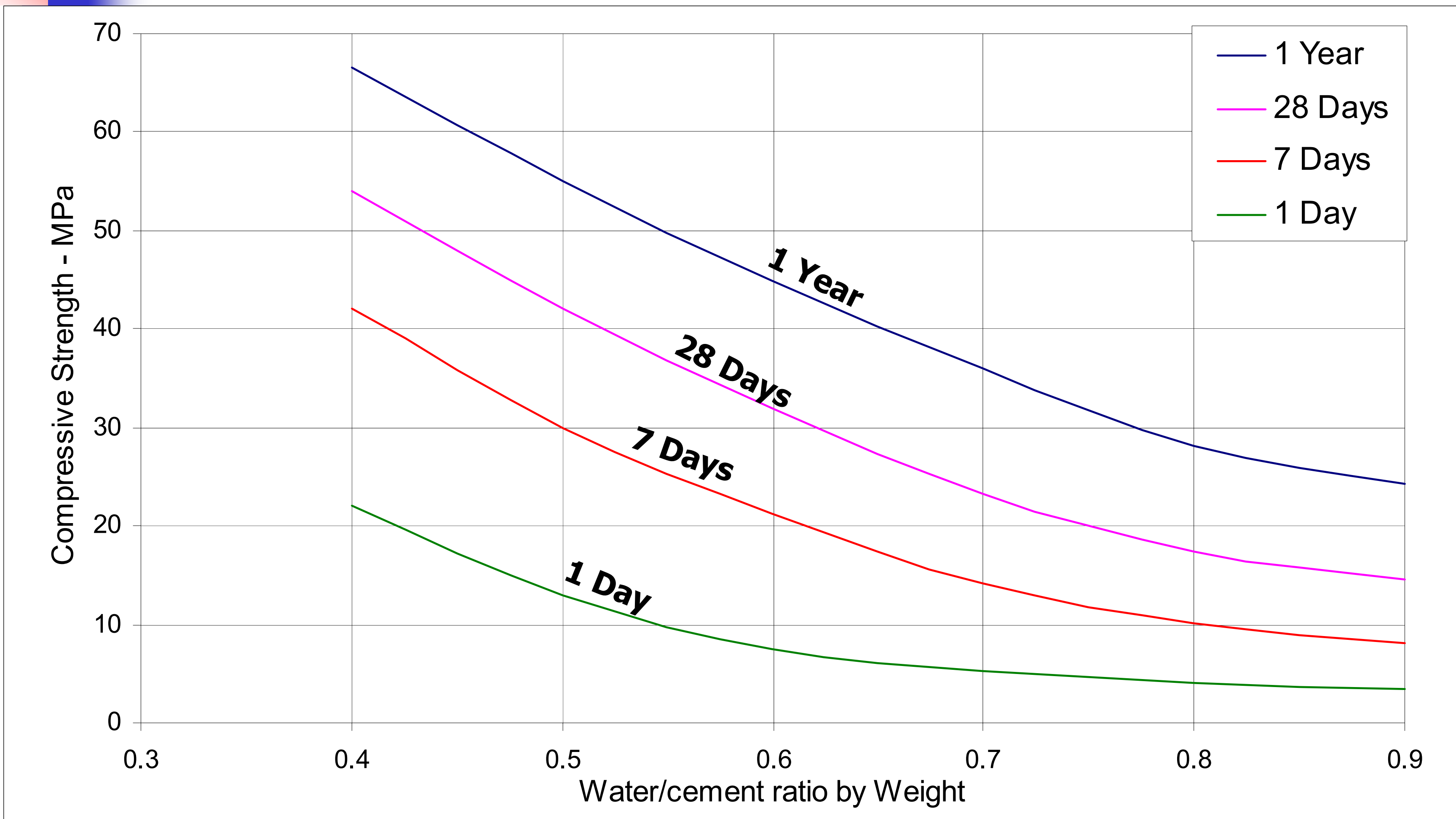
Water / Cement ratio



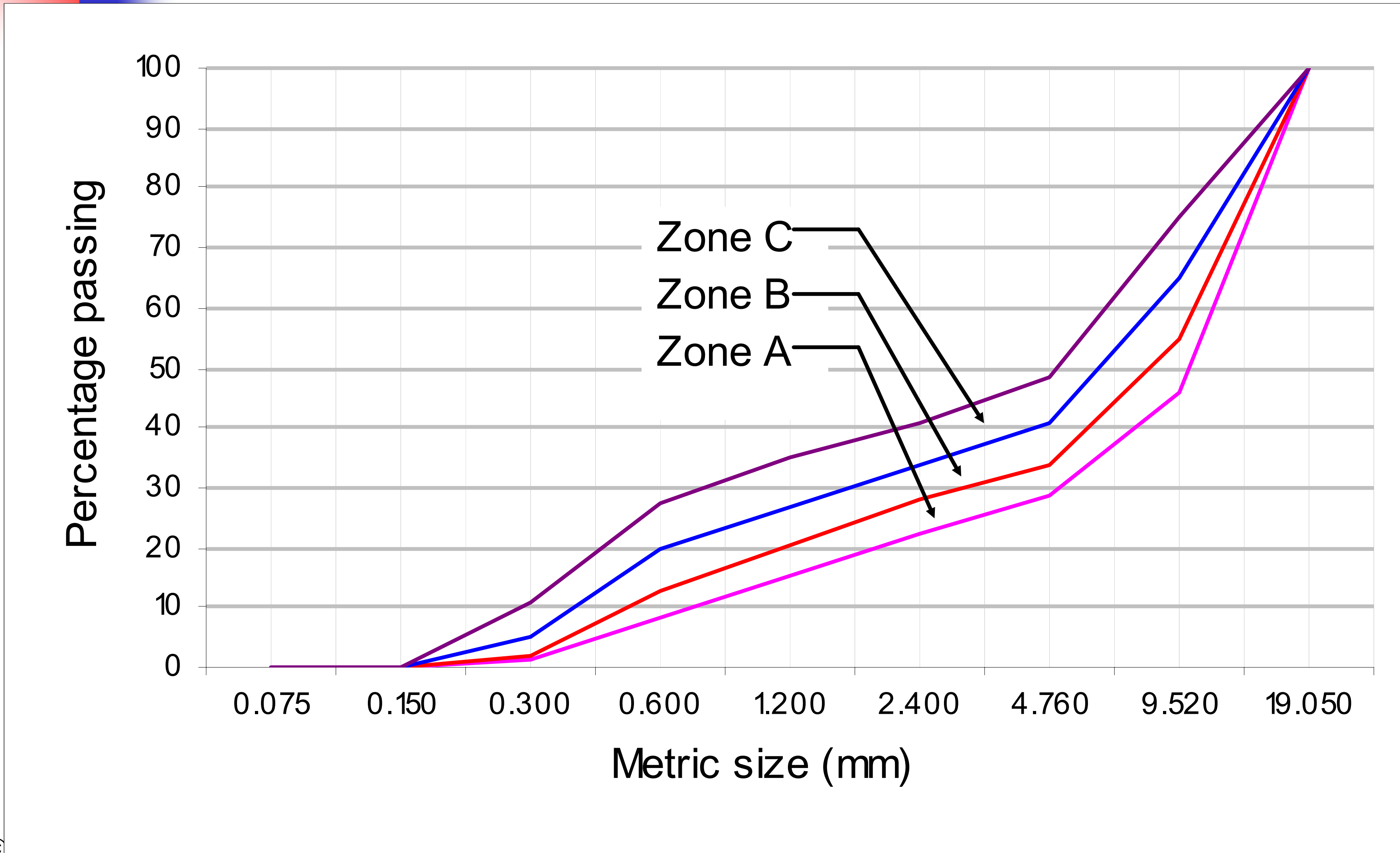
Relation between Compressive strength and Water/Cement Ratio for OPC of late 1950's



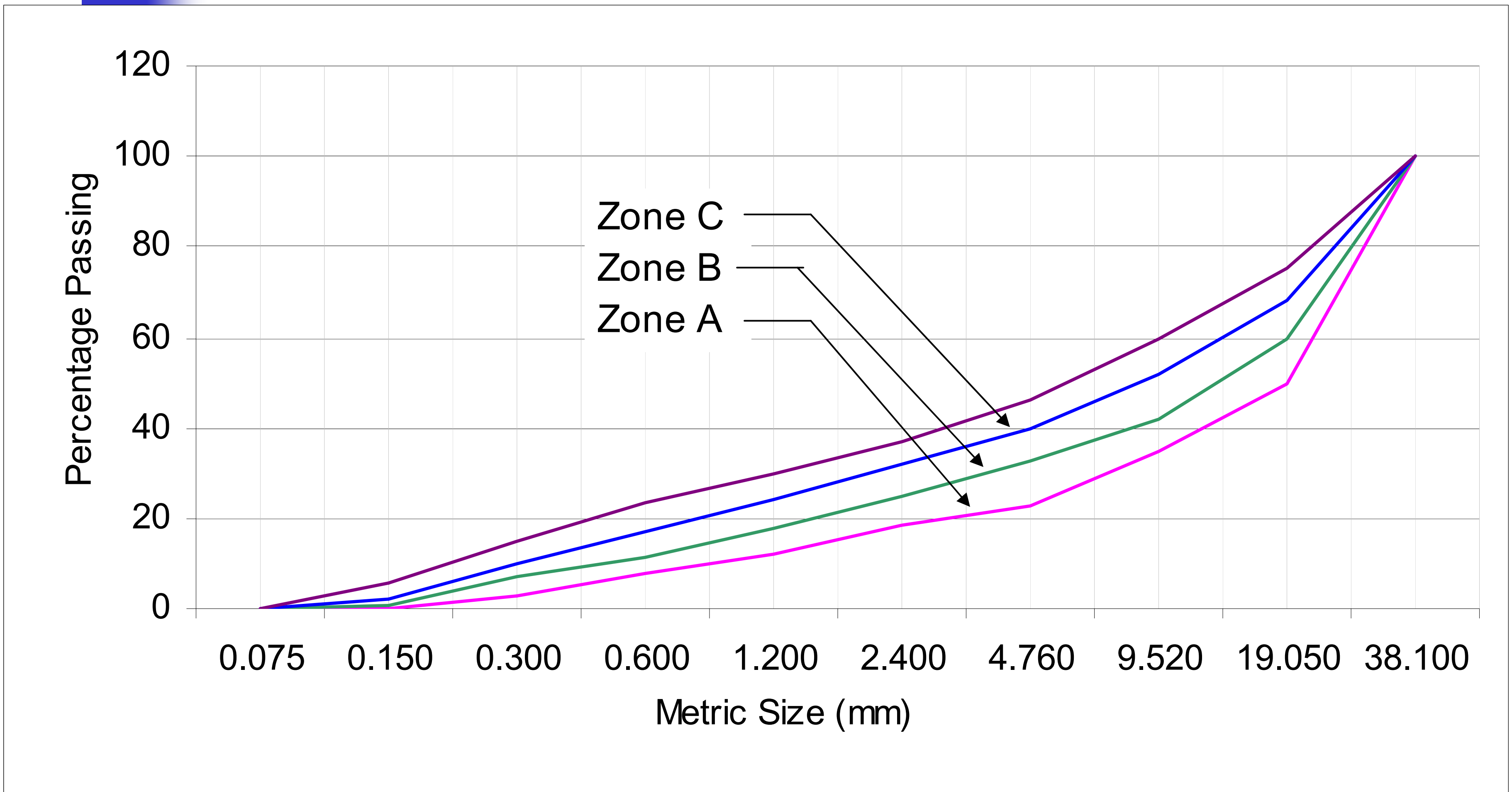
Relationship between Water/cement ratio and Compressive strength for OPC of late 1970's



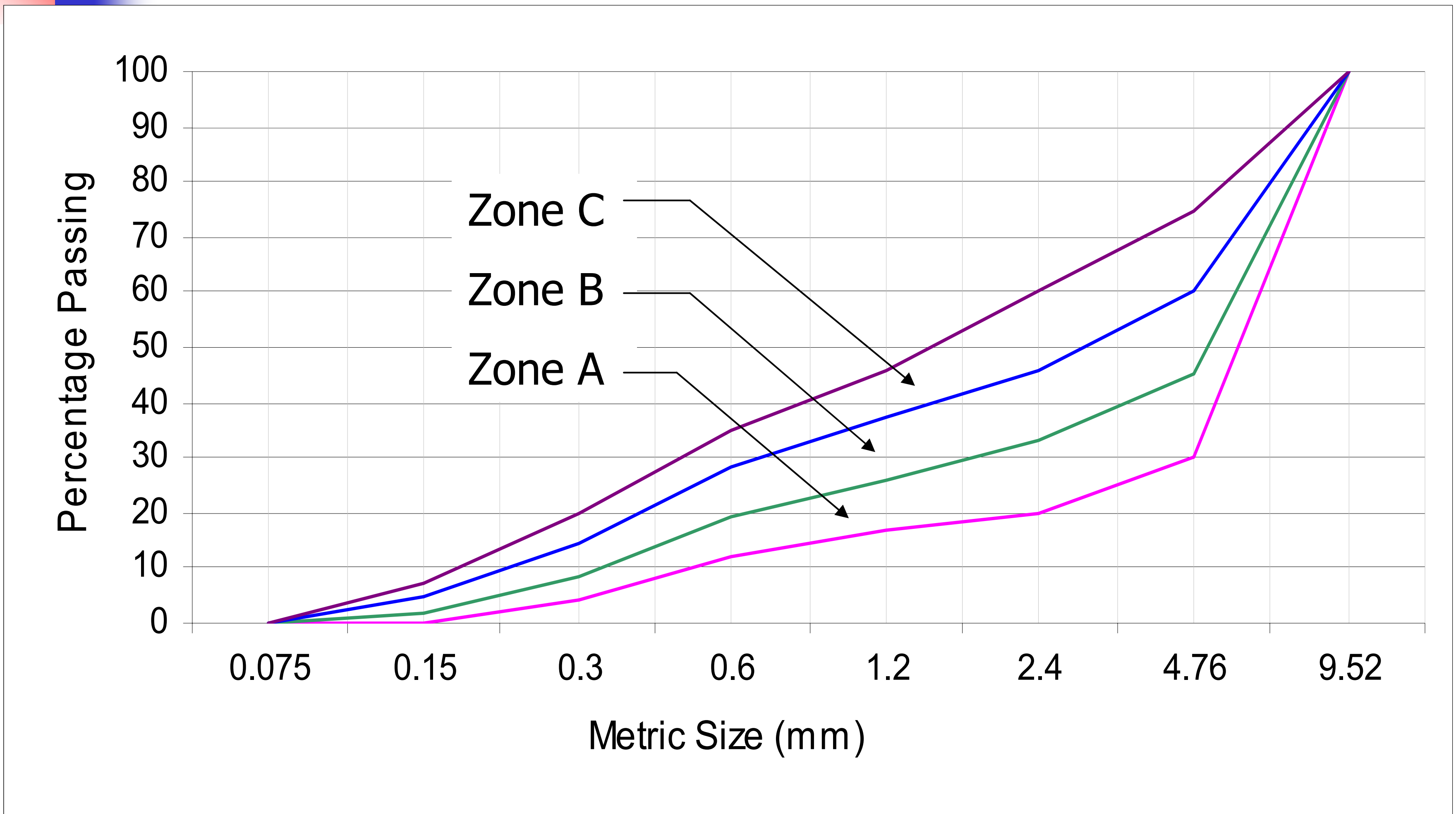
Road Note No. 4 type grading curves for 19.05 mm aggregate



Road Note No. 4 type grading curves for 38.1 mm aggregate.



McIntosh and Erntroy's type grading curves for 9.52mm aggregate



Aggregate/Cement Ratio (by weight) with different Gradings of 38.1mm Irregular Aggregate

Degree of Workability		Very low				Low				Medium				High			
Grading curve No. on Fig. 3.17		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Water/cement ratio by weight	0.35	4.0	3.9	3.5	3.2	3.4	3.3	3.2	2.9	2.9	2.8	2.6	2.5	2.7	2.5	2.3	2.3
	0.40	5.3	5.3	4.7	4.3	4.5	4.5	4.2	3.8	3.8	3.8	3.7	3.4	3.5	3.5	3.3	3.1
	0.45	6.5	6.5	5.9	5.3	5.6	5.6	5.3	4.8	4.6	4.7	4.6	4.3	4.1	4.4	4.3	4
	0.50	7.7	7.7	7.1	6.3	6.7	6.6	6.3	5.7	5.4	5.7	5.5	5.1	4.8	5.2	5.1	4.8
	0.55	-	-	8.1	7.3	7.6	7.6	7.2	6.6	6.2	6.5	6.3	5.8	x	5.9	6	5.5
	0.60			-	-	-	-	-	7.4	7.0	7.3	7.1	6.6	x	x	6.7	6.2
	0.65								8.1	7.8	8.1	7.8	7.2	x	x	7.3	6.9
	0.70								-	-	-	-	7.9	x	x	-	7.4
	0.75												-	x	x	-	8
	0.80													x	x	-	-

- Indicates that the mix was outside the range tested.

x Indicates that the mix would segregate.

These proportions are based on specific gravities of approximately 2.5 for the coarse aggregate and 2.6 for the fine aggregate.

Example

Mix design for road slab

- Minimum compressive strength (at 28 Days) = 28 MPa
- Method of compaction – Needle vibration
- Quality control – Good
- Workability – Very low
- Cement used – Ordinary Portland cement
- Aggregate shape – Irregular

Estimated relation between Minimum and Mean Compressive Strengths of Site Cubes with Additional Data on Coefficient of Variation

Degree of Control	Conditions	Minimum strength as a percentage of mean strength	Coefficient of Variation for the probability of a cube strength below the minimum occurring	
			once in 100	once in 200
Very Good	Weigh-batching; use of graded aggregates, moisture determinations on aggregates, etc. Constant supervision	75	10.7	9.7
Fair	Weigh -batching; use of two sizes of aggregate only; water content left to mixer-driver's judgenment. Occasional supervision	60	17.2	15.5
Poor	Inaccurate volume batching of all-in aggregate. No supervision	40	25.8	23.3

Steps in Mix design

- **Minimum strength =** 30 MPa
- **Calculation of Mean strength**

  mean strength = minimum strength / 0.75 (slide # 33)

  mean strength = 30 / 0.75 = 40 MPa
- **Determination of Water/Cement ratio**

  water/cement ratio = 0.48 (slide # 26)
- **Determination of Aggregate cement ratio**

  workability is very low and using water/cement ratio as 0.48 from slide # 31 we get,

  aggregate cement ratio = 7.2
- **Proportion**

  fine : 19.0 – 4.75 : 38.1 – 19.0 aggregates = 1 : 0.94 : 2.59

Steps in Mix design contd….

- Since the aggregate /cement ratio is 7.2, therefore the proportion of cement and aggregates is 1 : 1.59 : 1.50 : 4.11

- **Determination of cement content**

Materials	Sp. gravity
water	1.0
Cement	3.15
Coarse aggregate	2.50
Fine aggregate	2.60

Steps in Mix design contd….

Expression for calculation of cement content

$$\frac{W}{1000} + \frac{C}{1000 \times 3.15} + \frac{A_1}{1000 \times 2.60} + \frac{A_2}{1000 \times 2.50} = 1$$

where W, C, A_1, A_2 are the required weights of water, cement, fine aggregate, and coarse aggregate respectively

$$\frac{0.48 \times C}{1000} + \frac{C}{1000 \times 3.15} + \frac{1.59 \times C}{1000 \times 2.60} + \frac{(1.50 + 4.11) \times C}{1000 \times 2.50} = 1$$

Cement = 273.75 kg and hence,

Water = 0.48 x 273.75 = 131.4 kg

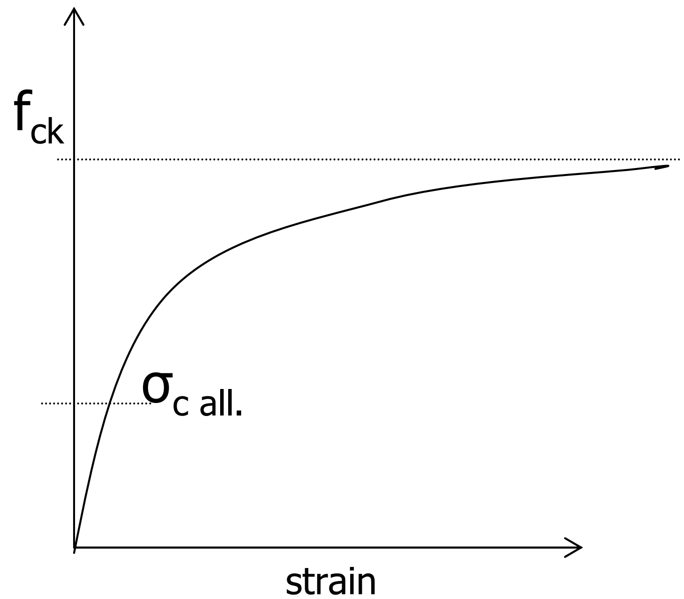
Fine aggregates = 1.59 x 273.75 = 435.26 kg

19.0 – 4.75 aggregates = 1.50 x 273.75 = 410.63 kg

38.1 – 19.0 aggregates = 4.11 x 273.75 = 1125.11 kg

Total = 2376.15 kg

Material graphs

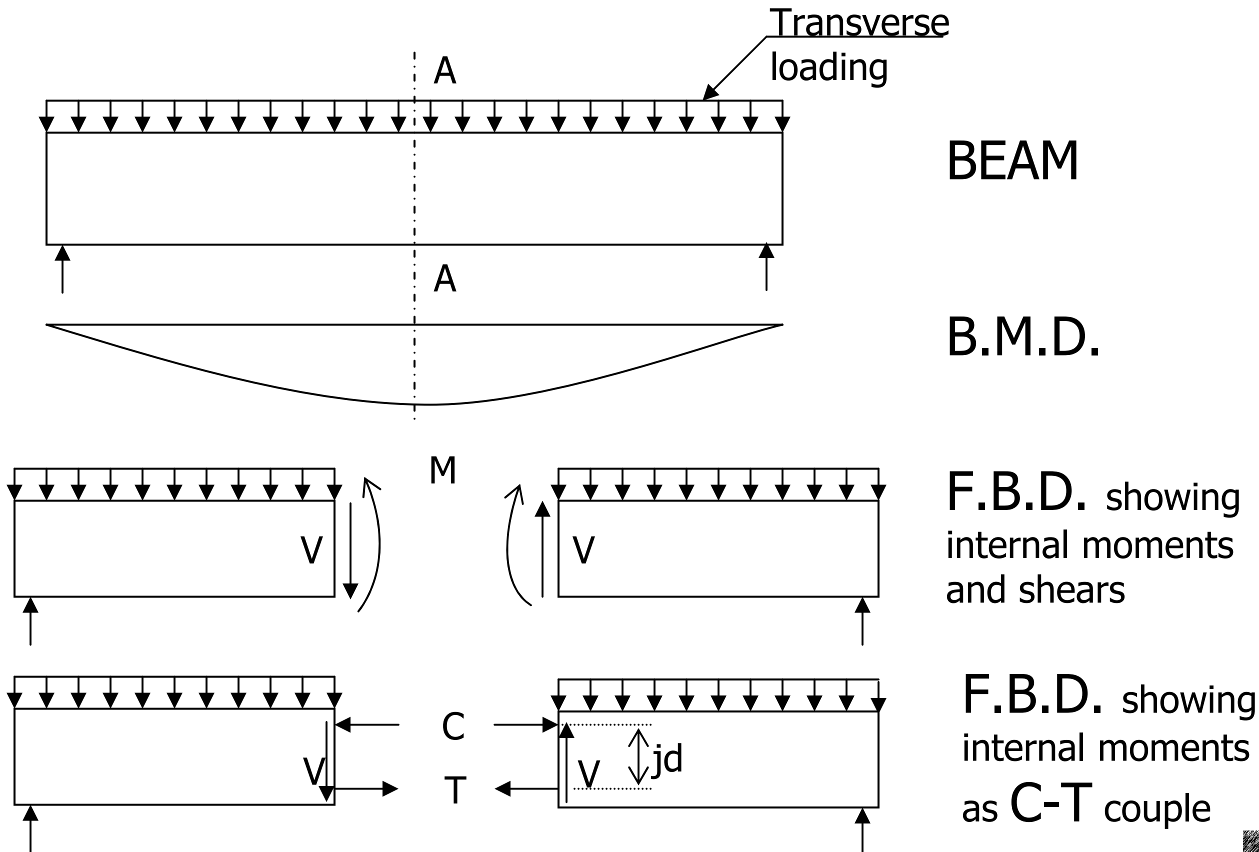


$$\sigma_{call.} = \frac{f_{ck}}{F.S.}$$

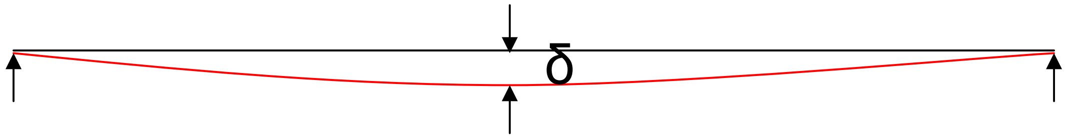
Structural Members

Flexural Member

Subjected to transverse loading and resists internal moments and shears.



Assumptions

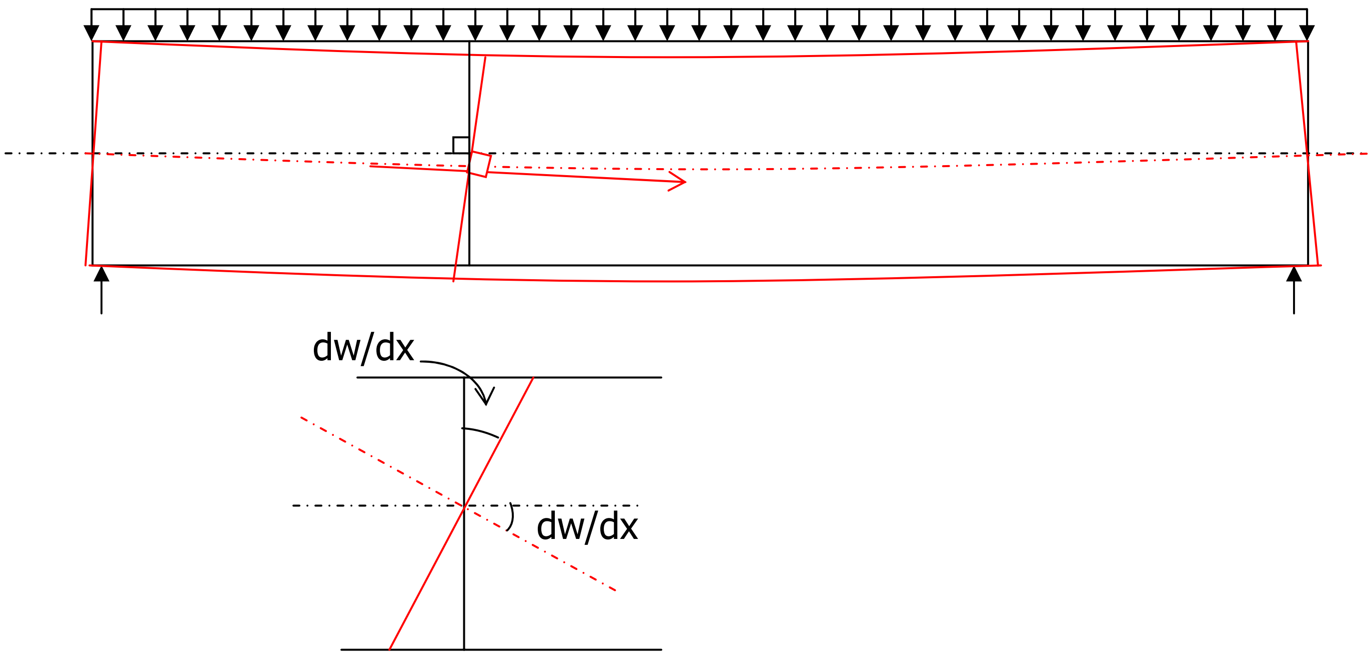


δ is very small.

- Length of the member remains same during bending; i.e. deformation is very small in comparison to the length.

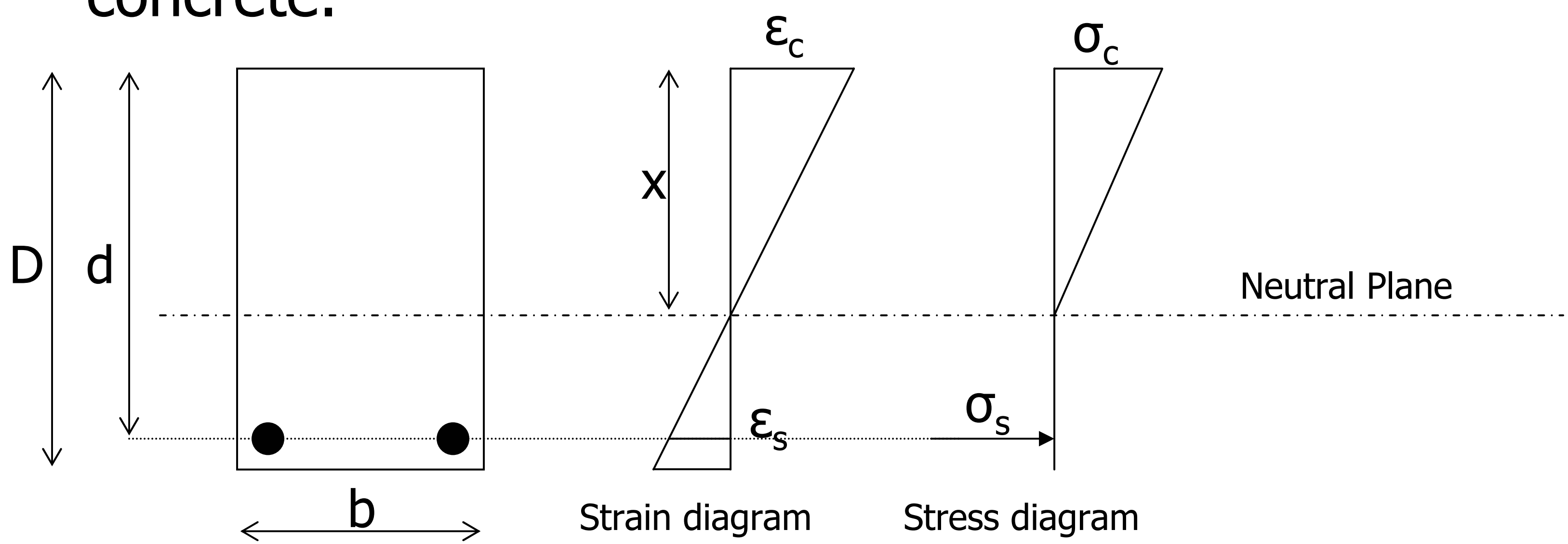
Assumptions…

- Plane sections remain plane during the process of bending (i.e. shear deformation is neglected)



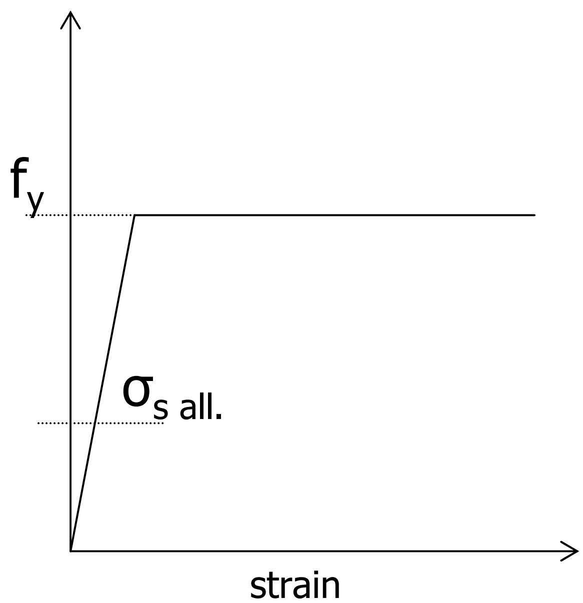
Assumptions…

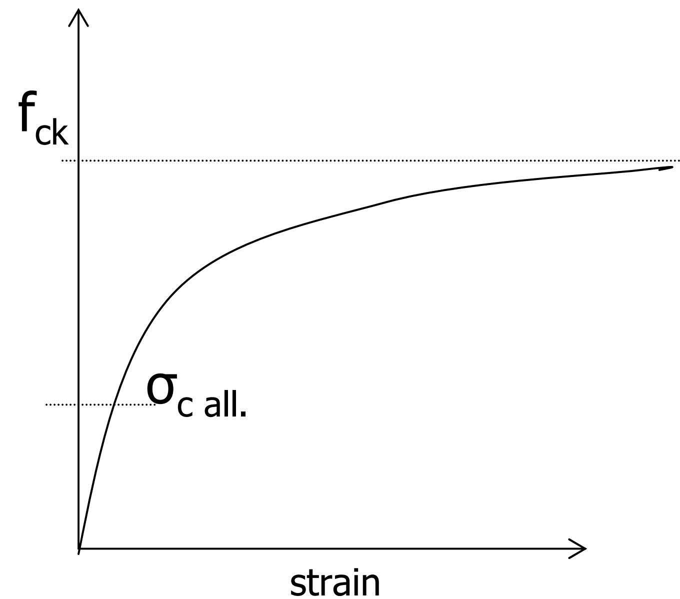
- All tensile stresses are taken by steel and none by concrete.



- No slippage between concrete and steel

- The stress-strain relationship of steel and concrete, under working loads, is a straight line.

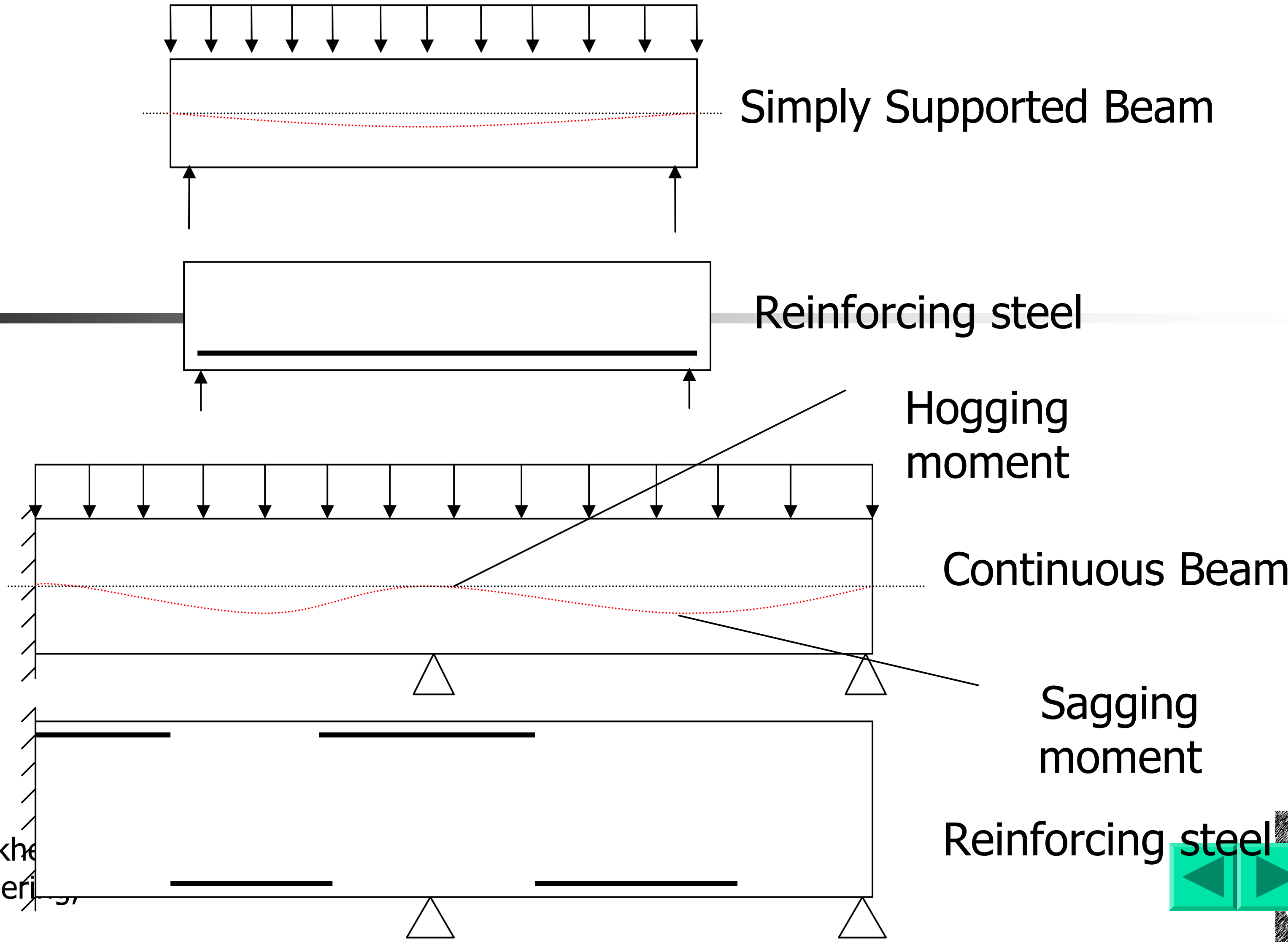




$$\sigma_{sall.} = \frac{f_y}{F.S.}$$

$$\sigma_{call.} = \frac{f_{ck}}{F.S.}$$

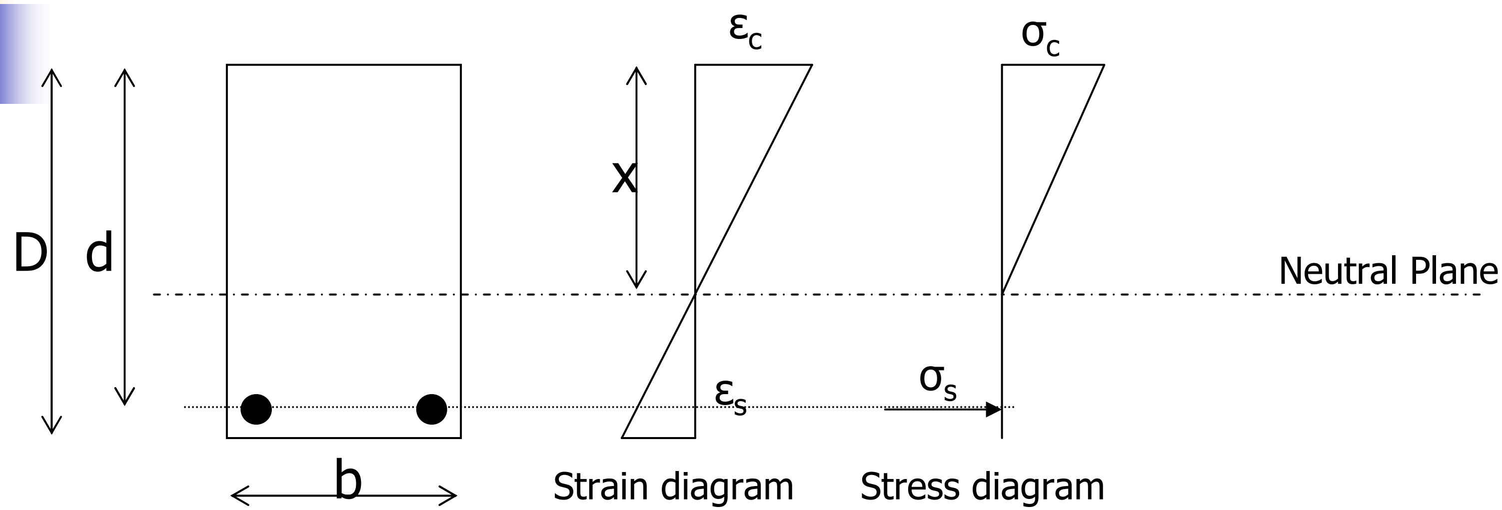
RCC Flexural Member



Modular ratio

$$m = \frac{E_s}{E_c} = \frac{280}{3\sigma_{c.all}}$$

- The modular ratio *m* has the value $280/(3\sigma_{c.all})$ where $\sigma_{c.all}$ is the allowable compressive stress (N/mm^2) in concrete due to bending.



d = Effective depth

x = Depth of Neutral axis

$\varepsilon_{c\ max}$ = Maximum compressive strain

$\varepsilon_{s\ max}$ = Maximum tensile strain

E_s = Young's modulus of steel

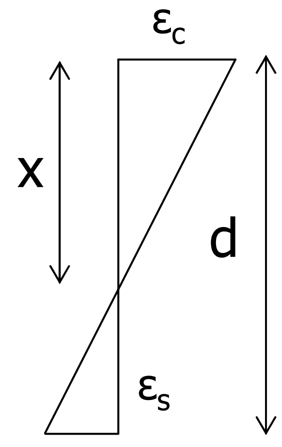
E_c = Young's modulus of concrete

m = E_s / E_c = Modular ratio

Compatibility Relationship:

$$\frac{\varepsilon_{c\,max}}{x} = \frac{\varepsilon_{s\,max}}{d - x}$$

$$\varepsilon_{s\max} = \frac{(d-x)}{x}\varepsilon_{c\max}$$



Strain diagram

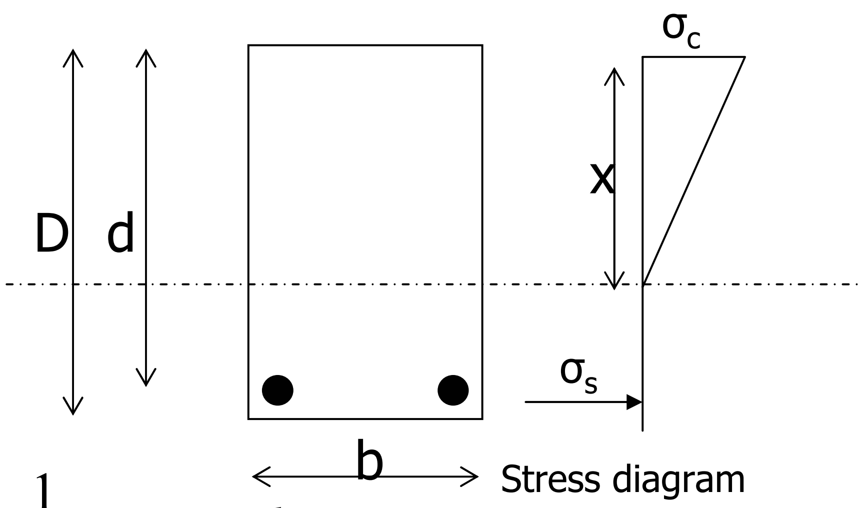
Constitutive Relationship :

$$\sigma_c = E_c \varepsilon_c \quad \& \quad \sigma_s = E_s \varepsilon_s$$

Modular Ratio = $m = \frac{E_s}{E_c}$

$$\sigma_s = mE_c\varepsilon_c$$

Equilibrium Equations :



1. $F_s = F_c$

But $$F_c = \int \sigma_c \, dA = \frac{1}{2} \sigma_{c\,max} \, x b$$

$$F_s = \sigma_s A_s$$

(Since the bar dia is small, we can take average stress σ_s.)

$$\frac{1}{2}\sigma_{c\max}xb = \sigma_s A_s$$

$$\frac{1}{2}E_c\varepsilon_{c\max}xb = mE_c\varepsilon_{s\max}A_s$$

$$\varepsilon_{c\max}xb = 2m\varepsilon_{s\max}A_s$$

$$\varepsilon_{c\max}xb = 2m\frac{(d-x)}{x}\varepsilon_{c\max}A_s$$

$$xb = 2m\frac{(d-x)}{x}A_s$$

$$x^2b = 2mdA_s - 2mxA_s$$

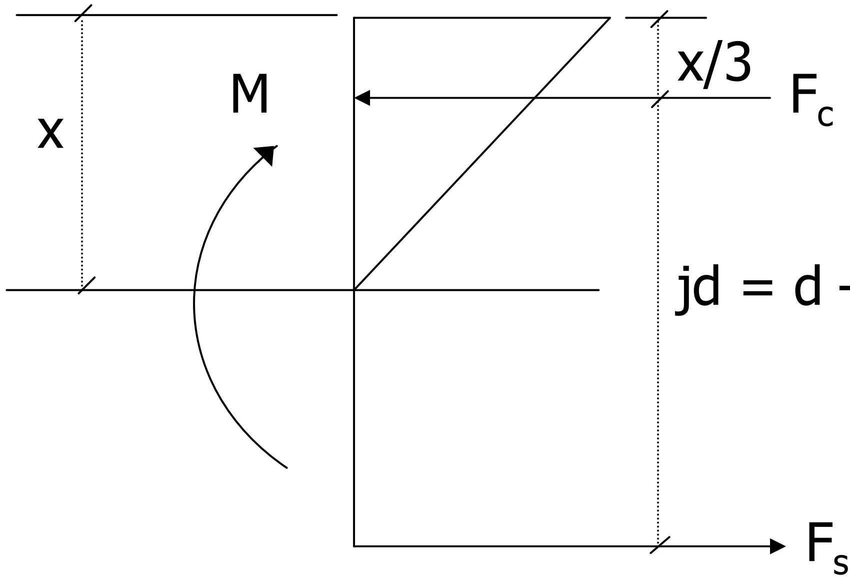
$$x^2b + 2mxA_s - 2mdA_s = 0$$

Therefore, $$x = \frac{-2mA_s \pm \sqrt{(2mA_s)^2 + 8mbdA_s}}{2b}$$, x < d

This is a property of cross section and materials.

Equilibrium eqns.

2. Taking moment about reinforcing steel,



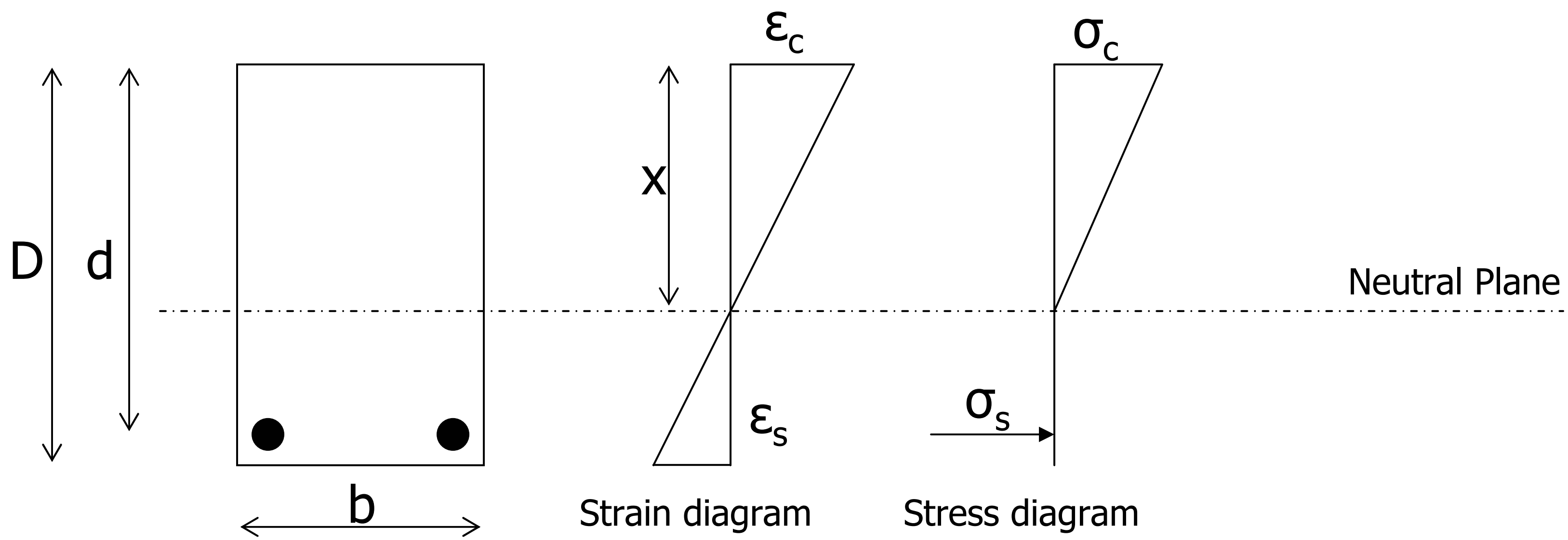
M = F_c jd = F_s jd

$$M = \frac{1}{2}\sigma_{c\max} xb(d - \frac{x}{3})$$

$$M = A_s \sigma_s (d - \frac{x}{3})$$

Balanced Section

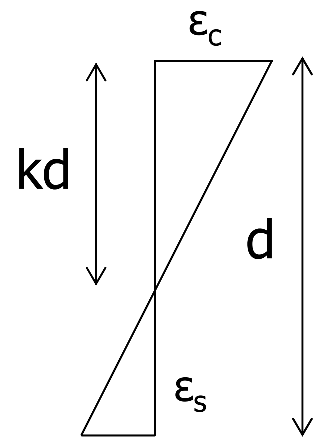
- Both steel and concrete fail simultaneously



Balanced section contd…

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d-x}$$

$$\frac{d-x}{x} = \frac{\varepsilon_s}{\varepsilon_c}$$



Strain diagram

$$Know,$$

$$m = \frac{280}{3*\sigma_{call}}$$

$$x_{bal} = kd$$

$$therefore,$$

$$\frac{d-kd}{kd} = \frac{\sigma_{sall}}{m*\sigma_{call}}$$

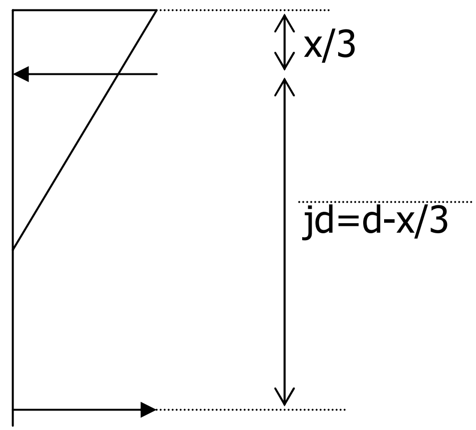
$$\frac{\sigma_{sall}}{93.33} = \frac{1}{k} - 1$$

$$\frac{\sigma_{sall}}{93.33} + 1 = \frac{1}{k}$$

therefore,

$$k = \frac{93.33}{\sigma_{sall} + 93.33}$$

k is the property of steel grade



$$\text{lever arm} = jd = d - \frac{x}{3}$$

$$j = 1 - \frac{k}{3}$$

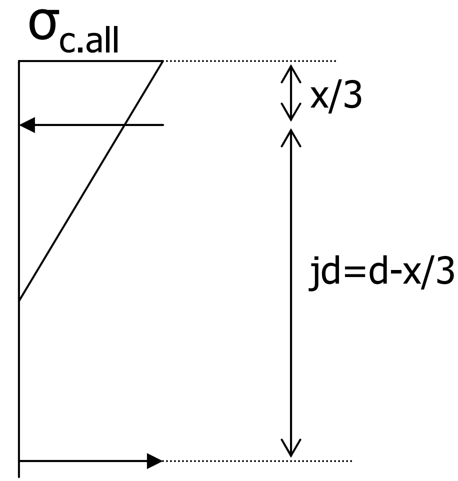
j is the property of steel grade

$$M_{all} = \frac{1}{2}\sigma_{call} x\, b\, (d - \frac{x}{3})$$

$$M_{all} = \frac{1}{2}\sigma_{call} k\, d\, b\, (d - \frac{kd}{3})$$

$$M_{all} = [\frac{k}{2}(1 - \frac{k}{3})]\ \sigma_{call}\ b\, d^2$$

Steel grade — $[\frac{k}{2}(1 - \frac{k}{3})]$; Concrete grade — σ_{call}; Cross section — $b\, d^2$



$$\frac{M_{all}}{b\, d^2} = R$$

R=Moment of resistance factor

depends on material properties

Also,

$$M_{all} = \sigma_{sall}\ A_s\ (d - \frac{kd}{3})$$

$$\frac{\mathrm{M_{all}}}{b\,d^2} = R$$

Also,

$$\mathrm{M_{all}} = \sigma_{sall}\,A_s\,(d\text{-}\frac{kd}{3})$$

$$A_s = \frac{M_{all}}{\sigma_s jd}$$

$$\frac{A_s}{bd} = \frac{M_{all}}{\sigma_s jbd^2}$$

$$p = \frac{1}{\sigma_s j}\frac{M_{all}}{bd^2}$$

Relation between p and M/bd^2 is dependent on material only

Design constants for Balanced Section

Steel / Concrete		Fe250				Fe415			
		σ_{sall} = 140 N/mm^2				σ_{sall} = 230 N/mm^2			
Grade	σ_{call}	k	j	R	pt	k	j	R	pt
M20	7.0	0.4	0.87	1.22	1.00	0.29	0.9	0.91	0.44
M25	8.5	0.4	0.87	1.48	1.21	0.29	0.9	1.11	0.54
M30	10.0	0.4	0.87	1.74	1.43	0.29	0.9	1.31	0.63

Design example

Given IS 456-2000

- Moment (M) = 20KN-m
- Steel Grade is Fe415; σ_{sall} = 230MPa refer table 22
- Concrete Grade is M20; σ_{call} = 7 MPa refer table 21

To Find

- Effective depth 'd'
- Area of steel 'A_{st}'

Solution

Assume b = 230mm

For Fe 415 grade steel and M20 grade concrete

R = 0.91 ; pt = 0.44

Now,

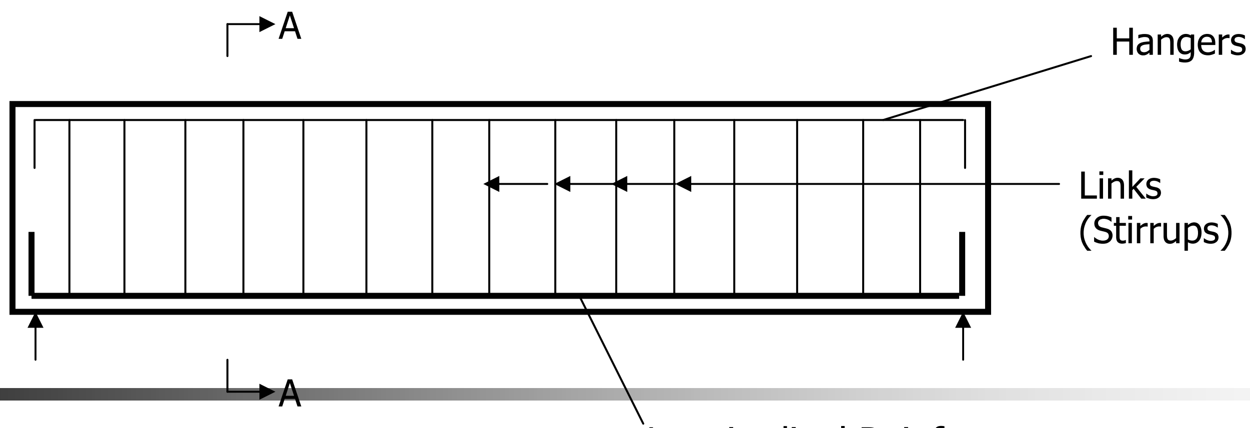
$d = \sqrt{(M_{all} / R*b)}$

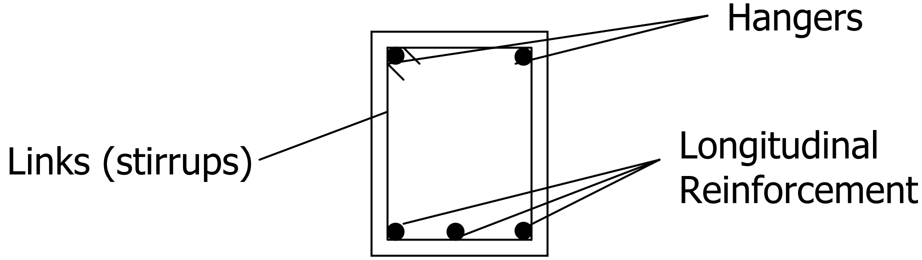
$= \sqrt{(20*10^6 / 0.91*230)}$

$= 309.122 \simeq$ <u>310 mm</u>

A_{st} = pt*b*d/100 = 0.44*230*310/100 = <u>313.72 mm^2.</u>

Reinforced Beam



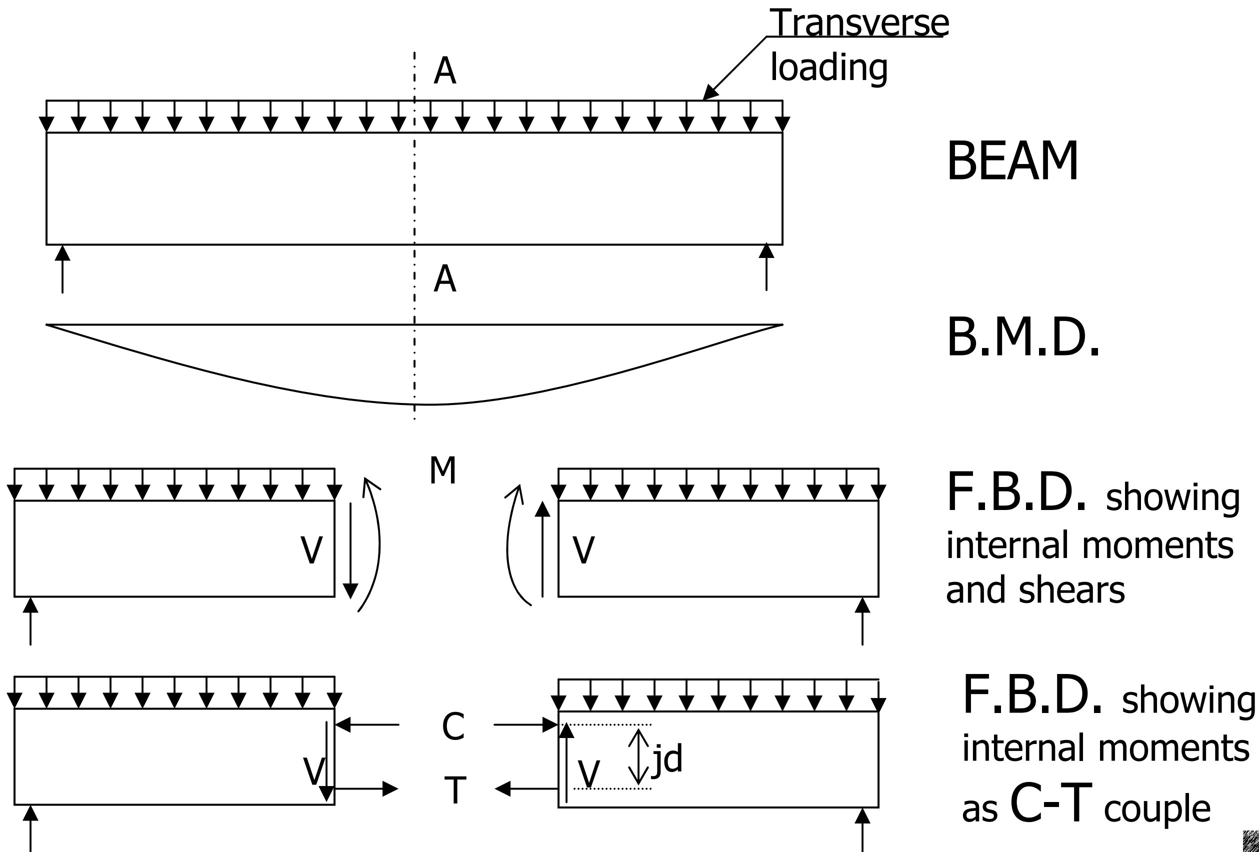


Section A-A

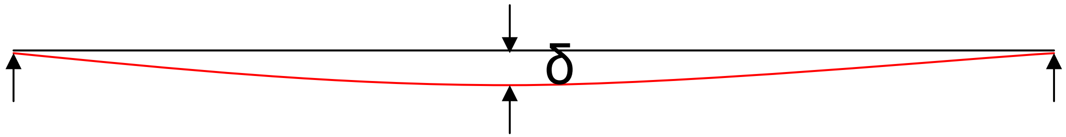
Structural Members

Flexural Member

Subjected to transverse loading and resists internal moments and shears.



Assumptions

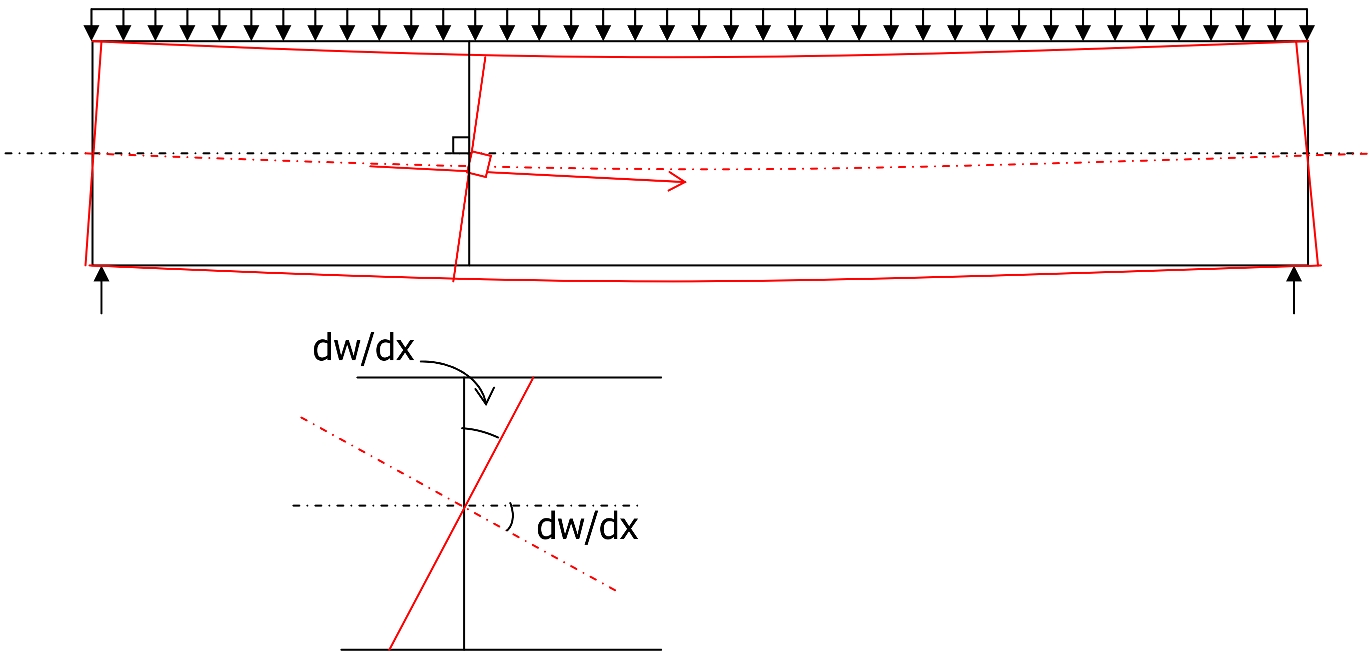


δ is very small.

- Length of the member remains same during bending; i.e. deformation is very small in comparison to the length.

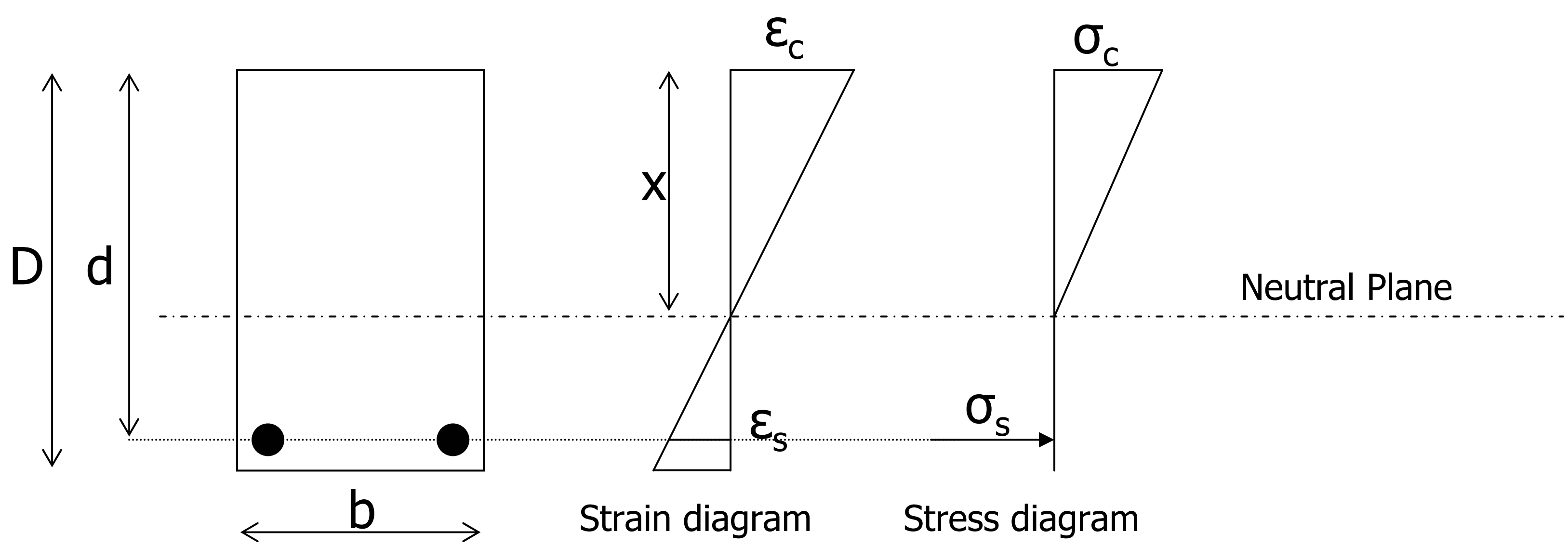
Assumptions…

- Plane sections remain plane during the process of bending (i.e. shear deformation is neglected)



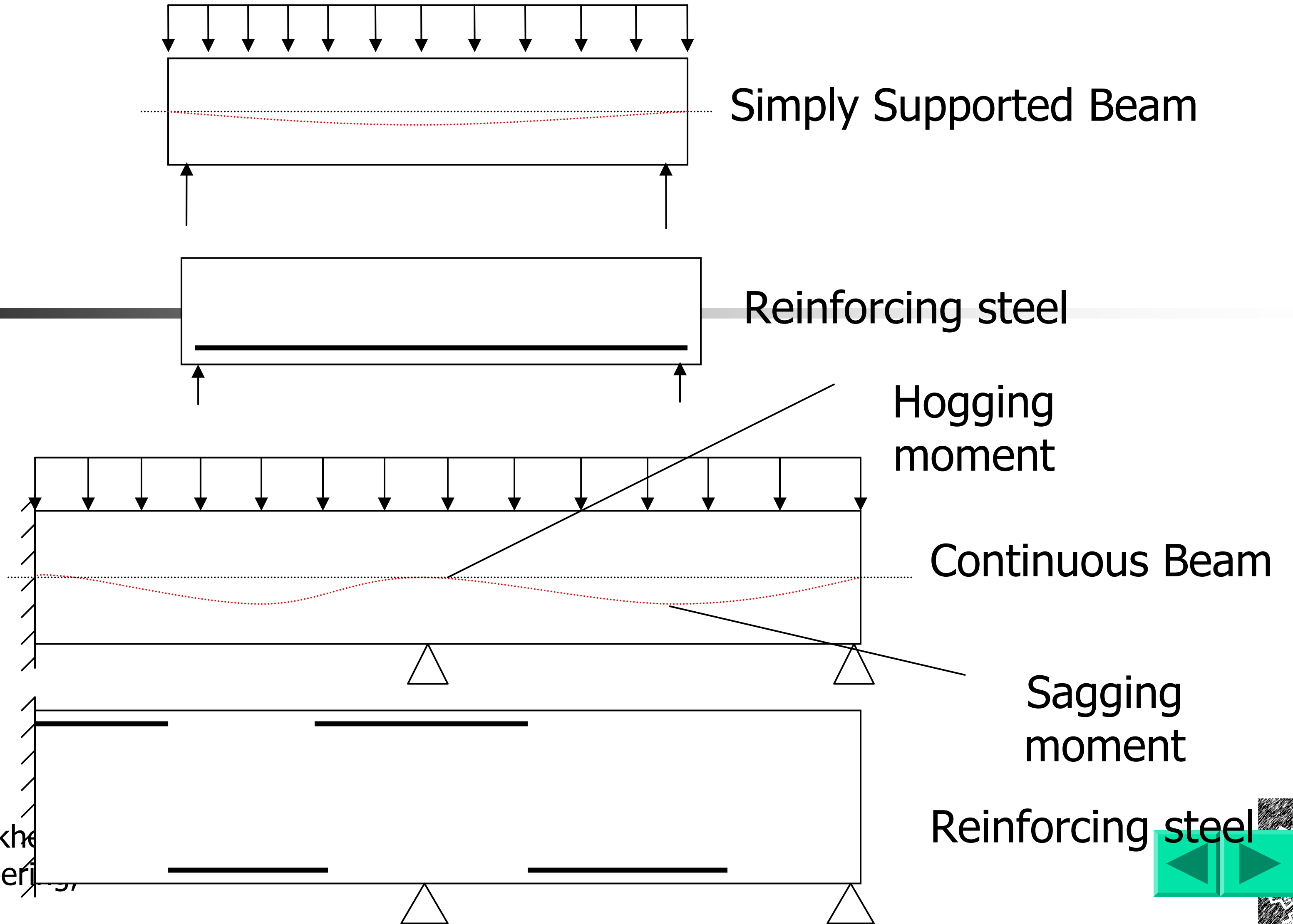
Assumptions…

- All tensile stresses are taken by steel and none by concrete.

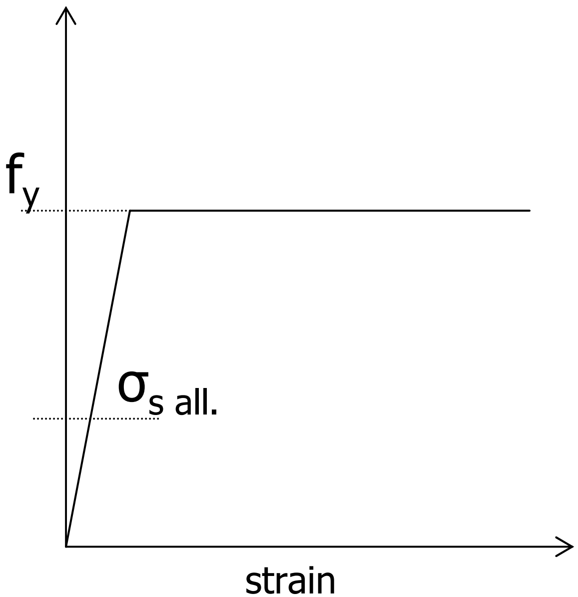


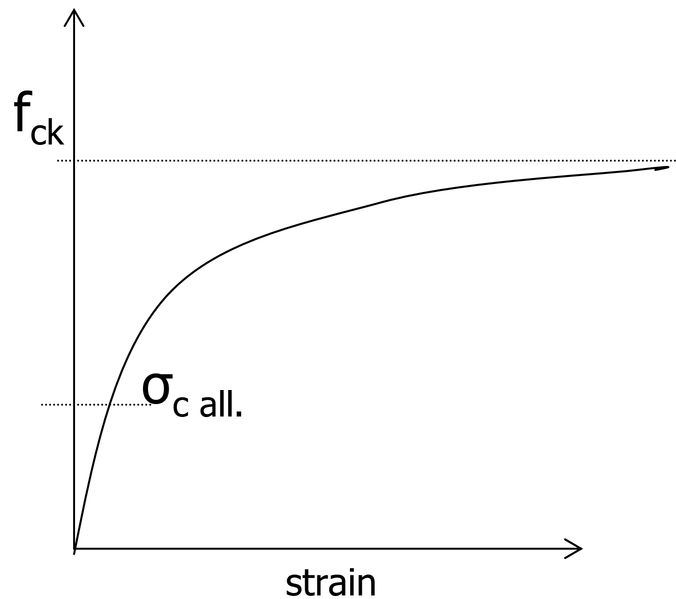
- No slippage between concrete and steel

RCC Flexural Member



- The stress-strain relationship of steel and concrete, under working loads, is a straight line.





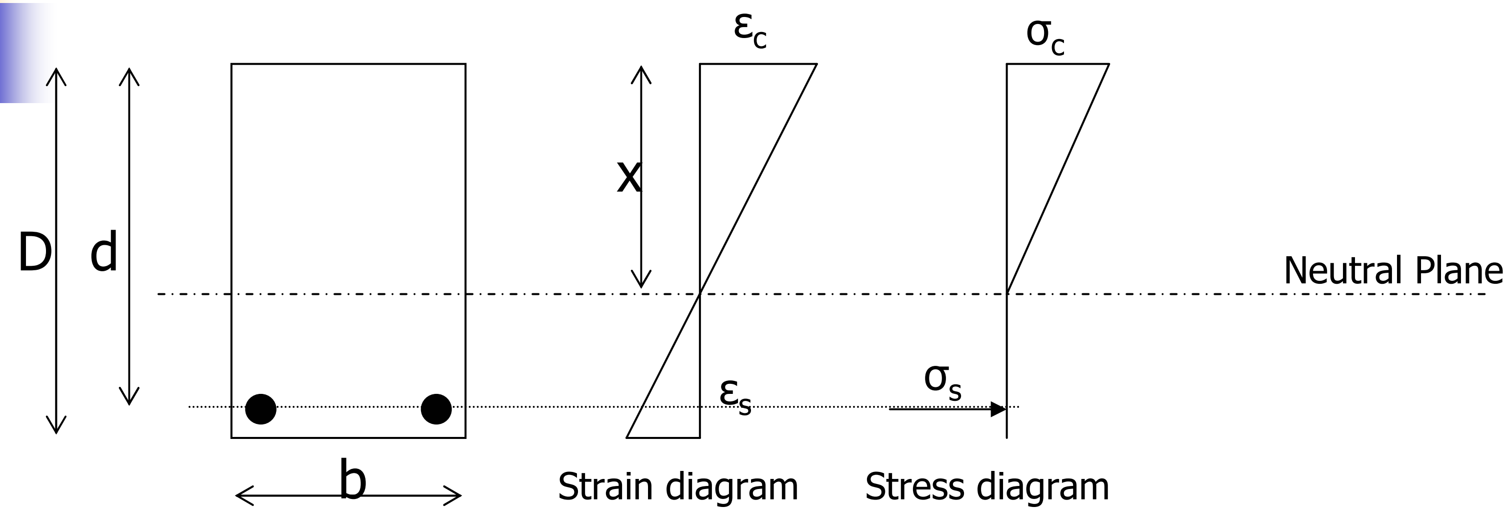
$$\sigma_{sall.} = \frac{f_y}{F.S.}$$

$$\sigma_{call.} = \frac{f_{ck}}{F.S.}$$

Modular ratio

$$m = \frac{E_s}{E_c} = \frac{280}{3\sigma_{c.all}}$$

- The modular ratio *m* has the value 280/(3$\sigma_{c.all}$) where $\sigma_{c.all}$ is the allowable compressive stress (N/mm^2) in concrete due to bending.



d = Effective depth

x = Depth of Neutral axis

$\varepsilon_{c\ max}$ = Maximum compressive strain

$\varepsilon_{s\ max}$ = Maximum tensile strain

E_s = Young's modulus of steel

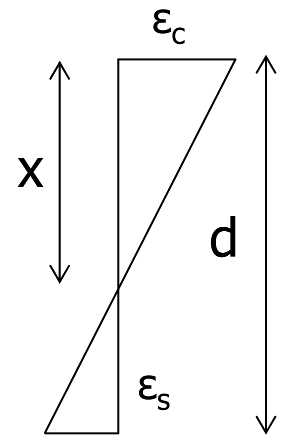
E_c = Young's modulus of concrete

m = E_s / E_c = Modular ratio

Compatibility Relationship:

$$\frac{\varepsilon_{c\,max}}{x} = \frac{\varepsilon_{s\,max}}{d - x}$$

$$\varepsilon_{s\,max} = \frac{(d-x)}{x}\varepsilon_{c\,max}$$



Strain diagram

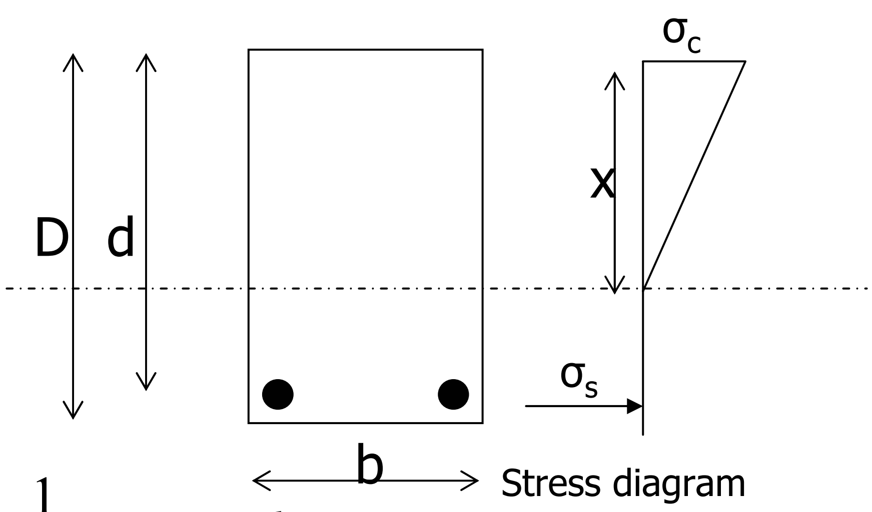
Constitutive Relationship :

$$\sigma_c = E_c \varepsilon_c \qquad \& \qquad \sigma_s = E_s \varepsilon_s$$

Modular Ratio = $m = \frac{E_s}{E_c}$

$$\sigma_s = mE_c\varepsilon_c$$

Equilibrium Equations :



1. $F_s = F_c$

But $$F_c = \int \sigma_c \, dA = \frac{1}{2}\sigma_{c\max} x b$$

$$F_s = \sigma_s A_s$$

(Since the bar dia is small, we can take average stress σ_s.)

$$\frac{1}{2}\sigma_{c\max} xb = \sigma_s A_s$$

$$\frac{1}{2}E_c \varepsilon_{c\max} xb = mE_c \varepsilon_{s\max} A_s$$

$$\varepsilon_{c\max} xb = 2m\varepsilon_{s\max} A_s$$

$$\varepsilon_{c\max} xb = 2m\frac{(d-x)}{x}\varepsilon_{c\max} A_s$$

$$xb = 2m\frac{(d-x)}{x}A_s$$

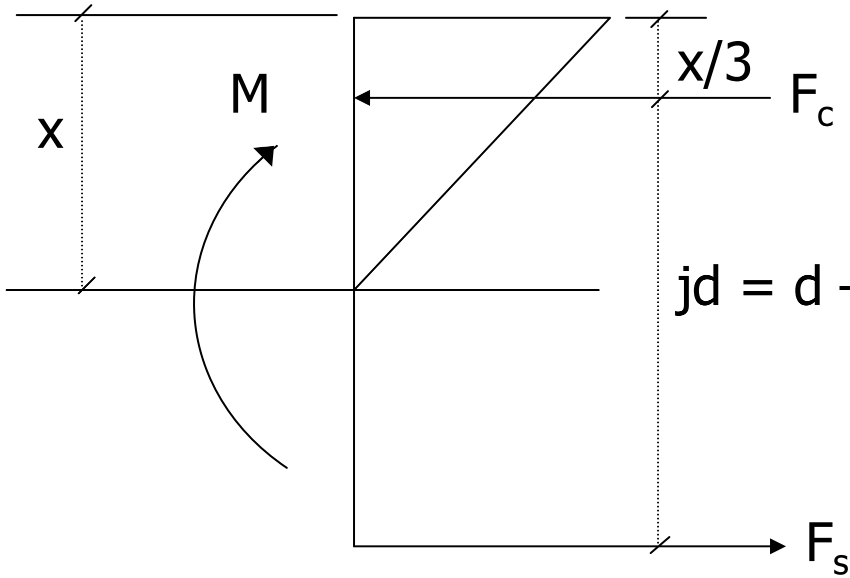
$$x^2 b = 2mdA_s - 2mxA_s$$

$$x^2 b + 2mxA_s - 2mdA_s = 0$$

Therefore, $$x = \frac{-2mA_s \pm \sqrt{(2mA_s)^2 + 8mbdA_s}}{2b}$$, $x < d$

This is a property of cross section and materials.

2. Taking moment about reinforcing steel,



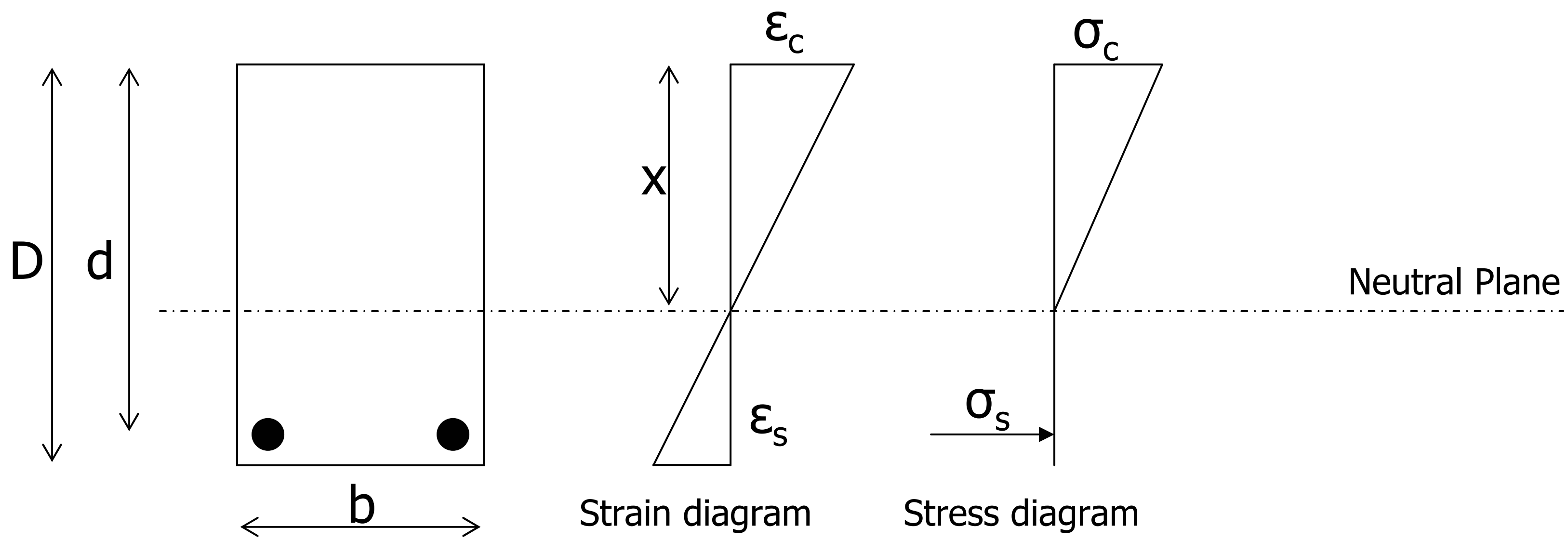
$M = F_c\, jd = F_s\, jd$

$$M = \frac{1}{2}\sigma_{c\max} x b\left(d - \frac{x}{3}\right)$$

$$M = A_s \sigma_s \left(d - \frac{x}{3}\right)$$

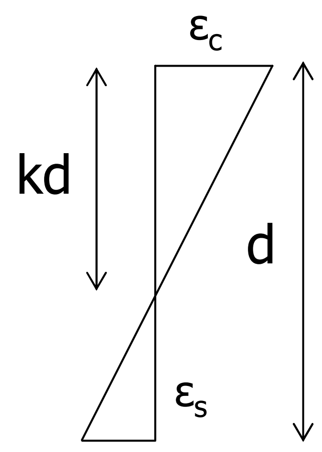
Balanced Section

- Both steel and concrete fail simultaneously



$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d-x}$$

$$\frac{d-x}{x} = \frac{\varepsilon_s}{\varepsilon_c}$$



Strain diagram

$Know,$

$$m = \frac{280}{3*\sigma_{call}}$$

$$x_{bal} = kd$$

$therefore,$

$$\frac{d-kd}{kd} = \frac{\sigma_{sall}}{m*\sigma_{call}}$$

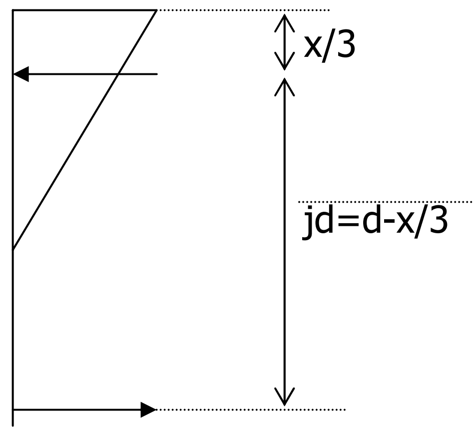
$$\frac{\sigma_{sall}}{93.33} = \frac{1}{k} - 1$$

$$\frac{\sigma_{sall}}{93.33} + 1 = \frac{1}{k}$$

therefore,

$$k = \frac{93.33}{\sigma_{sall} + 93.33}$$

k is the property of steel grade



$$\text{lever arm} = jd = d - \frac{x}{3}$$

$$j = 1 - \frac{k}{3}$$

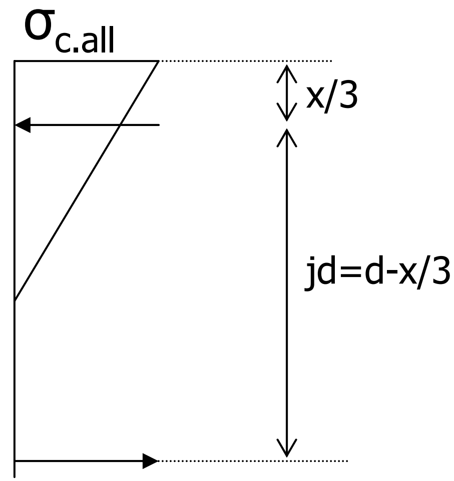
j is the property of steel grade

$$M_{all} = \frac{1}{2}\sigma_{call} x\, b\, (d - \frac{x}{3})$$

$$M_{all} = \frac{1}{2}\sigma_{call} k\, d\, b\, (d - \frac{kd}{3})$$

$$M_{all} = [\frac{k}{2}(1 - \frac{k}{3})]\ \sigma_{call}\ b\, d^2$$

Steel grade — Concrete grade — Cross section



$$\frac{M_{all}}{b\, d^2} = R$$

R=Moment of resistance factor

Also,

depends on material properties

$$M_{all} = \sigma_{sall}\ A_s\ (d - \frac{kd}{3})$$

$$\frac{M_{all}}{b\,d^2} = R$$

Also,

$$M_{all} = \sigma_{sall}\, A_s\, (d - \frac{kd}{3})$$

$$A_s = \frac{M_{all}}{\sigma_s jd}$$

$$\frac{A_s}{bd} = \frac{M_{all}}{\sigma_s jbd^2}$$

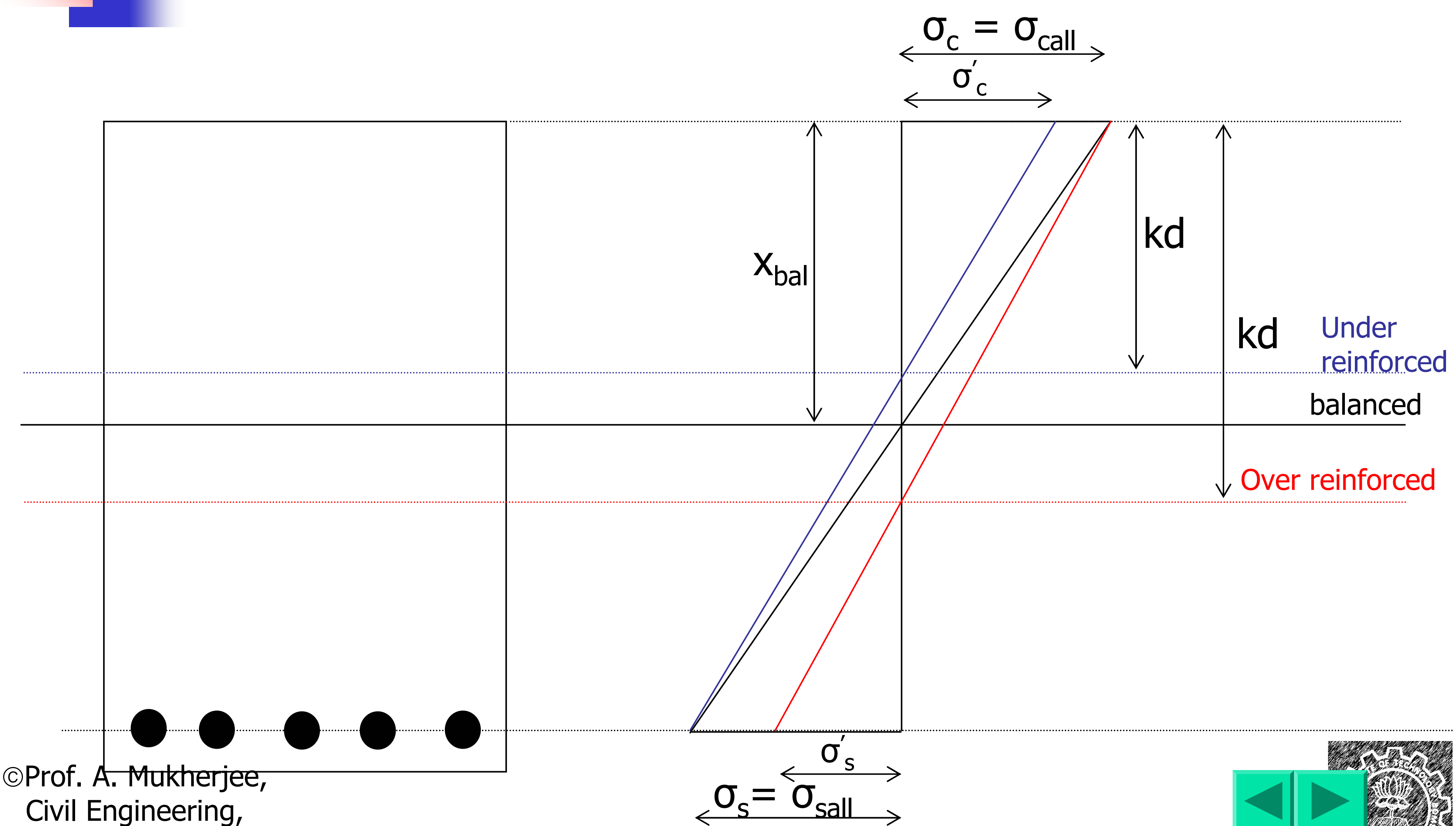
$$p = \frac{1}{\sigma_s j}\frac{M_{all}}{bd^2}$$

Relation between p and M/bd^2 is dependent on material only

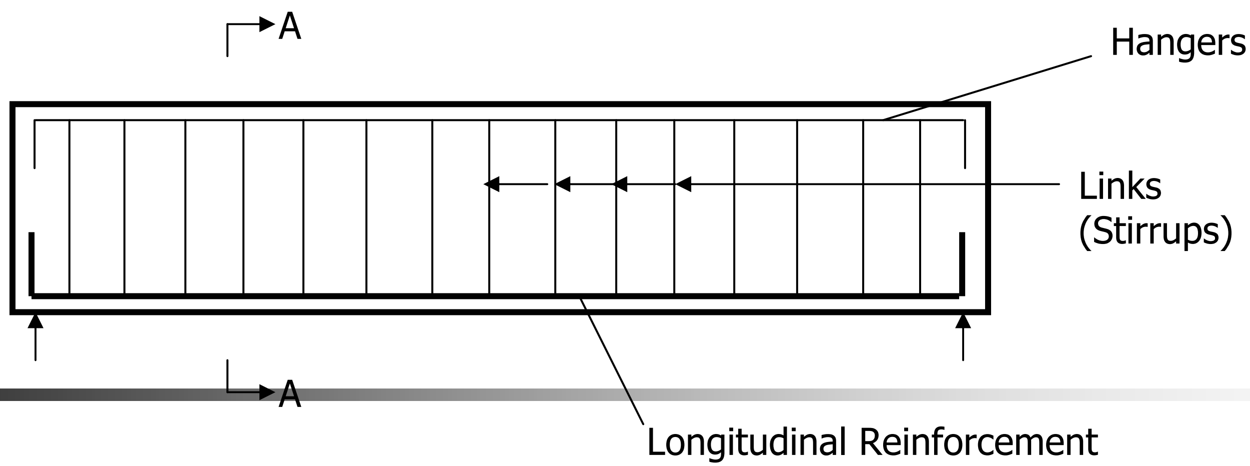
Design constants for Balanced Section

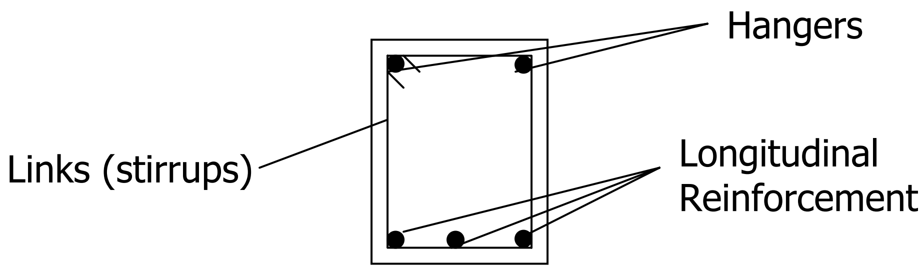
Steel / Concrete		Fe250				Fe415			
		σ_{sall} = 140 N/mm^2				σ_{sall} = 230 N/mm^2			
Grade	σ_{call}	k	j	R	pt	k	j	R	pt
M20	7.0	0.4	0.87	1.22	1.00	0.29	0.9	0.91	0.44
M25	8.5	0.4	0.87	1.48	1.21	0.29	0.9	1.11	0.54
M30	10.0	0.4	0.87	1.74	1.43	0.29	0.9	1.31	0.63

Under Reinforced, Over Reinforced and Balanced Section



Reinforced Beam





Section A-A

Design of Section

Given

- Moment (M) = 20KN-m
- Steel Grade is Fe415; σ_{sall} = 230MPa
- Concrete Grade is M20; σ_{call} = 7 MPa

To Find

- Effective depth 'd'
- Area of steel 'A_{st}'

IS 456-2000

refer table 22

refer table 21

Solution

Assume b = 230mm

For Fe 415 grade steel and M20 grade concrete

R = 0.91 ; pt = 0.44

Now,

$$d = \sqrt{M_{all} / (Rb)}$$

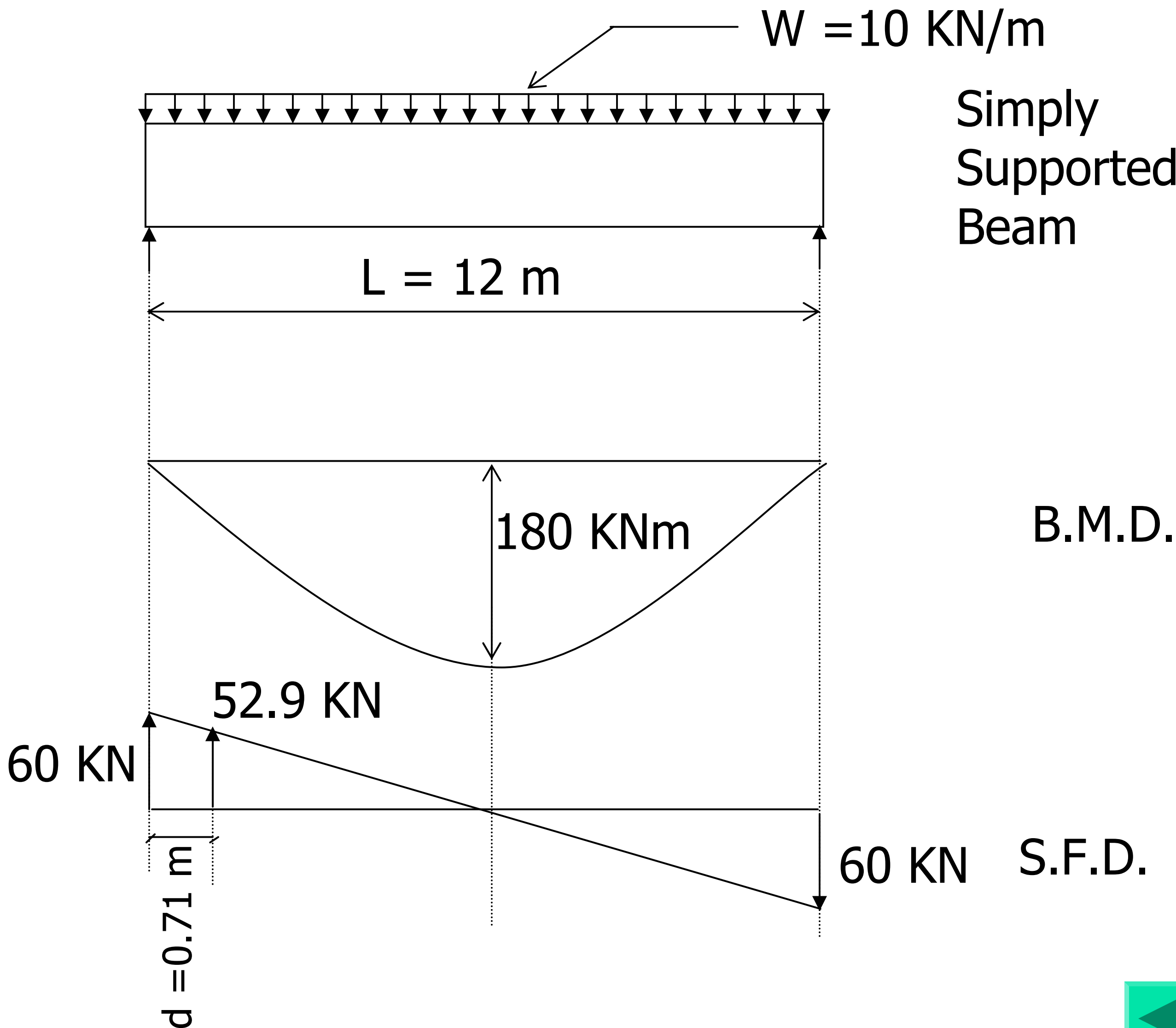
$$d = \sqrt{(20*106) / (0.91*230)}$$

$$d = 309.122 \approx 310 \text{ mm}$$

A_{st} = pt*b*d/100 = 0.44*230*310/100 = <u>313.72 mm²</u>.

<u>Provide 3 # 12</u> Φ (3×113=339 mm^2)

Design of Beam 1



Material Grade:

Concrete M20 and Steel Fe415

Permissible stresses:

Concrete = $\sigma_{call} = 7 \quad N/mm^2$ → Table 21

Steel = $\sigma_{sall} = 230 \quad N/mm^2$ → Table 22

IS – 456:2000

Design Constants:

R = 0.91

Pt_{bal} = 0.44

Calculation of Depth:

Assume b = 400 mm

M = R b d²

Therefore,

$$d_{req} = \sqrt{\frac{M}{R\ b}} = \sqrt{\frac{180\ x\ 10^6}{0.91\ x\ 400}}$$

$$d_{req} = 703\ mm$$

Say, 710 mm

Table 16

For moderate exposure

Clear cover=30 mm

Assuming effective cover = 50 mm

Therefore, Overall depth = D = 710 + 50 = 760 mm

Calculation of A_{st}

$$\sigma_{st} A_{st}\, jd = M$$

$$230 \times A_{st} \times 0.9 \times 710 = 180 \times 10^6$$

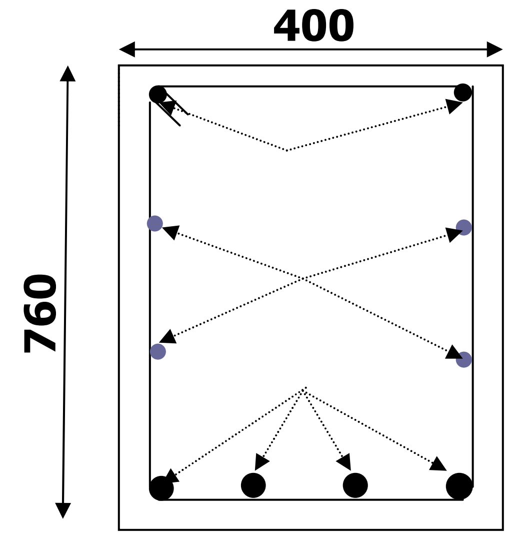
Therefore, $A_{streq} = 1224.74\ mm^2$

Provide 4 – 20Φ

A_{st} provided = $1256.63\ mm^2$

Clear gap bet. bars= (400-2×30-4×20)/3

=87mm > 50mm OK



Curtailment of Reinforcement

We will curtail 2 -20 dia bars.

Therefore, A_{st} = 628.32 mm^2

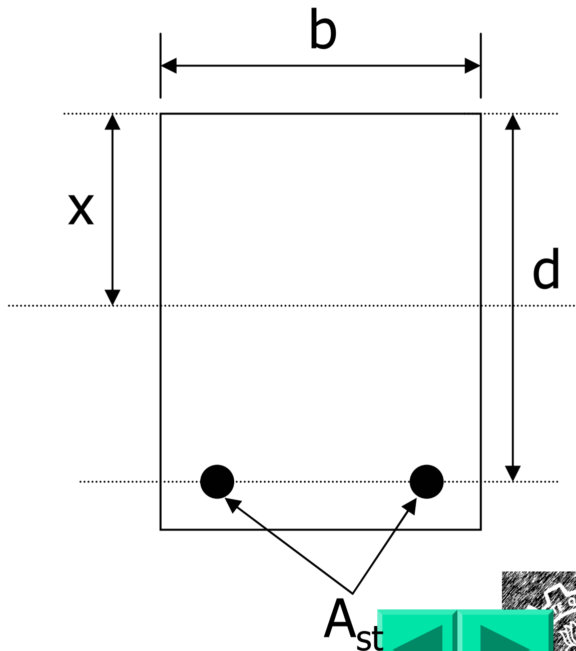
Moment Resisting Capacity of 2-20 dia bars

To Determine the depth of N.A.

Taking moment of effective areas about N.A.

$$b\frac{x^2}{2} = mA_{st}(d-x)$$

$$m = \frac{280}{3\ \sigma_{all}} = \frac{280}{3 \times 7} = 13.33$$



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IITB

$$400 \frac{x^2}{2} = 13.33 \times 628.32 \times (710 - x)$$

$$200 x^2 + 8375.5056x - 5.946608976 \times 10^6 = 0$$

$$\text{Therefore, } x = 152.76 \text{ mm}$$

Moment Resisting capacity of section

$$M' = \sigma_{st} A_{st} (d – x / 3)$$

$$= 230 \times 628.32 \times (710 – 152.76/3)$$

$$= 95.246 \text{ KN-m}$$

Theoretical point of curtailment (TPC) from Support

$$M' = 60\,y - W\,y^2/2$$

$$95.246 = 60y - 5y^2$$

$$5y^2 - 60y + 95.246 = 0$$

Solving, y = 10.12m and 1.88m

Actual point of curtailment (APC) shall extend beyond the TPC by distance

12 * bar diameter = 240 mm
Effective depth = 710 mm

} Whichever is greater | Clause 26.2.3.1

Design for Shear

Shear force at critical section = 52.9 KN → Clause 22.6.2.1

Percentage of tension reinforcement,

$$pt = \frac{A_{st}}{bd} \times 100$$

$$pt = \frac{628.32}{400 \times 710} \times 100 = 0.22$$

Therefore, τ_c = 0.21 N/mm² → Table 23

Shear strength of concrete = τ_c bd

= 0.21 x 400 x 710

= 59,640 N

> 52,900 N

Hence, minimum shear reinforcement is required.

Minimum of the following spacing shall be provided

1) 0.75 d = 0.75 x 710 = 532.5 mm

2) 300 mm

Clause 26.5.1.5

3) $\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y}$

Clause 26.5.1.6

Using 2-legged 8 Φ stirrups.

A_{sv} = 100.53 mm^2

$$S_v = \frac{0.87 f_y A_s}{0.4b} = \frac{0.87x415x100.53}{0.4x400}$$

$$S_v = 226.85mm$$

Provide 2 – legged 8 dia stirrups at 225 c/c.

Check for Deflection

Clause 23.2.1

span = 12 m > 10 m

Basic Value = 20 x 10/12 = 16.67

Modification Factor = 1.3

(Depends on area and stress of steel

in tension reinforcement)

Refer Fig. 4 of IS- 456:2000

Modified Basic Value = 16.67 x 1.3 = 21.67

L / d = 12 / 0.71 = 16.90 < 21.67

Side Face Reinforcement

Clause 26.5.13

Since the depth of the beam exceeds 750 mm, side face reinforcement shall be provided.

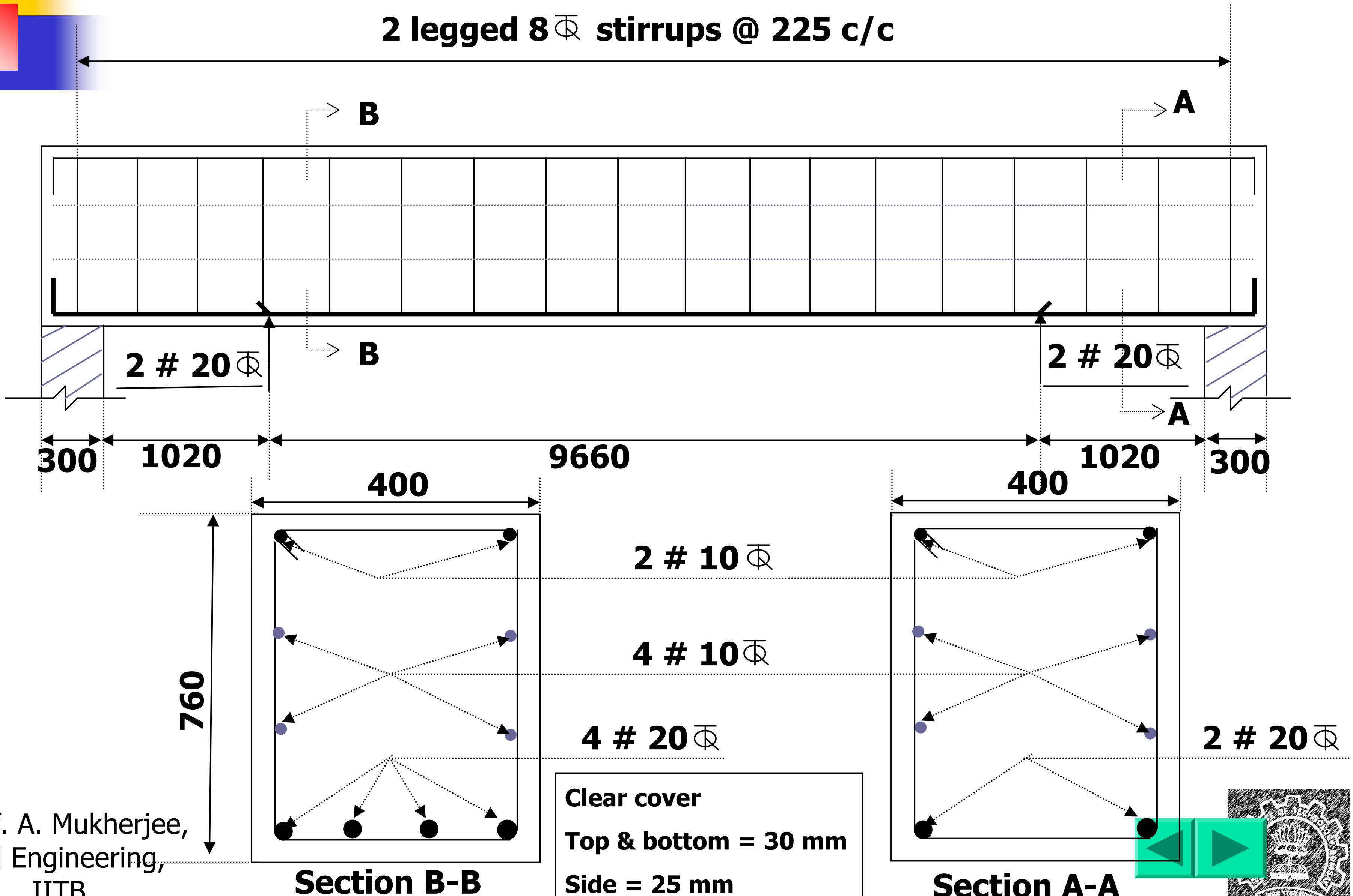
Total area of side face reinforcement = 0.1 % of the web area

$$A_{side} = 0.1 \times 760 \times 400 / 100$$
$$= 304 \text{ mm}^2$$

Provide 4 – 10 dia. bars.

Above reinforcement shall be provided equally on two faces at a spacing not exceeding 300 mm.

Reinforcement Details



Clear cover

Top & bottom = 30 mm

Side = 25 mm

Design of Beams 2

Design a fixed beam with concrete grade M20 and steel Fe415.

Effective span of beam= 10 m

Superimposed Load = 80 KN/m (including finishing load)

Width of beam= 500 mm (say)

Solution

Assume overall depth of beam=1500 mm (To calculate self wt of beam)

If required depth is more than assumed then revise the calculations.

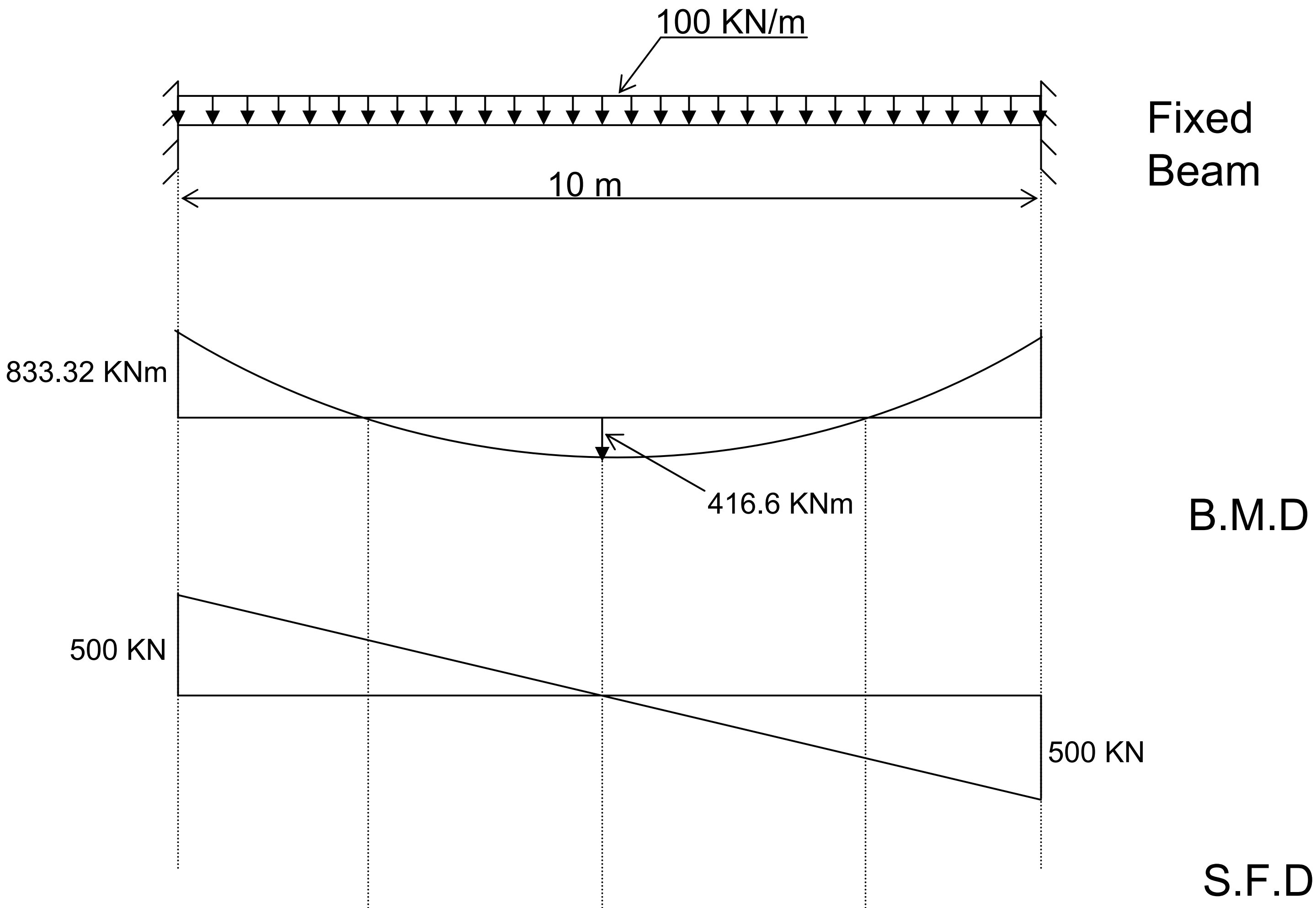
Loading:

Superimposed Load = 80 KN/m

Self weight = 25 x 0.5 x 1.5 = 18.75 KN/m

98.75 KN/m

Say 100 KN/m


100 KN/m
10 m
Fixed
Beam
833.32 KNm
416.6 KNm
B.M.D
500 KN
500 KN
S.F.D

Material Grade:

Concrete M20 and Steel Fe415

Permissible stresses:

Concrete = $\sigma_{call} = 7 \quad N/mm^2$ → Table 21

Steel = $\sigma_{sall} = 230 \quad N/mm^2$ → Table 22

IS – 456:2000

Design Constants:

R = 0.91

Pt_{bal} = 0.44

Calculation of Depth:

b = 500 mm

M = R b d^2

Therefore,

$$d_{req} = \sqrt{\frac{M}{R\ b}} = \sqrt{\frac{833.33\ x\ 10^6}{0.91\ x\ 500}}$$

$$d_{req} = 1353\ mm$$

Say, 1360 mm

Table 16

For moderate exposure

Clear cover=30 mm

Assuming effective cover = 80 mm

Therefore, Overall depth = D = 1360 + 80 = 1440 mm

< 1500 mm assumed initially

Calculation of A_{st} at support

$M = 833.34$ KNm.

$$\sigma_{st} A_{st} jd = M$$

$$230 \times A_{st} \times 0.9 \times 1360 = 833.34 \times 10^6$$

Therefore, $A_{streq} = 2960.14 \text{ mm}^2$

<u>Provide 4 -25 Φ and 4 - 20</u> Φ

i.e. Area of steel = 4*491 + 4*314

= 3220 mm^2

Calculation of A_{st} at midspan

M = 416.6 KNm.

$$\sigma_{st} A_{st} jd = M$$

$$230 \times A_{st} \times 0.9 \times 1360 = 416.6 \times 10^6$$

Therefore, $A_{streq} = 1480 \text{ mm}^2$

<u>Provide 5 -20</u> Φ

i.e. Area of steel = 5*314

= 1570 mm²

Curtailment of midspan Reinforcement

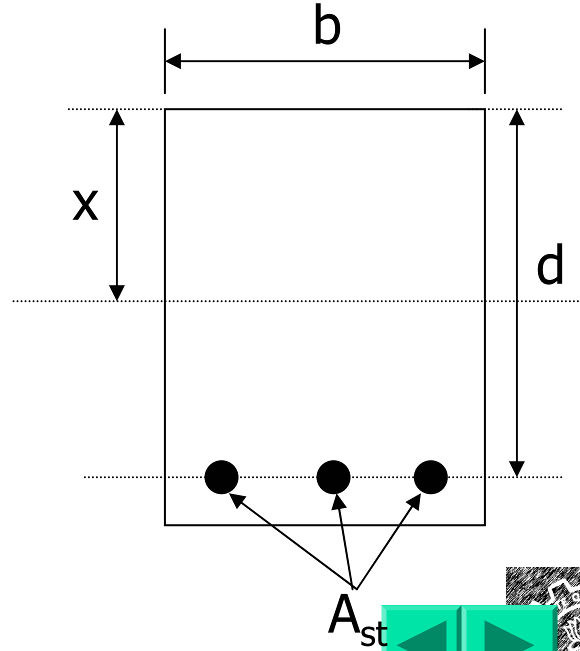
We will curtail 2 -20 dia bars.

Therefore, A_{st} = 628.32 mm^2

Moment Resisting Capacity of 3-20 dia bars

To Determine the depth of N.A.

Taking moment of effective areas about N.A.



$$b\frac{x^2}{2} = mA_{st}(d-x)$$

$$m = \frac{280}{3\ \sigma_{all}} = \frac{280}{3 \times 7} = 13.33$$

$$500\ \frac{x^2}{2} = 13.33 \times 3 \times 314 \times (1360 - x)$$

$$250\ x^2 + 12556.86x - 17.077329 \times 10^6 = 0$$

Therefore, $x = 237.4506$ mm

Moment Resistance capacity of section

$$M' = \sigma_{st} A_{st} (d - x / 3)$$

$$= 230 \times 628.32 \times (1360 - 237.45/3)$$

$$= 185.10 \text{ KN-m}$$

Theoretical point of curtailment (TPC) from Support

$$M' = 500\, y - 100\, y^2/2 - 833.33$$

$$1018.43 = 500y - 50y^2$$

$$50y^2 - 500y + 1018.43 = 0$$

Solving, y = 2.8m and 7.2m

Actual point of curtailment (APC) shall extend beyond the TPC by distance

12 x bar diameter = 300 mm
Effective depth = 1360 mm

} Whichever is greater

Clause 26.2.3.1

Design for Shear

Shear force at critical section = 364 KN → Clause 22.6.2.1

Percentage of tension reinforcement,

$$pt = \frac{A_{st}}{bd} \times 100$$

$$pt = \frac{3220}{500 \times 1360} \times 100 = 0.47$$

Therefore, $\tau_c = 0.29$ N/mm^2 → Table 23

Shear strength of concrete = τ_c bd

Vc = 0.29 x 500 x 1360

= 197.2 KN

< 364 KN

Hence shear reinforcement is required.

Design shear Vs = V – Vc

= 364 – 197.2

= 166.8 KN

Assuming 2 Legged 8 Φ stirrups

$$S_v = \frac{\sigma_{sv} \; A_{sv} \; d}{V_s} = \frac{230 \; x \; 100 \; x \; 1360}{166.8 \; x \; 10^3}$$

$$S_v = 187.5mm$$

ANNEX- B
B-5.4

1) 0.75 d = 0.75 x 1360 = 1020.0 mm

2) 300 mm

3) $$s_v = \frac{0.87 \times 415 \times 100}{0.4 \times 500} = 180.5mm$$

Therefore, provide 8 Φ 2- Legged Stirrups @ 180 c/c.

As shear force goes on reducing towards centre, we can increase the spacing of stirrups in the middle zone.

We will provide 2 legged 8 ɸ stirrups @ 300 c/c.

Shear carrying capacity of nominal stirrups,

$$V_n = \frac{230 \times 100 \times 1360}{300}$$

$$V_n = 104.26 \ KN$$

Area of tension reinforcement in mid span = 1570 mm^2

$$P_t = \frac{A_{st}}{bd} \times 100$$

$$P_t = \frac{1570}{500 \times 1360} \times 100 = 0.23 \ \%$$

Table 23

Therefore, τ_c = 0.212 N/mm2

Shear carrying capacity of section,

$= V_n + V_c$

$= \tau_c bd + V_n$

$= 0.212 \times 500 \times 1360 + 104260$

$= 247$ KN

Distance from centre where this SF will be reached = 247/100=2.47m

We will provide 2 Legged 8 Φ stirrups @ 300 c/c in middle 4 m zone.

Side Face Reinforcement

Total area of side face reinforcement =

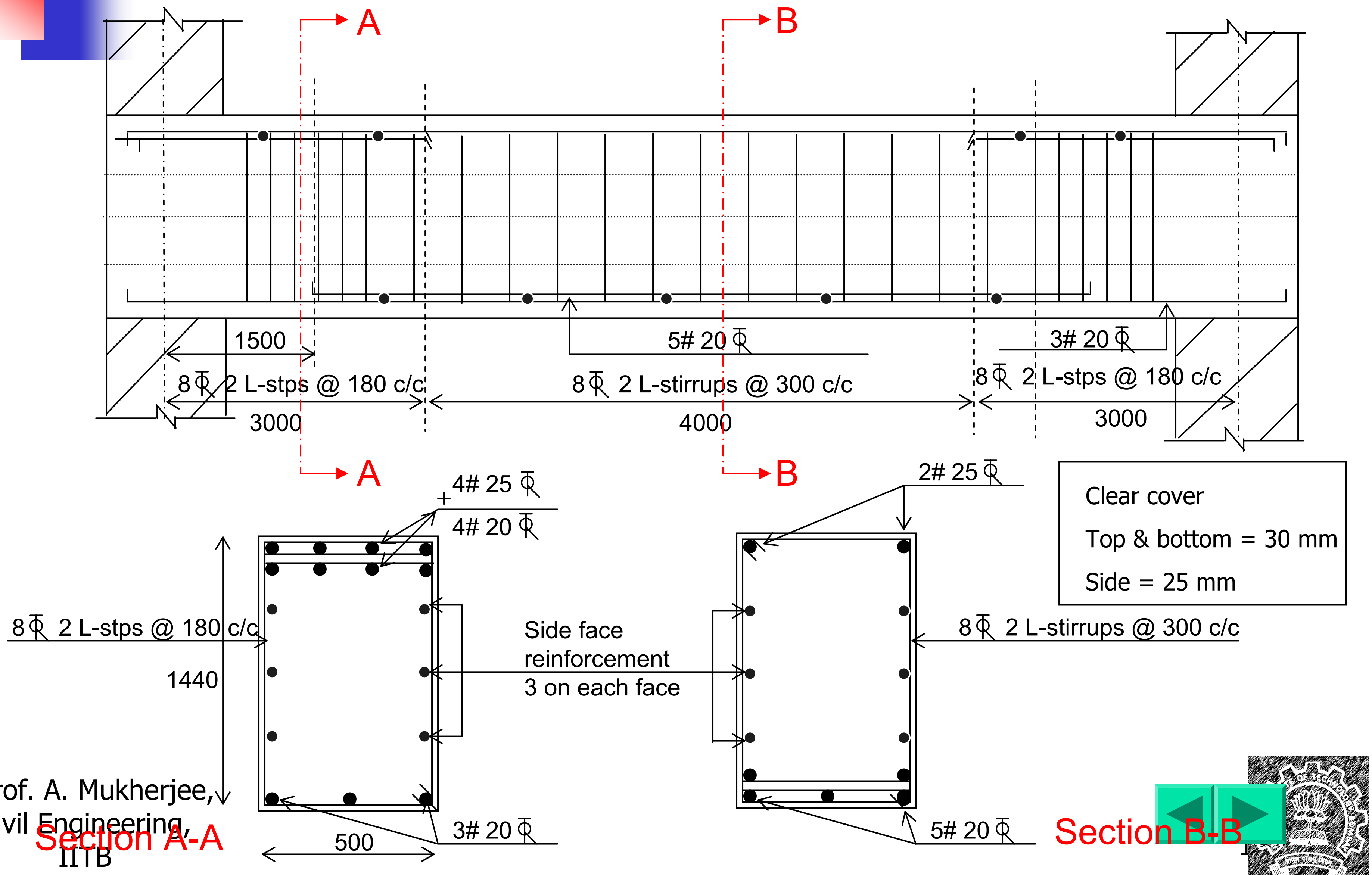
0.1 % of the web area

$A_{side} = 0.1 \times 1440 \times 500 / 100$

$= 720$ mm2

Provide 6 – 12 dia. bars.

Reinforcement Details



Uncertainties in Design

- Loads
- Materials
- We have so far limited the maximum stress in the materials to take care of both

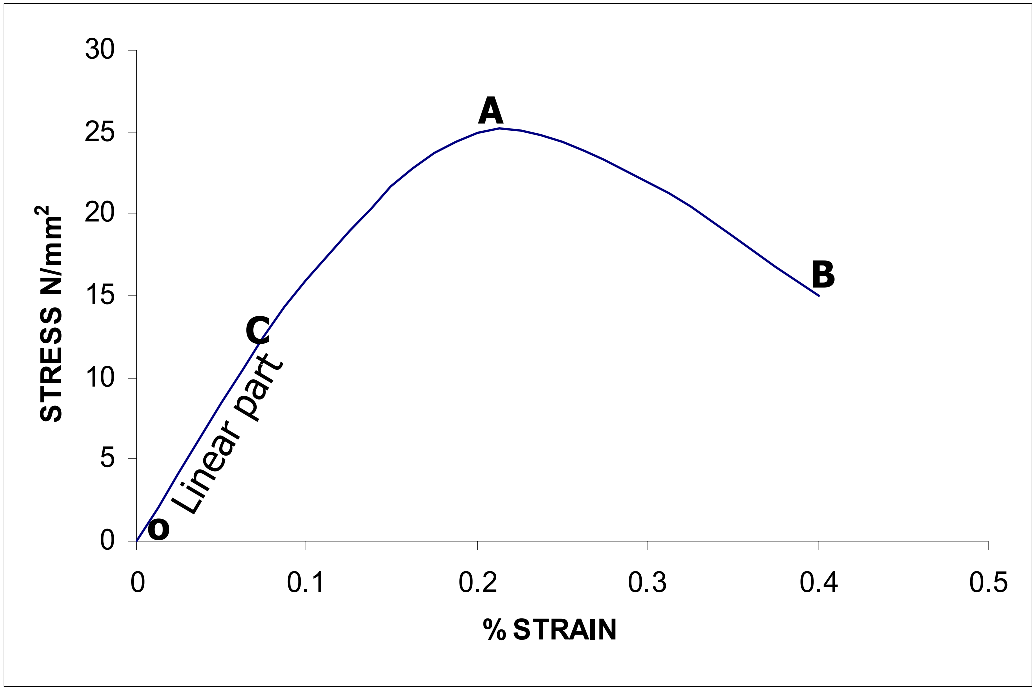
$$\sigma_{all} = \frac{\sigma_{ult}}{FS}$$

- Not a justifiable approach

Working Stress Method

- Concrete and steel assumed to behave elastically.

The allowable stresses are obtained by dividing the limiting stresses of material by factor of safety.



Typical Stress strain curve for concrete

Limitations of Working Stress Method

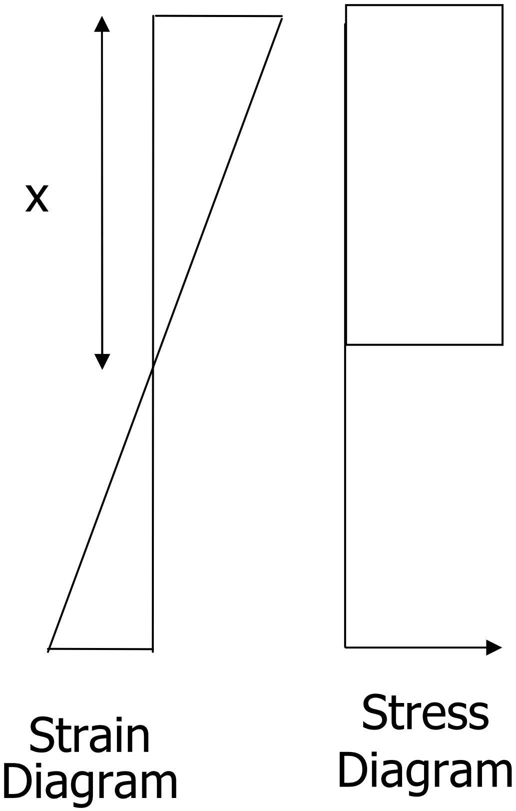
- Ignores uncertainties in different types of load.
- Does not use the full range of strains in the material.
- Therefore, disregards the nonlinear part of the material curve.
- Considers failure as a function of stress while it is a function of strain.
- Stress as a measure of safety does not give true margin of safety against failure. A stress factor of safety 3 for concrete does not mean that the member will fail at a load three times the working load.
- The structure must carry loads safely. It is logical to use the method based on load causing failure.
- The additional load carrying capacity of the structure due to redistribution of moment can not be accounted for.
- Produces conservative designs.

Ultimate Load Method

- The loads are enhanced by load factor-
- Design load = LF * service load
- The method uses total stress-strain curves of the material.
- Strain based failure.
- LF=3 means the structure has 3 times more capacity
- Since the method considers the plastic region of the stress-strain curve also, it utilizes the reserve capacity of the member.
- Uncertainty in load only is considered. Uncertainty in material is ignored.

Ultimate Load Method

- Utilization of large reserve strength in plastic region and of the ultimate strength of the members results in slender section lead to excessive deformation and cracking.



Limit State Method

- Combines the concept of ultimate load method and working stress method. Partial factor of safety on materials and load factor on loads.
- Uncertainties of both materials and loads can be considered realistically
- Different limit states are considered – collapse, servicability and durability.

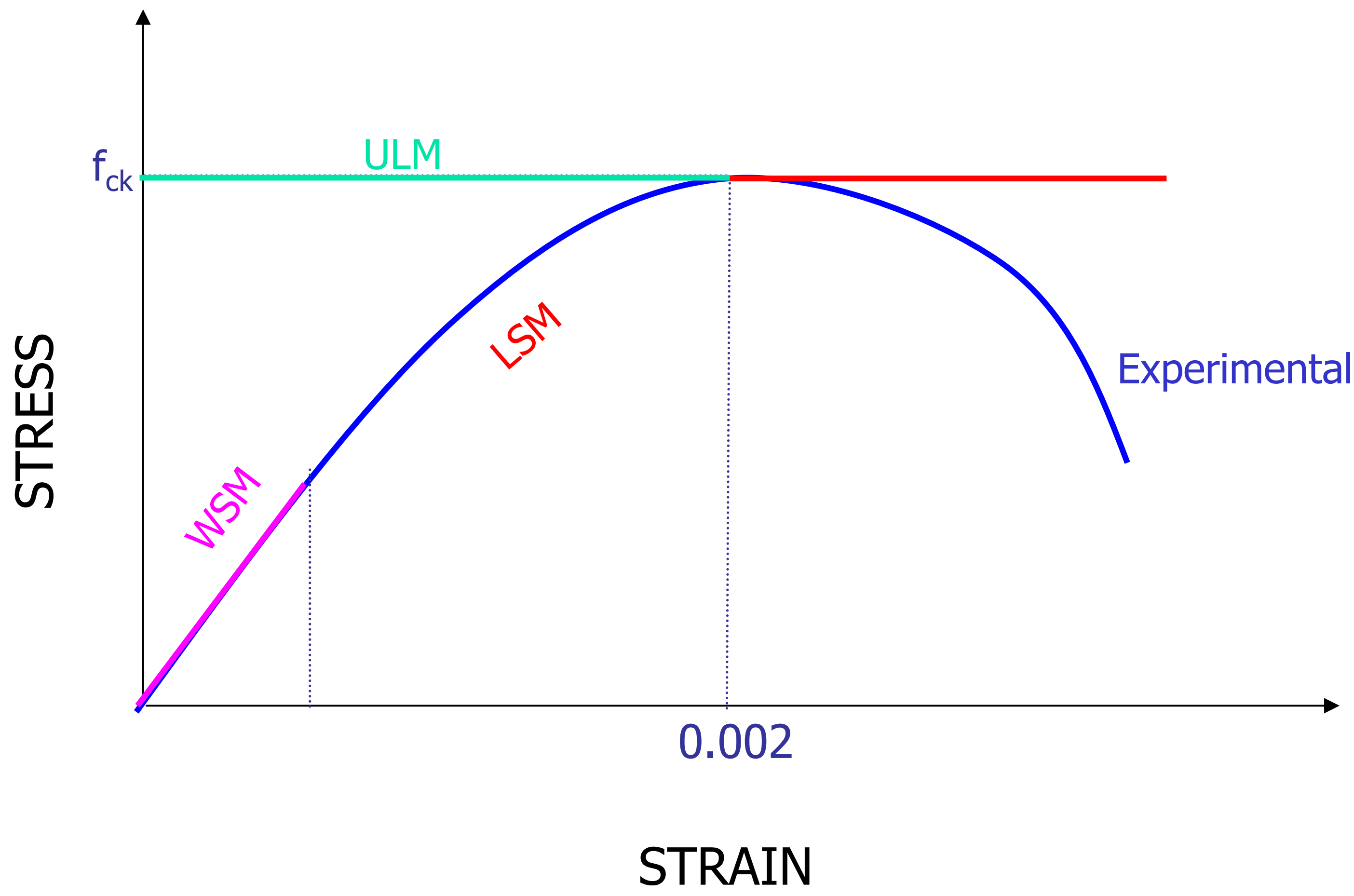
Methods of Design

1. Working Stress Method (WSM) – factor on material properties

2. Ultimate Load Method (ULM) – factor on loads
   Design load = LF * expected load

3. Limit State Method (LSM) – partial factors on both

Stress-strain curves for Concrete



Limit States

The structure must be fit to perform its function satisfactorily during its service life span. The condition or the state at which the structure, or part of a structure becomes unfit for its use is called Limit State.

<u>Three types of Limit States:</u>

- Limit States of Collapse
  - i. Flexure ii. Compression
  - iii. Shear iv. Torsion
- Limit States of Serviceability
  - i. Deflection ii. Cracking
- Limit States of Durability – time dependent deteriorations- creep, fatigue, diffusion

Partial Safety Factors γ_f for Loads

Table 18 – IS 456 :2000

Load Combinations	Limit State of Collapse			Limit State of Serviceability		
	DL	IL	WL	DL	IL	WL
DL + IL	1.5	1.5	-	1.0	1.0	-
DL + WL	1.5 or 0.9*	-	1.5	1.0	-	1.0
DL + IL + WL	1.2	1.2	1.2	1.0	0.8	0.8

Notes:

1. While considering earthquake effects, substitute EL for WL.
2. For the limit states of serviceability, the values of γ_f given in this table are applicable for short term effects. While assessing the long term effects due to creep the dead load and that part of the live load likely to be permanent may only be considered.

* This value to be considered when stability against overturning or stress reversal is critical.

Partial Safety Factor γ_m for Material Strength

Clause 36.4.2

- Accounts for construction faults, workmanship and supervision.
- When assessing the strength of a structure or structural member for the limit state of collapse, the values of partial safety factor, γ_m should be taken as 1.5 for concrete and 1.15 for steel.
- When assessing the deflection, the material properties such as modulus of elasticity should be taken as those associated with the characteristic strength of the material.

Limit State of Collapse: Flexure

Clause 38.1

- Assumptions

1. Plane sections normal to the axis of the member remain plane during bending. This means that the strain at any point on the cross section is directly proportional to the distance from the neutral axis.
2. The maximum strain in concrete at the outermost compression fibre is 0.0035.
3. The tensile strength of concrete is ignored.

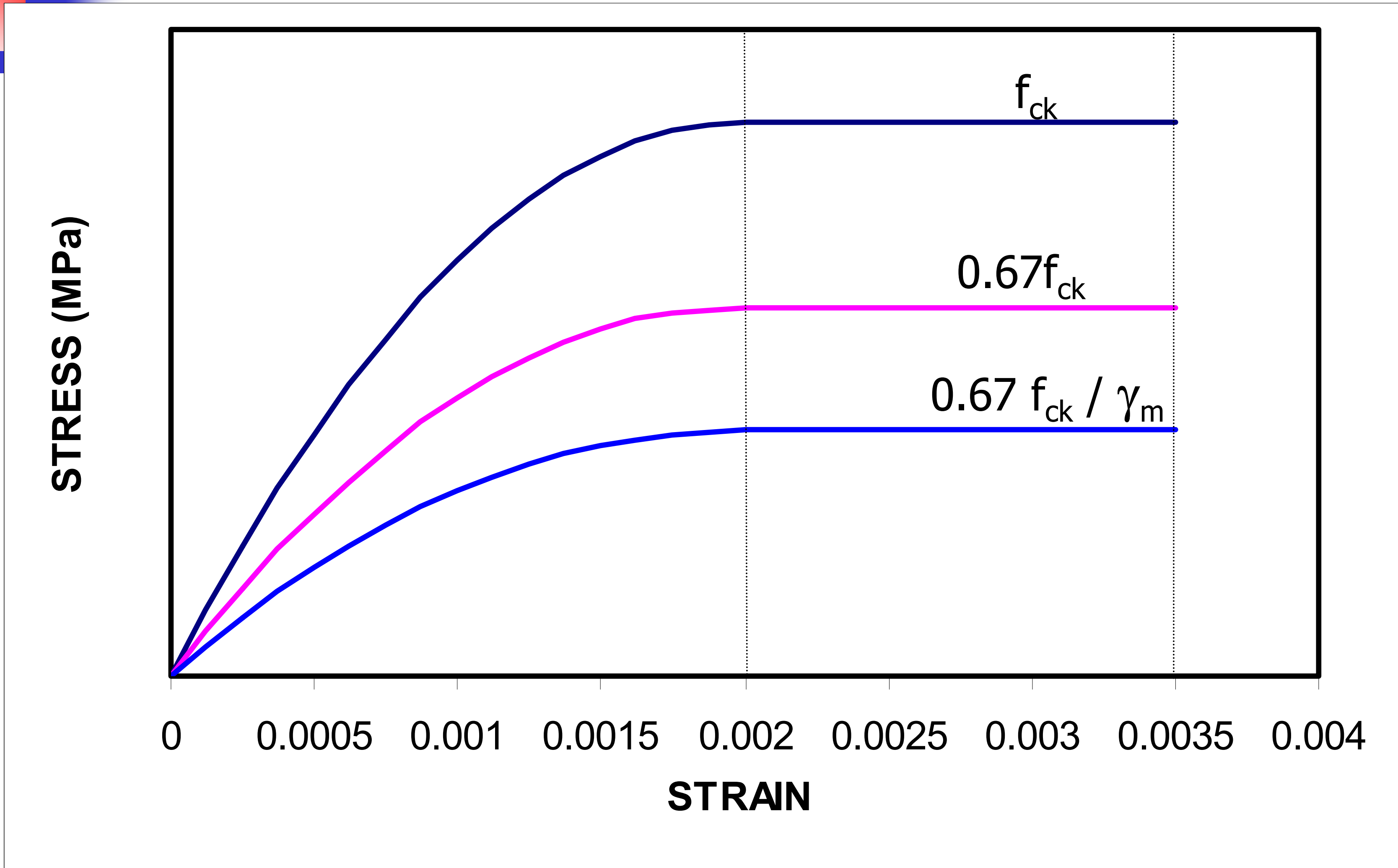
Assumptions contd….

- The strain in the tension reinforcement is to be not less than

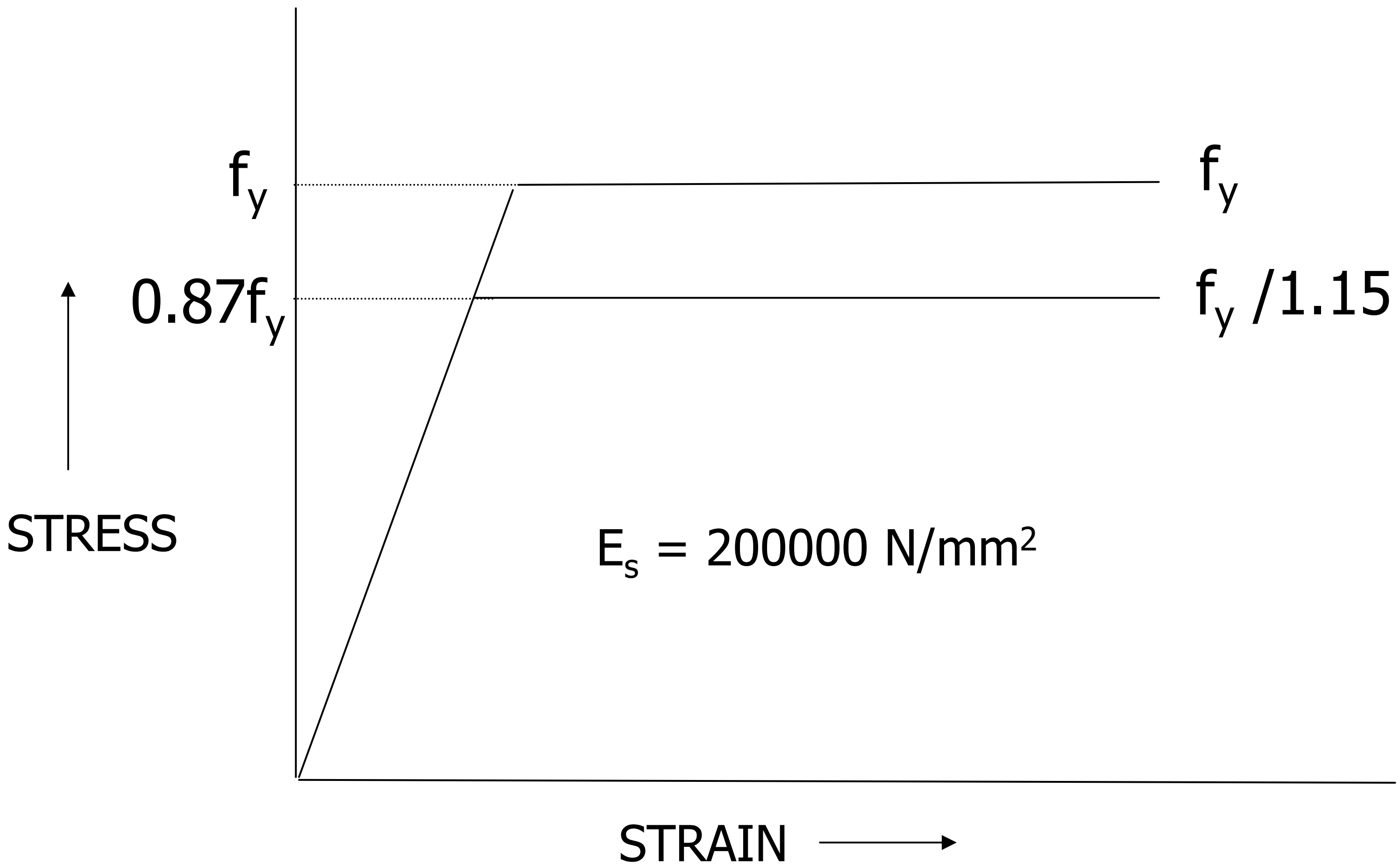
$$\frac{0.87\ f_y}{E_s} + 0.002$$

This assumption is intended to ensure ductile failure, that is, the tensile reinforcement has to undergo a certain degree of inelastic deformation before the concrete fails in compression.

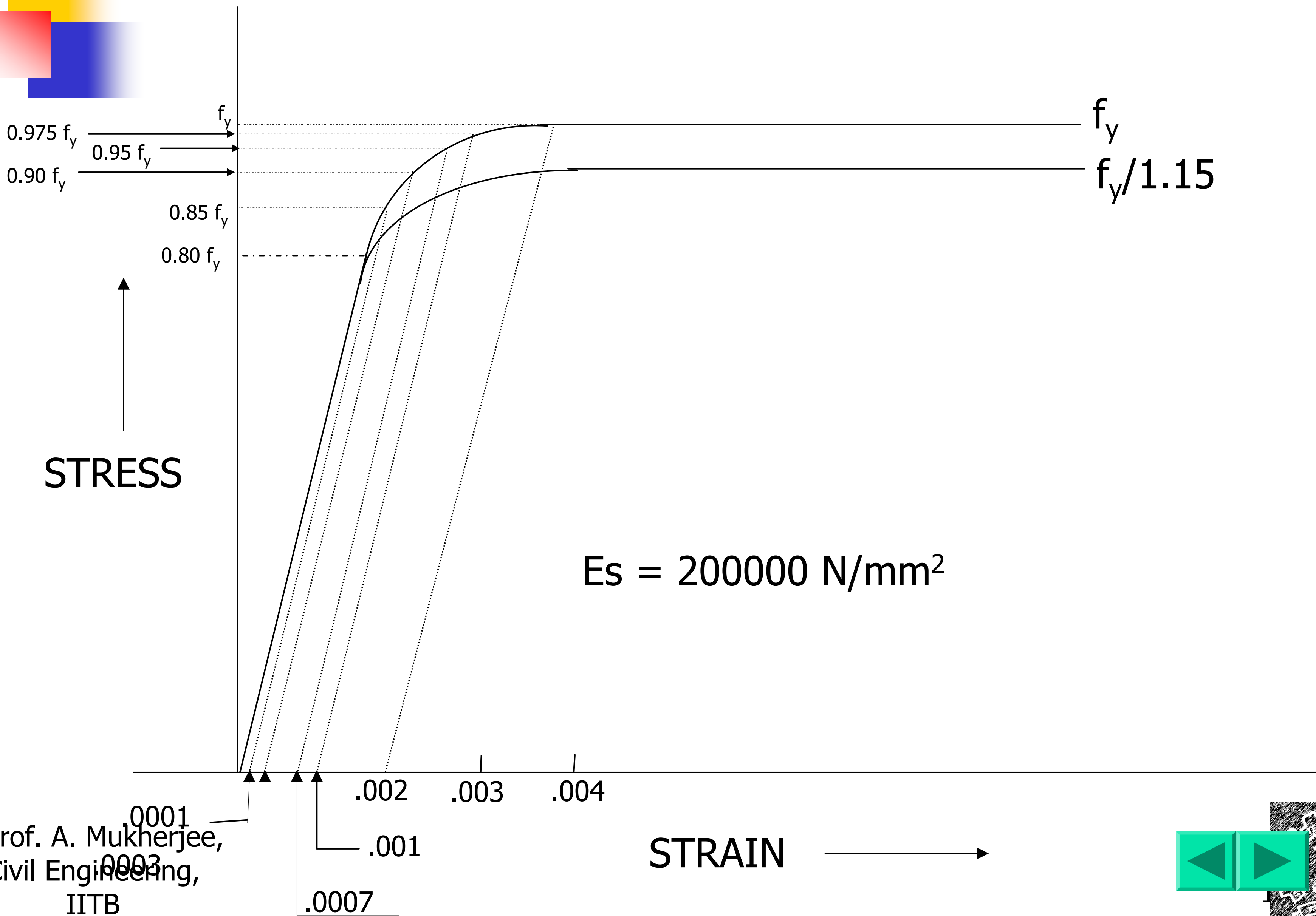
Idealized Stress-Strain Curve for Concrete



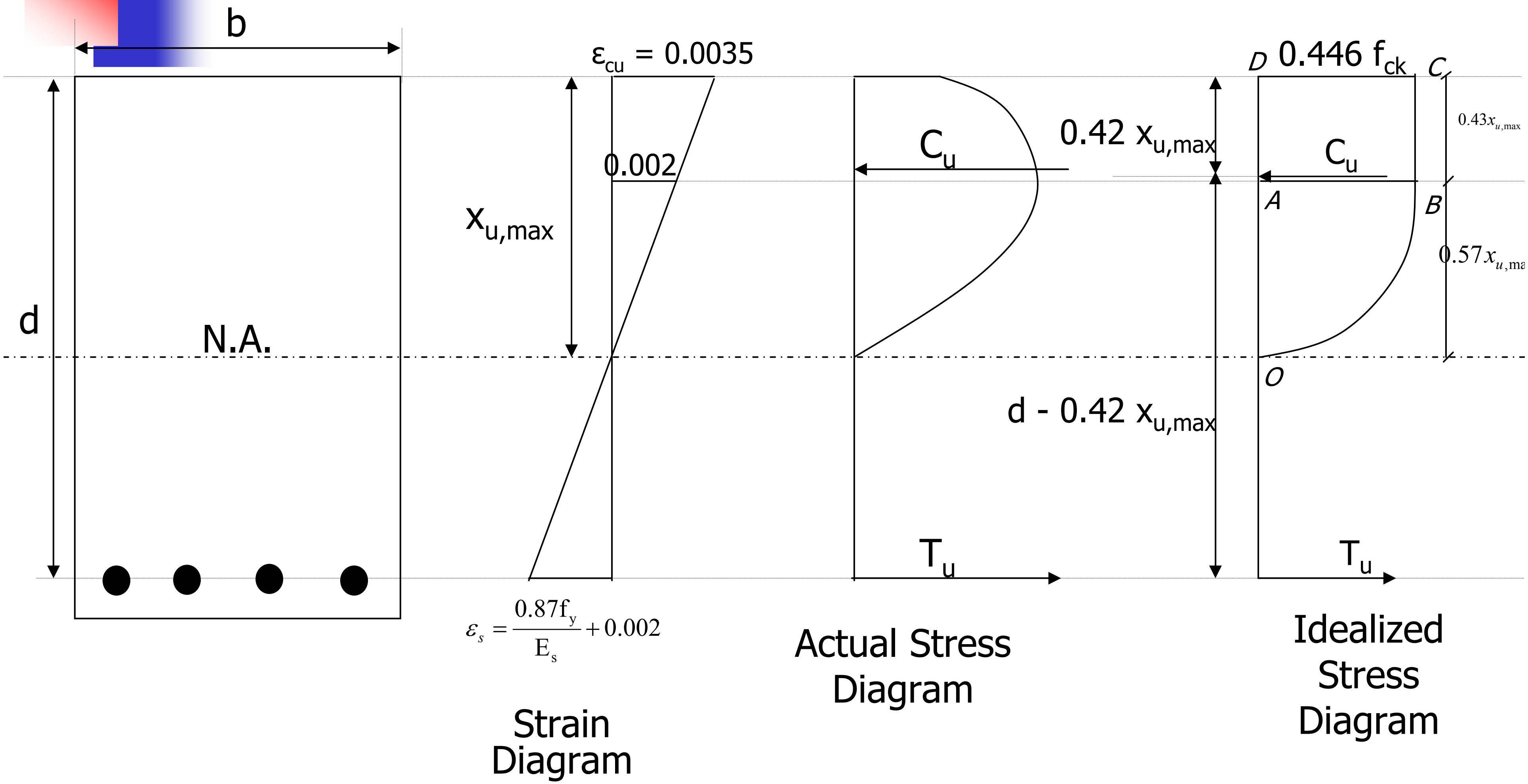
Stress- strain curve for Mild Steel



Stress –strain curve for Deformed Bars



Stress Block Parameters



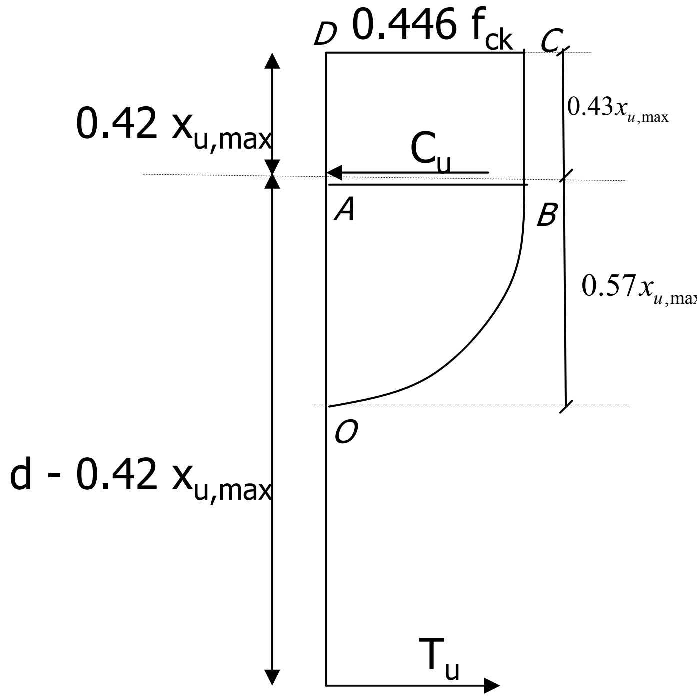
Stress Block Parameters contd….

Area of the stress block = Area of rectangle ABCD

+ Area of parabola OAD

$= 0.191\, f_{ck}\, x_{umax} + 0.169\, f_{ck}\, x_{umax}$

$= 0.36\, f_{ck}\, x_{umax}$



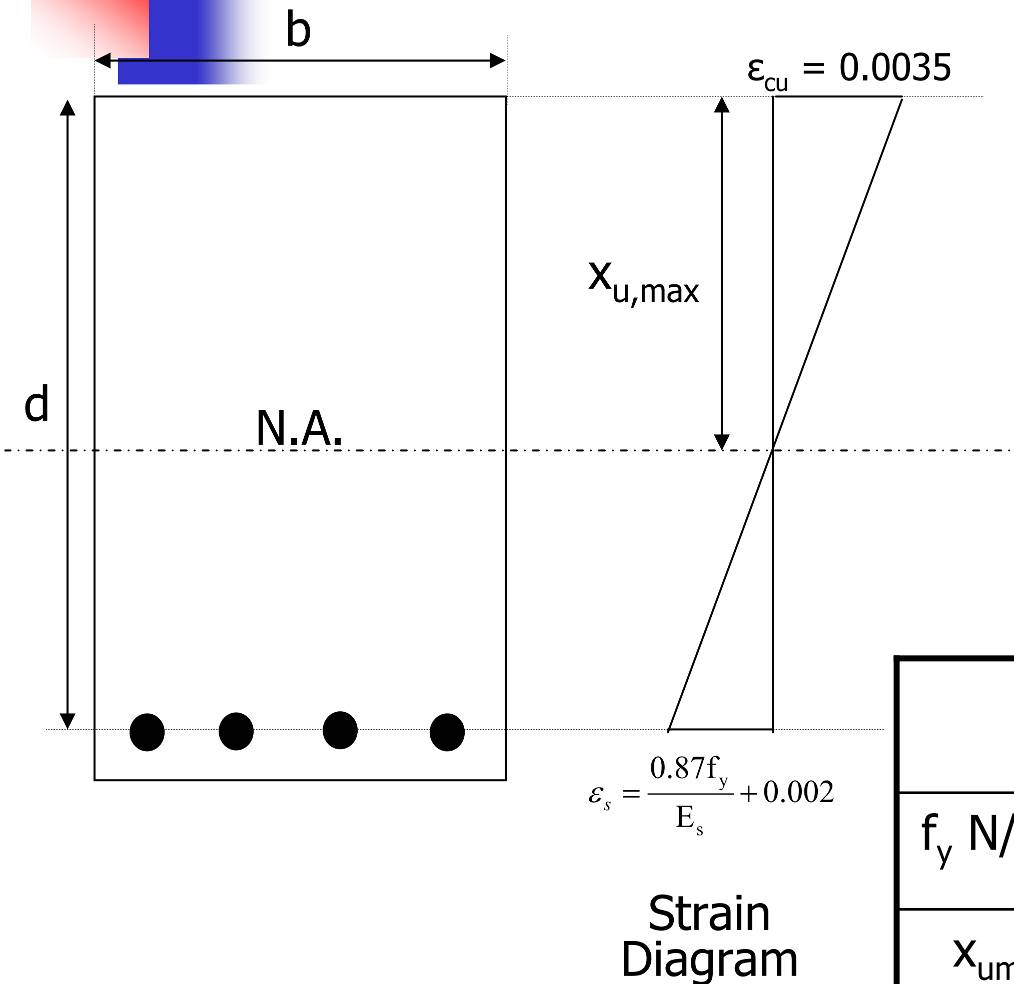
Idealized Stress Diagram

Distance of centroid of stress block from the compression face,

$$\bar{x} = \frac{0.191 f_{ck} x_{u\max} \times 0.214 x_{u\max} + 0.169 f_{ck} x_{u\max} \times 0.643 x_{u\max}}{0.36 f_{ck} x_{u\max}}$$

$$= 0.42 x_{u\max}$$

Maximum Depth of Neutral Axis



From Strain Diagram,

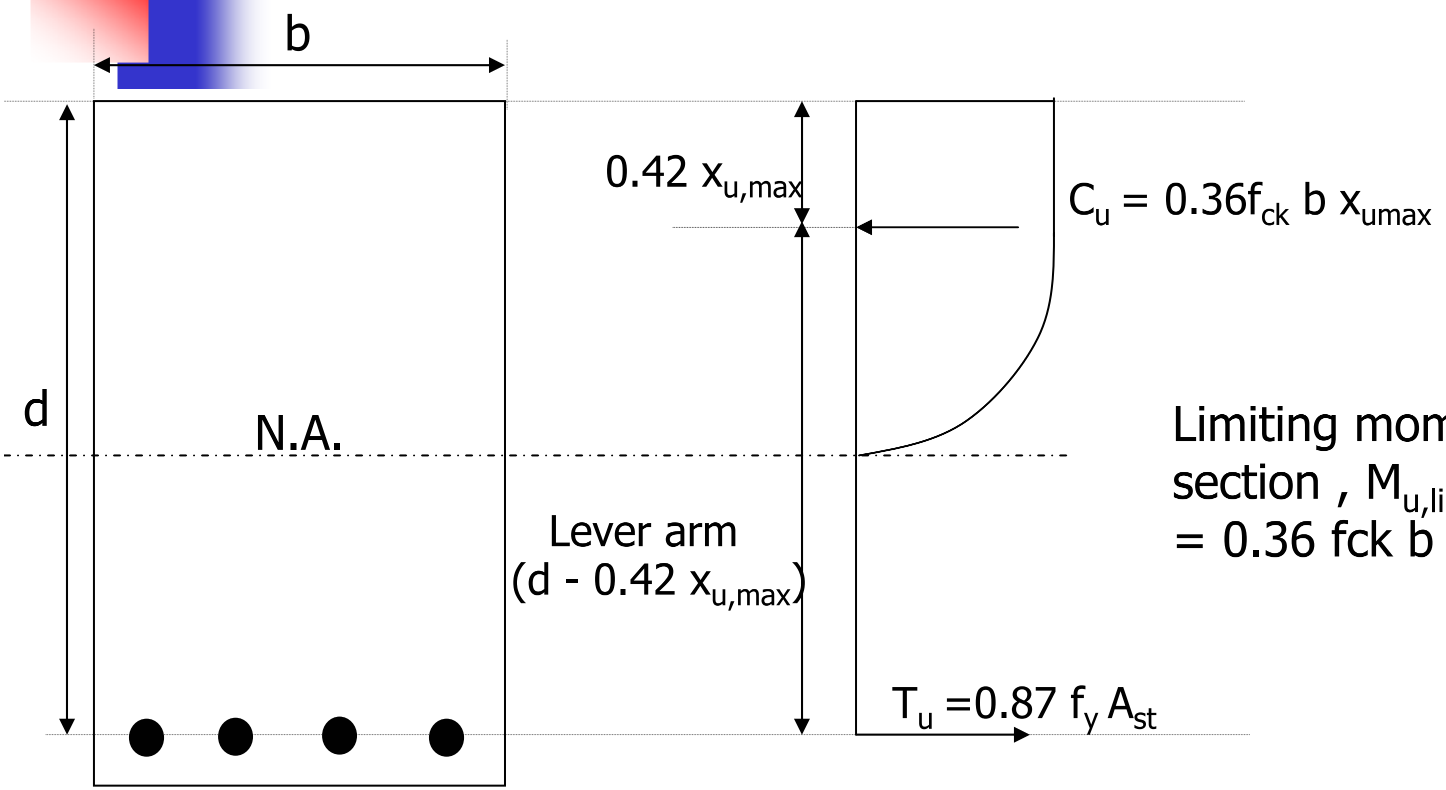
$$\frac{x_{u\max}}{d - x_{u\max}} = \frac{0.0035}{\left(0.002 + 0.87 f_y E_s\right)}$$

$$\frac{x_{u\max}}{d} = \frac{0.0035}{\left(0.0055 + 0.87 f_y E_s\right)}$$

X_{umax} depends on grade of steel.

Values of x_{umax}/d for different grades of steel			
f_y N/mm²	250	415	500
x_{umax}/d	0.531	0.479	0.456

Singly Reinforced Rectangular Section



Limiting moment of resistance of section , $M_{u,lim}$

$= 0.36\ fck\ b\ x_{umax}\ (d - 0.42\ x_{umax})$

$T_u = C_u$, gives

$0.87\ f_y\ A_{st} = 0.36\ f_{ck}\ b\ x_{umax}$

$p_{t,lim} = (A_{st}/bd) \times 100 = (0.36\ f_{ck}\ b\ x_{umax}/0.87\ f_y\ bd) \times 100$

Limiting moment of resistance and reinforcement index for singly reinforced rectangular sections

f_y N/mm^2	250	415	500
$\frac{M_{u,lim}}{f_{ck} bd^2}$	0.149	0.138	0.133
$\frac{p_{t,lim} f_y}{f_{ck}}$	21.97	19.82	18.87

Limiting moment of resistance factor $M_{u,lim}/bd^2$, for singly reinforced rectangular sections

fck, N/mm^2	fy, N/mm^2		
	250	**415**	**500**
15	2.24	2.07	2.00
20	2.98	2.76	2.66
25	3.73	3.45	3.33
30	4.47	4.14	3.99

Maximum percentage of tensile reinforcement $p_{t,lim}$ for singly reinforced rectangular sections

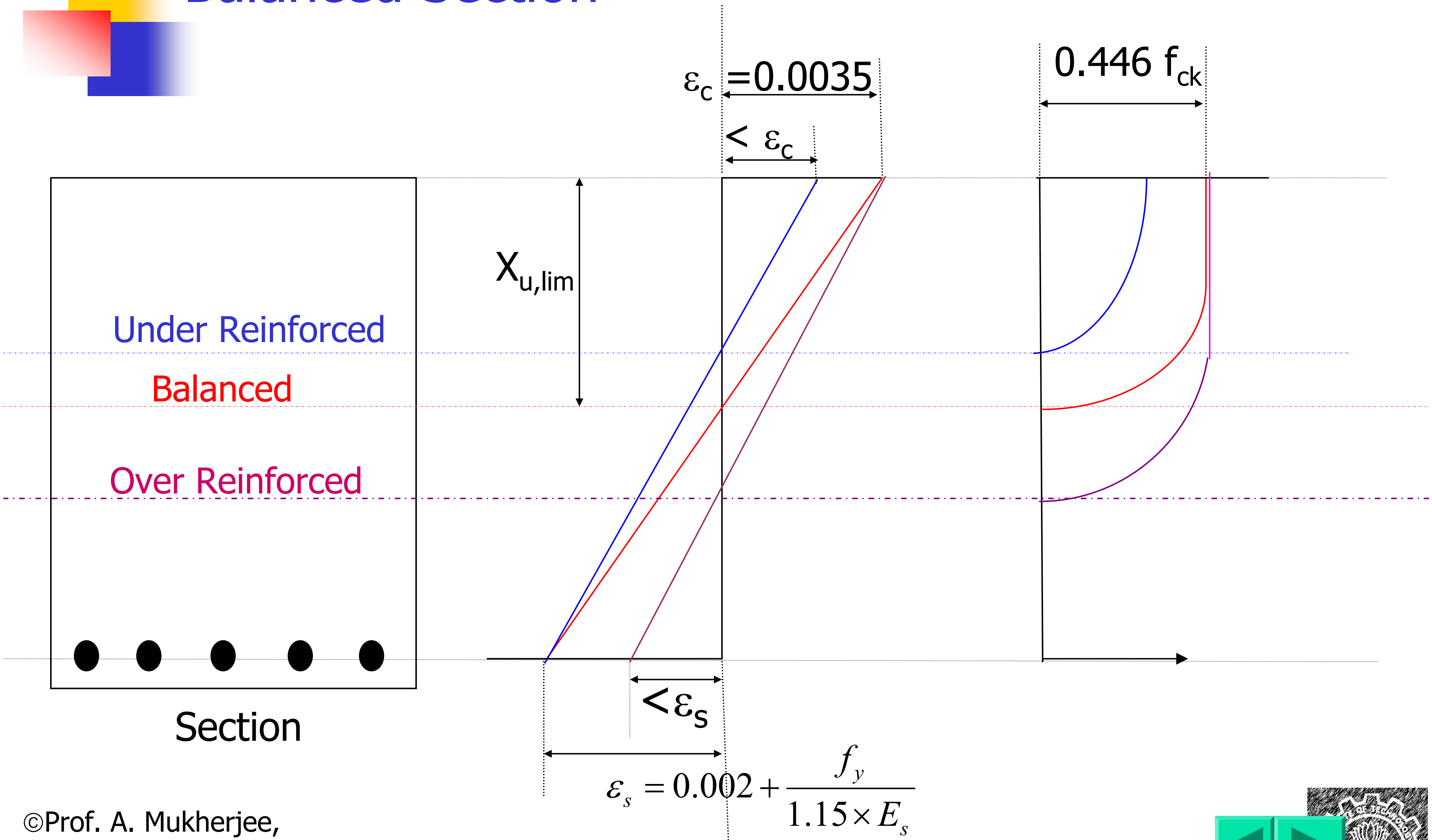
fck, N/mm²	fy, N/mm²		
	250	**415**	**500**
15	1.32	0.72	0.57
20	1.76	0.96	0.76
25	2.20	1.19	0.94
30	2.64	1.43	1.13

Singly Reinforced Section

Under Reinforced Section

- $p_t < p_{t,lim}$
- Steel yields before the concrete crushes in compression
- Since $A_{st} < A_{st,max}$, $x_u < x_{u,max}$
- Failure is characterized by substantial deflection and excessive cracking giving ample warning of impending failure.

Under Reinforced , Over Reinforced and Balanced Section



Analysis Problem

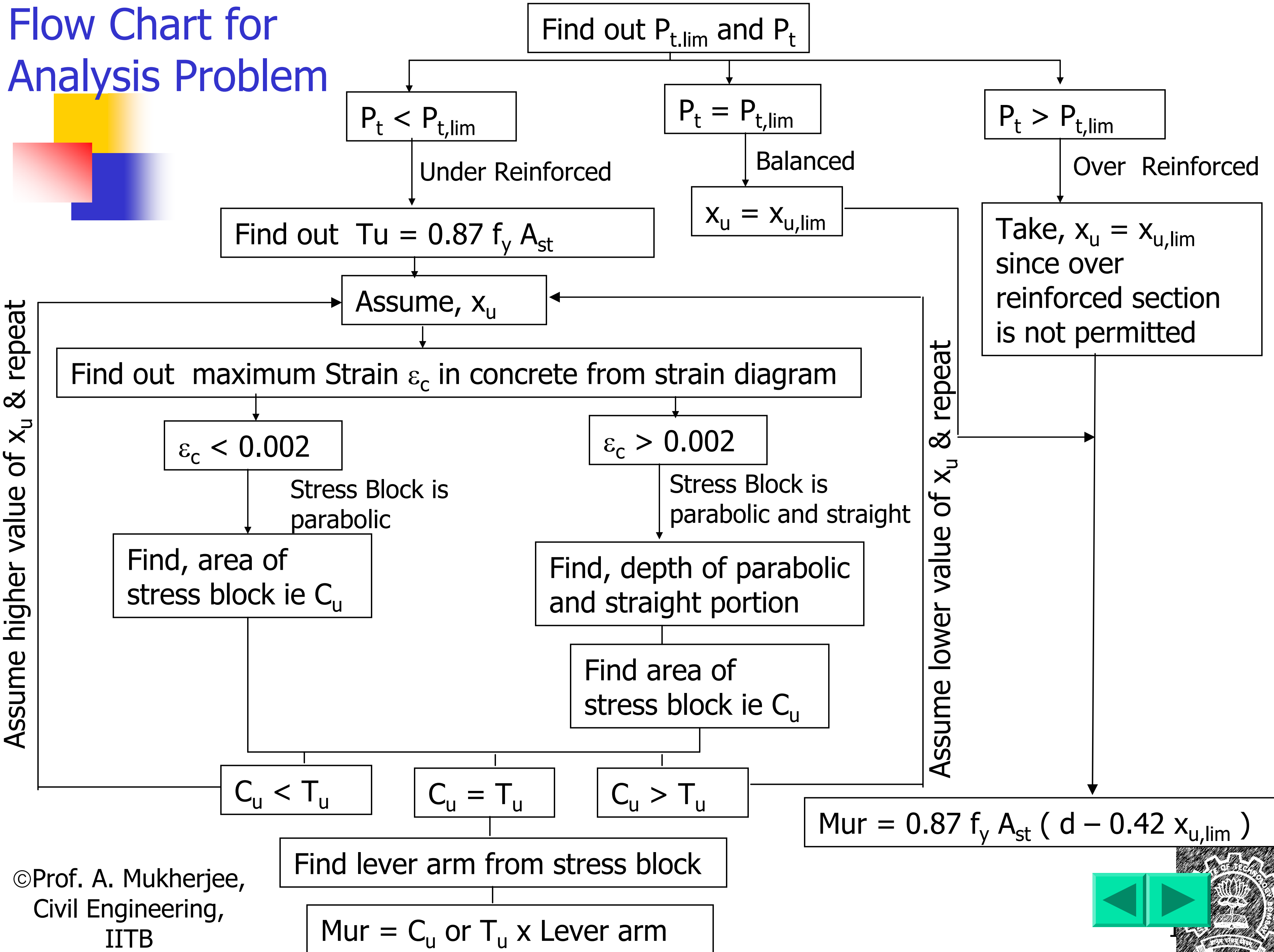
Given: Material Properties (f_{ck} and f_y)

Cross section properties and A_{st}

To Find: Moment of Resistance M_{ur} or

Allowable load

Flow Chart for Analysis Problem



Example 1:

- A RC beam of rectangular section 230mm wide and 400 mm deep is reinforced with 4 bars of 12mm diameter provided with an clear cover of 25mm. Calculate the ultimate moment of resistance of the section and the maximum uniformly distributed super-imposed load this beam can carry if it is simply supported over a span of 3.5m. The material used are concrete grade M20 and steel grade Fe415.

Example 1: contd…..

Given:

f_{ck} =20 N/mm², f_y=415 N/mm², b=230mm, D=400mm, Ast=4-#12 = 4 x 113 = 452 mm², L=3.5m

Effective depth = d = D – d′ = 400 -31 = 369mm

Pt=452X100/(230X369)=0.532<0.96, under-reinforced

T_u = 0.87 fy Ast

= 0.87 x 415 x 452 = 163194.6 N

$X_{u.max}$ = 0.479X369=176.5 mm

Assume, x_u = 100 mm

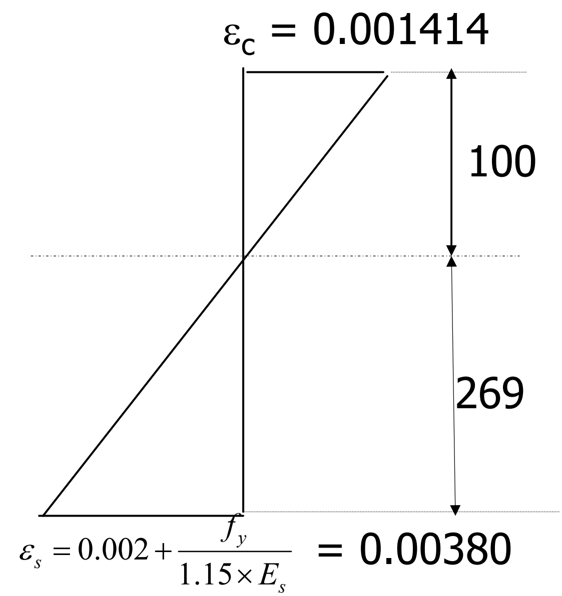
ε_c = 0.0038 x 100 / 269

= 0.001414

Since, ε_c < 0.002, Stress block is parabolic.

Stress corresponding to strain 0.001414

= 8.167 N/mm²



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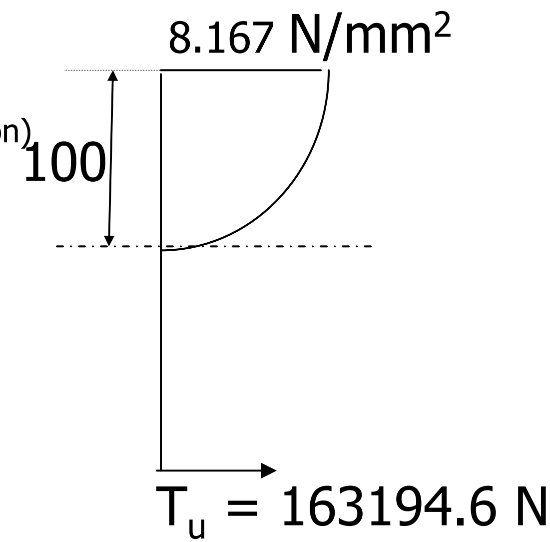
Example 1: contd…..



Area of Stress Block (ie area under stress curve x width of section)

= 111044.72

C_u = 111044.72 N < T_u

Assume higher value of x_u.

Assume, x_u = 122.5 mm

εc = 0.0038 x 122.5 / 246.5

= 0.00189

Since, ε_c < 0.002, Stress block is parabolic.

Stress corresponding to strain 0.00189

= 8.9066 N/mm2

Example 1: contd.....

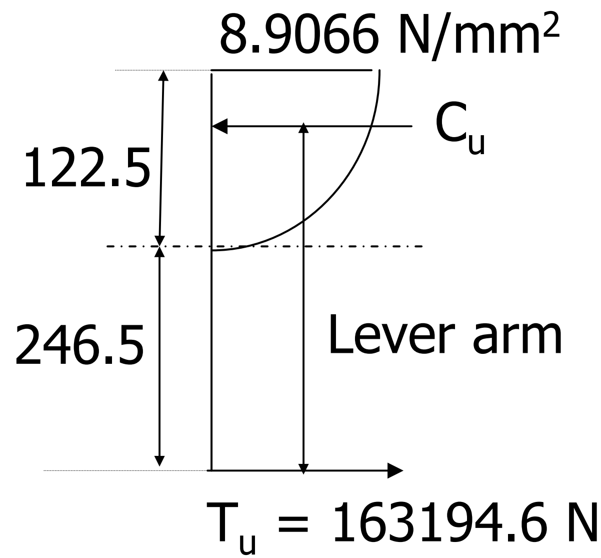
Area of Stress Block

$= 708.51 \times 230$

$C_u = 162957.3\ N \sim T_u$

Lever arm = 77+ 246.5

= 323.5 mm



Moment of Resistance,

Mur = C_u x Lever arm

= 162957.3 x 323.5

= 52.72 kN-m

For Simply Supported Beam, $M_u = w_u L^2/ 8$

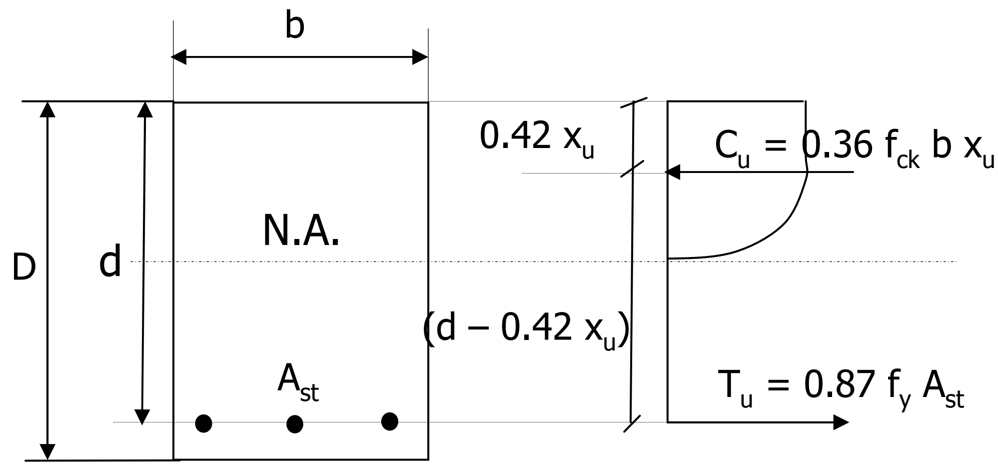
$52.72 = w_u (3.5)^2/8$, $w_u = 34.43$ kN/m

Safe load, $w_{safe} = w_u / 1.5 = 22.95$ kN/m

Allowable superimposed Load = $w_{safe} - w_{dead}$

= 22.95 – 25 x 0.23 x 0.4 = 20.65 kN/m

Analysis Problem
(Simplified Approach)



1.Depth of Neutral Axis

Equilibrium of Internal Forces; $C_u = T_u$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$\frac{x_u}{d} = \frac{0.87 f_y}{0.36 f_{ck}}\left(\frac{A_{st}}{bd}\right) = \frac{0.87 f_y}{0.36 f_{ck}}\left(\frac{p_t}{100}\right)$$

If $x_u > x_{u,lim}$, then take $x_u = x_{u,lim}$

2.Ultimate Moment of Resistance

Mur = C_u x Lever arm = Tu x Lever arm

Lever arm = d – 0.42x_u

Therefore, Mur = 0.36 f_{ck} b x_u (d – 0.42 x_u)

Or = 0.87 f_y A_{st} (d – 0.42 x_u)

Example 1: (Using Simplified Approach)

- Given:

f_{ck} =20 N/mm², f_y=415 N/mm², b=230mm, D=400mm,
Ast=4-#12 = 4 x 113 = 452 mm², L=3.5m

Effective depth = d = D – d′ = 400 -31 = 369mm

Depth of Neutral Axis = x_u= $\frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 452}{0.36 \times 20 \times 230}$ = 98.54 mm

Balanced depth of N.A. = $x_{u,max}$ = 0.479 d = 0.479 x 369 = 176.7 mm

Since $x_u < x_{u,max}$, section is under-reinforced.

M_{ur} = 0.87 fy Ast (d – 0.42 x_u)

= 0.87 x 415 x 452 (369 – 0.42 x 98.54)

= 53.46 KN-m

Example 1: (Using Simplified Approach)

Contd…

For a simply supported beam,

$M_u = w_u L^2/8$

$53.46 = w_u (3.5)^2/8$

Therefore,

w_u =34.91 kN-m

Safe Load, $w = w_u/1.5$

=23.27 kN-m

Dead load = 0.23 x 0.4 x 25 = 2.3 kN-m

Allowable superimposed load = 23.27 -2.3

= 20.97kN-m

Example 2:

A rectangular beam simply supported at its ends carries a uniformly distributed superimposed load of 25 kN/m over a simply supported span of 6m. The width of beam is 300mm. The characteristic strength of concrete is 20 N/mm2 and that of steel is 500N/mm^2. Design smallest section of the beam. (By LSM)

Assume overall depth of beam = L /10 = 6000/10 = 600 mm.

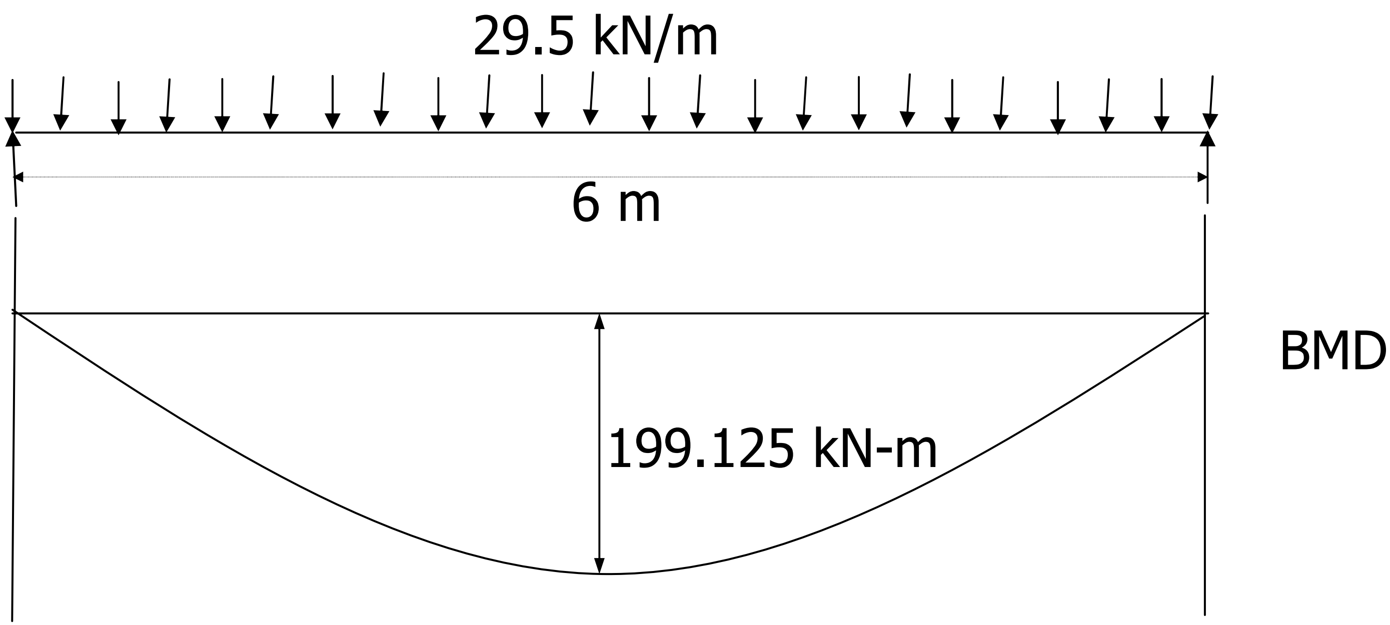
Dead Load = 0.6 x 0.3 x 25 = 4.5 kN/m

Superimposed Load = 25 kN/m

Total = 29.5 kN/m

Example 2:

contd…



$M_{max} = wL^2/8 = 29.5 \times 6^2 / 8 = 132.75$ kN-m

Factored moment $M_u = 1.5 \times 132.75 = 199.125$ kN-m

Example 2:

contd…

For M20 and Fe500,

Mu,lim/bd2 = 2.66 → Table D SP-16

$$d_{req} = \sqrt{\frac{M_{u,\lim}}{Rb}}$$

$$d_{req} = \sqrt{\frac{199.125\times10^{6}}{2.66\times300}}$$

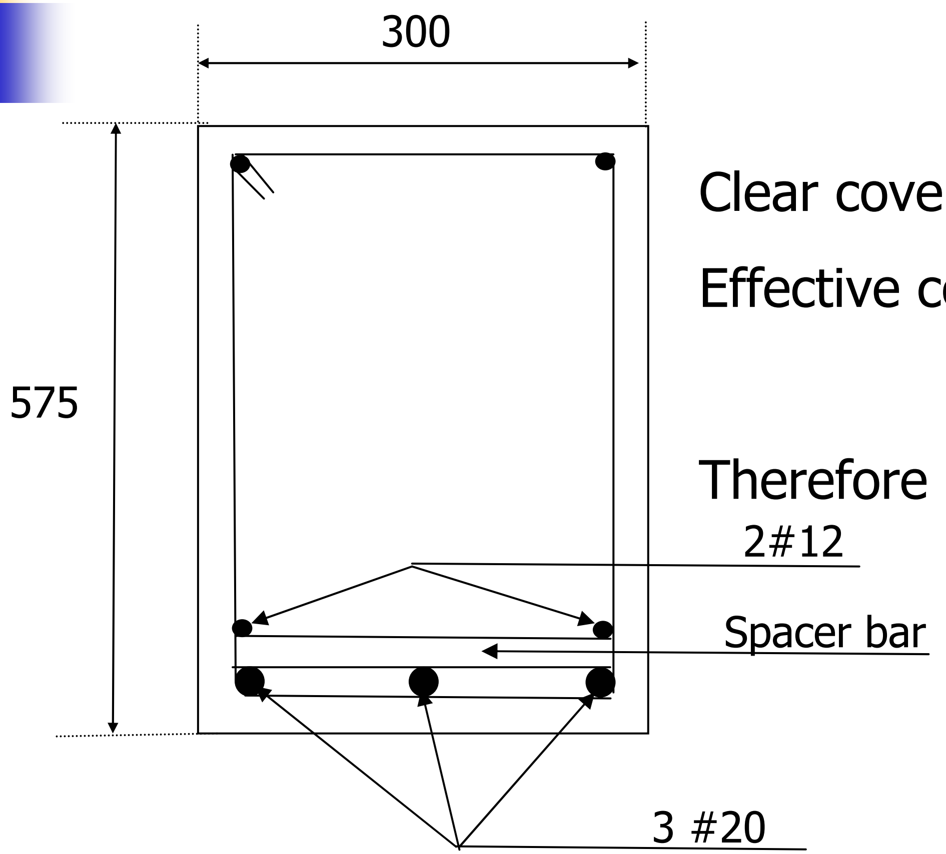
$$d_{req} = 500mm$$

$P_{t,lim}$ = 0.76 → Table E SP-16

A_{st} = 0.76 x 300 x 500 /100 =1140 mm^2

Provide 3 # 20 Tor and 2 # 12 Tor bars.

Example 2: contd…



Clear cover = 30 mm

Effective cover = 30 + 8 + 20 +25/2

=70.5 mm

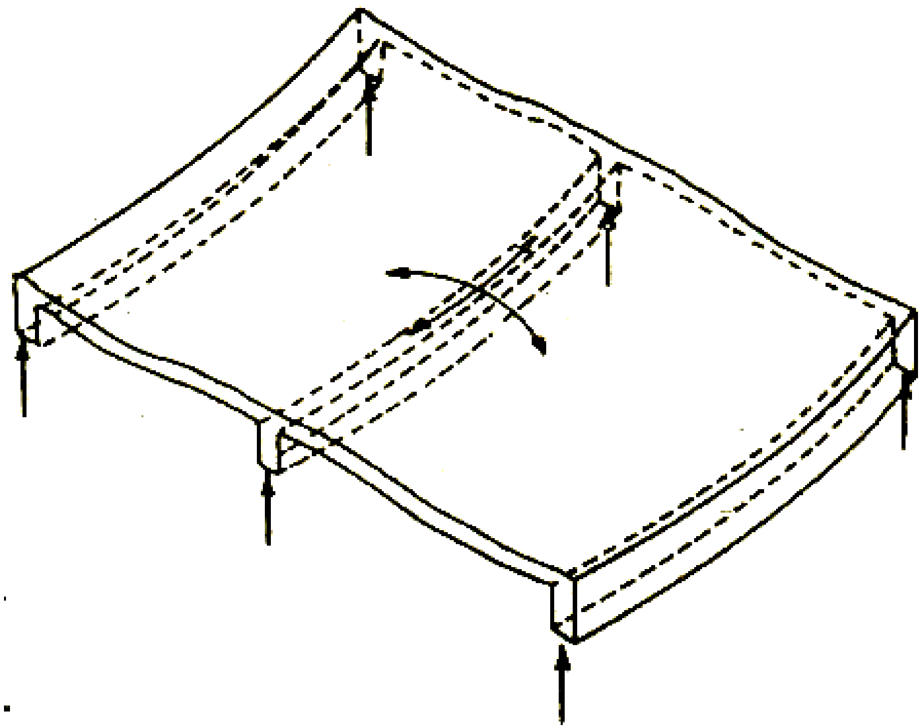
Therefore D = 500 + 70.5 =570.5mm

say 575 mm

< 600 mm

Assumed initially

Flanged Sections (T and L beams)



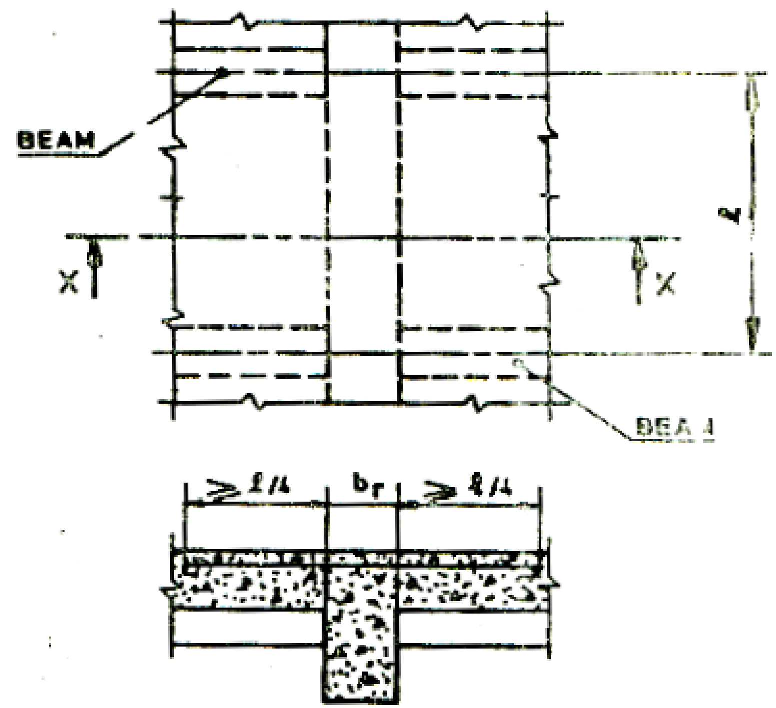
•Slab acts along with the beam in resisting compressive forces.

•Flange provides the compressive resistance and the web provides shear resistance and stiffness.

Requirements for T-beams and L-beams (Clause 23.1.1)

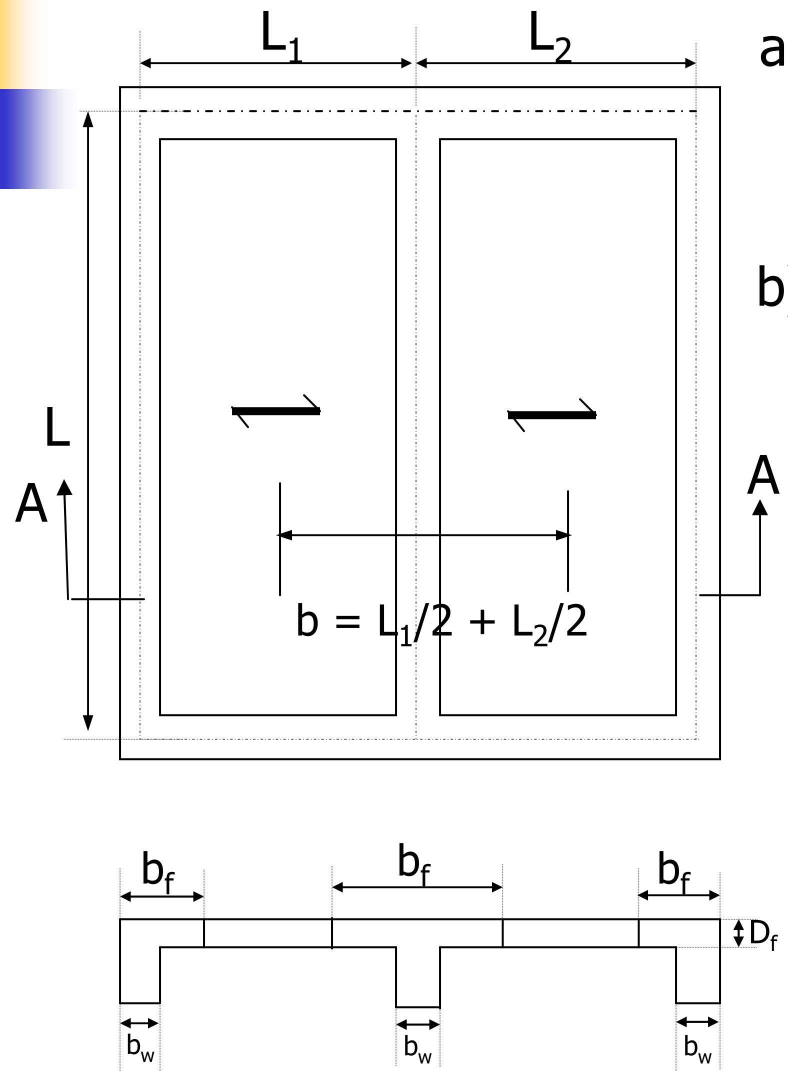
a) The slab shall be cast integrally with the web, or the web and the slab shall be effectively bonded together in any other manner; and

b) If the main reinforcement of the slab is parallel to the beam, transverse reinforcement shall be provided as shown in fig. below. Such reinforcement shall not be less than 60 percent of the main reinforcement at mid span of slab.



SECTION XX

Effective width of flange: (Clause 23.1.2)



Section A-A

a) For T-beams:

$$b_f = \frac{l_0}{6} + b_w + 6D_f \leq b(\text{actual width})$$

b): For L-Beams

$$b_f = \frac{l_0}{12} + b_w + 3D_f \leq b \text{ (actual width)}$$

C) For isolated beams, the effective flange width shall be obtained as below but in no case greater than actual width.

$$\text{For T-beam, } b_f = \frac{l_0}{\left(\frac{l_0}{b}\right) + 4} + b_w$$

$$\text{For L-beam, } b_f = \frac{0.5 l_0}{\left(\frac{l_0}{b}\right) + 4} + b_w$$

Note: For continuous beams and frames l_0 may be assumed as 0.7 times effective span.

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Properties of Flanged Section

- Depending upon depth of N.A. (x_u) in relation to depth of flange thickness (D_f) following cases arise.

Case 1
$x_u < D_f$
(N.A. lies inside the flange.)

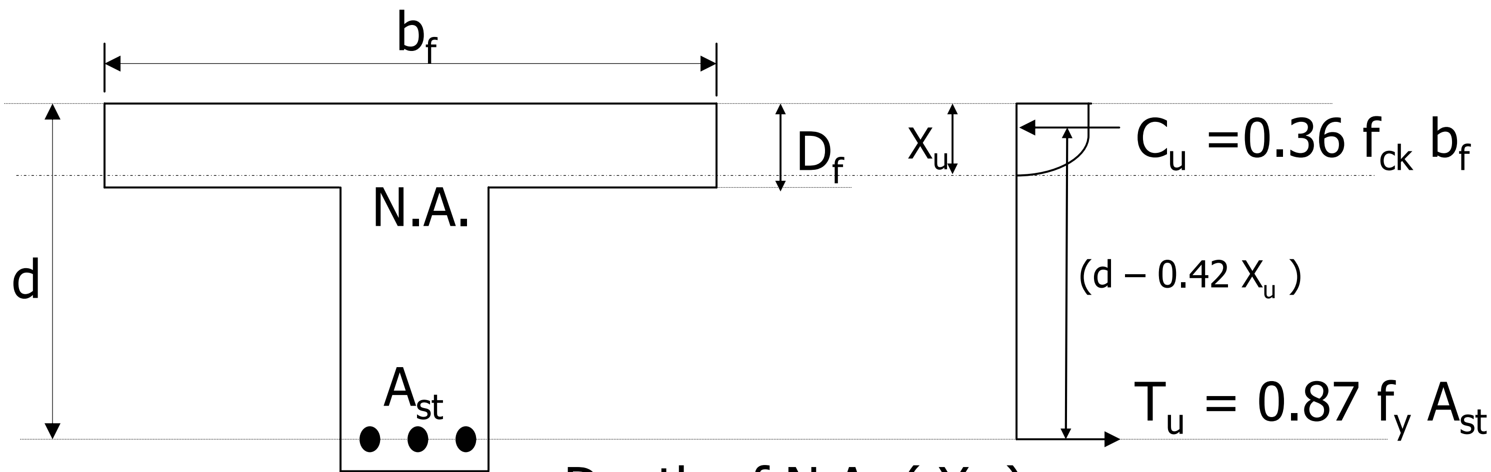
Case 2
$x_u < D_f$
(N.A. lies outside the flange.)

Case 2a
$3x_u/7 > D_f$
(Rectangular part of the stress block greater than D_f)

Case 2b
$3x_u/7 < D_f$
(Rectangular part of the stress block less than D_f)

Case 1: Neutral axis lying inside the flange ie $X_u < D_f$

In this case flanged beam can be considered as a rectangular beam of width $b = b_f$ and expression for X_u, M_{ur} and A_{st} for singly reinforced beam can be used by replacing b by b_f.



Depth of N.A. (X_u)

From equilibrium condition, $C_u = T_u$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

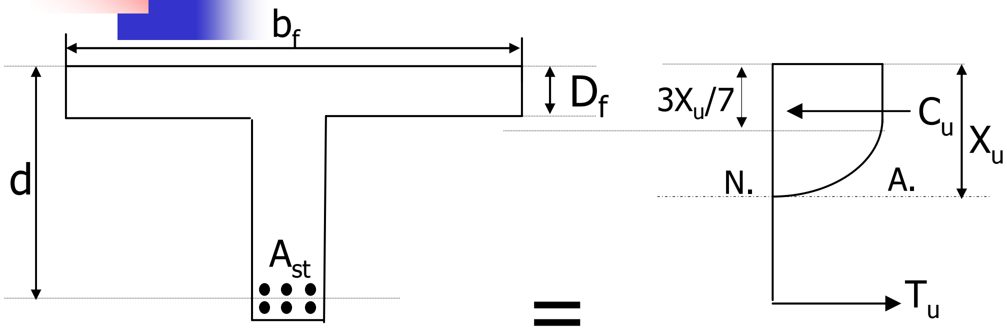
Moment of Resistance (M_{ur}) :

$$M_{ur} = 0.36 f_{ck} b_f x_u (d - 0.42 x_u) \quad \text{OR}$$

$$M_{ur} = 0.87 f_y A_{st} (d - 0.42 x_u)$$

Case 2: N.A. lying in the web ie $X_u > D_f$

Case 2a: $3X_u/7 > D_f$

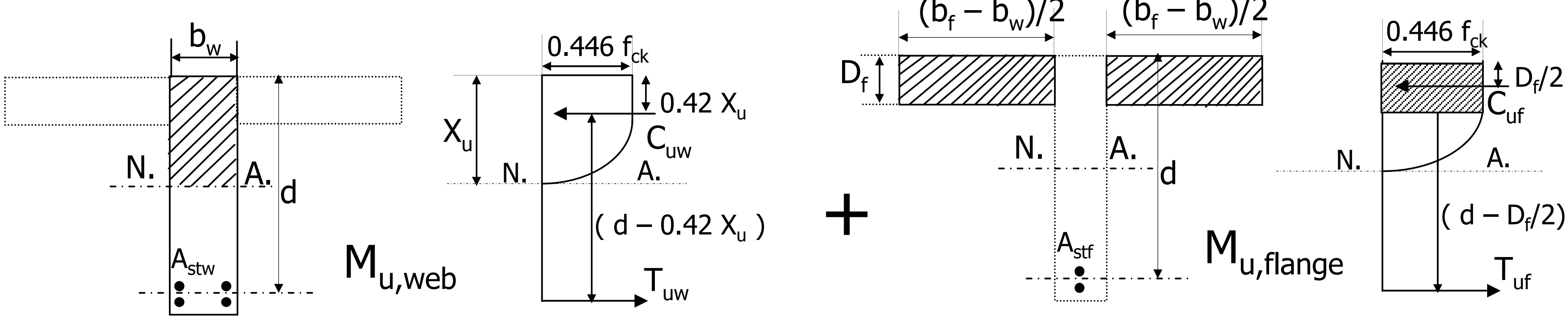


Depth of N.A. (X_u) :

For equilibrium, $C_u = T_u$

$$0.36 f_{ck} b_w x_u + 0.446 f_{ck}(b_f - b_w) D_f = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st} - 0.446 f_{ck}(b_f - b_w) D_f}{0.36 f_{ck} b_w}$$



Moment of Resistance (M_{ur}) : $M_{u,web} + M_{u,flange}$

$$M_{ur} = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck}(b_f - b_w) D_f (d - \frac{D_f}{2})$$

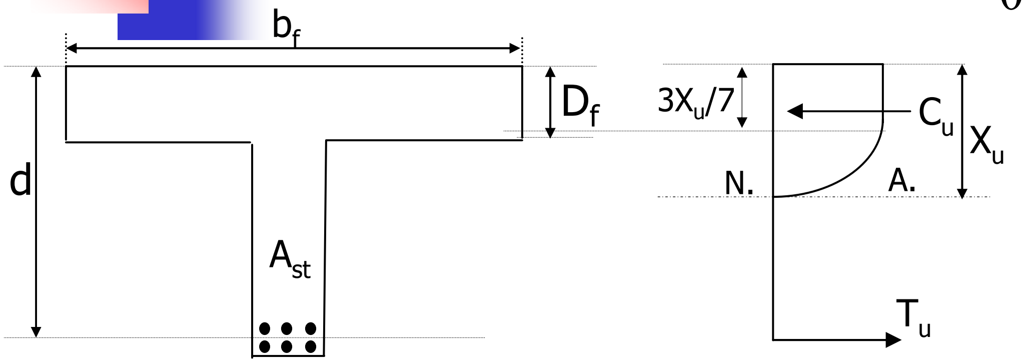
Case 2b: $3X_u/7 < D_f$

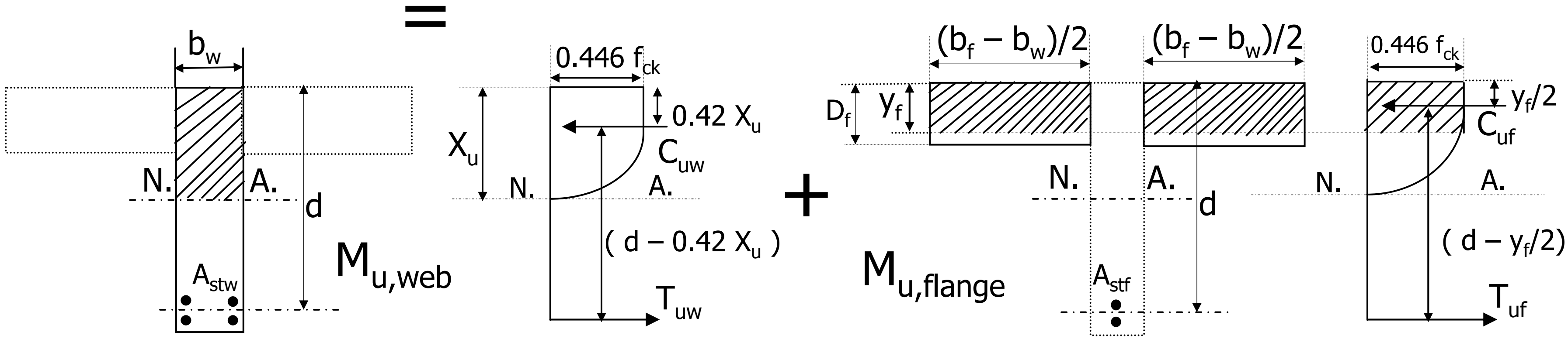
Depth of N.A. (X_u) :
For equilibrium, $C_u = T_u$

$$0.36 f_{ck} b_w x_u + 0.446 f_{ck}(b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$where, y_f = 0.15 x_u + 0.65 D_f \le D_f$$

$$x_u = \frac{0.87 f_y A_{st} - 0.446 \times 0.65 f_{ck} D_f (b_f - b_w)}{0.36 f_{ck} b_w + 0.446 \times 0.15 f_{ck} (b_f - b_w)}$$





Moment of Resistance (Mur) : $M_{u,web}$ + $M_{u,flange}$

$$M_{ur} = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck}(b_f - b_w) y_f (d - \frac{y_f}{2})$$

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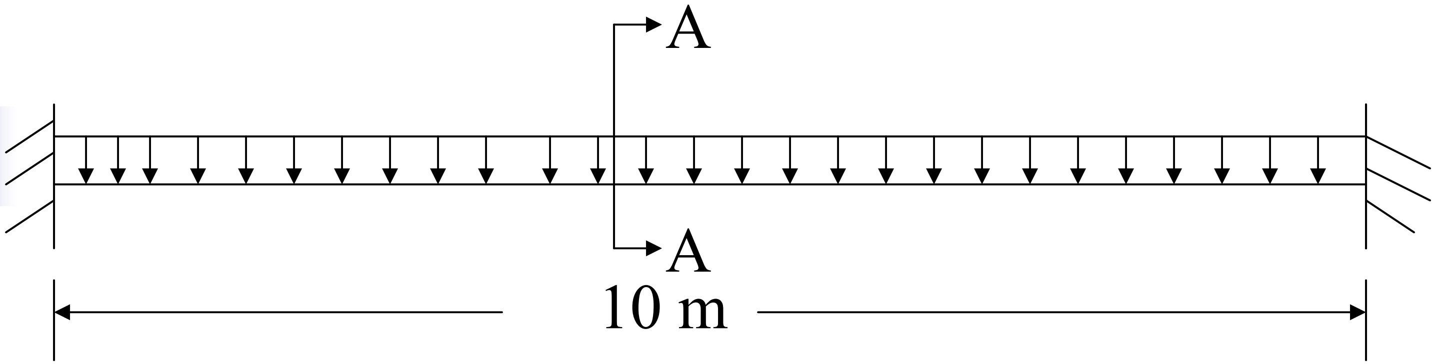
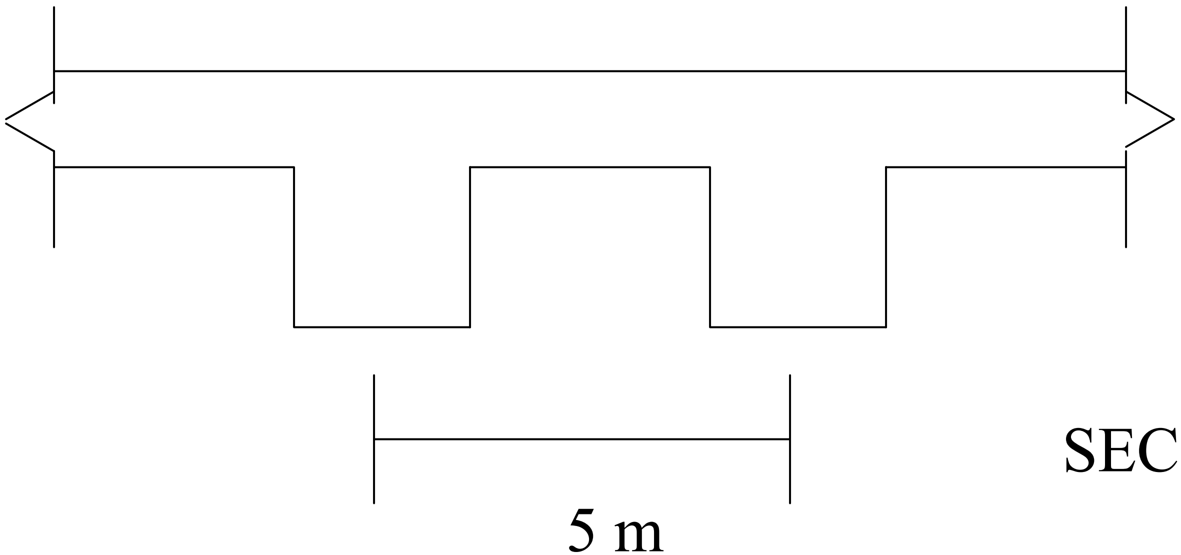
Design Example

- Beam Span : 10m (15-R/10)
- Beam end conditions: Fixed
- Beam width : 500mm (your choice)
- Spacing of Beams : 5m c/c
- Slab thickness : 150mm
- Concrete grade : M20 (Your grade)
- Reinforcements : Fe415
- Imposed Load
  on Slab : 12.5 kN/sqm (10kN/sqm)





SECTION A-A

Imposed load on slab = 12.5 kN/m^2

Slab thickness = 0.15 m

Load from slab = (12+0.15*25)kN/m^2

= 16.35 kN/m^2

Load on beam = 16.35*5=81.75 kN/m

Assumed beam depth 1200mm

Self weight of beam web = (1.2-0.15)*0.5*25

= 13.125 kN/m

Total load = (81.75+13.125) kN/m

= 94.875 kN/m

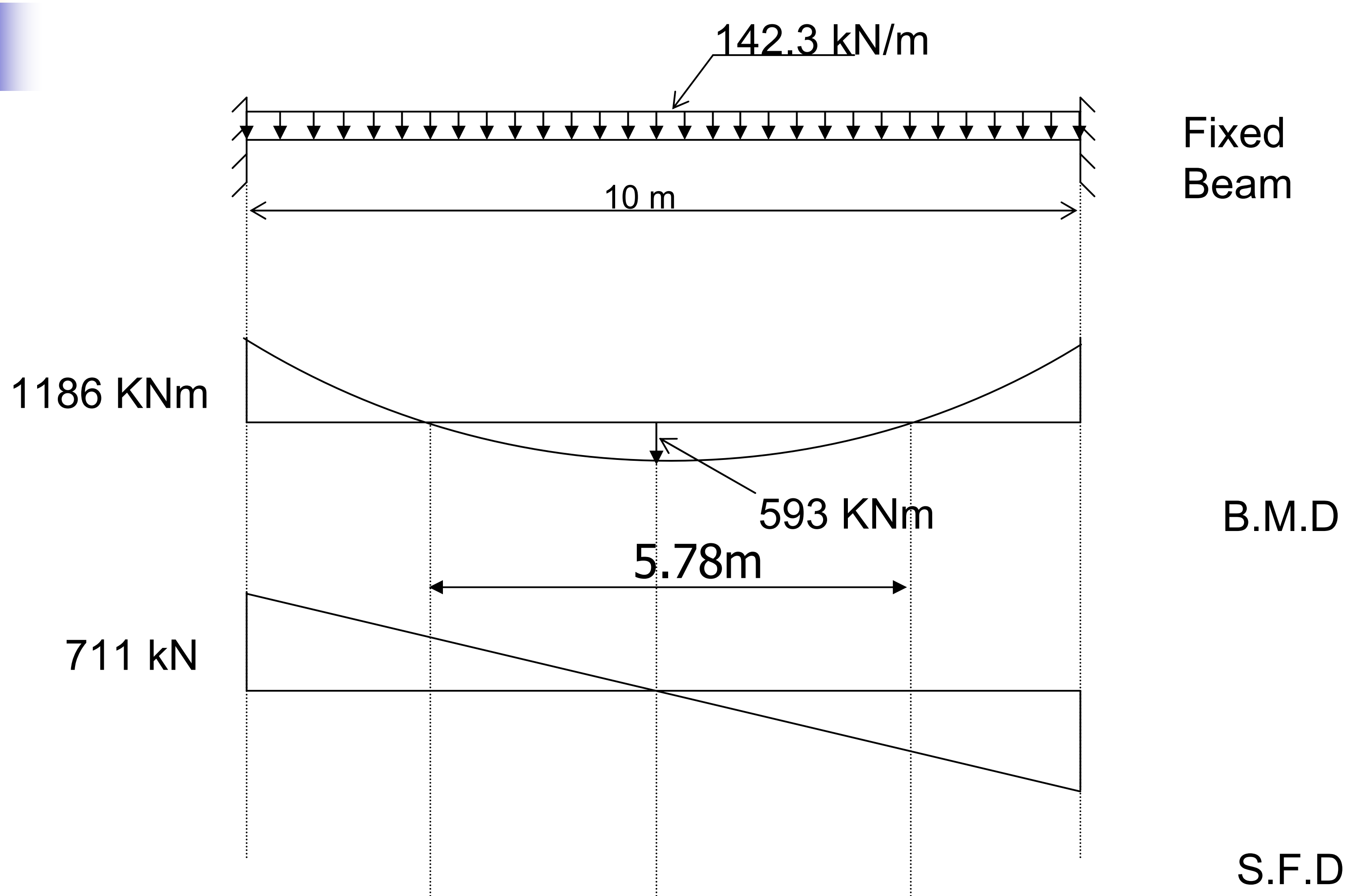
Factored load =1.5*94.875 = 142.3 kN/m

Maximum sagging moment at span (M_u)= 593 kN-m

For T beams, from

$$b_f=L_0/6+b_w+6D_f$$

Cl 23.1.2(a) page-37


142.3 kN/m
10 m
Fixed
Beam
1186 KNm
593 KNm
5.78m
B.M.D
711 kN
S.F.D

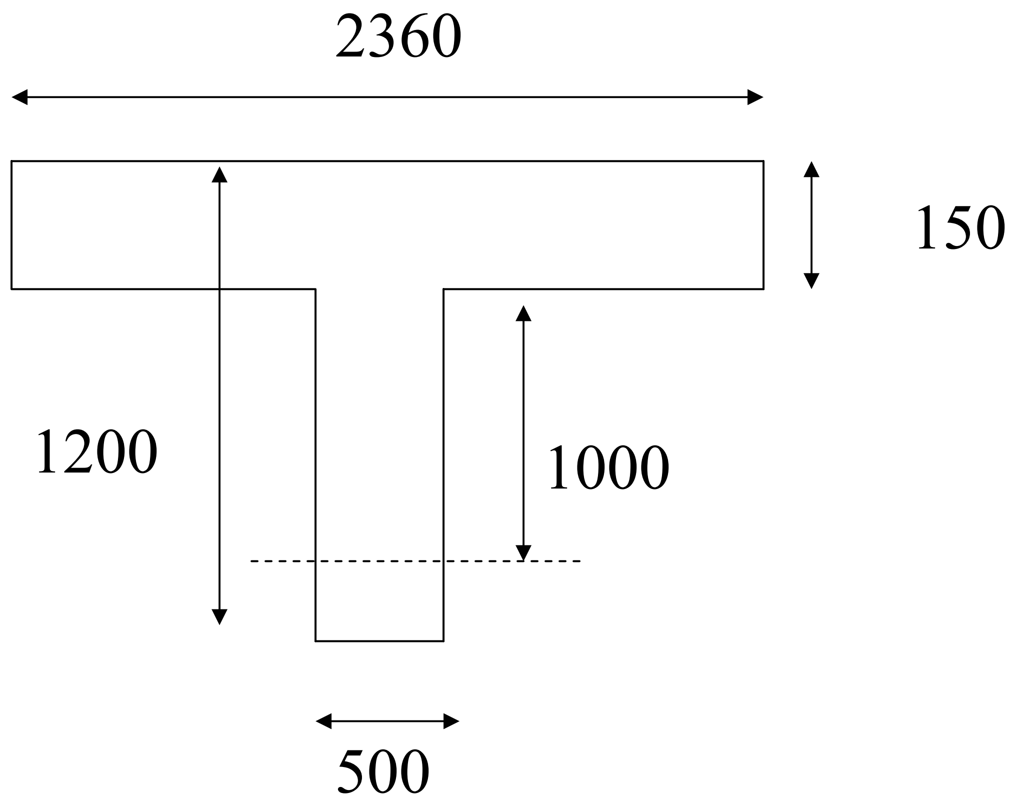
$$b_f = L_0/6 + bw + 6D_f$$

L_0=Distance between points of zero moments=5.78 m

b_w=500 mm

D_f=150 mm

b_f=5.78/6+500+6*150=2360 mm



Let 50 mm be clear cover,

Effective depth = 1150 mm

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- $M_{u\ res} = 0.36 * D_f * (d - 0.42 * D_f) * b_f * f_{ck}$
  $= 2770.546$ kNm

Since $M_{u\ res} > M_{u\ load}$

Hence, $x_u < D_f$. Therefore, neutral axis in flange,

Hence Beam acts as a Rectangular Beam and not as a Tee Beam.

$$A_{st,req} = \frac{0.5 \times f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 \times M_u}{f_{ck} b d^2}}\right) bd$$

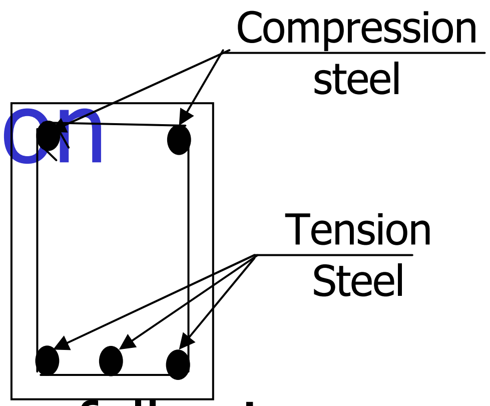
$$A_{st,req} = \frac{0.5 \times 20}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 594 \times 10^6}{20 \times 23601150^2}}\right) 2360 \times 1150$$

$$A_{st,req} = 1447.34 mm^2$$

- Provide 4# 25

A_{st}, provided = $1570\,mm^2 > 1447.3\,mm^2$

Doubly Reinforced Section



Doubly Reinforced Section is required under the following circumstances

- Sectional dimensions are restricted by headroom considerations and strength of singly reinforced section is inadequate.
- If high bending moment exists over a relatively short length of the beam only (e.g. over supports of a continuous beam.)
- To increase the stiffness of the section
- For member subjected to reversal of stresses

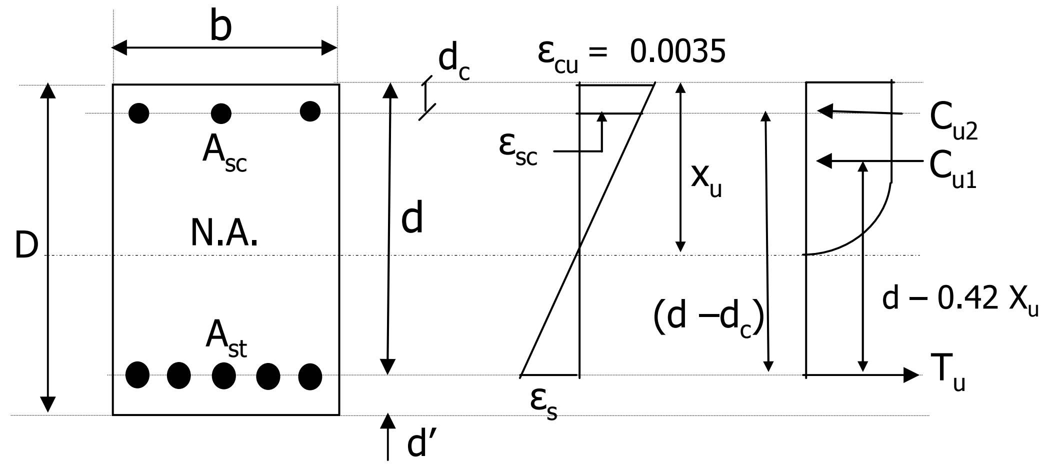
Properties of Doubly Reinforced Section

Depth of N.A. (X_u)

Equating total compression (in concrete and compression steel) with total tension (in steel) ,

$$C_u = T_u$$

$$\underbrace{0.36 f_{ck}\, b\, x_u}_{C_{u1}} + \underbrace{(f_{sc} - f_{cc})\, A_{sc}}_{C_{u2}} = \underbrace{0.87 f_y\, A_{st}}_{T_u}$$



f_{cc} = compressive stress in concrete at the level of compression steel (for simplification f_{cc} may be ignored or may be taken as $0.45 f_{ck}$)

fsc = stress in the compression steel corresponding to ε_{sc}. It can obtained from the strain diagram, and is given by : $\varepsilon_{sc} = 0.0035(1 - d_c/x_u)$

For mild steel (Fe250), $f_{sc} = \varepsilon_{sc} E_s =< 0.87 f_y$

For HYSD bars, the values of f_{sc} are obtained from stress-strain diagram of HYSD Bars corresponding to values of ε_{sc} for different ratios d_c/d.

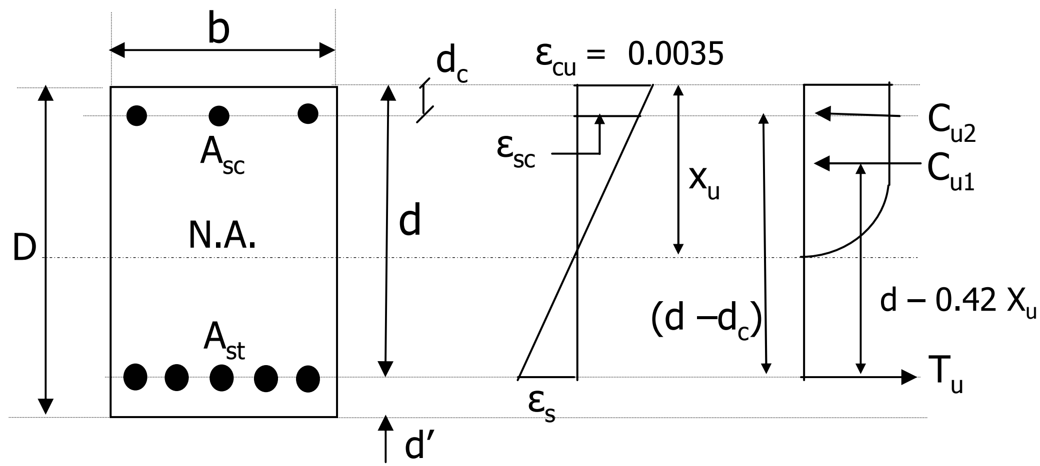
Stress in compression reinforcement f_{sc} N/mm^2 in doubly reinforced section with HYSD Bars

fy, N/mm2	d_c/d			
	0.05	**0.10**	**0.15**	**0.20**
415	355	353	342	329
500	424	412	395	370

Properties of Doubly Reinforced Section

Moment of Resistance (Mur):

The ultimate moment of resistance is obtained by taking moments of Cu1 (concrete) and Cu2 (compression steel) about centroid of tension steel.



$$M_{ur} = M_{u1} + M_{u2}$$

$$= 0.36\, f_{ck}\, b\, X_u\, (d - 0.42\, X_u) + (f_{sc} - f_{cc})\, A_{sc}\, (d - d_c)$$

Note: In design, the section is kept balanced to make full utilization of moment of resistance of concerte. Therefore, $M_u = M_{ur} = M_{u1} + M_{u2}$

where, $M_{u1} = M_{ur,max}$ (ie $M_{u,lim}$) of a singly reinforced balanced section

and $M_{u2} = M_u - M_{ur,max}$, this moment will be resisted by compression steel.

Properties of Doubly Reinforced Section

1. Area of tension steel A_{st}:

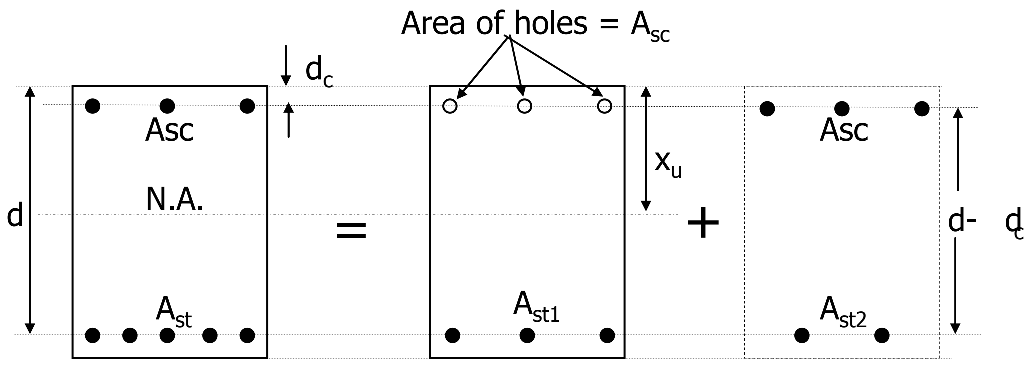
$M_u = M_{ur} = Mu_1 + Mu_2$

For section I resisting moment Mu1,

$M_{u1} = 0.87\, f_y\, A_{st1}\, (d - 0.42\, X_u)$

In design problem, the section is kept balanced to make full utilization of concrete.

Hence, $M_{u1} = M_{u,lim}$

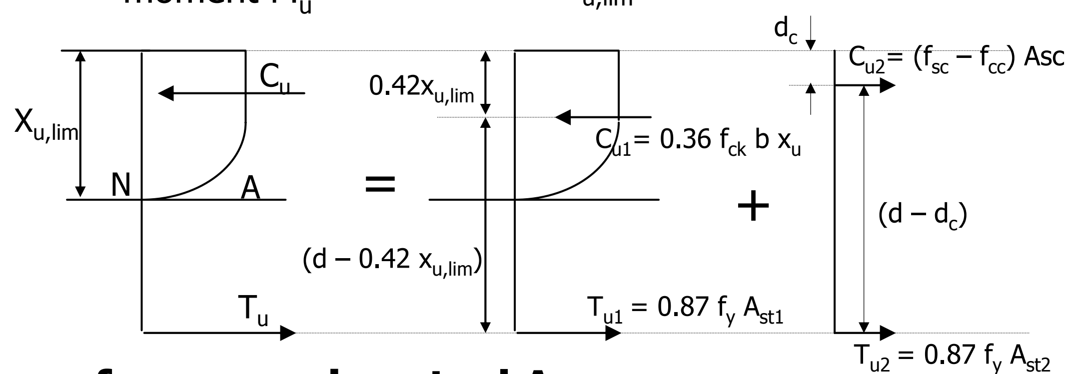


Section Subjected to moment M_u

Sec I: Resisting moment M_{u1} ie $M_{u,lim}$

Sec II: Resisting moment M_{u2}

$$A_{st1} = \frac{M_{u,\,lim}}{0.87 f_y \left(d - 0.42 x_{u,\,lim}\right)}$$



For section II resisting moment M_{u2},

$M_{u2} = M_u - M_{u1} = 0.87\, f_y\, A_{st2}\, (d - d_c)$

$$A_{st2} = \frac{M_{u2}}{0.87 fy \left(d - d_c\right)}$$

Total tension steel $A_{st} = A_{st1} + A_{st2}$

2. Area of compression steel A_{sc}

By equilibrium, $C_{u2} = T_{u2}$

$(f_{sc} - f_{cc})\, A_{sc} = 0.87\, f_y\, A_{st2}$

$$A_{sc} = \frac{0.87 fy A_{st2}}{\left(f_{sc} - f_{cc}\right)} \cong \frac{0.87 f_y A_{st2}}{f_{sc}}$$

Design Example

Design a fixed beam with concrete grade M20 and steel Fe415.
Effective span of beam= 10 m
Live Load = 85 kN/m
Take width of beam= 450 mm ,
Thickness of slab = 120mm ,
c/c distance between beams = 3000 mm

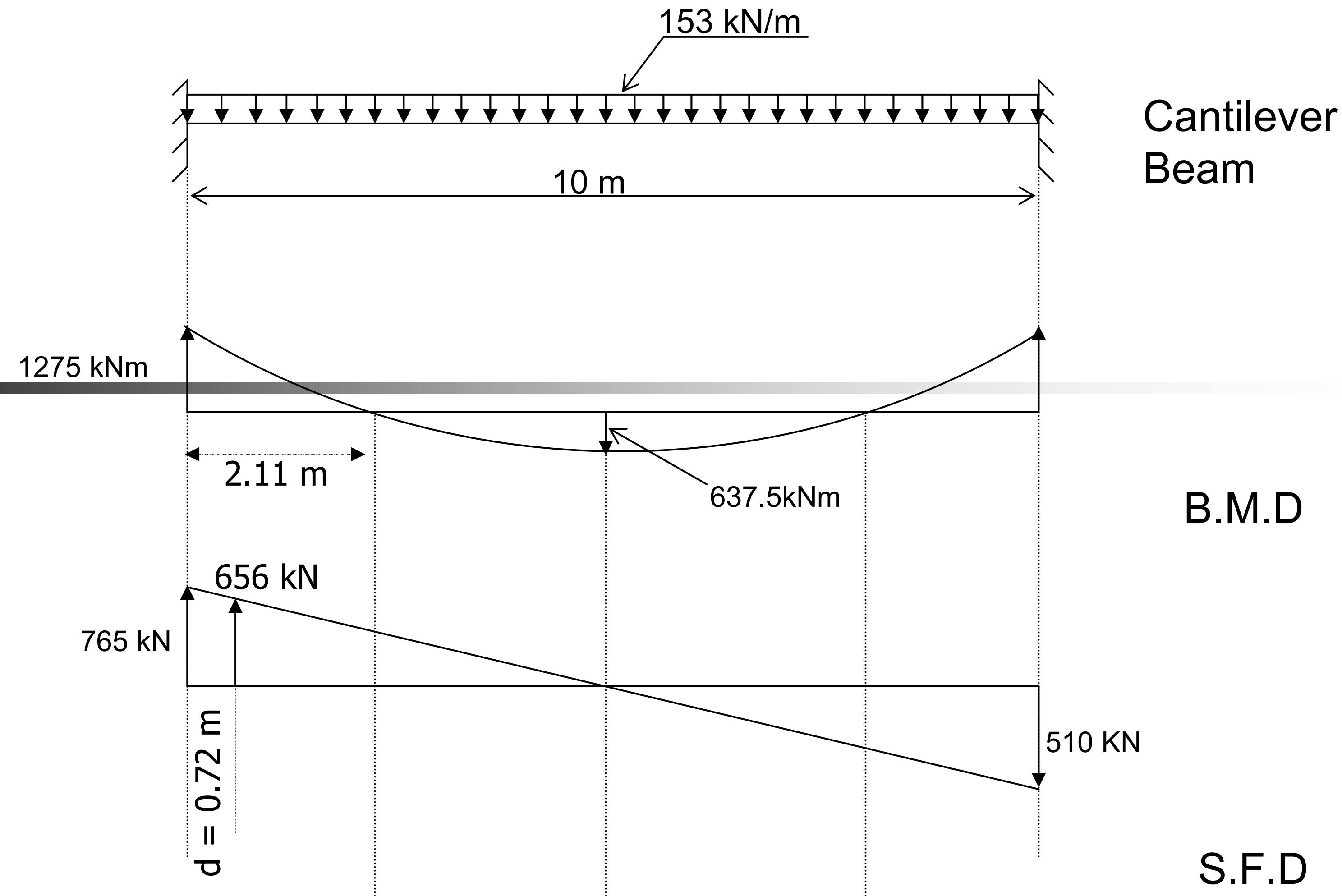
Solution

Assume overall depth of beam=800 mm (To calculate self wt of beam)

Loading:

Superimposed Load	= 85	kN/m
Slab Load = 25 x 0.12 x 3.0	= 9	kN/m
Beam load= 25 x (0.8 – 0.12) x .450	= 7.65	kN/m
Total	101.65 kN/m ~ 102	kN/m

Factored load = 1.5*101.65 = 152.5kN/m ~ 153kN/m


153 kN/m
10 m
Cantilever
Beam
1275 kNm
2.11 m
637.5kNm
B.M.D
656 kN
765 kN
d = 0.72 m
510 KN
S.F.D

Material Grade:

Concrete M20 and Steel Fe415

Maximum B.M. at support = $WL^2/12 = 153 \times 10^2/12$

$= 1275$ kN-m

Maximum B.M. (at midspan) = $WL^2/24 = 153 \times 10^2/24$

$= 637.5$ kN-m

We will design centre section as T-beam and support section as doubly reinforced beam.

Design of T-beam (at centre): M_u = 637.5 kN-m

Effective flange width $b_f = L_0/6 + bw + 6D_f$

L_0=Distance between points of zero moments=5.78 m

b_w=450 mm

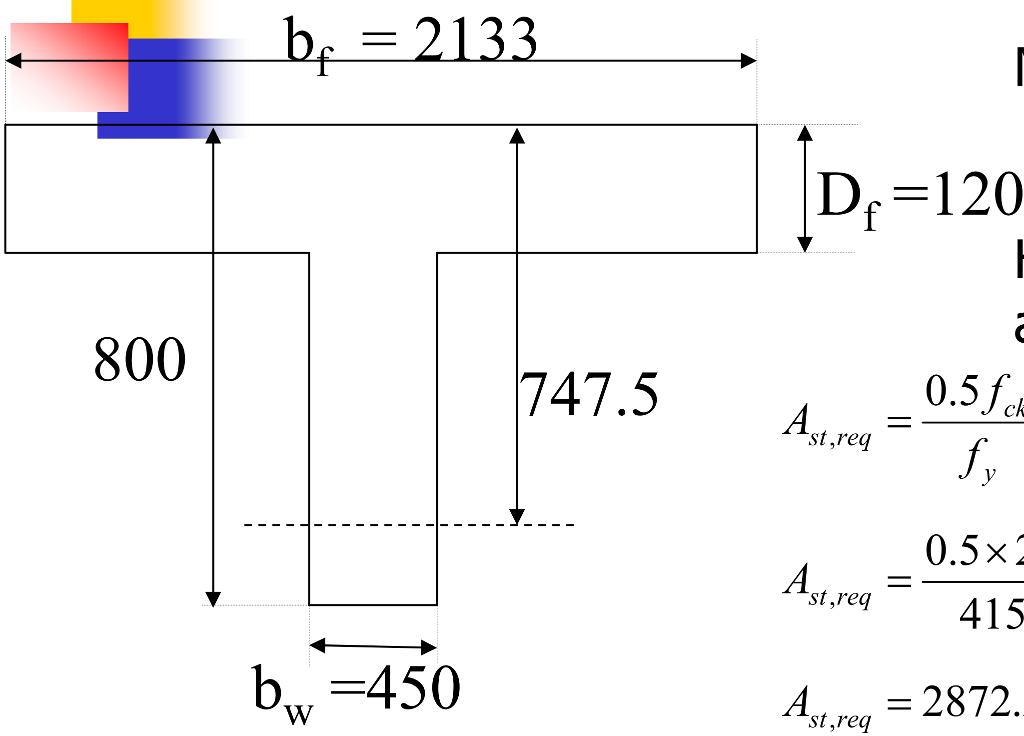
D_f=120 mm

b_f=5.78/6+450+6*120=2133 mm < b =3000 mm

Assuming 25mm dia bars.

Effective cover d' = 30 + 10 + 12.5 = 52.5 mm

Effective depth d = 800 – 52.5 = 747.5 mm



$M_{u,flange}$ = 0.36 f_{ck} b_f D_f (d – 0.42 D_f)

= 1284.7 kN-m > M_u

Hence N.A. lies in flange and section acts as a rectangular section.

$$A_{st,req} = \frac{0.5 f_{ck}}{f_y}\left(1-\sqrt{1-\frac{4.6 M_u}{f_{ck} b d^2}}\right) bd$$

$$A_{st,req} = \frac{0.5\times 20}{415}\left(1-\sqrt{1-\frac{4.6\times 637.5\times 10^6}{20\times 450\times 747.5^2}}\right) 450\times 747.5$$

$$A_{st,req} = 2872.20 mm^2$$

Provide 6 # 25 dia bars.

$A_{st,prov}$ = 2940 mm^2

Design of doubly reinforced section (at support) : M_u= 1275 kN-m

For M20 and Fe415 , R_u = 2.76 and P_t = 0.96

D = 800 mm , b = 450 mm

As we need doubly reinforced section, higher effective cover will be assumed. Say d' = 80 mm

Effective depth d = 800 – 80 = 720 mm

Mu,lim = $R_u bd^2$ = 2.76 x 450 x 720^2 = 643.82 kN-m < M_u

The beam must be doubly reinforced

Calculation of tension steel A_{st} :

Total tension steel $A_{st} = A_{st1} + A_{st2}$

$$A_{st1} = \frac{M_{u.\lim}}{0.87 \times f_y \times (d - 0.42 x_{u.\lim})}$$

$$A_{st1} = \frac{643.82 \times 10^6}{0.87 \times 415(720 - 0.42 \times 344.88)}$$

For Fe415

$X_{u,lim}$ = 0.479 d

$$A_{st1} = 3100.4 mm^2$$

$$A_{st2} = \frac{M_u - M_{u,\lim}}{0.87 \times f_y (d - d_c)}$$

d_c = effective cover to compression reinforcement

$$A_{st2} = \frac{631.18 \times 10^6}{0.87 \times 415 (720 - 50)}$$

$$A_{st2} = 2609.2 mm^2$$

Total tension steel $A_{st} = A_{st1} + A_{st2}$

$= 3100.4 + 2609.2 = 5709.6 \text{ mm}^2$

Provide 12# 25 dia bar. $A_{stprov} = 5880 \text{ mm}^2$

Calculation of compression steel A_{sc}:

$$A_{sc} = \frac{0.87 f_y A_{st2}}{f_{sc}}$$

f_{sc} = stress in compression steel which can be calculated from (d_c/d)

$d_c/d = 50 / 720 = 0.0694 \longrightarrow f_{sc} = 354\ N/mm^2$

Refer Table F of SP 16

$$A_{sc} = \frac{0.87 \times 415 \times 2609}{354}$$

$$A_{sc,req} = 2661 mm^2$$

Provide 6 # 25 dia Bars.

$As_{t,provd} = 2940\ mm^2$

Curtailment of Support Reinforcement

As per Clause 26.2.3.4,

At least one-third of the total reinforcement provided for negative moment at the support shall extend beyond the point inflection for a distance not less than the effective depth of the member or 12Φ or one-sixteenth of the clear span whichever is greater.

Therefore, A_{st} required to extend = 5880/3 = 1960 mm^2

We will curtail 8# 25 dia bars. $A_{st,avaliable}$ = 1960 mm^2

Required to extend by distance,

Effective depth d = 745 mm

12 Φ = 12 x 25 = 300 mm

Clear span/16 = (10,000 - 400)/16 = 600 mm

(assuming support width = 400 mm)

} Whichever is greater

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Design for Shear

Factored Shear force at critical section V_u= 656 kN → Clause 22.6.2.1

ζ_{cmax} = V_u/bd = 2.023 N/mm^2 < 2.8 N/mm^2 → Clause 40.2.3

Percentage of tension reinforcement,

$$P_t = \frac{A_{st}}{b \times d} \times 100 = \frac{5880}{450 \times 720} \times 100$$

$$P_t = 1.8\%$$

Therefore, τ_c = 0.758 N/mm^2 → Clause 40.2.1

Shear strength of concrete = τ_c bd

V_c = 0.758 x 450 x 720

= 245.6 KN

< 655.5KN

Hence shear reinforcement is required.

166.8 x 10

Design shear $V_s = V_u - V_c$

$= 655.5 - 245.6$

$= 409.9$ KN

Provide 2- legged 12 Φ stirrups.

$$S_v = \frac{0.87 f_y A_{sv} d}{V_s} = \frac{0.87 \times 415 \times 226 \times 720}{409.9 \times 10^3} = 143.32 mm$$

Clause 40.4 a)

Therefore, provide 12 Φ 2- Legged Stirrups @ 140 c/c.

Minimum spacing requirement for shear reinforcement

1) 0.75 d = 0.75 x 720= 540 mm

2) 300 mm

Clause 26.5.1.5

3) $$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y}$$

Clause 26.5.1.6

Using 2-legged 12 Φ stirrups.

A_{sv} = 226 mm²

$$S_v = \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87 \times 415 \times 226}{0.4 \times 450}$$

$$S_v = 453mm$$

As shear force goes on reducing towards the centre, we can increase the spacing of stirrups in the middle zone.

We will provide 2 legged 12Φ stirrups @ 300 c/c.

Provision 26.5.1.6 need not be complied with when the maximum shear stress calculated is less than half the permissible value and in members of minor structural importance such as lintels.

Percentage of tension steel in midspan,

$$P_t = \frac{A_{st}}{b_w d} \times 100 = \frac{2940}{450 \times 745.5} \times 100 = 0.87\%$$

τ_c =0.589 N/mm^2 → Table 19

Shear resisting capacity of section = τ_c b d

= 0.589 x 450 x 745.5

V_u' = 197.59 kN

Distance corresponding to shear force $V_u'/2$ (ie 98.79 kN)

$$98.79 = 1.5 \times (510 - 102\, y)$$

$$y = 4.35 \text{ m}$$

We will provide 2 Legged 12 ϕ stirrups @ 300 c/c in middle 1.3 m zone.

No Side face reinforcement is required as depth of web in a beam is less than 750 mm.

(Depth of web = 800 – 120 = 680 mm)

Check for Deflection

span = 10 m

Basic Value = 26

Modification Factor = 1.1

(Depends on area and stress of steel in tension reinforcement)

Reduction Factor = 0.8

(Depends on ratio of b_w/b_f)

Modified Basic Value = 26 x 1.1 x 0.8 = 22.88

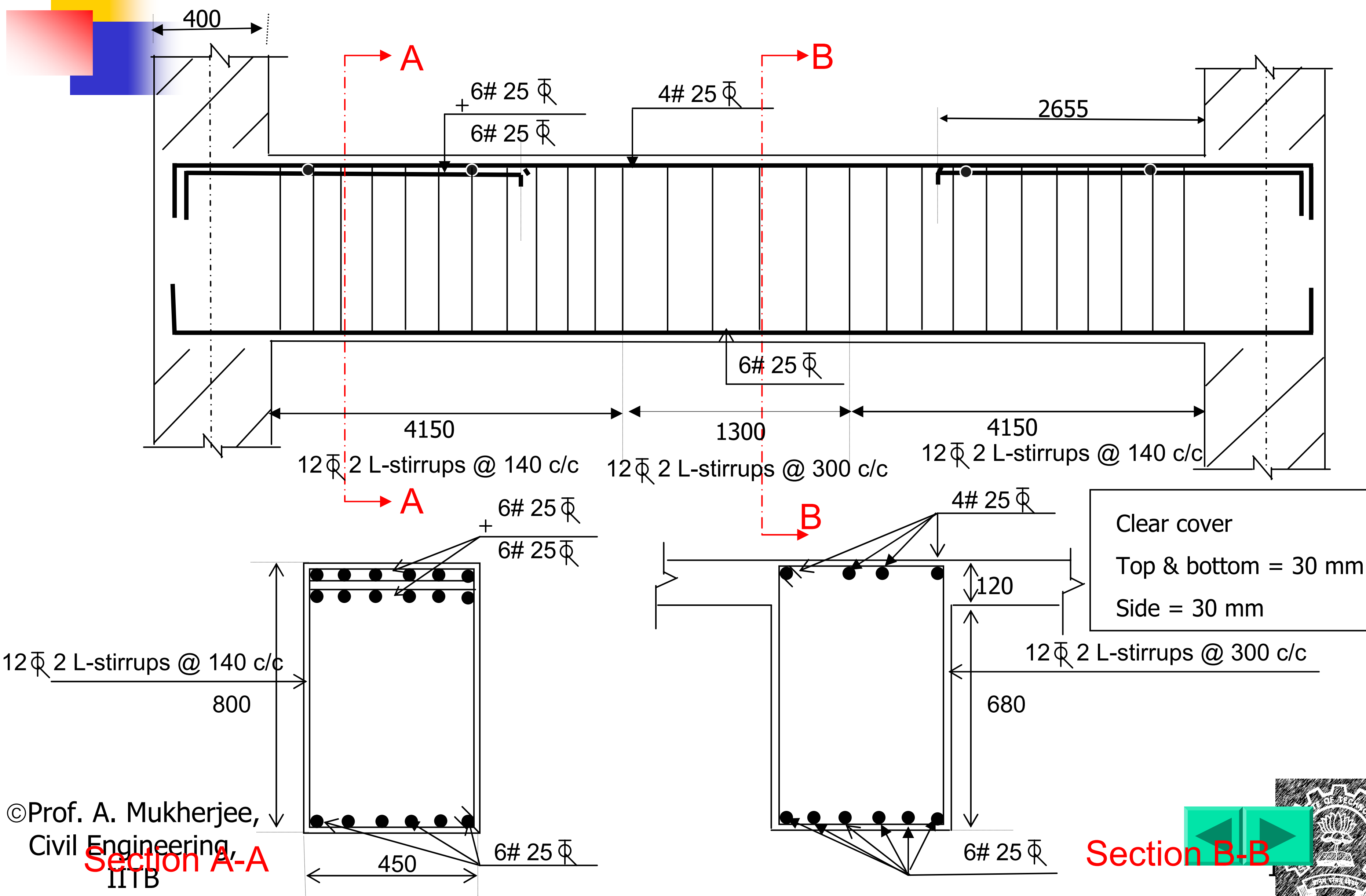
L / d = 10 / 0.745 = 13.42 < 22.88

Clause 23.2.1

Refer Fig. 4 of IS- 456:2000

Refer Fig.6 Clause 23.2.1e)

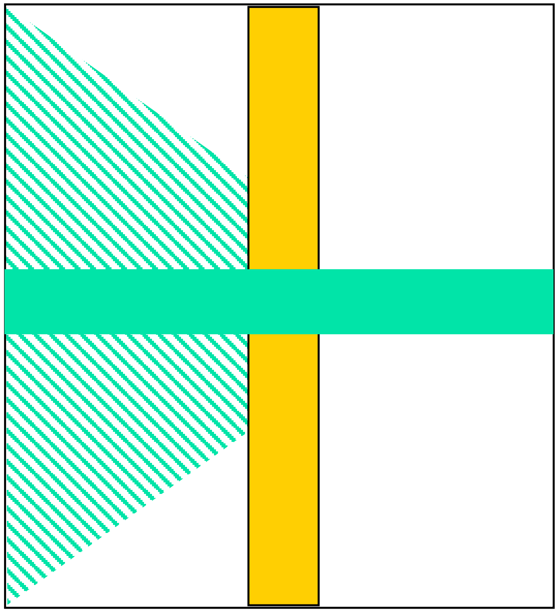
Reinforcement Details

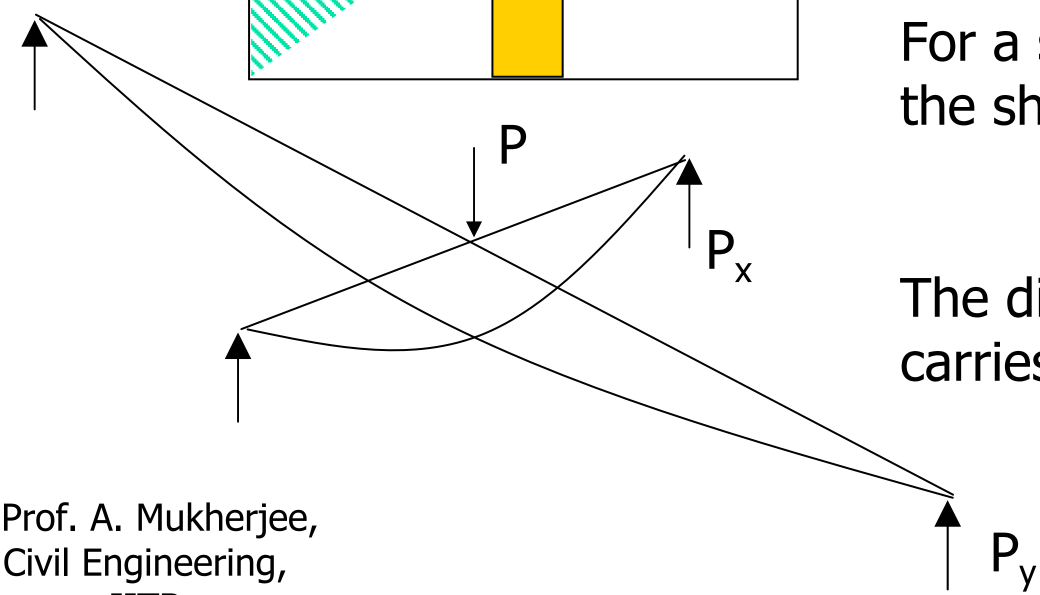


Slabs

- Slab is a planer member supporting a transverse load.
- Slabs transfer the load to the supporting beams in one or two directions.
- Slabs behave primarily as flexural members and the design is similar to that of beam.
- In slab, the shear stresses are usually low and hence shear reinforcement is rarely required.
- The depth of slab is governed by the deflection criteria.

Distribution of loads





$P=2P_x+2P_y$

$$\delta = \frac{P_x L_x^3}{24EI} = \frac{P_y L_y^3}{24EI}$$

$$L_x < L_y \longrightarrow P_x >> P_y$$

$$\frac{L_y}{L_x} = 2 \longrightarrow \frac{P_x}{P_y} = 8$$

For a slab supported on four edges the shorter span carries higher load

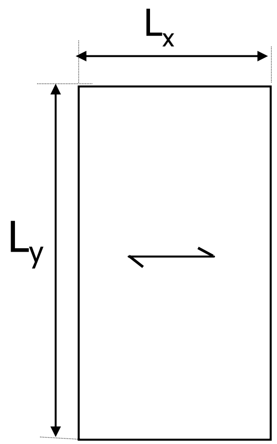
The direction with higher stiffness carries higher load

Classification of Slabs

One way Slab

$L_y/L_x > 2$ (One way action)

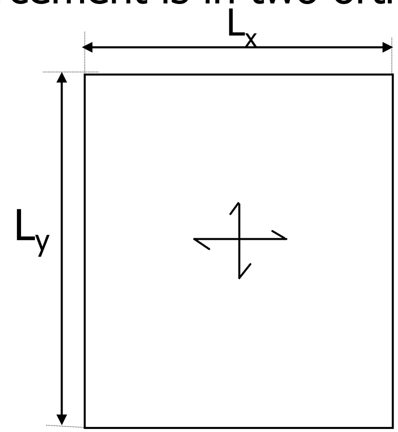
Main Reinforcement is in one direction.



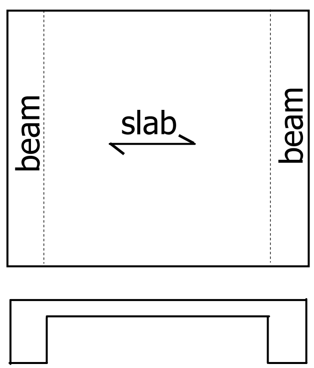
Two way Slab

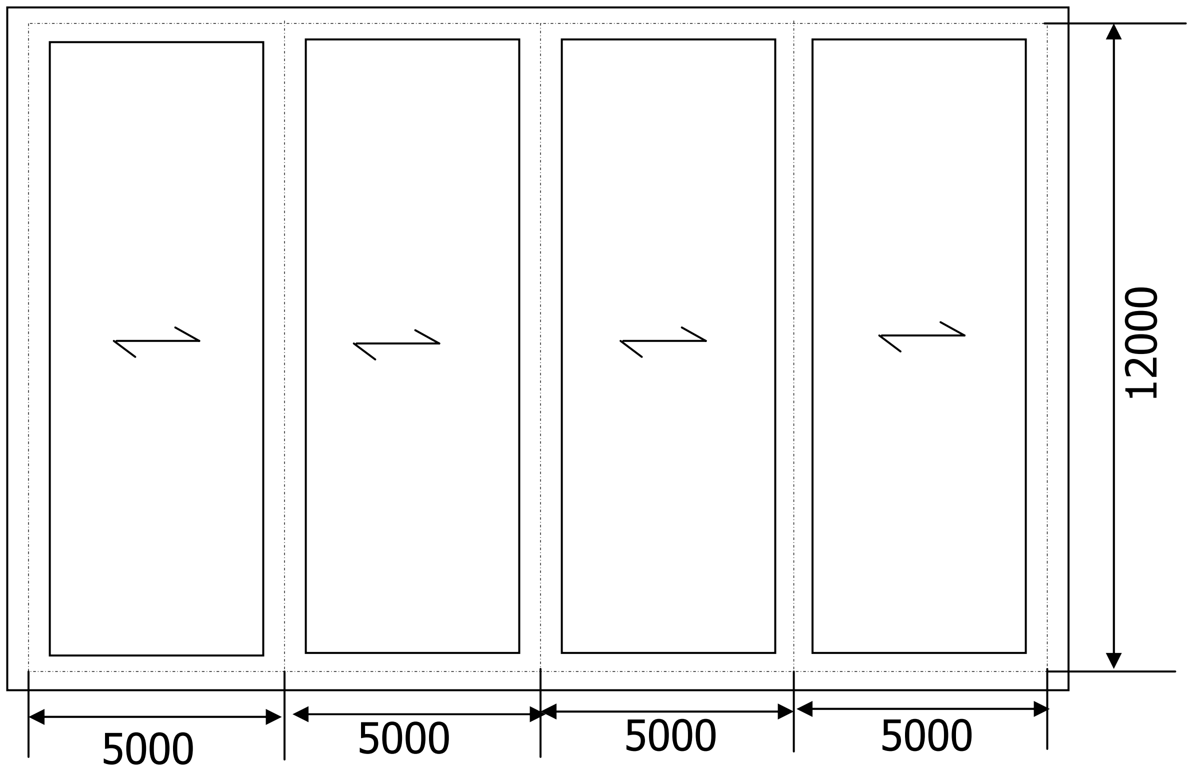
$L_y/L_x < 2$ (Two way action)

Reinforcement is in two orthogonal directions



2. When the slab is supported on two opposite parallel edges then it spans only in one direction.





Slab-beam arrangement.

Width of beam = 300 mm,

Live load = 5 kN/m^2, Floor finish Load = 1 kN/m^2

Design slab and show reinforcement details.

L_x = 5000 mm (Shorter dimension of slab)
L_y = 12000 mm (Longer dimension of slab)
L_y/L_x = 12000 / 5000 = 2.4 > 2.0 , Hence one way slab.

Trial depth (From deflection criteria):

Basic (L_x/d) ratio = 26

Assuming modification factor = 1.25

Allowable (L_x/d) ratio = 26 x 1.25 = 32.5

Therefore, d = 5000 / 32.5 = 153 mm

Assuming effective cover = 25 mm

Overall depth D = 153 + 25 = 178 mm   Say 175 mm

Calculation of Loads:

Dead Load = 25 x 0.175 = 4.375 kN/m^2

Finish Load = 1 kN/m^2

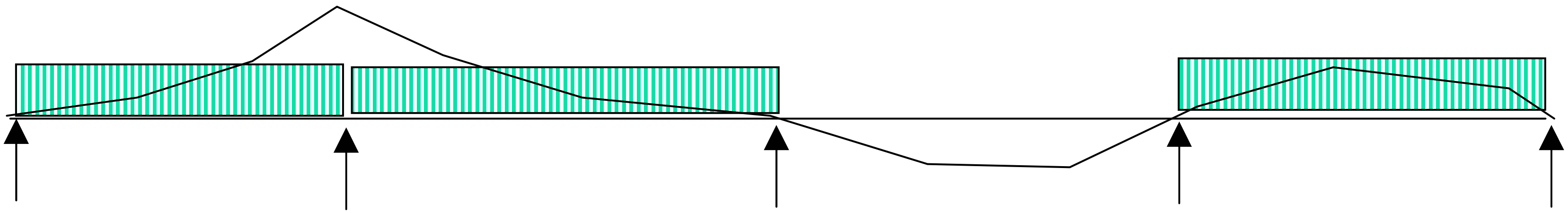
Total Dead Load = 5.5 kN/m^2

Live Load = 5 = 5.0 kN/m^2

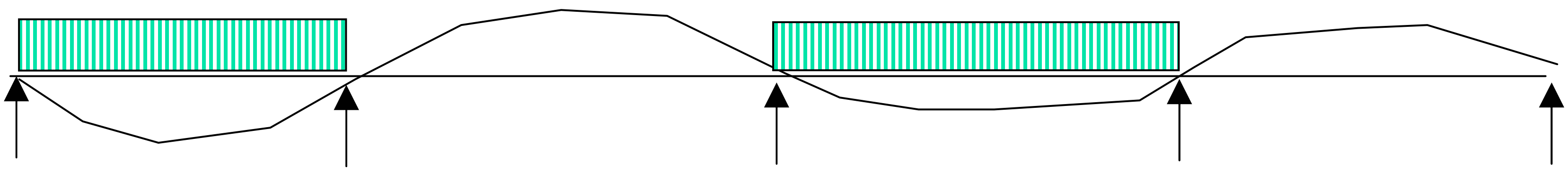
Factored dead load = W_{ud} = 1.5 x 5.375 = 8.1 kN/m^2

Factored live load = W_{uL} = 1.5 x 5.0 = 7.5 kN/m^2

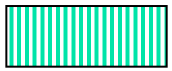
Influence lines for continuous beams



BM at Penultimate Support



BM at end span



Imposed loads for maxm. BM

Calculation of B.M. (Refer Table = 12 , Clause 22.5.1)

Type of Load	Span Moment		Support Moment	
	Near middle of end span	At middle of interior span	At support next to end support	At interior support
D. L. = 8.1 kN/m^2	(1/12) x 8.1 x 5^2 = 16.88 kNm/m	(1/16) x 8.1 x 5^2 =12.65 kN m/m	- (1/10) x 8.1 x 5^2 =- 20.25 kN m/m	- (1/12) x 8.1 x 5^2 =- 16.88 kN m/m
L. L. = 7.5 kN/m^2	(1/10) x 7.5 x 5^2 = 18.75 kN- m/m	(1/12) x 7.5 x 5^2 = 15.62 kN- m/m	- (1/9) x 7.5 x 5^2 =- 20.83 kN- m/m	- (1/9 x 7.5 x 5^2 =- 20.83 kN m/m
Total	35.63 kN-m/m	28.27 kN-m/m	41.08 kN-m/m	37.71 kN-m/m
Depth ‵ d ‵ from BM	-	-	122 mm < 150 mm	-
$A_{st, reqd}$ (Ref Table 41 , SP16)	**12 Φ @ 150 mm c/c**	**12 Φ @200 mm c/c**	**12 Φ @ 130 mm c/c**	**12 Φ @ 140 mm c/c**

The spacing of main reinforcement shall not exceed
i) 3d =3 x150 = 450 mm or ii) 300 mm
whichever is smaller.

Clause 26.3.3 b)

Calculation of S.F. (Refer Table = 13 , Clause 22.5.1 and 22.5.2)

Type of Load	At End Support	At support next to end support		At all other interior support
		Outer Side	Inner Side	
D.L. =8.1 kN/m^2	0.4 x 8.1 x 5.0 = 16.2 kN/m	0.6 x 8.1 x 5.0 = 24.3 kN/m	0.55 x 8.1 x 5.0 = 22.28 kN/m	0.5 x 8.1 x 5.0 =20.25 kN/m
L.L = 7.5 kN/m^2	0.45 x 7.5 x 5.0 =16.88 kN/m	0.6 x 7.5 x 5.0 = 22.5 kN/m	0.6 x 7.5 x 5.0 = 22.5 kN/m	0.6 x 7.5 x 5.0 = 22.5 kN/m
Total ' V_u '	33.08 kN/m	46.8 kN/m	44.78 kN/m	42.75 kN/m
$\zeta_v = V_u/d$	0.22 N/mm^2	0.31 N/mm^2	0.30 N/mm^2	0.29 N/mm^2
P_t	*0.25 %	0.58 %	0.58%	0.53%
ζ_c (Table 19)	0.36 N/mm^2	0.50 N/mm^2	0.50 N/mm^2	0.49 N/mm^2
ζ_c k (Clause 40.2.1.1)	1.25 x 0.36 =0.45 N/mm^2 $> \zeta_v$	1.25 x 0.50 =0.625 N/mm^2 $> \zeta_v$	1.25 x 0.50 =0.625 N/mm^2 $> \zeta_v$	1.25 x 0.49 =0.62 N/mm^2 $> \zeta_v$

Hence, Shear reinforcement is not required.

* Half steel is curtailed.

Distribution Steel: (Clause 26.5.2.1)

For deformed bars 0.12% (of total C/S area) reinforcement shall be provided.

A_{st} = 0.12 x 1000 x 175 /100 = 210 mm^2

Using 8 ₹ bars (area = 50 mm^2)

Spacing = 1000 x 50/210 = 238 mm (< 5d or 450 mm)

Clause 26.3.3 b)

Provide 8₹ bars @ 230 c/c.

Check for Deflection

Clause 23.2.1

span = 5 m

Basic Value = 26

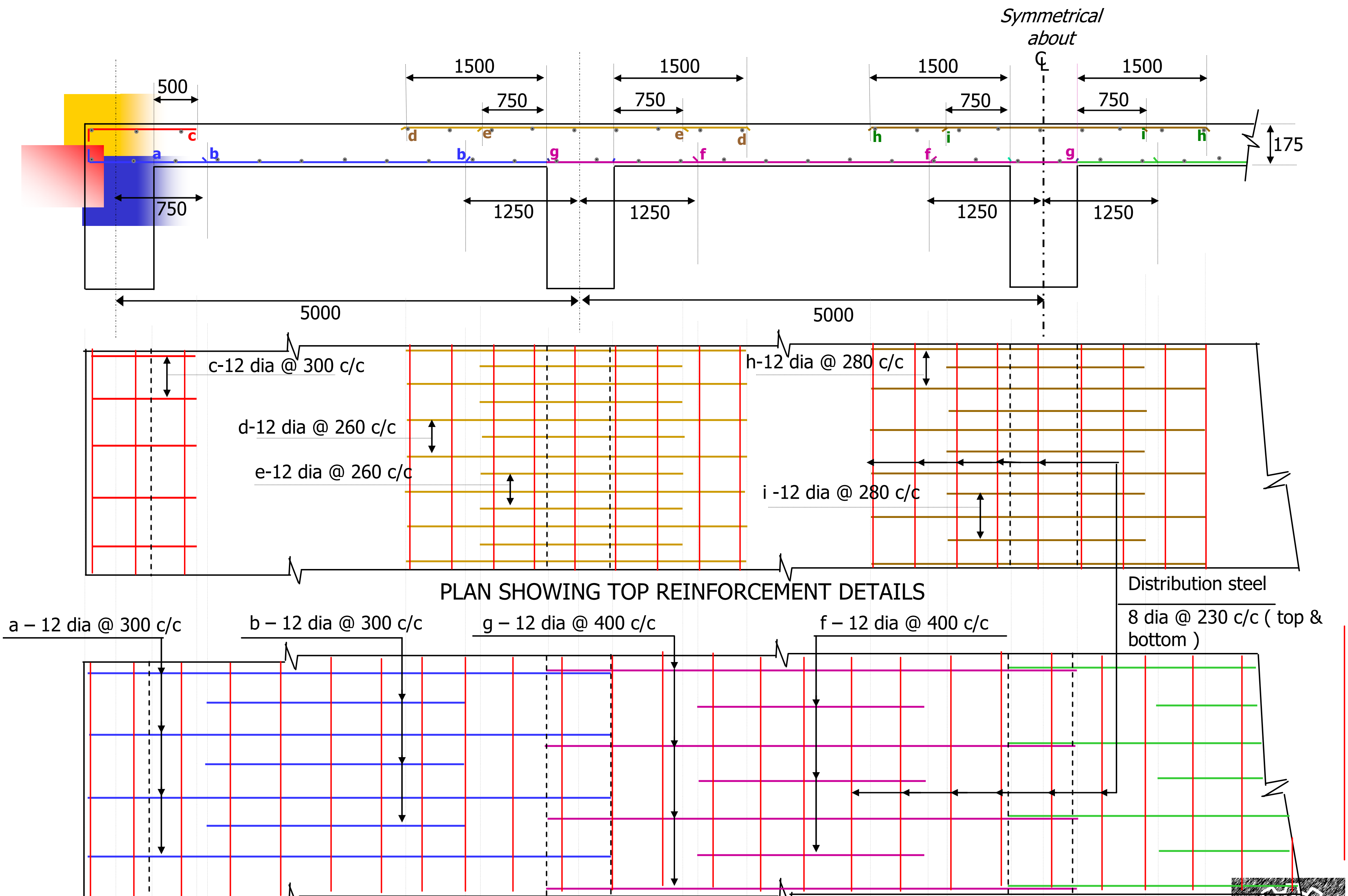
Modification Factor = 1.25

(Depends on area and stress of steel
in tension reinforcement , P_t =0.5)

Refer Fig. 4 of IS- 456:2000

Modified Basic Value = 26 x 1.25 = 32.5

L / d = 5000 / 150 = 33.3 ~ 32.5



Clear cover = 20 mm

Two Way Slabs (Annexure D , IS 456:2000)

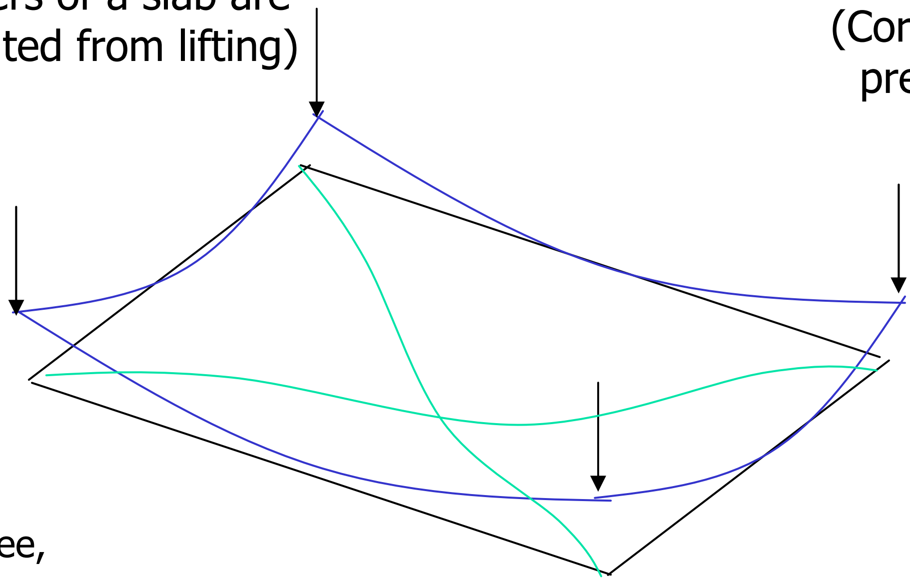
$L_y/L_x < 2$

Restrained Slabs

(Corners of a slab are prevented from lifting)

Simply Supported Slabs

(Corners of a slab are not prevented from lifting)



Restrained Two Way Slabs (D-1 , IS 456:2000)

D-1-1 The maximum bending moments per unit width in a slab are given by the following equations:

$$M_x = \alpha_x w L_x^2 \text{ and } My = \alpha_y w L_x^2$$

Where,

M_x, M_y = moments on strips of unit width spanning L_x and L_y respectively.

w = total design load per unit area.

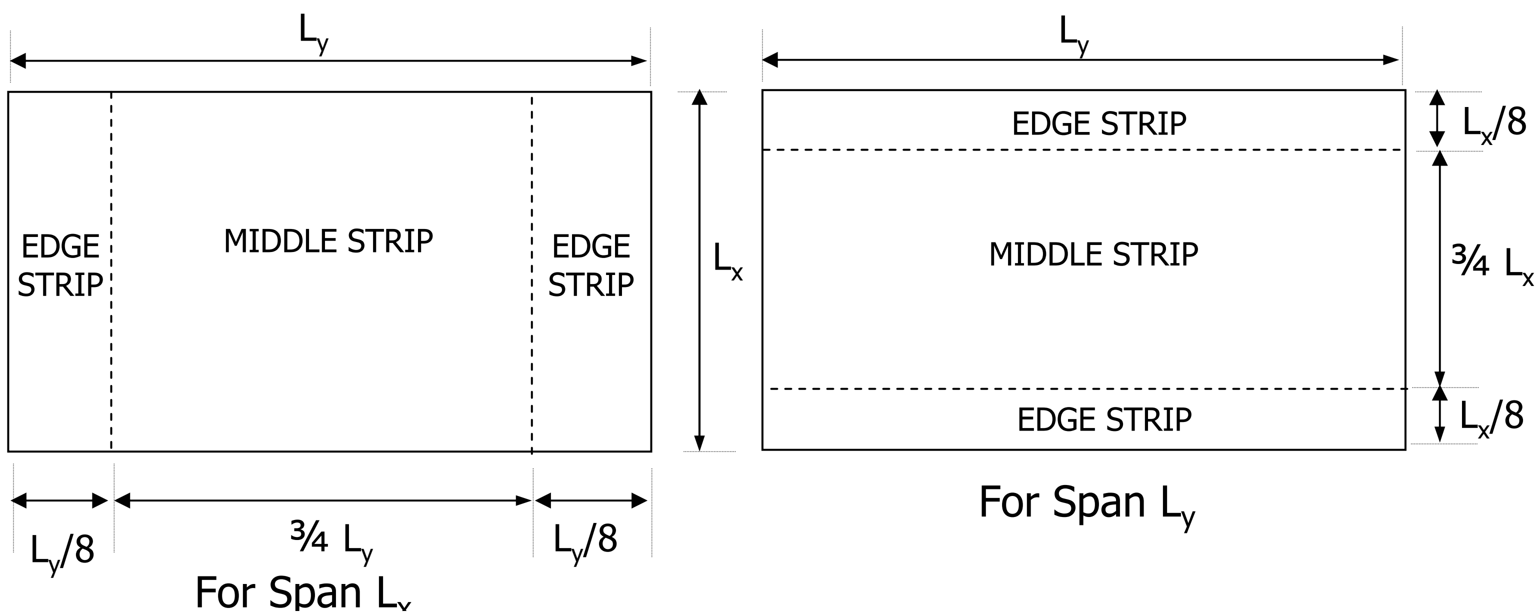
Lx and Ly = Lengths of the shorter span and longer span respectively.

α_x and α_y are coefficients given in table 26 (IS 456:2000)

Restrained Two Way Slabs

Provision of Reinforcement:

- Slabs are divided in each direction into middle strips and edge strips. The middle strip being three-quarters of the width and each edge strip one-eight of the width.



- Maximum moments calculated as per clause D-1-1 apply only to the middle strips and no redistribution shall be made.

Restrained Two Way Slabs

Provision of Reinforcement contd..

- Tension reinforcement provided at mid-span in the middle strip shall extend in the lower part of the slab to within 0.25L of a continuous edge, or 0.15L of a discontinuous edge.

- Over the continuous edges of a middle strip, the tension reinforcement shall extend in the upper part of the slab a distance of 0.15L from the support, and at least 50 percent shall extend a distance of 0.3L.

- At a discontinuous edge, negative moments may arise. They depend on the degree of fixity at the edge of the slab but, in general, tension reinforcement equal to 50 percent of that provided at mid-span extending 0.1L into the span will be sufficient.

- Reinforcement in edge strip, parallel to that edge, shall comply with the minimum given in clause 26.5.2.1 and requirements for torsion given in i to iii.

Restrained Two Way Slabs

Provision of Reinforcement contd..

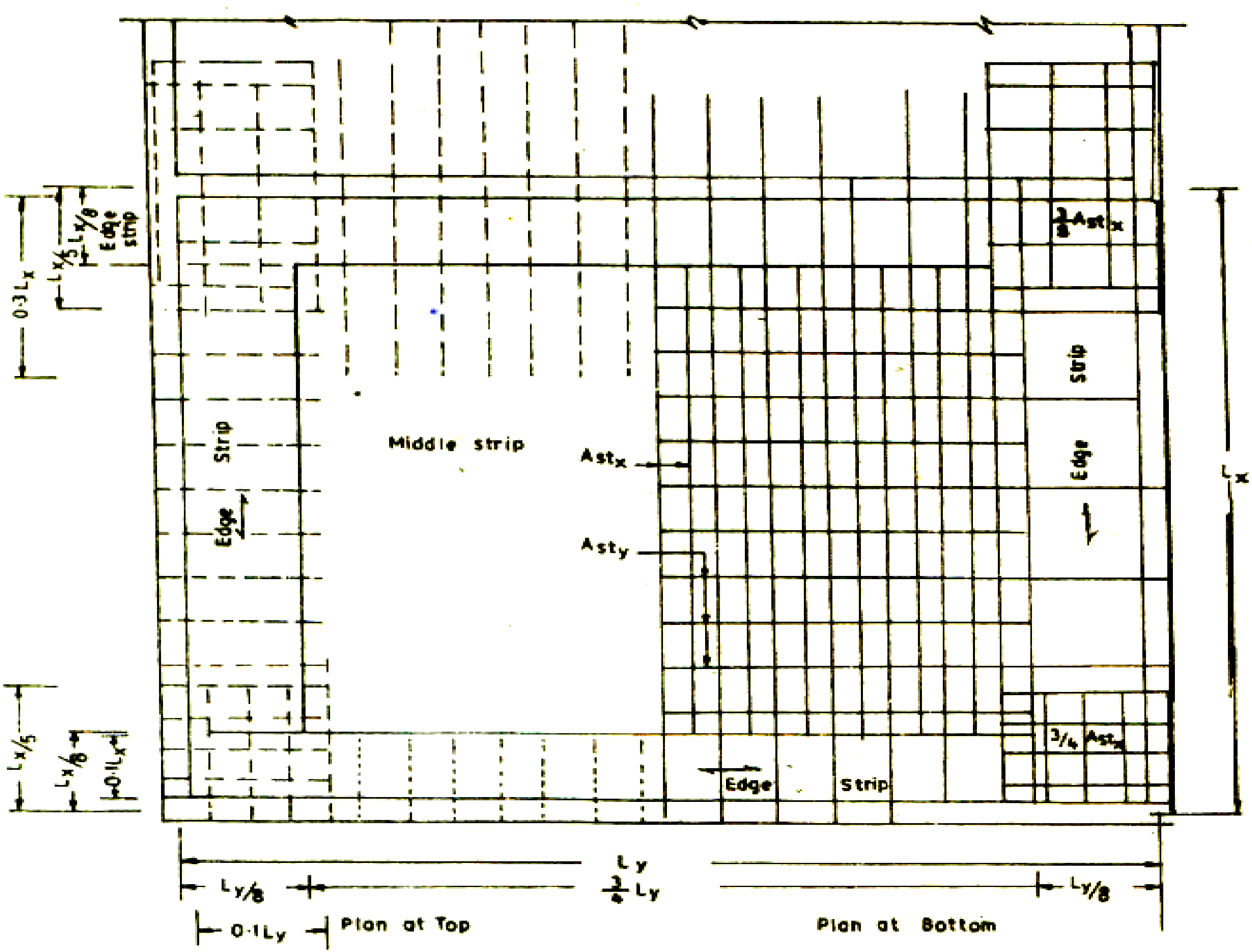
i. Torsion reinforcement shall be provided at any corner where the slab is simply supported on both edges meeting at that corner. It shall consist of top and bottom reinforcement, each with layers of bars placed parallel to the sides of the slab and extending from the edges a minimum distance of one-fifth of the shorter span. The area of reinforcement in each of these four layers shall be three-quarters of the area required for the maximum mid-span moment in the slab.

ii. Torsion reinforcement equal to half that described above shall be provided at a corner contained by edges over only one of which the slab is continuous.

iii. Torsion reinforcements need not be provided at any corner contained by edges over both of which the slab is continuous.

Reinforcement Detailing for restrained two way slab

Annexure D , D-1.22 to D-1.10 ,
IS 456:2000



Simply supported Two Way Slabs (D-2 , IS 456:2000)

- When simply supported slabs do not have adequate provision to resist torsion at corners and to prevent the corners from lifting, the maximum moments per unit width are given by the following equation:

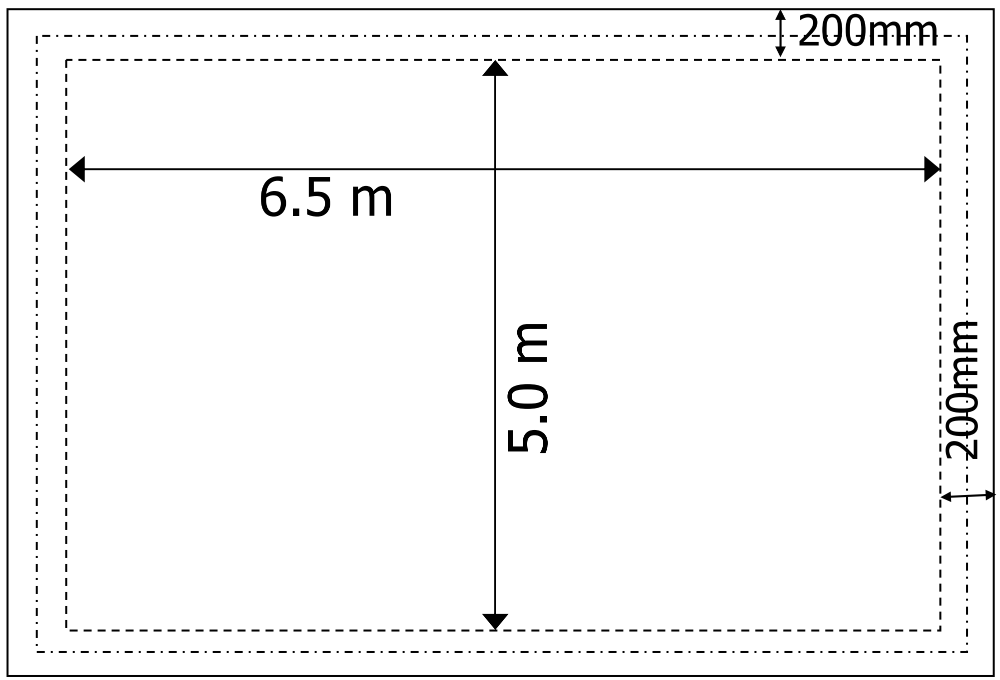
$$M_x = \alpha_x w L_x^2 \text{ and } My = \alpha_y w L_x^2$$

α_x and α_y are coefficients given in table 27 (IS 456:2000)

- At least 50 percent of the tension reinforcement provided at mid-span should extend to the supports. The remaining 50 percent should extend to within $0.1L_x$ or $0.1L_y$ of the support, as appropriate.

Design Example

Design a R.C. slab for a room measuring 6.5m x 5 m. The slab is to be cast monolithically over the beams with corners held down. The width of the supporting beams is 200mm. The slab carries superimposed load of 3kN/m^2. Use M20 grade of concrete and Fe415 steel.



Effective span, L_x = 5000 + 200/2 + 200/2 = 5200 mm
L_y = 6500 + 200/2 + 200/2 = 6700 mm

Note: Effective span = c/c distance between support or clear span + d , whichever is smaller. (Clause 22.2 a)
Here effective span is taken as c/c distance between support.

L_x = 5200 mm (Shorter dimension of slab)
L_y = 6700 mm (Longer dimension of slab)
L_y/L_x = 6700 / 5200 = 1.29 < 2.0 , Hence two way slab.

Trial depth (From deflection criteria):
Basic (L_x/d) ratio = 20
Assuming modification factor = 1.25
Allowable (Lx/d) ratio = 20 x 1.25 = 25
Therefore, d = 5200 / 25 = 208 mm
Assuming effective cover = 25 mm
Overall depth D = 208 + 25 = 233 mm Say 225 mm
Therefore, effective depth ‵ d ‵ = 200 mm

Calculation of Loads:

Consider 1m width of slab ie b = 1000 mm

Dead Load = 25 x 0.225 = 5.625 kN/m

Live Load = 3 x 1 = 3.0 kN/m

Total Load = 8.625 kN/m

Ultimate load = W_u = 1.5 x 8.625 = 12.94 kN/m

Ly / Lx = 1.29 , Four edges discontinuous

(Refer table 26 , IS 456:2000)

Span	α	M	' d ' from BM consideration	A_{st}
Short span	α_x =0.0783	$M_{ux} = \alpha_x w_u L_x^2$ =27.40 kN-m	99.63 mm < 200 mm	8 dia @ 130 c/c
Long Span	α_y =0.056	$M_{uy} = \alpha_y w_u L_x^2$ =19.6 kN-m	-	8 dia @ 180 c/c

Distribution Steel: (Clause 26.5.2.1)

For deformed bars 0.12% (of total C/S area) reinforcement shall be provided.

$$A_{st} = 0.12 \times 1000 \times 225 / 100 = 270 \text{ mm}^2$$

Using 8 bars (area = 50 mm^2)

Spacing = 1000 x 50/270 = 185 mm (< 5d or 450 mm)

Provide 8 dia. bars @ 180 c/c.

Check for Shear:

(a) Long discontinuous edge

$V_{u,max} = w_u L_x [\beta / (2\beta + 1)]$ where $\beta = L_y/L_x = 1.29$

$= 24.25$ kN

$\zeta_u = 0.12$ N/mm^2

Area of tension steel = 385 mm^2

$P_t = 0.1925$ %

$\zeta_{c,\ perm} = 0.32$ N/mm^2 $> \zeta_u$

Hence shear reinforcement is not required.

(b) Short discontinuous edge

$V_{u,max} = w_u L_x /3$

$= 22.43$ kN

$\zeta_u = 0.112$ N/mm^2

Area of tension steel = 278 mm^2

$P_t = 0.12$ %

$\zeta_{c,\ perm} = 0.28$ N/mm^2 $> \zeta_u$

Hence shear reinforcement is not required.

Check for Deflection

Clause 23.2.1

span L_x= 5.2 m

Basic Value = 20

Modification Factor = 1.4

(Depends on area and stress of steel
in tension reinforcement , P_t =0.1925)

Refer Fig. 4 of IS- 456:2000

Modified Basic Value = 20 x 1.4 = 28

L / d = 5200/ 200 = 26 < 28

Torsion Steel:

All the edges are discontinuous edges.

Area of steel @ midspan=A_{stx} = 385 mm^2

Torsion reinforcement = $0.75A_{stx}$

= 289 mm^2

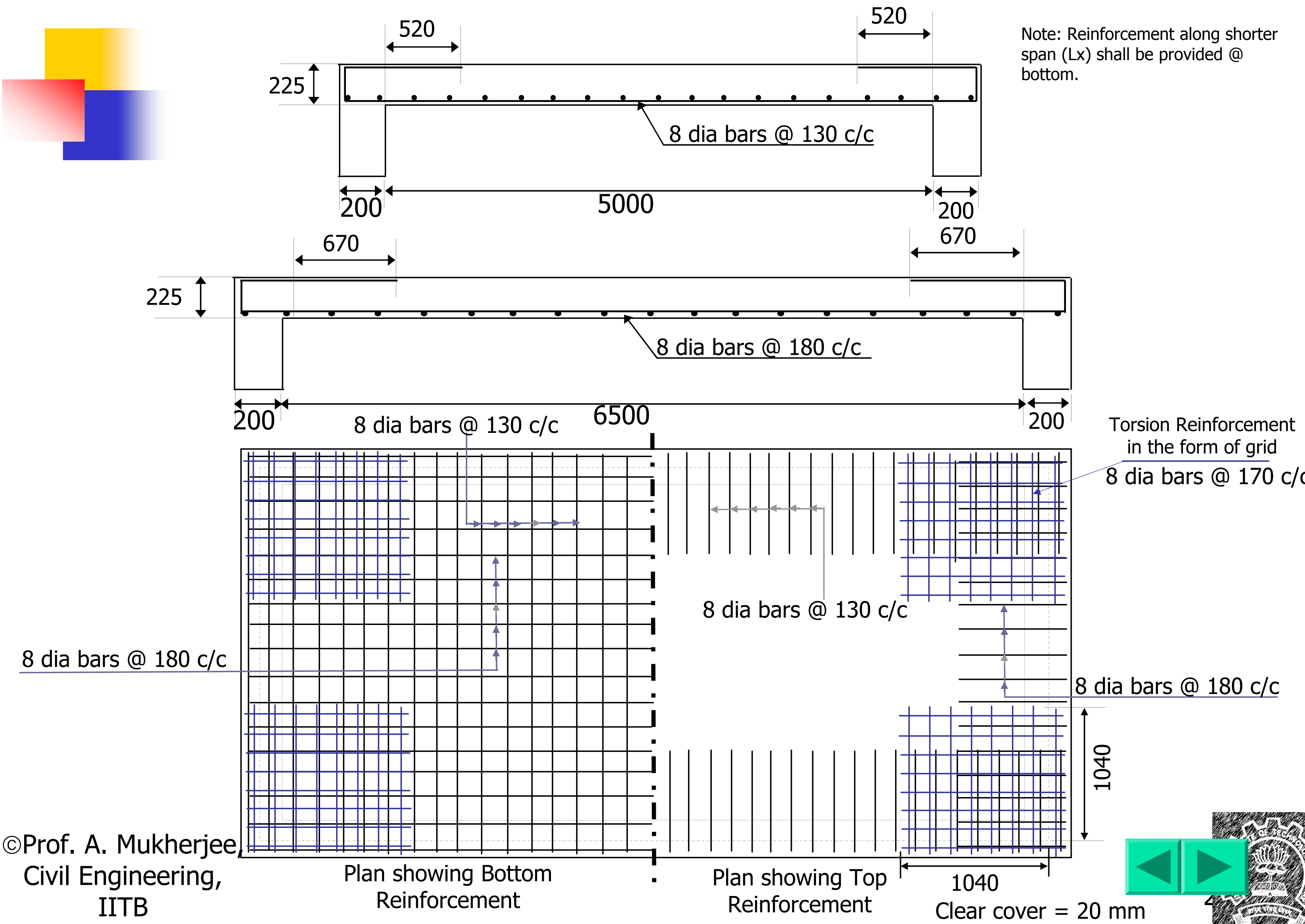
This reinforcement shall be provided in the form of grid and should be extended from the edges for a distance $L_x/5$ =1040 mm

Using 8 dia bars.

Spacing = 50 x 1000/289 = 173 mm

Provide 8 dia bars at 170 c/c.

Reinforcement Detailing



Note: Reinforcement along shorter span (Lx) shall be provided @ bottom.

Design of Compression members

- Structural element subjected to axial compressive forces (almost every time moment is also be present) is called compressive member. Like,
  - Columns
  - Struts
  - Inclined members
  - Shear walls


Interior concrete column
construction continues Level D
NOV 7 2002

Definitions according to code

- *Clause* 25.1.1 – Column or strut is a compression member, effective length (*explained later*) of which exceeds three times the least lateral dimension.
- *Clause* 26.5.3.1 h – Pedestal is the compression member, the effective length of which does not exceed three times the least lateral dimension.

Capacity computation of short column under axial loading

- Under pure axial loading conditions, the design strength of a short column is obtainable as,

$$P_0 = C_C + C_S$$
$$= f_{cc}Ac + f_{sc}A_{sc}$$
$$\Rightarrow P_0 = f_{cc}A_g + (f_{sc} - f_{cc})A_{sc}$$
$$P_0 = f_{cc}A_g + (f_{sc} - f_{cc})A_{sc}$$

$$f_{sc} = 0.870\ f_y \text{ for Fe 250}$$
$$0.790\ f_y \text{ for Fe 415}$$
$$0.746\ f_y \text{ for Fe 500}$$

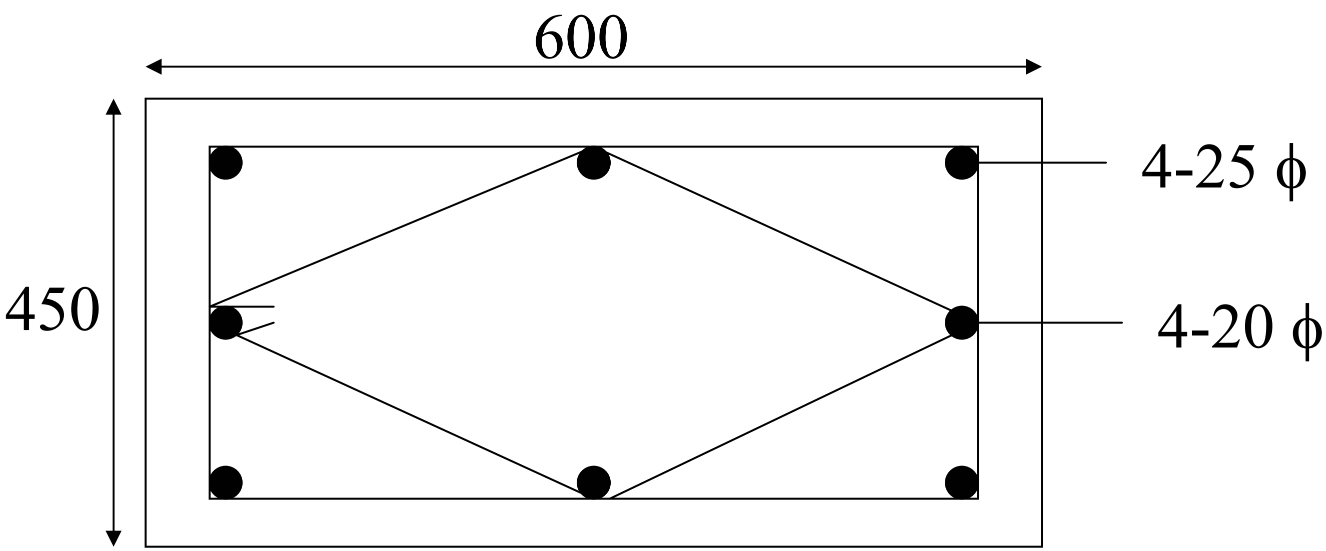
Where,

A_g = gross area of cross-section = $A_c + A_{sc}$

A_{sc} = total area of longitudinal reinforcement = ΣA_{si}

A_c = net area of concrete in the section= A_g - A_{sc}

Lets take a column of 600 X 450 with the following reinforcement. Now compute its axial load carrying capacity.



Here A_g=600*450 mm ^{2} (steel neglected)

Now steel provided

4-25 ϕ at corners : 4 *491 = 1964 mm ^{2}

4-20 ϕ additional : 4* 314 = 1256 mm ^{2}

Total longitudinal steel provided = 3220 mm ^{2}

Since $P_0 = f_{cc}A_g+(f_{sc}-f_{cc})A_{sc}$

P_o=).446*20*600*450 + (0.79*415-0.446*20)*3220

=3435

KN

This is the factored capacity. Hence load carrying capacity of this column is 4260 KN.

Code requirements for reinforcement and detailing

Clause 26.5.3.1

- Longitudinal reinforcement shall not be less than 0.8 % nor more than 6 % (4% is actually recommended) of the gross sectional area of the column.

- Minimum number of longitudinal bars provided in a column shall be 4 in rectangular and 6 in circular columns(12 mm dia min. bar)

Code requirements for reinforcement and detailing (contd.)

- Spacing of longitudinal bars measured along the periphery of the column shall not exceed 300mm
- In pedestals in which longitudinal reinforcement is not taken in account in strength calculations, nominal longitudinal reinforcement not less than 0.15 percent of the cross-sectional area shall be provided.

Transverse Reinforcement

Clause 26.5.3.2

- All longitudinal reinforcement in a compression member must be enclosed within transverse reinforcement, comprising either lateral ties (with internal angles 135 ^{0}) or spirals.
- The pitch of transverse reinforcement shall not be more than the least of following:

  i) The least lateral dimension

  ii) Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied.

  iii) 300 mm

Transverse Reinforcement (contd.)

- The diameter lateral ties shall not be less than ¼ of diameter of largest longitudinal bar and in no case less than 6 mm. (In your code it is misprinted as 16 mm, please correct it)

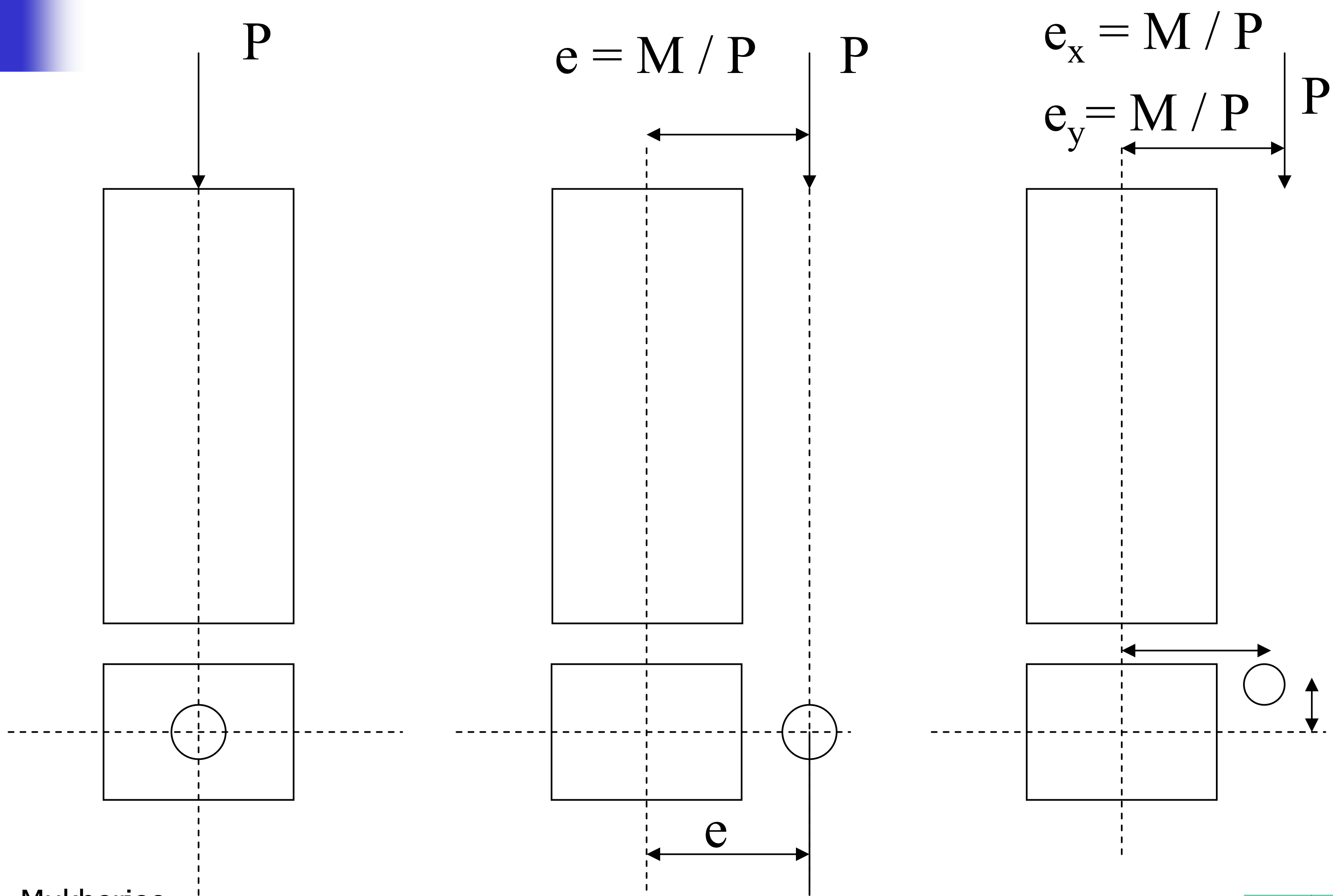
Helical reinforcement

- Pitch – Helical reinforcement shall be regular formation with the turns of the helix spaced evenly and its ends shall be anchored properly by providing one and a half extra turns of the spiral bar.
  - The diameter and pitch of the spiral may be computed as in last slide except when column is designed to carry a 5 % overload, in which case,

    Pitch $< =$ min (75 mm, core diamtere/6)

    Pitch $= >$ max (25 mm, 3 * diameter of bar forming the helix)

Classification - Based on loading



Axial loading

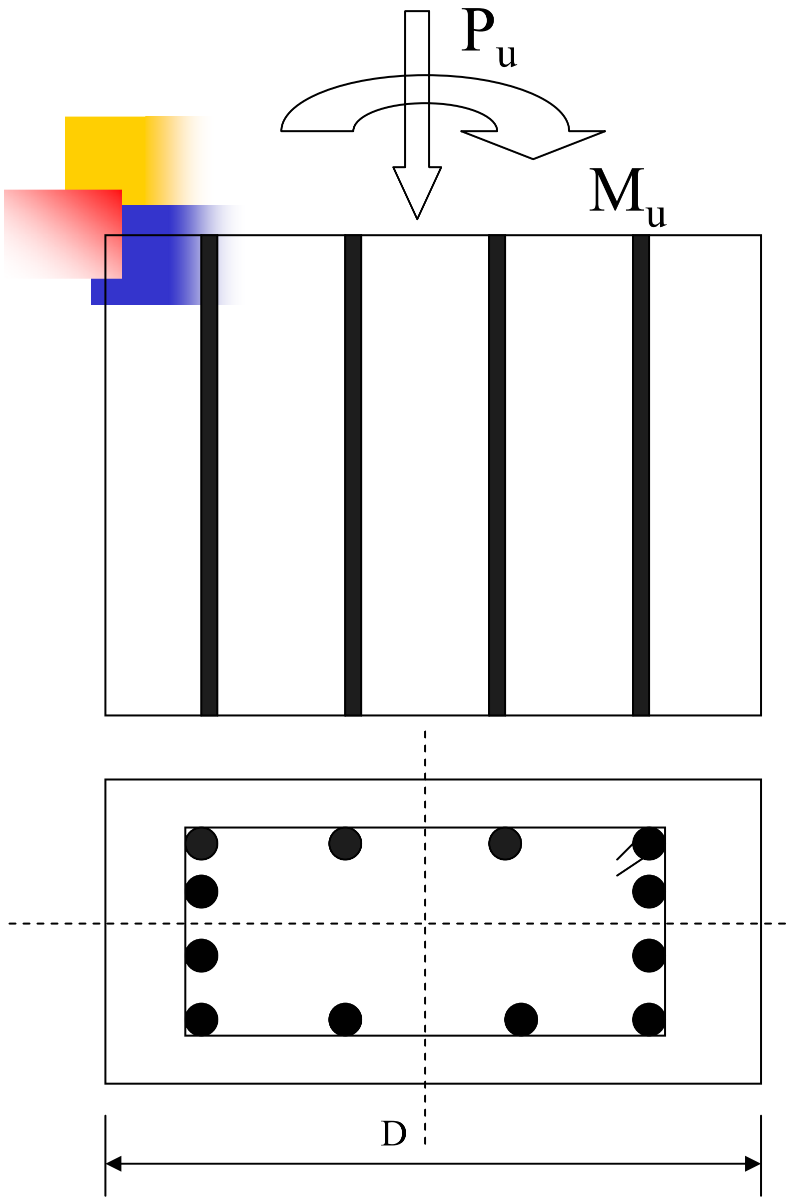
Uniaxial eccentric loading

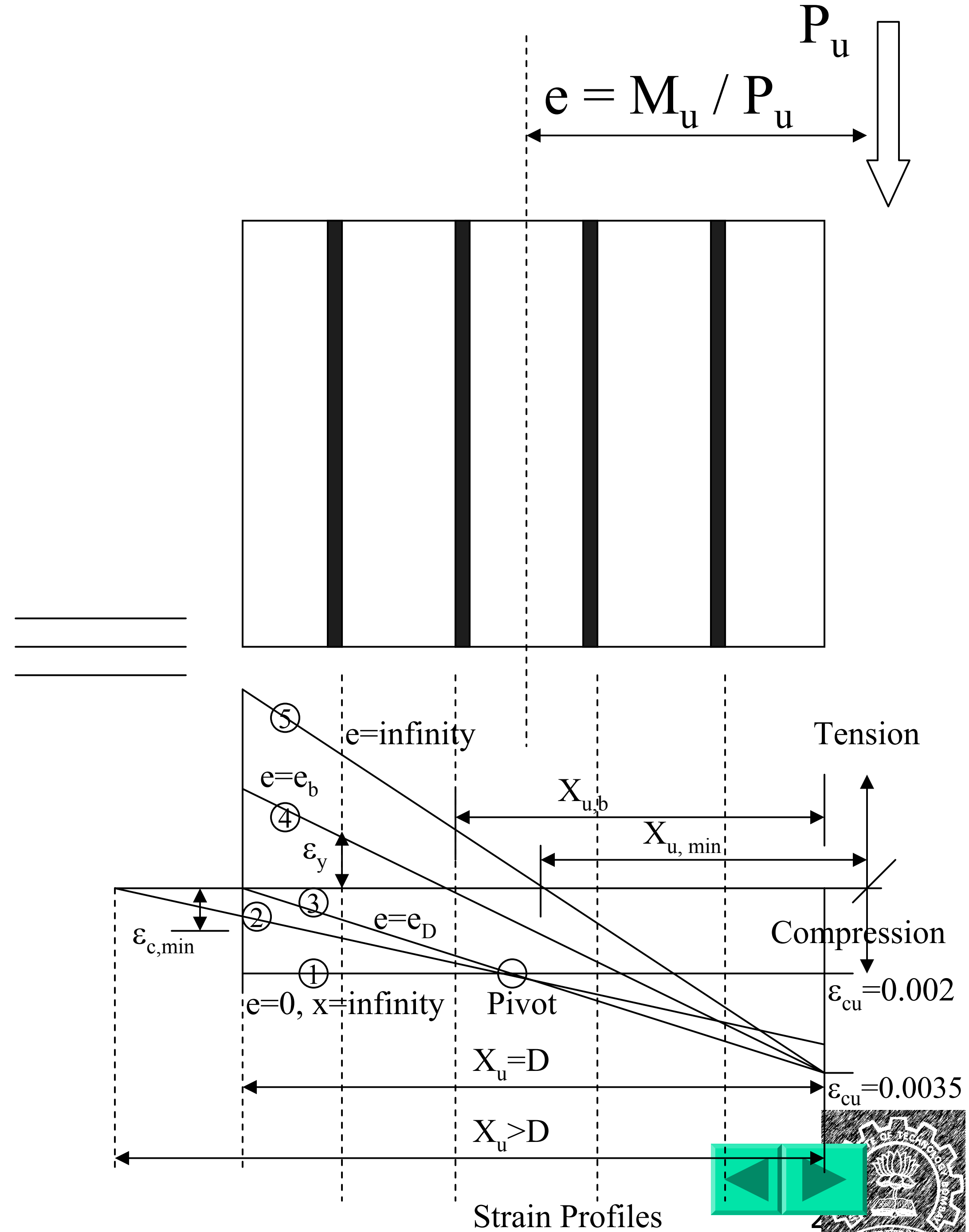
Biaxial eccentric loading

Design of short columns under compression with uniaxial bending

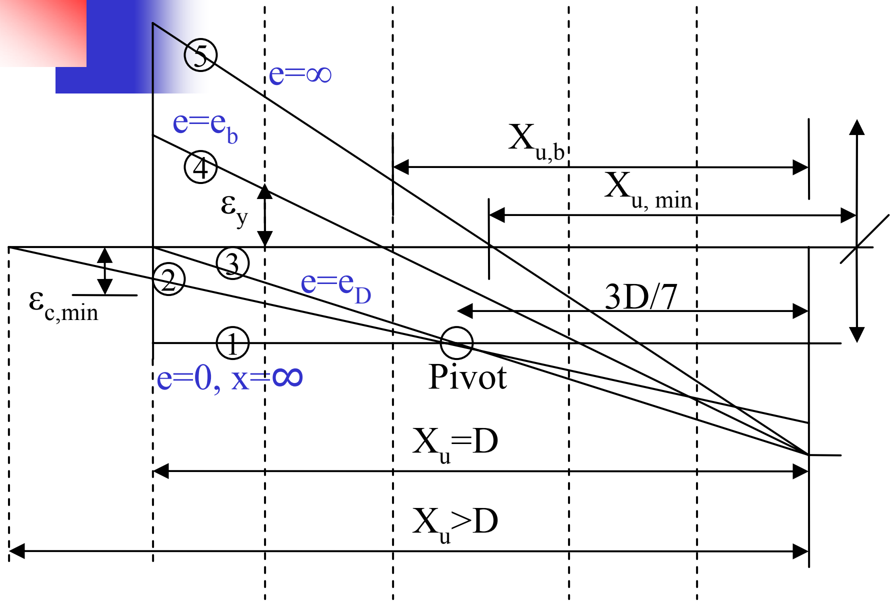
Here the column is subjected to axial compression combined with uniaxial bending (bending in major or minor axis).

This is equivalent to axial load applied at an eccentricity $e= M_u / P_u$ with respect to


Pu
Mu
D


Pu
e = Mu / Pu
e=infinity
Tension
e=eb
Xu,b
Xu, min
εy
εc,min
e=eD
Compression
εcu=0.002
e=0, x=infinity
Pivot
Xu=D
εcu=0.0035
Xu>D
Strain Profiles

Strain Profiles



4. This refers to ultimate limit state wherein the yielding of the outermost steel on tension side and the attainment of maximum compressive strain in concrete (0.0035) at the highly compressed edge of the column occur simultaneously. ($e=e_b$)

1. The strain corresponding to e=0 (M_u=0) is limited to ε_{cu}=0.002 at the limit state of collapse in compression.

5. This is equivalent to pure flexure (P_u=0) and at the limit state of collapse the strains is specified as ε_{cu}=0.0035.

Strain profile for within above limiting cases is non-uniform and assumed to be linearly varying across the section.

2. This occurs when the entire section is in compression and NA lies outside the section (X_u>D), the code limits the strain as ε_c==0.0035-0.75$\varepsilon_{c,min}$

3. This limiting condition occurs when the resultant neutral axis coincides with the edge farthest removed from the highly compressed edge, i.e. $X_u=D$, correspondingly $e=e_D$.

Axial-Load Moment Interaction

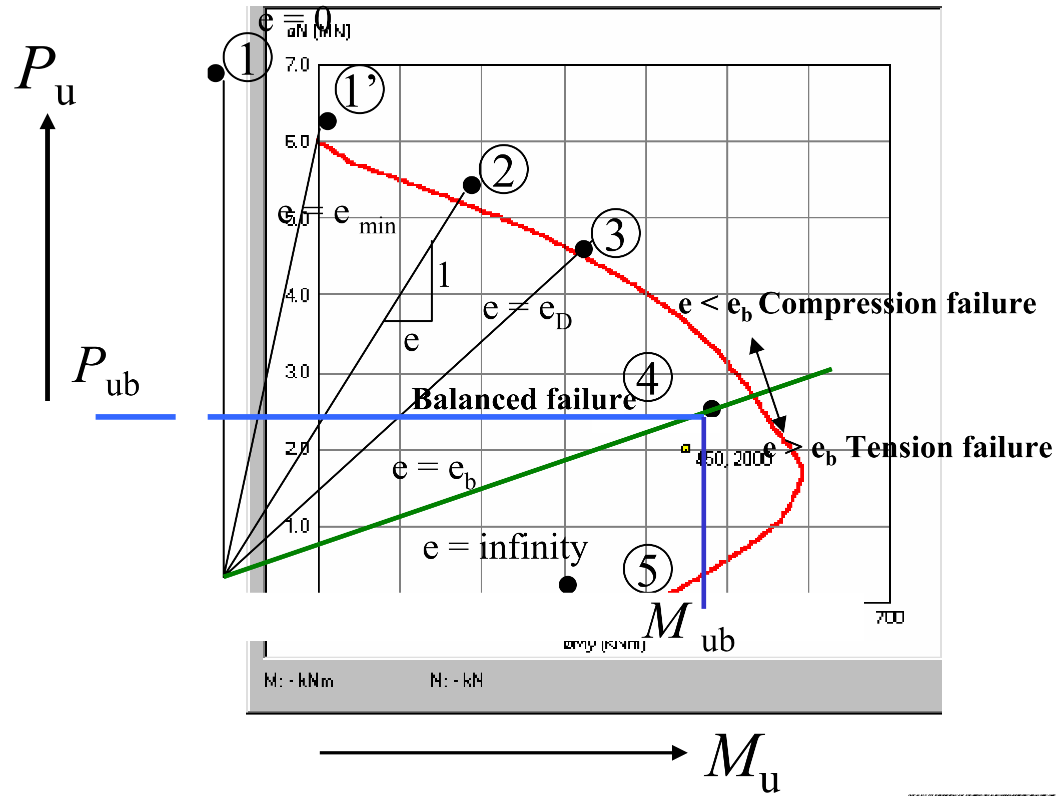
The design strength of uniaxial eccentrically loaded short Column depends on axial compression component (P_{ur}) and Corresponding moment Component ($M_{ur} = P_{ur} * e$).

$$P_{ur} = C_c + C_s \text{ and } M_{ur} = M_c + M_s$$

Thus given an arbitrary value of e, it is possible to arrive at the design strength but only after first locating NA which can be achieved by considering moments of forces C_c and C_s about the eccentric line of action of P_{ur}, but the expression for C_c and C_s in terms of X_u are such that, in general, it will not be possible to obtain a closed-form solution in terms of e. The relation is highly nonlinear, requiring a trial-and-error solution.

Interaction Envelope

Interaction curve is a complete graphical representations of the design strength of a uniaxially eccentrically loaded column of a given proportions. If load p is applied on a short column with an eccentricity e, and if this load is gradually increased till the ultimate limit state is defined, and the ultimate load at failure is given by P_{uR} and the corresponding moment M_{uR}, then the coordinates M_{ur}, P_{uR} form the unique point on the interaction diagram.
(Refer SP : 16 Chart 27-62)



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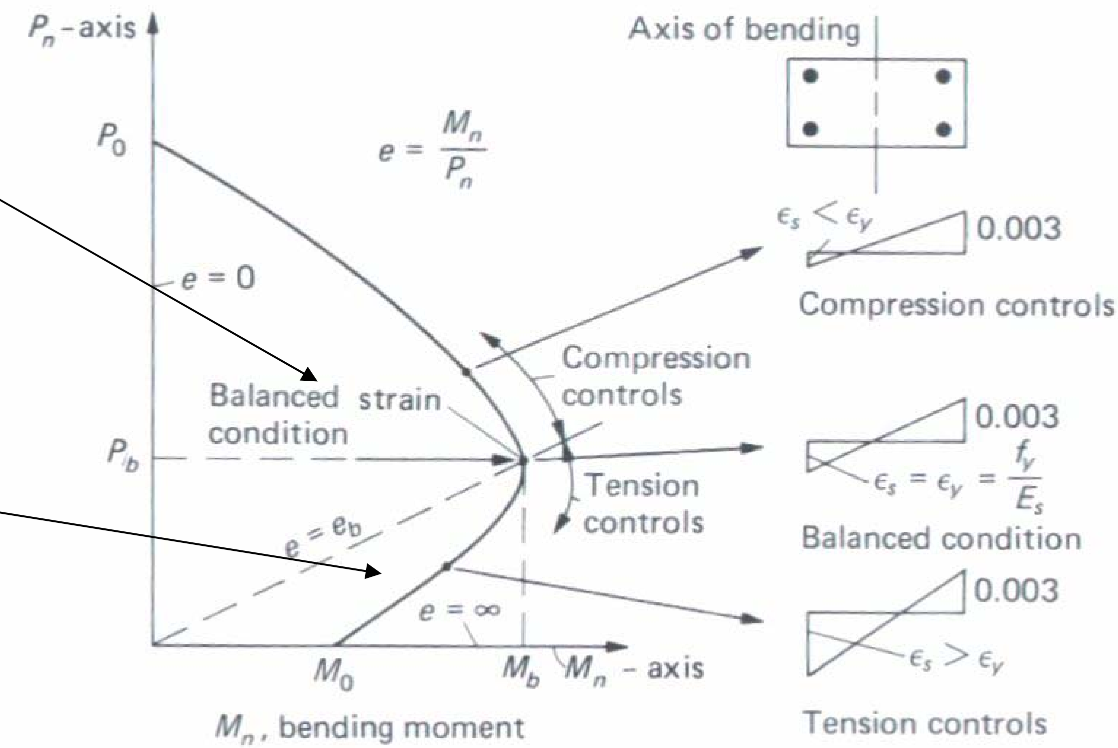
Behavior under Combined Bending and Axial Loads

Interaction Diagram Between Axial Load and Moment (Failure Envelope)

Concrete crushes before steel yields

Steel yields before concrete crushes



Note: Any combination of P and M outside the envelope will cause failure.

Analysis for design strength

Generalized expression for the resultant force in concrete (C_c) as well as its moment (M_c) with respect to the centroidal axis of bending may be derived as follows,

$$C_c = a\, f_{ck}\, b\, D$$

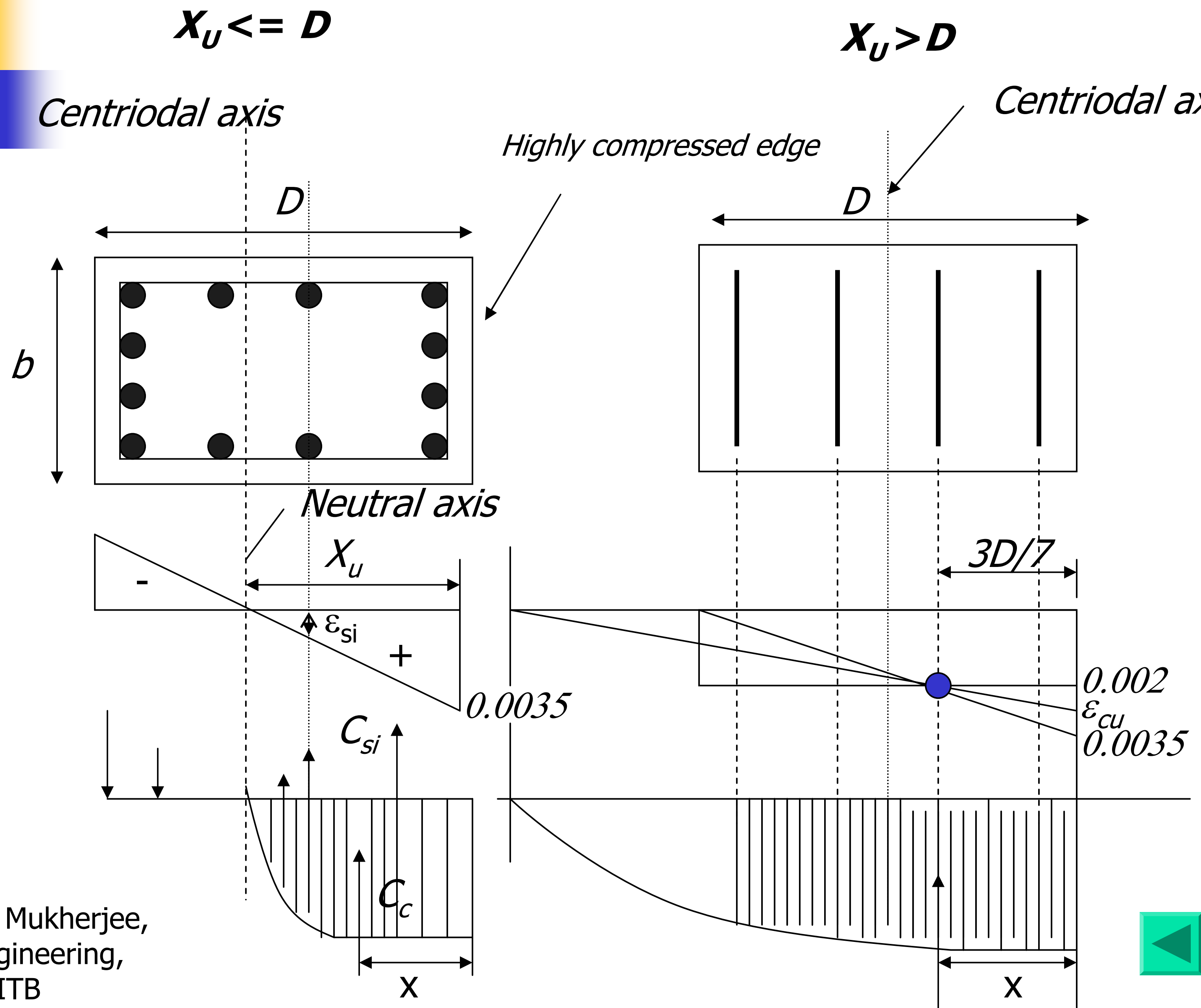
$$M_c = C_c\,(D/2 - x)$$

Where,

a = stress block area factor

x= distance between highly compressed edge and the line of action of Cc (centriod of stress block area)

Analysis for design strength of rectangular section



By simple integration, it is possible to derive expression for a and x for case (a) $X_u <= D$ and for the case (b) $X_u > D$

$$a = 0.362\, x_u / D \qquad \text{for } x_u <= D$$

$$ 0.447(1 - 4g/21) \qquad \text{for } x_u > D$$

$$x = 0.416\, x_u \qquad \text{for } x_u <= D$$

$$= (0.5 - 8g/49)\{D/(1 - 4g/21)\} \quad \text{for } x_u <= D$$

$$g = 16 / (7x_u/D - 3)^2$$

Similarly expression for the resultant force in the steel as well as its moment with respect to the centroidal axis of bending is easily obtained as

$$C_s = \sum_{i=1}^{n} (f_{si} - f_{ci}) A_{si}$$

$$M_s = \sum_{i=1}^{n} (f_{si} - f_{ci}) A_{si} y_i$$

where,

A_{si} = area of steel in the ith row (of n rows)

y_i = distance of ith row from the centroidal axis, measured positive in the direction towards the highly compressed edge

f_{si} = design stress in the ith row

ε_{si} = strain in the ith row obtainable from strain compatibility condition

f_{ci} = design compressive stress level in concrete

$$f_{ci} = 0 \qquad \text{if } \varepsilon_{si} <= 0$$

$$= 0.447\, f_{ck} \qquad \text{if } \varepsilon_{si} => 0.002$$

$$= 0.447 f_{ck}[2(\varepsilon_{si}/0.002)-(\varepsilon_{si}/0.002)^2] \quad \text{otherwise}$$

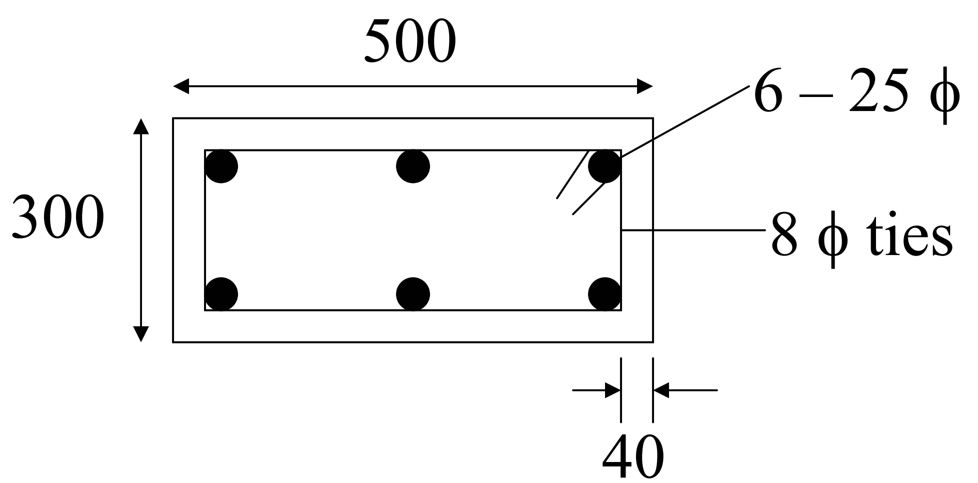
Also using similar triangle

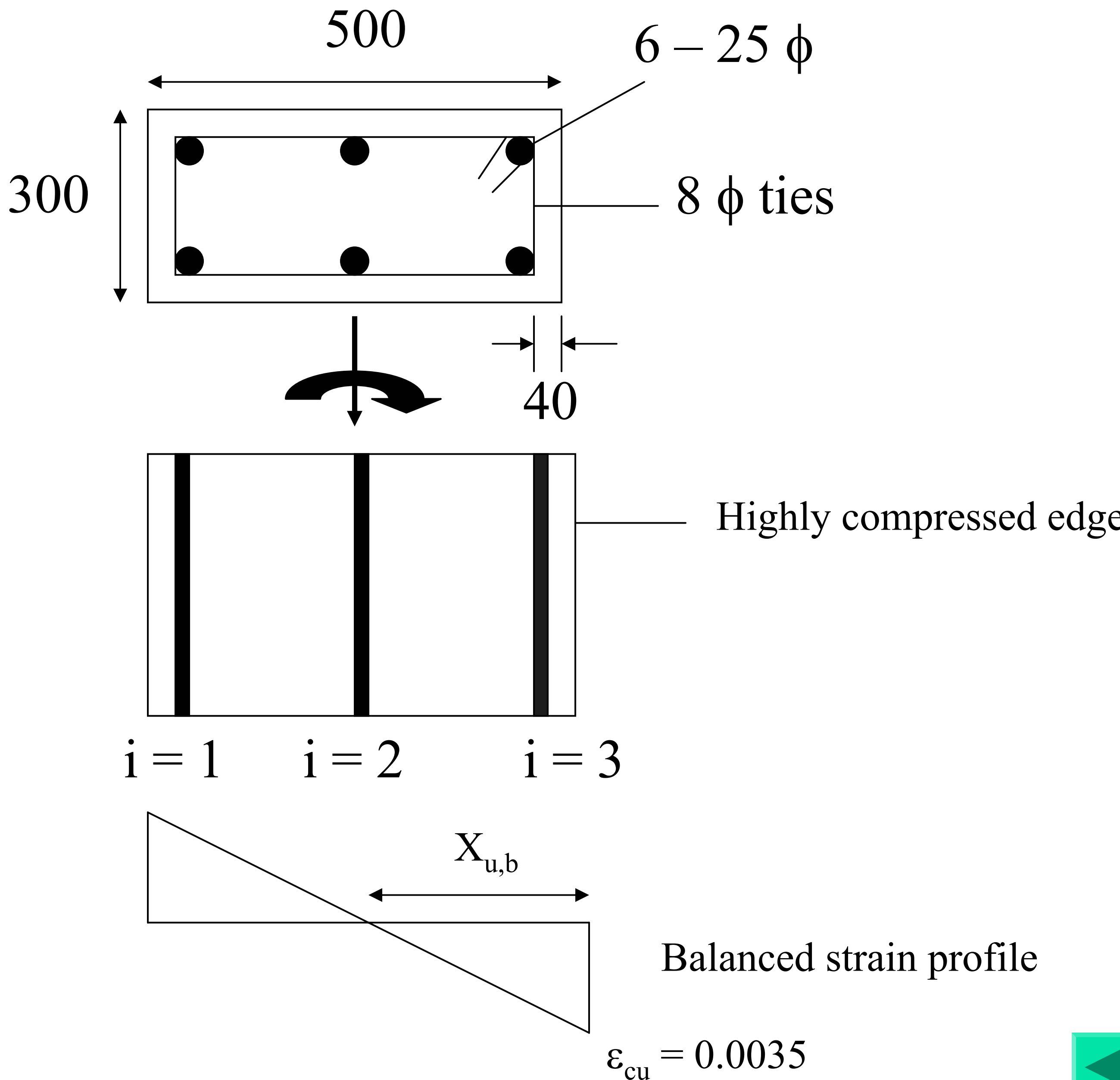
$$\varepsilon_{si}=0.0035[(x_u-D/2+y_i)/x_u] \quad \text{for } x_u <= D$$

$$=0.002\left[1+\frac{y_i - D/14}{x_u - 3D/7}\right] \qquad \text{for } x_u > D$$

Example
Computation of design strength

For the column section 500 X 300, Determine strength components corresponding to condition of balanced failure. Assume M25 concrete and FE 415 steel. Consider loading capacity with respect to the major axis alone.




500
6 – 25 ϕ
300
8 ϕ ties
40
Highly compressed edge
i = 1
i = 2
i = 3
X u,b
Balanced strain profile
εcu = 0.0035

$A_{s1} = A_{s2} = A_{s3} = 2*491 = 982\ mm^2$
$y_1 = -189.5$ mm, $y_2 = 0$ mm and $y_3 = 189.5$ mm

- Neutral axis depth - $x_{u,b}$

$\varepsilon_y = 0.87*415/2*10^5 + 0.002 = 0.003805$

By similar triangles,

$$x_{u,b} = \frac{0.0035*(500-60.5)}{0.0035+0.003805} = 210.06mm(< D/2 = 250mm)$$

- Strains in steel

$\varepsilon_{s1} = (-)\ \varepsilon_y = -0.003805$ (tensile)

$\varepsilon_{s2} = -\ 0.0035*(250-210.3)/210.6 = 0.000655$ (tensile)

$\varepsilon_{s3} = 0.0035*(210.6-60.5)/210.6 = 0.002495$ (compression) > 0.002

Similarly calculating stresses in steel:

f_{s1}=0.87*f_y = -360.9 MPa

f_{s2}=$E_s \varepsilon_{s2}$=(2*10^5)*(-)0.000581=-131MPa

f_{s3}=342.8+[(249.5-241)/(276-241)]*(351.8-342.8)=345 MPa

Design strength component in axial compression $P_{ub,x}$

C_c = 0.362*25*300*210.6=571779 N

$C_s = \Sigma C_{si} = \Sigma (f_{si}-f_{ci})A_{si}$

= [(-360.9)+(-131)+(345-0.447*25)]*982=-155.230 kN

Hence $P_{ub,x} = C_c + C_s$ = 571.8-155.23 = 416.6 kN

Design strength component in flexure $M_{ub,x}$

$$M_{ub,x} = M_c + M_s$$

$$M_c = C_c(0.5\,D - 0.416\,x_u)$$

= 571.8*(250-.416*210.6)=92.85 kNm

$$M_s = \Sigma\, C_{si} y_i$$

=(-360.9)*(-189.5)+(-131)*0+(345-0.447*25)(189.5)]*982=129.3 kNm

$$M_{ub,x} = M_c + M_s$$

= 92.85+129.3=221.15 kNm

Example - Design Problem

Using the interaction diagram given in SP 16, design the longitudinal reinforcement in a rectangular reinforced concrete column of size 300*600 subjected to a factored load of 1400 kN and a factored moment of 280 kNm with respect to the major axis. Assume M 20 concrete and Fe 415 steel.

As D=600 mm, the spacing between the corner bars will exceed 300 mm, hence inner rows of bars have to be provided to satisfy detailing requirement. Assume equal reinforcement on all four sides.

(*clause* 26.5.3.1 g).

Assuming an effective cover d` =60 mm

Therefore d`/D = 60/600=0.1

$p_u = P_u / f_{ck} bD = (1400*10^3)/(20*300*600) = 0.389$

$m_u = M_u / f_{ck} b^2 = (280*10^6)/(20*300*600)^2 = 0.130$

Referring to chart 44 (d'/D=0.10) of SP:16,

$P_{reqd} = 0.11*20 = 2.2$

Hence,

$A_{s,reqd} = 2.2 *300* 600/100 = 3960 \text{ mm}^2$

Detailing of longitudinal reinforcement

The design chart used have equal reinforcement on all 4 sides.

Hence provide 2 – 28 ϕ in outermost rows and 4 – 22 ϕ in two inner rows.

Total area provided = 1232*2+1520=3984 mm ^{2}

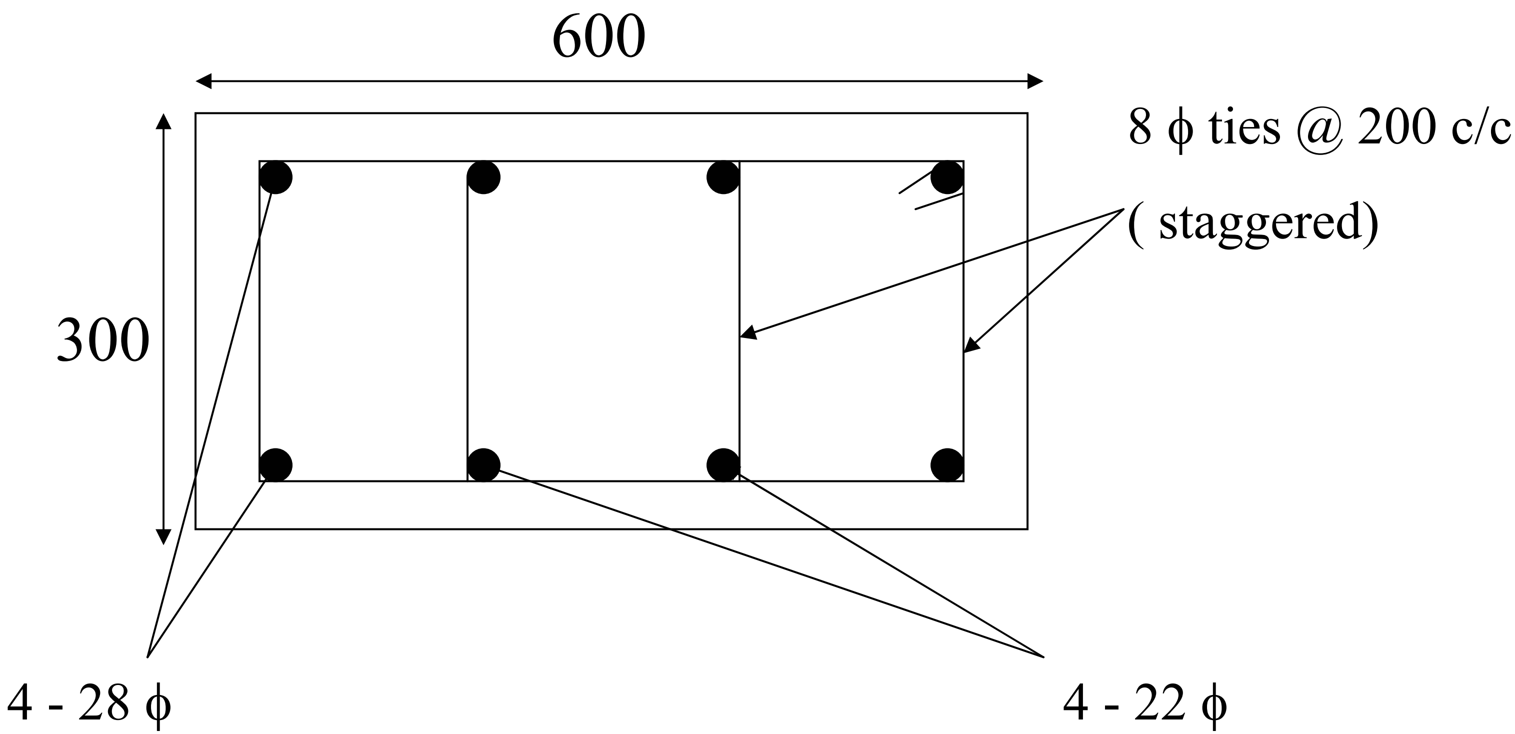
Thus area provided > area reqd. – OK

Check : Bar diameter < thickness/8 – OK (clause 26.2.2)

Assuming 8 mm ties,

effective cover = 40+8+14=62 = 60 – OK

Detailing



Short columns under axial compression with biaxial bending

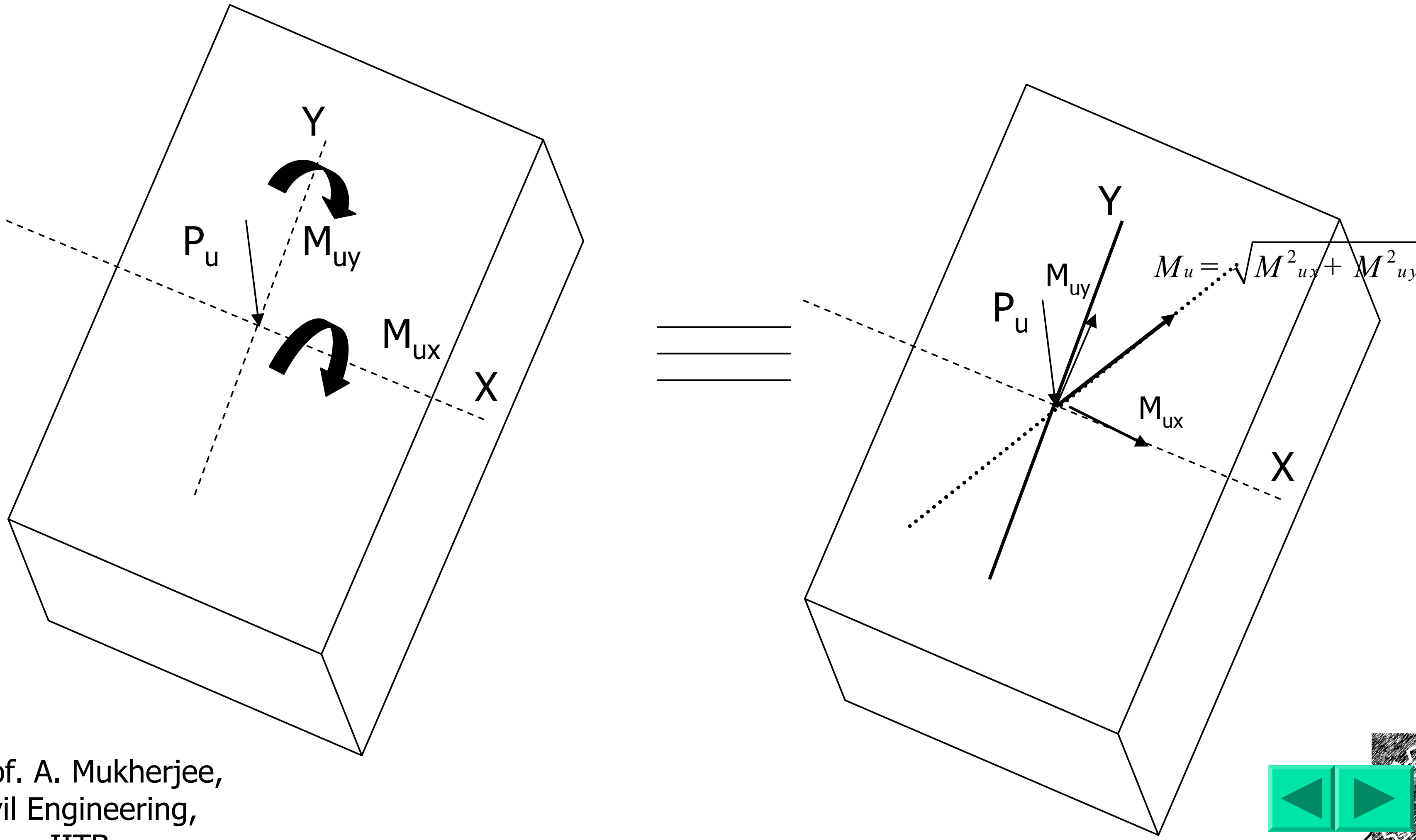
The factored moments M_{ux} and M_{uy} on a column can be resolved into a single moment M_u, which acts about an axis inclined to the two principal axes

$$M_u = \sqrt{M^2_{ux} + M^2_{uy}}$$

The resultant eccentricity e=M_u/P_u may be obtained as

$$e = \sqrt{e^2_{ux} + e^2_{uy}}$$

Resultant moment

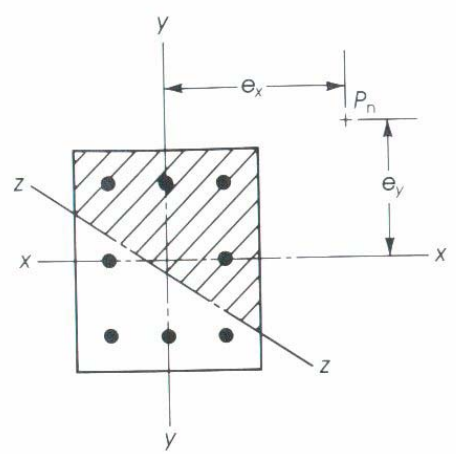


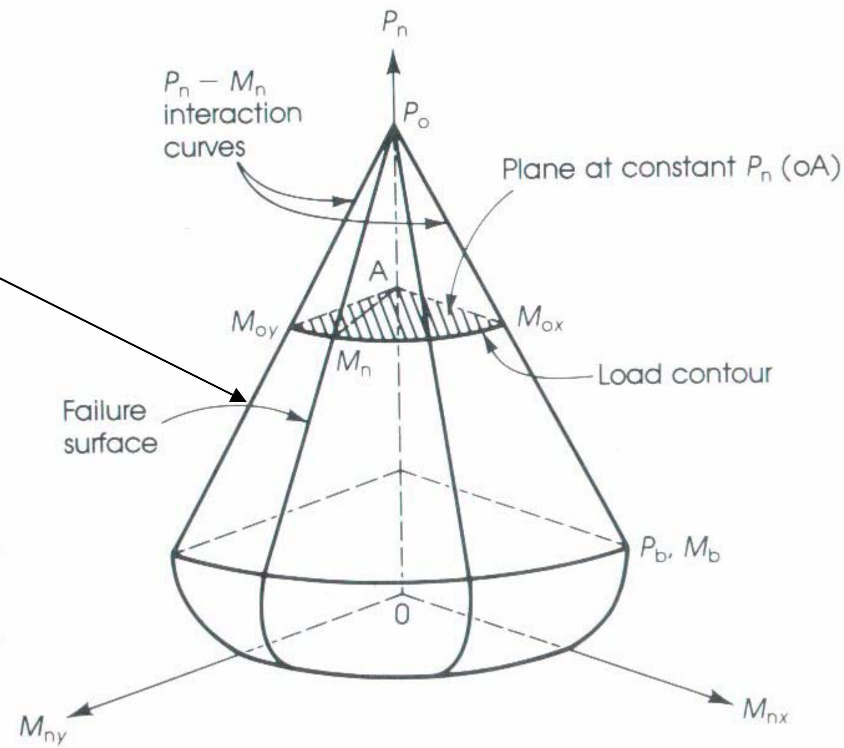
Interaction envelope for biaxially loaded column

The envelops is generated as the envelope of a number of design interactions curves for different axes of bending. The interaction diagram surface can be regarded as a failure surface within which the region is safe and any point (P_u, M_{ux}, M_{uy}) that lies outside the surface is unsafe.

Interaction curve for Bi-axial Bending and Axial Load

Unaxial bending about y-axis





Interaction between uniaxial moments:

$$\left[\frac{M_{ux}}{M_{ux1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy1}}\right]^{\alpha_n} \leq 1$$

M_{ux} and M_{uy} denote the factored biaxial moments acting on the column, and Mux1 and Muy1 denote the uniaxial moment capacities with reference to major and minor axes respectively, under an accompanying axial load $P_u = P_{uR}$

α_n depends on the P_u. For low axial loads it is 1 and for high loads it is 2. In between it is related as

$$P_{uz} = 0.45 f_{ck} A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}$$

where,

P_{uz}= P_u normalized with the maximum axial load capacity

Design of biaxially loaded column

Example

A column of 400 X 400, in the ground floor of a building is subjected to factored loads:

P_u=1300kN, M_{ux}=190kNm and M_{uy}=110kNm

The unsupported length of the column is 3.5m.

Design the reinforcement in the column, assuming M25 concrete and Fe 415 steel.

Here,

$D_x = D_y = 400$ mm, $l = 3500$ mm, $P_u = 1300$ kN,

$M_{ux} = 190$ kNm, $M_{uy} = 110$ kNm

Assuming an effective length factor k=0.85 —— (table 28)

Effective length can be calculated as,

$l_{ex} = l_{ey} = 0.85*3500 = 2975$ mm

Eccentricity= $l_{ex}/D_x = l_{ey}/D_y$

$= 2975/400 = 7.44 < 12$ —————— (*Clause* 25.1.2)

Hence the column is a short column.

Checking for minimum eccentricities

e_x=190000/1300=146 mm

e_y=110000/1300=84.6 mm

Minimum eccentricity:

$$e_{x,min} = e_{y,min} = 3500/500+400/30$$

$$= 20.3 \text{ mm} > 20 \text{ mm}$$ (*Clause* 25.4)

As the minimum eccentricities are less than the applied eccentricities, no modification are needed for M_{ux} and M_{uy}.

Longitudinal reinforcement for trial section

Designing for uniaxial eccentricity with P_u=13000kN and $M_u = 1.15\sqrt{M^2_{ux} + M^2_{uy}}$

We have considered a moment of 15 % in excess of the resultant moment for a trial section. Assuming d` =60 mm,

d`/D=60/400=0.15

$$\frac{P_u}{f_{ck}bD} = \frac{1300*1000}{25*400^2} = 0.325$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{252*10^6}{25*400^3} = 0.157$$

Refer to chart 45 (SP:16)

p/f_{ck}=0.14

hence, p_{reqd}=0.14*25=3.5 less than 4% OK

(*clause* 26.5.3.1)

$A_{s,reqd}$=3.5*400²/100=5600 mm ²

Provide 12- 25 ϕ thus, $A_{s,\ provided}$ =5892 mm ²

$P_{provided}$=3.68 % => p/f_{ck} =3.68/25 = 0.147

Assuming a clear cover of 40 mm and 8 mm ties

d` =40+8+25/2=60.5 mm

d`/D=60.5/400=0.15

Referring to chart 45 , $\frac{M_{ux1}}{f_{ck} b D^2} = 0.165$

$$M_{ux1} = M_{uy1} = 0.165*25*400^3 = 264kNm$$

which is greater than M_{ux} = 190 kNm & M_{uy} = 110 kNm

Calculating P_{uz} and α_n,

$$P_{uz} = 0.45 f_{ck} A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}$$

$$P_{uz} = (0.45*25*400^2) + (0.75*415 - 0.45 - 25)*5892 = 3568kN$$

P_u/P_{uz}=1300/3568=0.364 (lies between 0.2 and 0.8)

α_n=1 + (0.364-0.2)/(0.8-0.6)*1=1.273

(*clause* 39.6)

Check under biaxial loading

We need to check that

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \leq 1 \qquad \text{———————} \qquad (\textit{clause}\ 39.6)$$

$$= \left(\frac{190}{264}\right)^{1.273} + \left(\frac{110}{264}\right)^{1.273}$$

$$= 0.658 + 0.328$$

$$= 0.986 < 1.0$$

Hence the trail section is safe under the applied Loading.

Transverse reinforcement

The minimum diameter ϕ_t and maximum spacing s_t of the lateral ties are given as

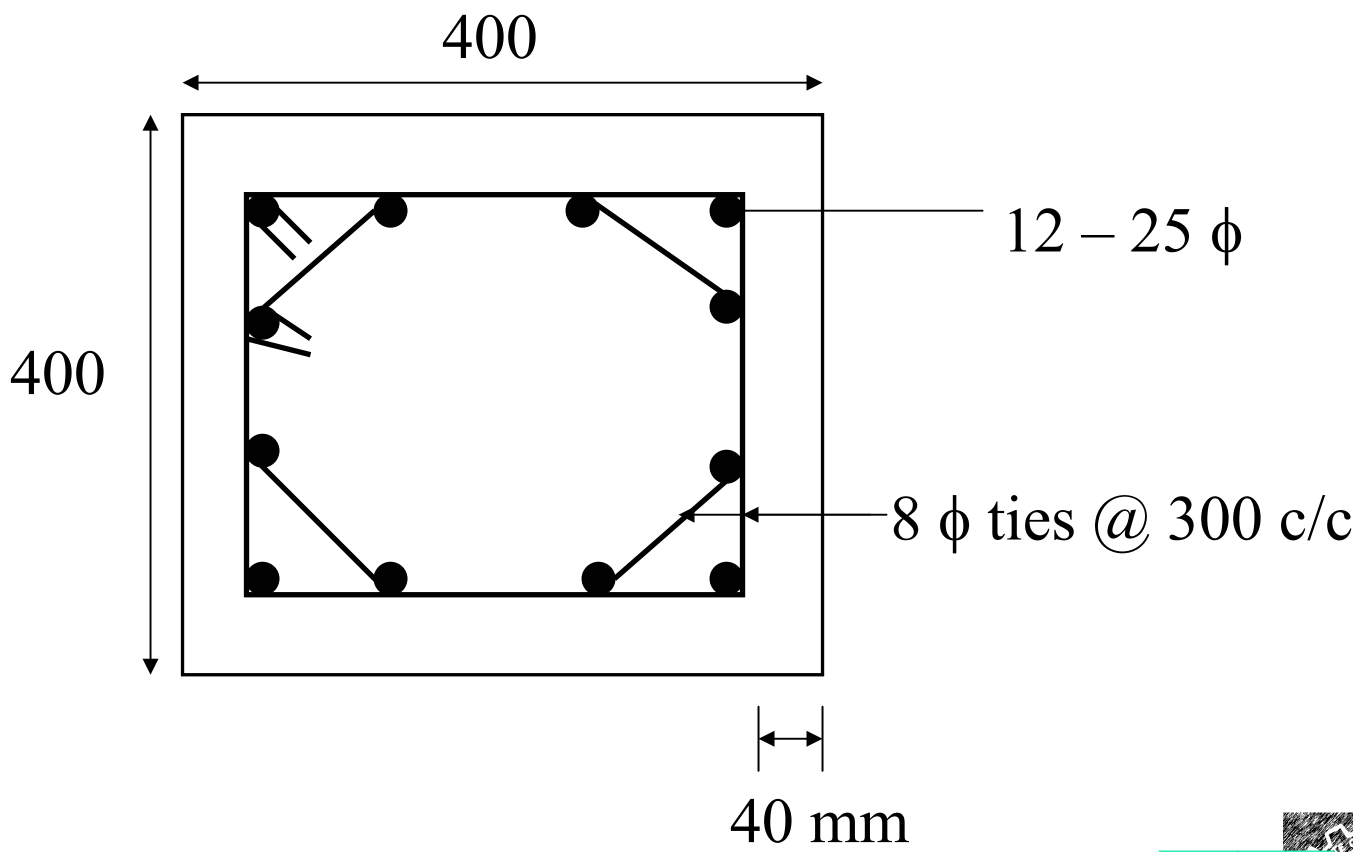
ϕ_t =maximum (25/4 mm , 6 mm) =8 mm

s_t=minimum (D=400 mm, 16*25 mm, 300 mm)

(Clause 26.5.3.2)

Hence provide 8 ϕ ties @ 300 mm c/c.

Detailing

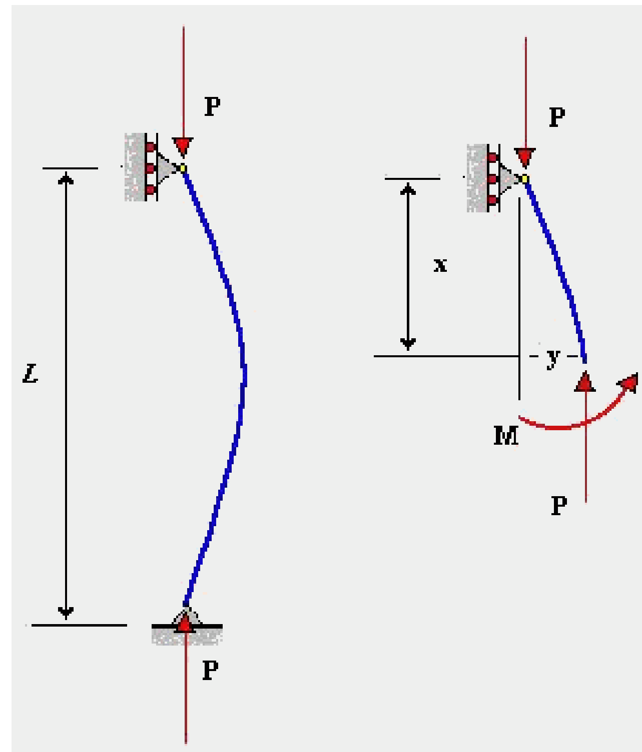


Euler buckling load

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{L_e^2 A}$$

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e / r)^2}$$



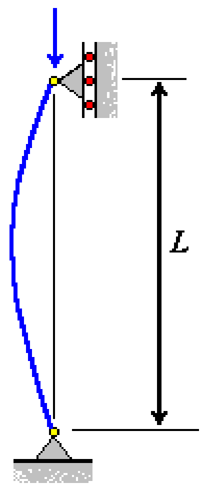
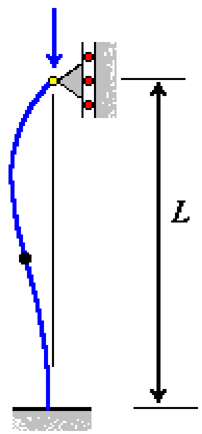
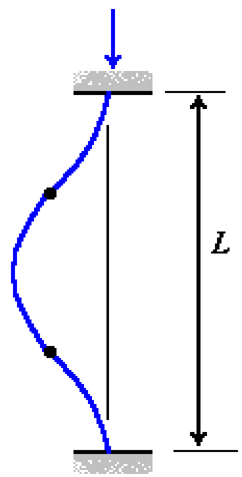
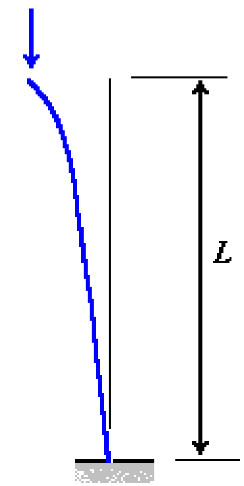
Pinned – Pinned column

Slenderness ratio

The factor L_e/r in denominator is defined as the slenderness ratio. This provides a measure of probability of column buckling.

Long columns fails in buckling under relatively low axial loads while short columns fail by crushing with the material reaching its ultimate strength.

Effective Length (*clause* 25.2)

(a) Pinned - pinned column	(b) Fixed - pinned column	(c) Fixed - fixed column	(d) Fixed - free column
			
$L_e = L$	$L_e = 0.699L$	$L_e = 0.5L$	$L_e = 2L$
K = 1	K = 0.699	K = 0.5	K = 2

Code recommendations for *k* factor for various boundary conditions (*Annex E - 3*)

Column braced against side sway:

a) both end fixed rotationally = 0.65

b) one end fixed and other pinned = 0.80

c) both end free rotationally = 1.00

Columns unbraced against sideway

a) both end fixed rotationally = 1.20

b) one end fixed and other partially fixed = 1.5

c) one end fixed and the other free = 2.00

How to determine whether column is braced or unbraced (*Annex E -2*)

- To determine whether a column is a no sway or a sway column, stability index Q may be computed as given below,

$$Q = \Sigma P_u \Delta_u / H_u h_s$$

Where,

ΣP_u = sum of axial loads on all column in the storey

Δ_u = elastically computed first order lateral deflection

H_u= total lateral force acting within the storey and

H_s = height of the storey

If Q<= 0.04, then the column in the frame may be taken as no sway column, otherwise he column will be considered as sway column.

Code requirements on slenderness limits

- *Clause* 25.3.1 – The unsupported length between end restraints shall not exceed 60 times the lest lateral dimension of a column.
- *Clause* 25.3.2 – If, in any given plane, one end of a column is unrestrained, its unsupported length, l, shall not exceed, $100b^2/D$.

Code requirements on minimum eccentricities

Clause 25.4

All columns shall be designed for minimum eccentricity,

$$e_{min\ x} = \min\ (L/500 + D/30,\ 20)\ mm$$
$$e_{min\ y} = \min\ (L/500 + b/30,\ 20)\ mm$$

Design of short column under axial loading

- Under pure axial loading conditions, the design strength of a short column is obtainable as,

$$P_0 = C_C + C_S$$

$$= f_{cc} Ac + f_{sc} A_{sc}$$

$$\Rightarrow P_0 = f_{cc} A_g + (f_{sc} - f_{cc}) A_{sc}$$

$$P_0 = f_{cc} A_g + (f_{sc} - f_{cc}) A_{sc}$$

$$P_u = 0.447 f_{ck} A_g + (f_{sc} - 0.447 f_{ck}) A_{sc}$$

$f_{sc} = 0.870\ f_y$ *for Fe 250*

$0.790\ f_y$ *for Fe 415*

$0.746\ f_y$ *for Fe 500*

Where

A_g = gross area of cross-section = $A_c + A_{sc}$

A_{sc} = total area of longitudinal reinforcement = ΣA_{si}

A_c = net area of concrete in the section= $A_g - A_{sc}$

- As explained earlier code requires all columns designed for “minimum eccentricities” in loading. When the minimum eccentricity as per *clause* 25.4 does not exceed 0.05 times the lateral dimension, the member may be designed by the following equation (*clause 39.3*):

$$P_{uo} = 0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc}$$

More about spiral columns

- In spiral column substantial ductility is achieved prior to the collapse of the column. The concrete in the core remains laterally confined by the helical reinforcement even after outer shell of concrete spalls off. Hence code permits 5 % increase in the estimation of strength beyond P_{uo} provided the following requirement is satisfied by the spiral reinforcement.

$$\rho_s \geq 0.36(A_g/A_{core}-1)f_{ck}/f_{sy}$$

$$\rho_s \geq 0.36(A_g/A_{core}-1)f_{ck}/f_{sy}$$

where,

ρ_s = Volume of spiral reinforcement / Volume of core (*per unit length of the column*)

A_{core} = total area of concrete core, measured outer-to-outer of the spirals

A_g = gross area of cross section;

f_{sy} = characteristic (yield) strength of spiral

Design of short column under axial loading

Example:

Design the reinforcement in a column of size 450 mm X 600 mm, subjected to an axial load of 2000kN under service dead and live loads. The column has an unsupported length of 3.0m and is braced against sideway in both direction. Use M20 concrete and Fe 415 steel.

Check for slenderness

$l_x = l_y$=3000 mm

D_y = 450mm, D_x = 600 mm

Slenderness $ratio_x = l_{ex}/D_x = k_x$*3000/600=5$k_x$

Slenderness $ratio_y = l_{ey}/D_y = k_y$*3000/450=6.67$k_y$

Since the column is braced in both directions, k_x and k_y are less than unity, and hence the column is short column in both direction.

- Check for minimum eccentricity

$e_{x,min}$ = 3000/500+600/30=26.0 mm (20.0 mm)

$e_{y,min}$ = 3000/500+450/30=21.0 mm (20.0 mm)

Now,

$0.05D_x = 0.05*600 = 30.0$ mm > e_{xmin} = 26.0 mm

$0.05\ D_y = 0.05*450 = 22.5$ mm > e_{ymin} = 21.0 mm

Hence, *clause 39.3* can be used for short axially loaded members in compression.

Factored load $P_U = 2000 * 1.5 = 3000$kN

$$P_u = 0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc}$$

$$3000*1000 = 0.4*20*(450*600)+(0.67*415-0.4*20)A_{sc}$$

$$\Rightarrow A_{sc} = 3111 \text{ mm}^2$$

Hence provide,

4-25 ϕ at corners : 4 *491 = 1964 mm ^{2}

4-20 ϕ additional : 4* 314 = 1256 mm ^{2}

Total steel provided = 3220 mm ^{2}

=> P = (100*3220)/(450*600)=1.192>0.8 (minimum reinforcement)

Lateral ties

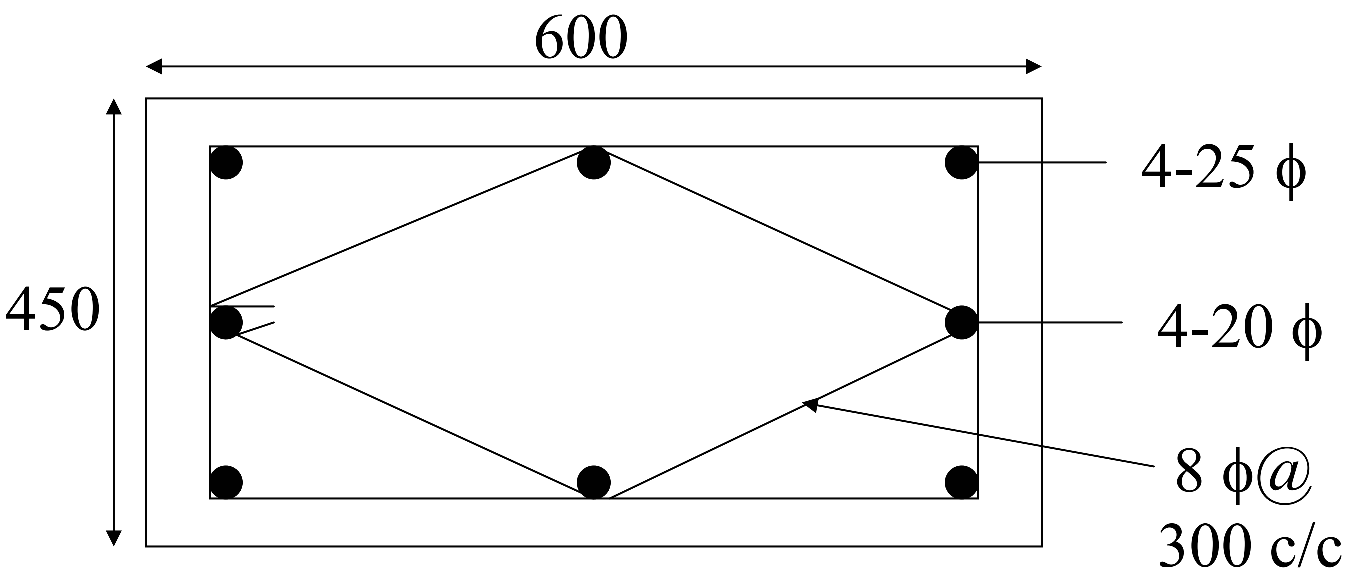
Tie diameter ϕ_t > max(25/4 mm,6 mm)

Hence let us provide 8mm diameter bar

Tie spacing s_t= min(450 mm,16*20 mm,300 mm)

Hence provide 8 ϕ ties @ 300 mm c/c

Detailing of reinforcement in short axially loaded column.

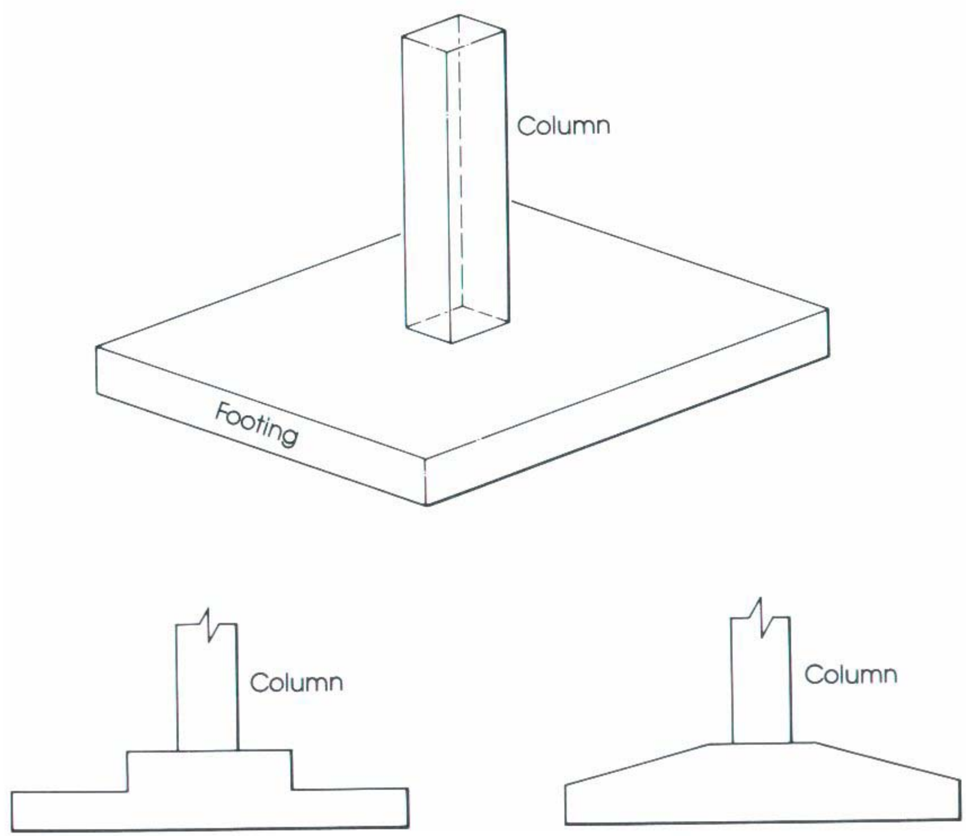


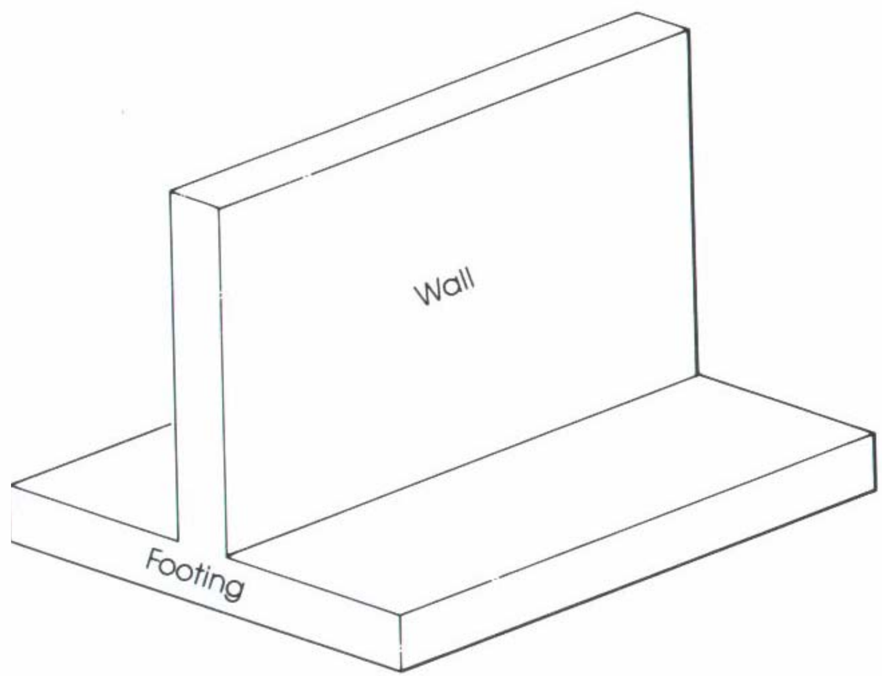
Footings

Definition

Footings are structural members used to support columns and walls and to transmit and distribute their loads to the soil in such a way that the load bearing capacity of the soil is not exceeded, excessive settlement, differential settlement,or rotation are prevented and adequate safety against overturning or sliding is maintained.

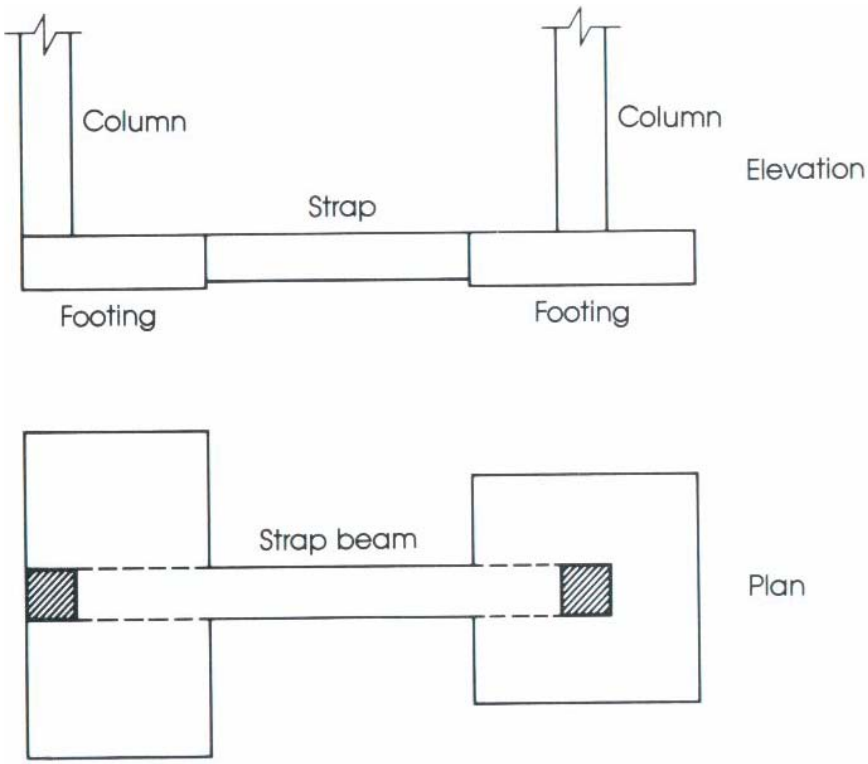
Types of Footings

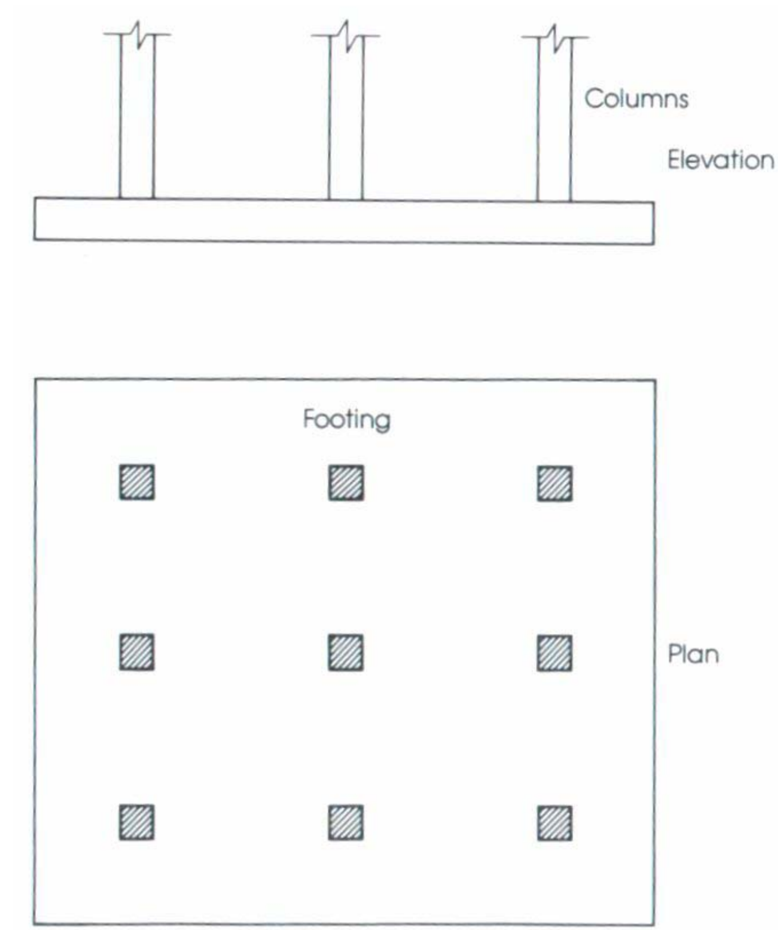




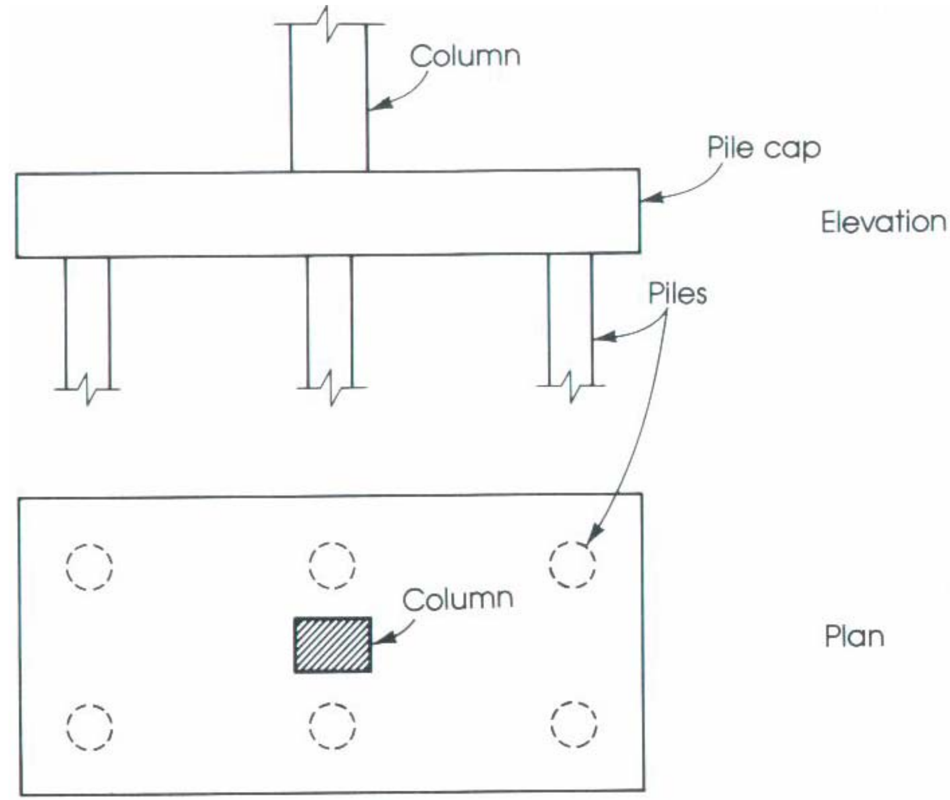
Wall footing.

Types of Footings

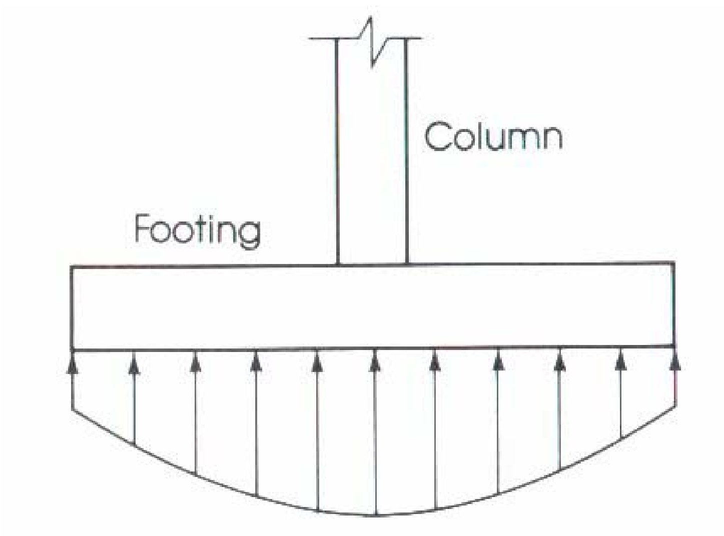




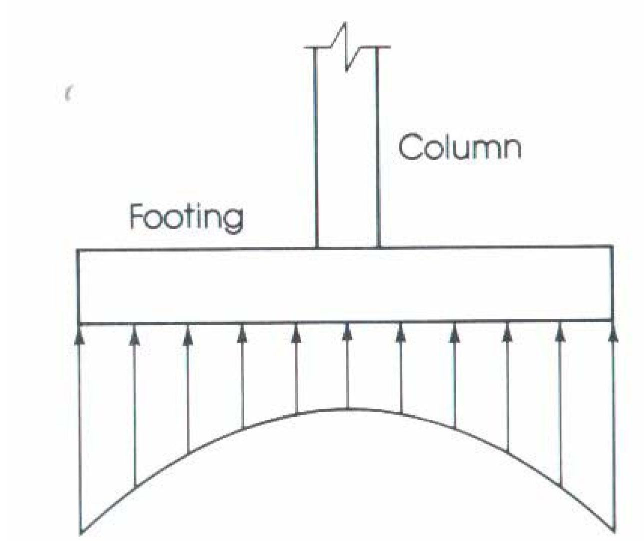
Types of Footings



Distribution of Soil Pressure

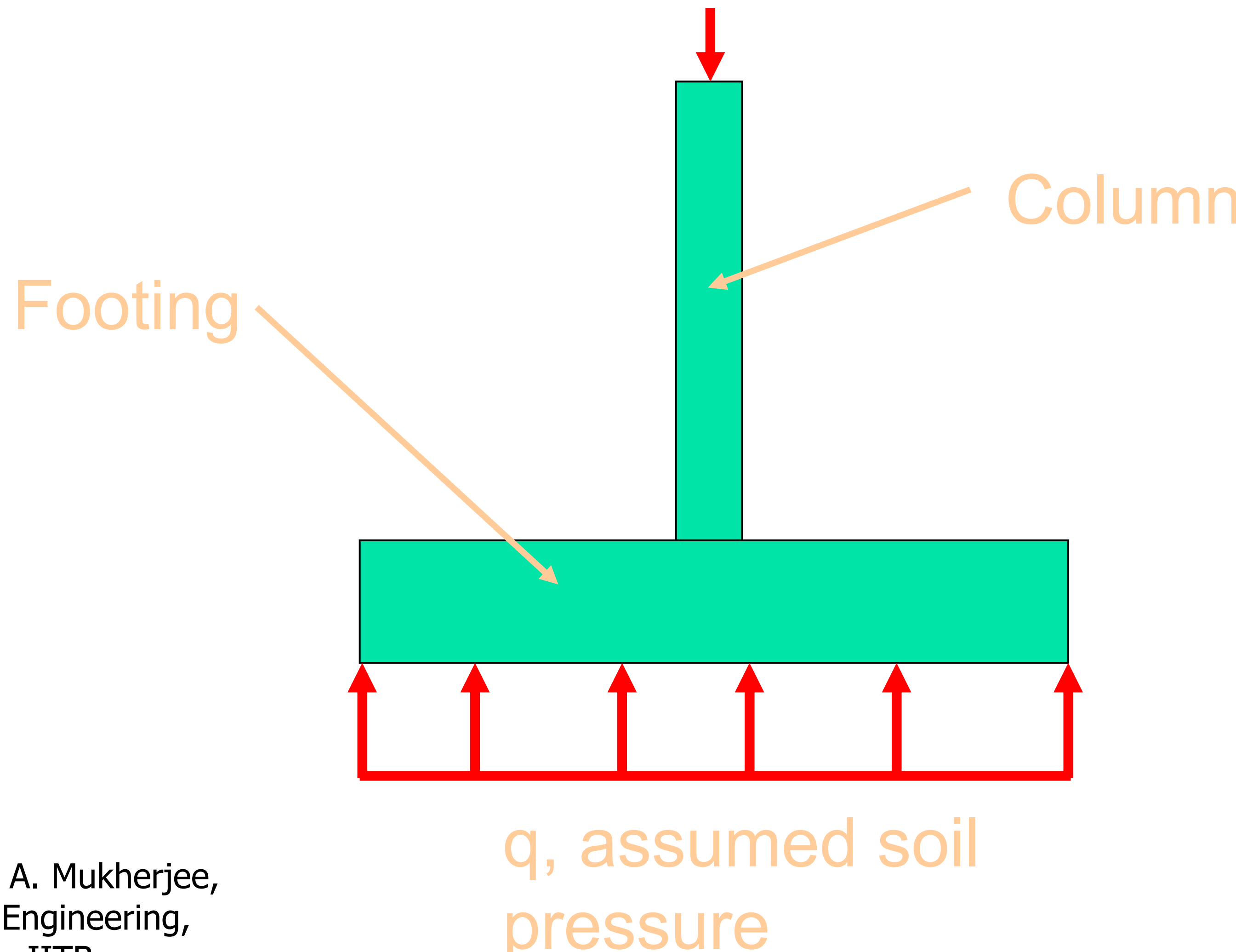


Soil pressure distribution in cohesionless soil.

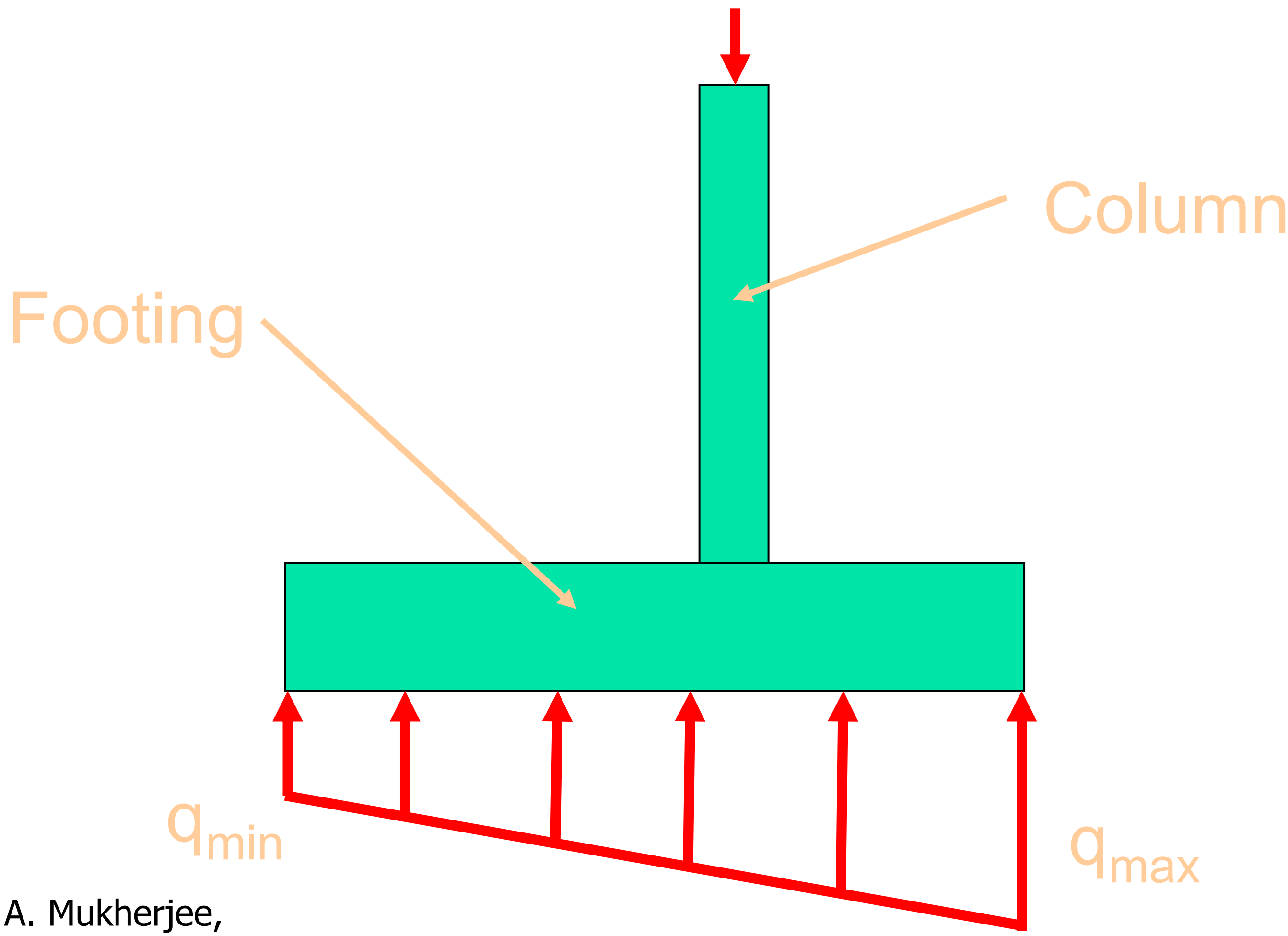


Soil pressure distribution in cohesive soil.

Isolated Footings



Eccentric Loading



Design Considerations

Footings must be designed to carry the column loads and transmit them to the soil safely while satisfying code limitations.

1. The area of the footing based on the allowable soil bearing capacity

2. Two-way shear or punching shear.

3. One-way shear

4. Bending moment and steel reinforcement required

Size of Footings

The area of footing can be determined from the actual external loads such that the allowable soil pressure is not exceeded.

$$\text{Area of footing} = \frac{\text{Total load (including self weight)}}{\text{allowable soil pressure}}$$

Strength design requirements

$$q_u = \frac{P_u}{\text{area of footing}}$$

Design of two-way shear

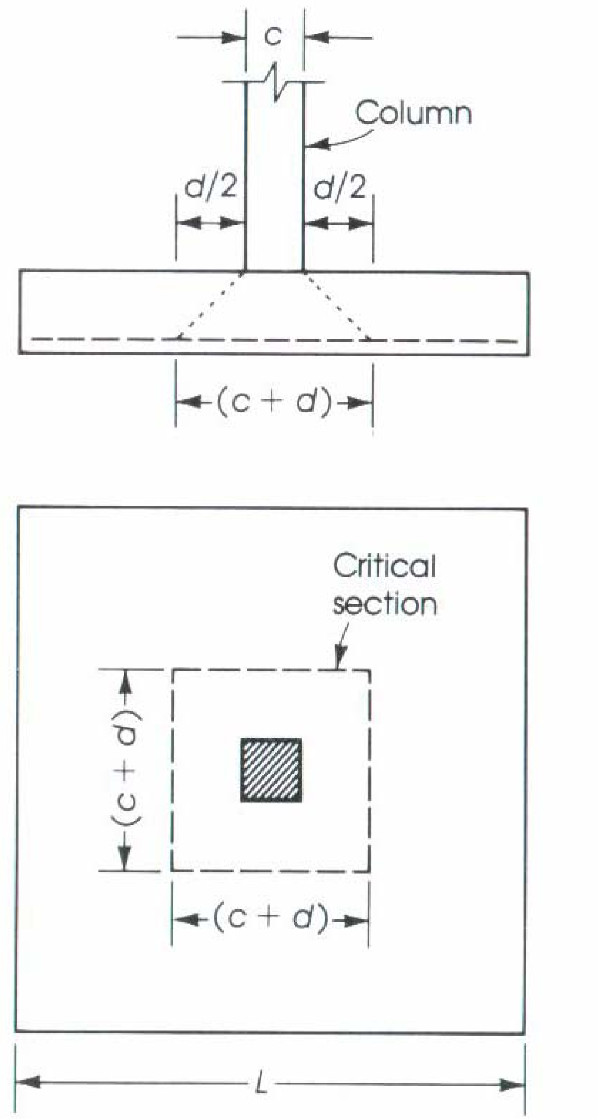
1. Assume d.
2. Determine b_0.

$b_0 = 4(c+d)$

for square columns where one side = c

$b_0 = 2(c_1+d) + 2(c_2+d)$

for rectangular columns of sides c_1 and c_2.

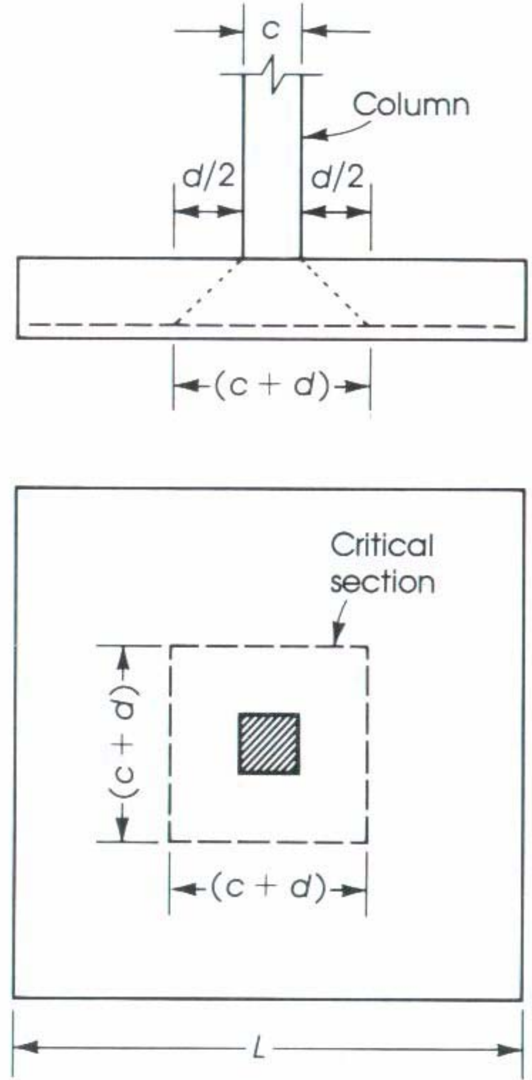


Design of two-way shear

3. The shear force V_u acts at a section that has a length $b_0 = 4(c+d)$ or $2(c_1+d) + 2(c_2+d)$ and a depth d; the section is subjected to a vertical downward load P_u and vertical upward pressure q_u.

$$V_u = P_u - q_u (c+d)^2 \quad \text{for square columns}$$

$$V_u = P_u - q_u (c_1+d)(c_2+d) \quad \text{for rectangular columns}$$

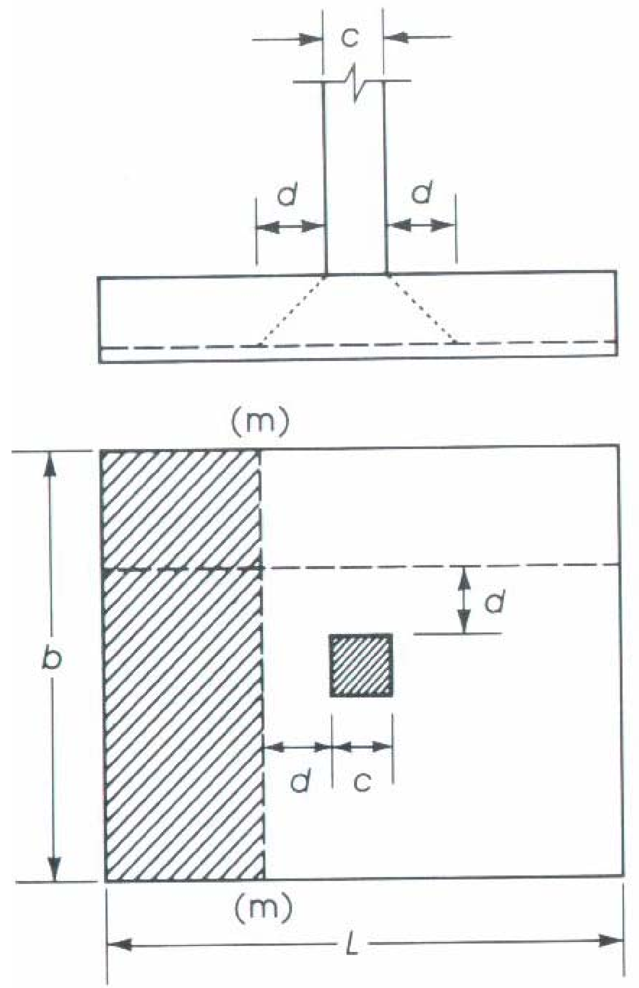


Design of one-way shear

The ultimate shearing force at section m-m can be calculated

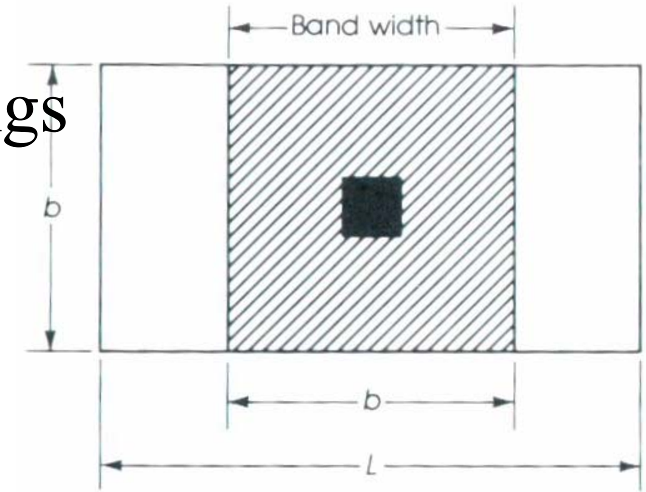
$$V_u = q_u\, b\left(\frac{L}{2} - \frac{c}{2} - d\right)$$

If no shear reinforcement is to be used, then d can be checked



Flexural Strength and Footing reinforcement

The reinforcement in one-way footings and two-way footings must be distributed across the entire width of the footing.

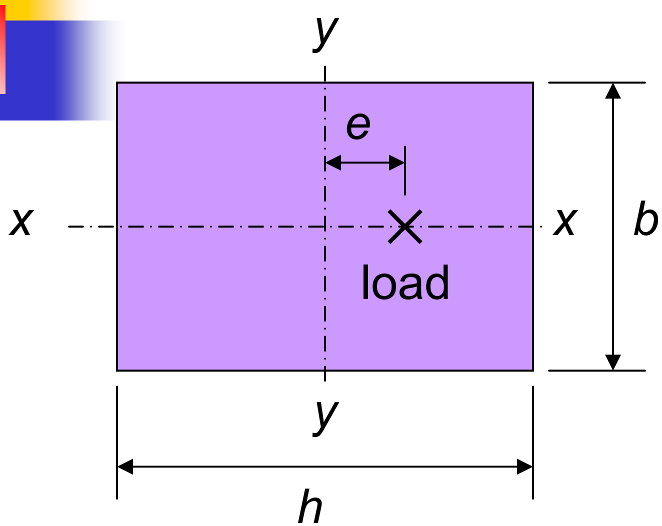


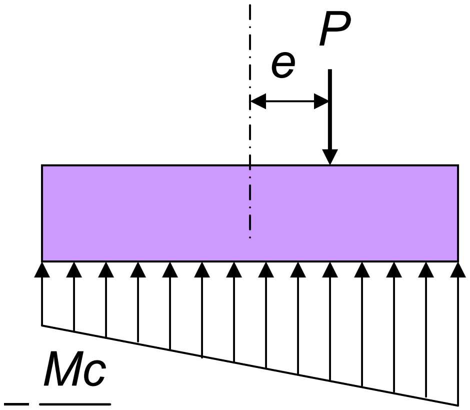
$$\frac{\text{Reinforcement in band width}}{\text{Total reinforcement in short direction}} = \frac{2}{\beta + 1}$$

where

$$\beta = \frac{\text{long side of footing}}{\text{short side of footing}}$$

Eccentrically Loaded Footings





$$p_{\min} = \frac{P}{A} - \frac{Mc}{I}$$

$$p_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

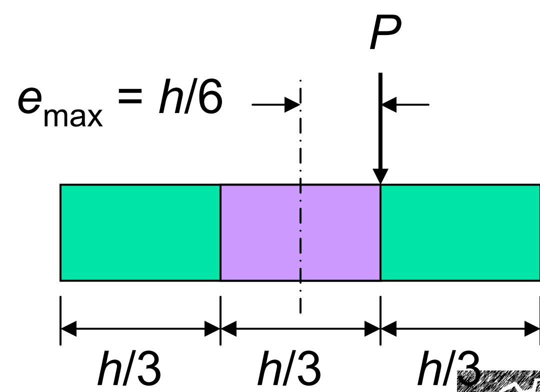
Tensile stress cannot be transmitted between soil and concrete.

For full compression, setting $p_{min} = 0$,

$$\frac{P}{A} = \frac{Mc}{I} = \frac{Pec}{I} \longrightarrow e = \frac{I}{Ac}$$

For rectangular footing of length h and width b,

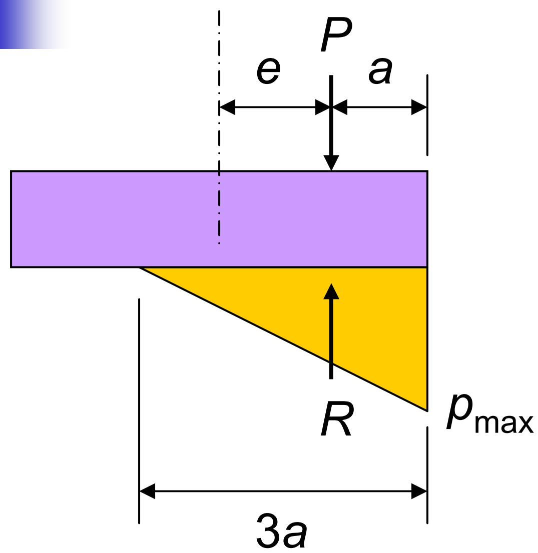
$$e = \frac{I}{Ac} = \frac{bh^3/12}{bh(h/2)} = \frac{h}{6}$$



Middle Third

Large eccentricity of load $e > h/6$

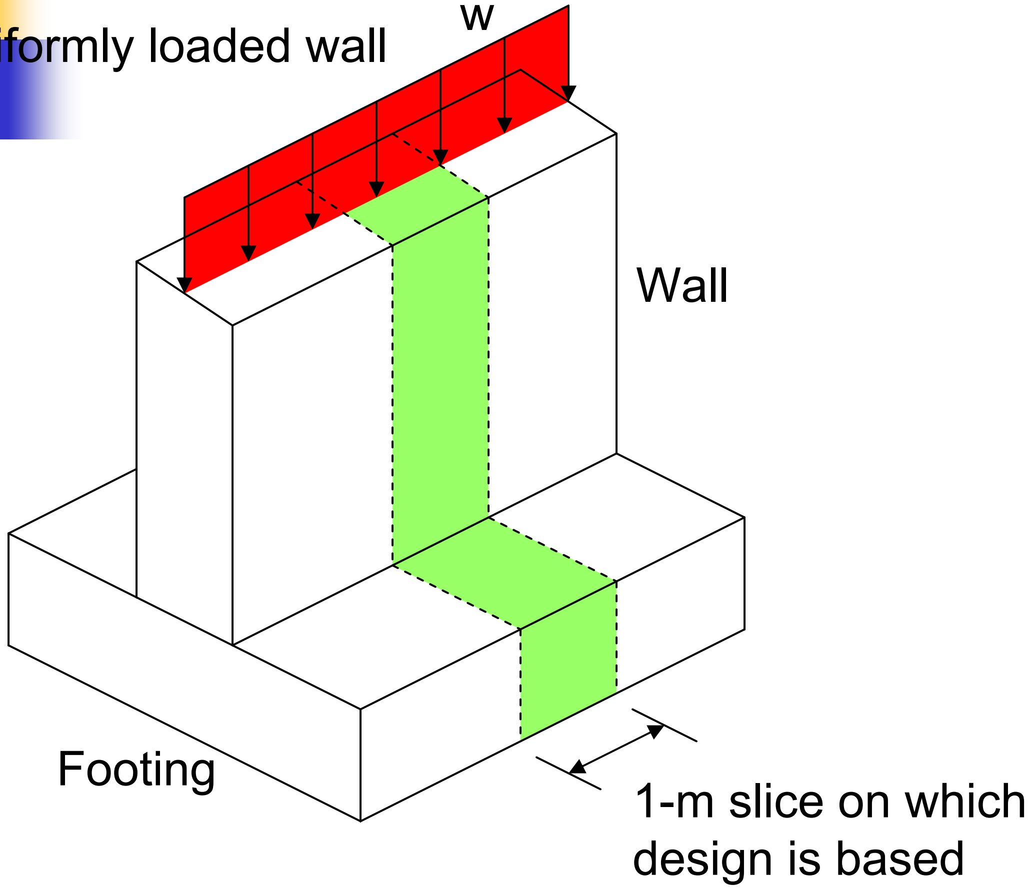
Centroid of soil pressure concurrent with applied load

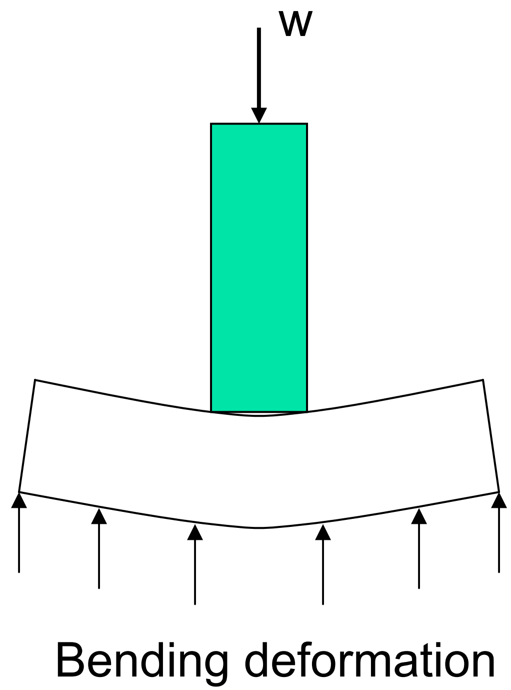


$$R = \frac{1}{2}(3ab)p_{max} = P \longrightarrow p_{max} = \frac{2P}{3ab}$$

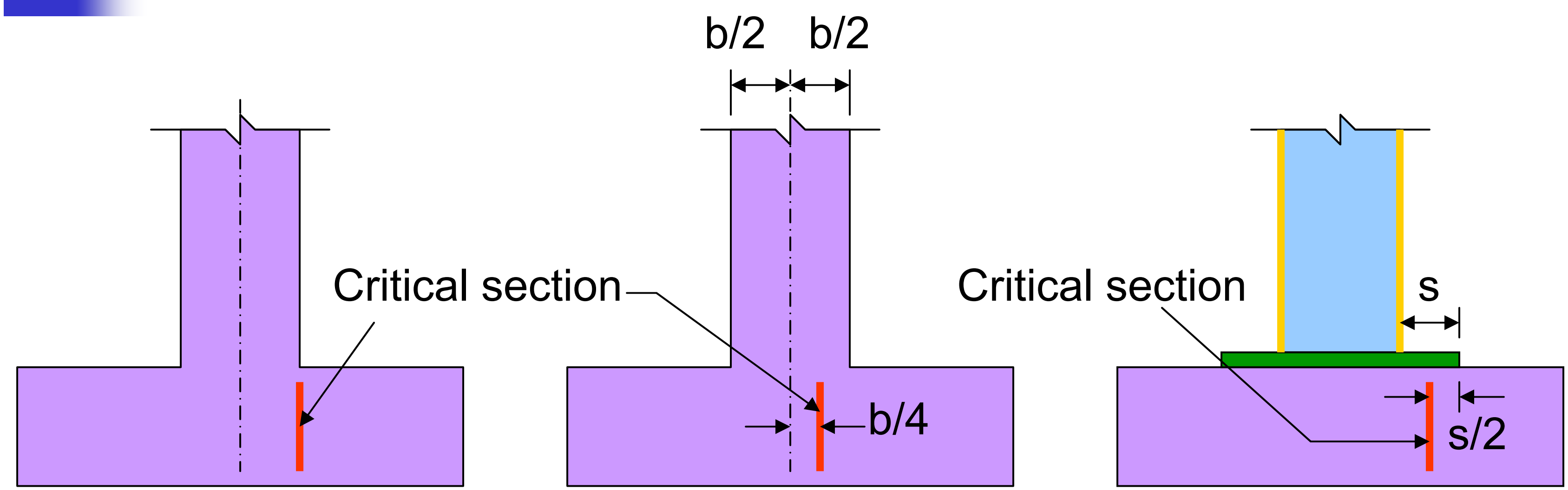
where $a = h/2 - e$

Wall Footings





Critical Section for Moment in Isolated Footings

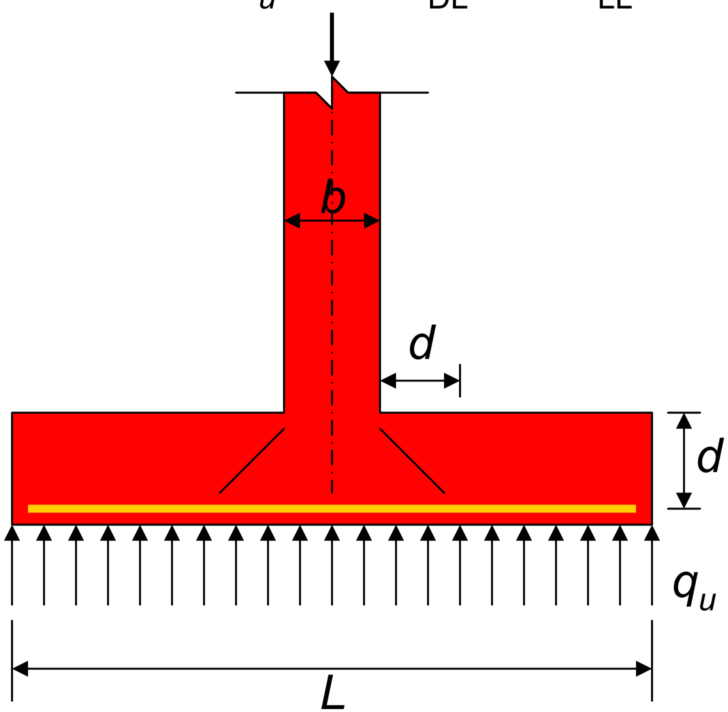


Concrete column, pedestal or wall

Masonry wall

Column with steel base plate

Moment and Shear in Wall Footings



Required $L = (w_{DL}+w_{LL})/q_a$

q_a = Allowable soil pressure, t/m^2

Factored wall load = w_u t/m

Factored soil pressure, $q_u = (w_u)/L$

$$M_u = \frac{1}{2} q_u \left(\frac{L-b}{2}\right)^2 = \frac{1}{8} q_u (L-b)^2$$

$$V_u = q_u \left(\frac{L-b}{2} - d\right)$$

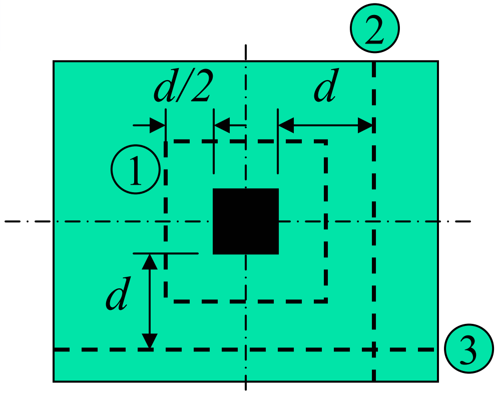
Min t = 15 cm for footing on soil, 30 cm for footing on piles

Min A_s = $(14 / f_y)$ (100 cm) d

Column Footings

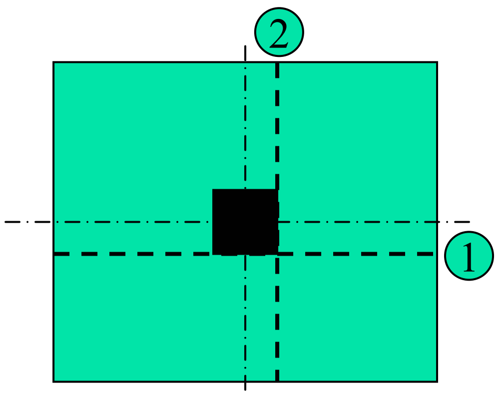
Weight of footing ≈ 4-8 % of column load

Critical section for shear



1. Punching shear
2. Beam-shear short direction
3. Beam-shear long direction

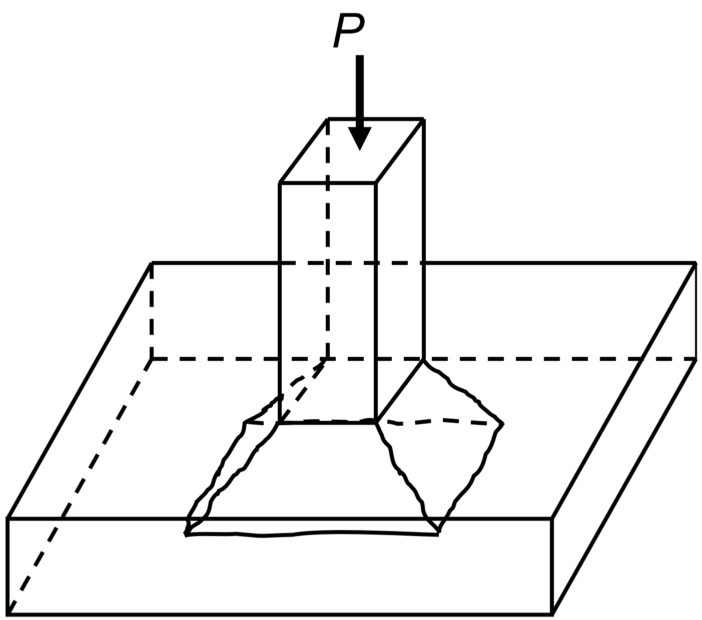
Critical section for moment

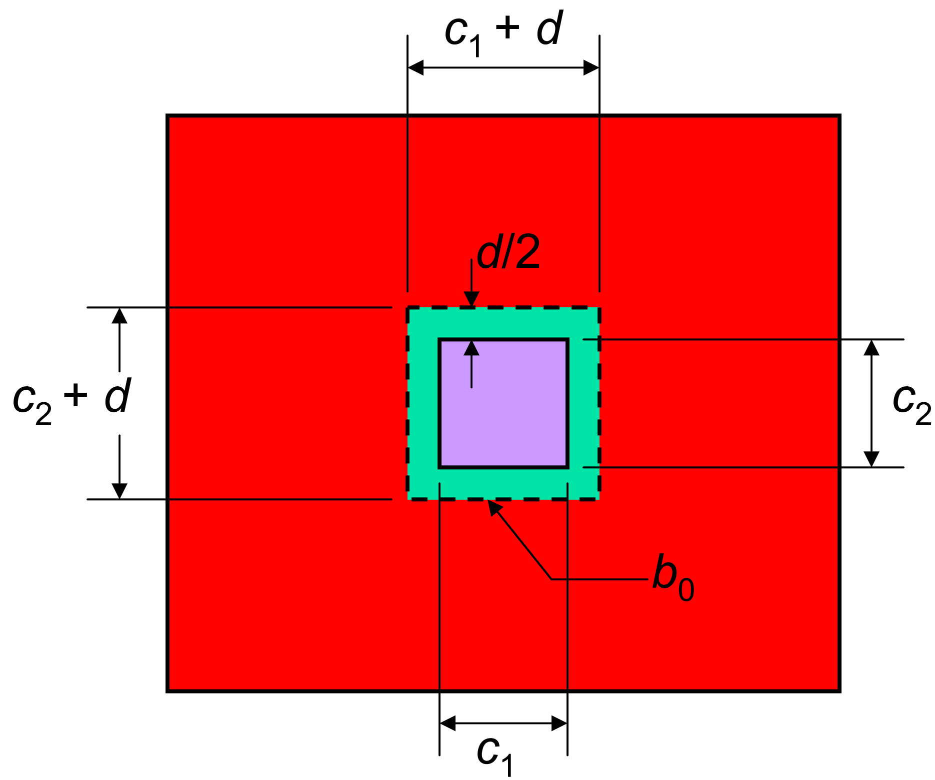


1. Moment short direction
2. Moment long direction

Two-Way Action Shear (punching-shear)

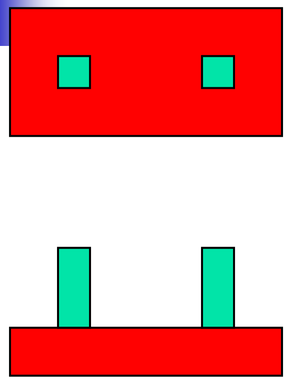
On perimeter around column at distance $d/2$ from face of column





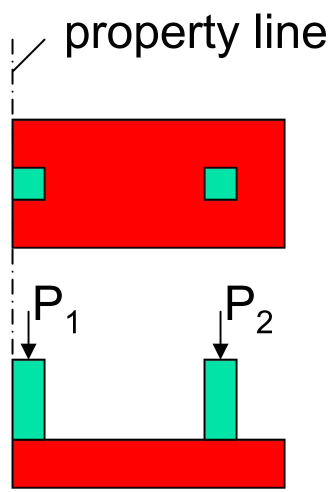
Combined Footings

Centroid of load resultant and footing must coinside



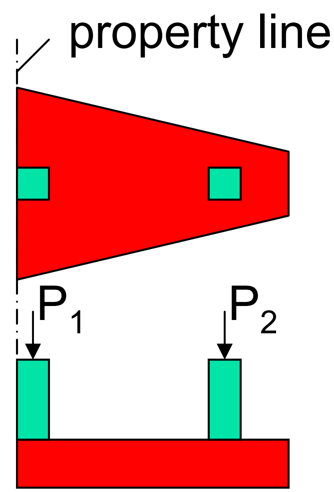
2 footings close

to each other



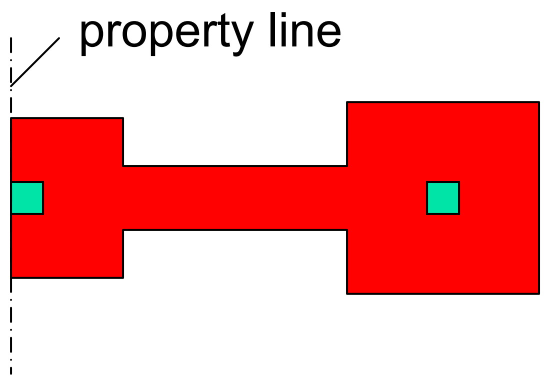
P_1 close to property

line and $P_2 > P_1$



If $1/2 < P_2/P_1 < 1$

use trapizoidal footing



If $P_2/P_1 < 1/2$, use strap combined footing

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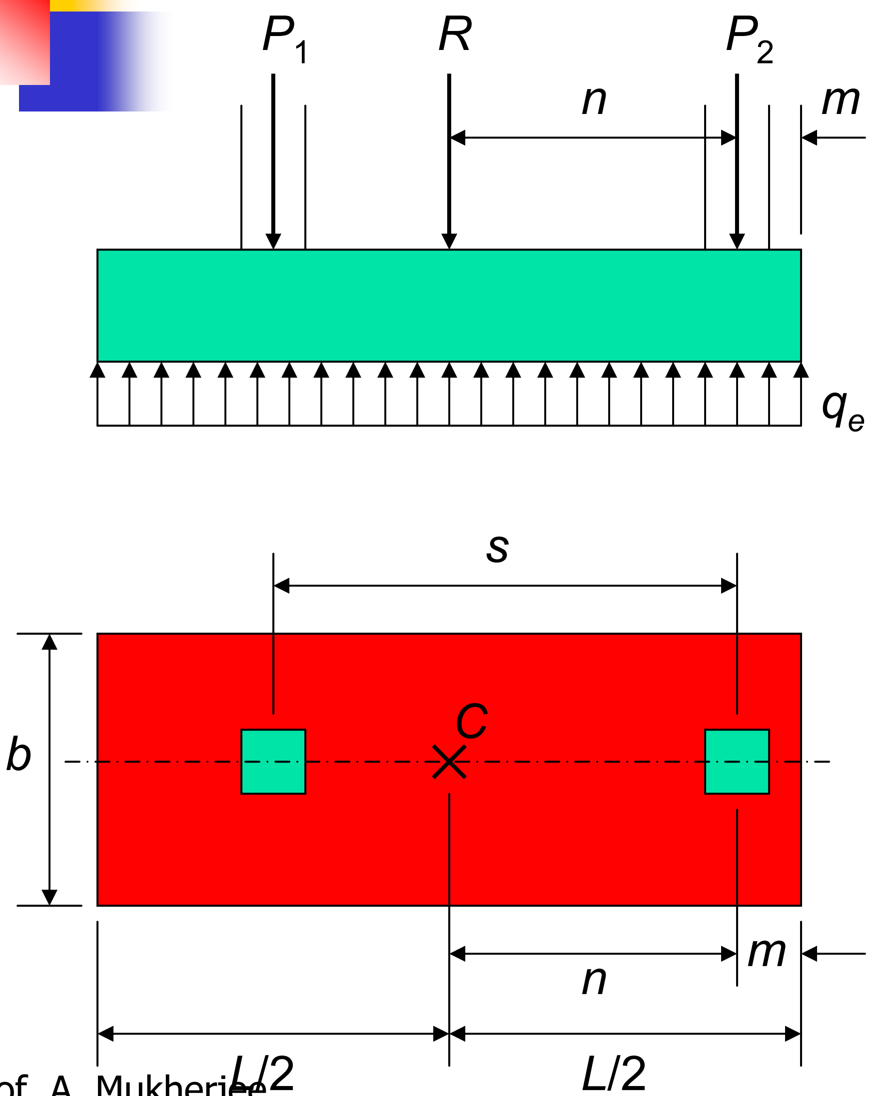
Civil Engineering,
IITB

Centroid of Combined Footings



(1) Compute centroid *C*

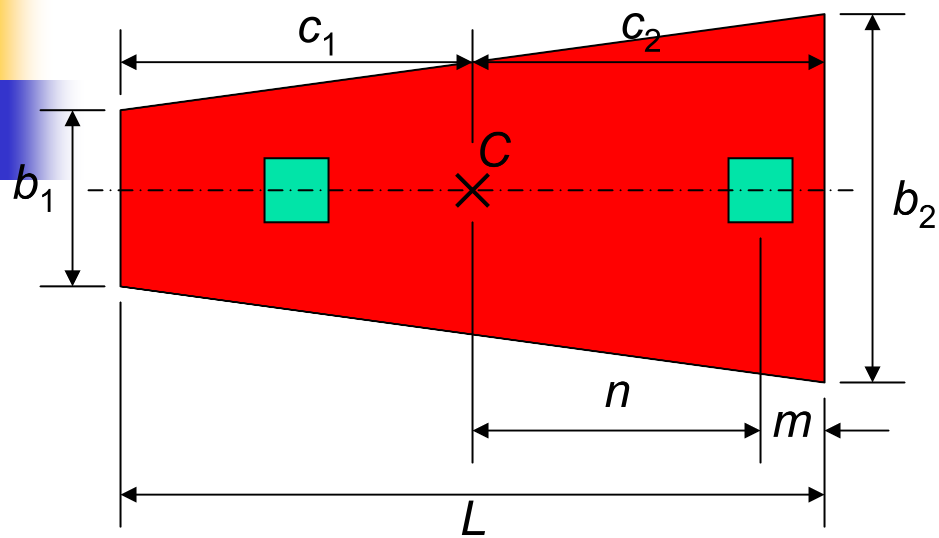
$$n = P_1 s \,/\, (P_1 + P_2) = P_1 s \,/\, R$$

(2) Footing area

$$L = 2\,(m + n)$$

$$b = R \,/\, (q_e L)$$

q_e = allowable soil pressure

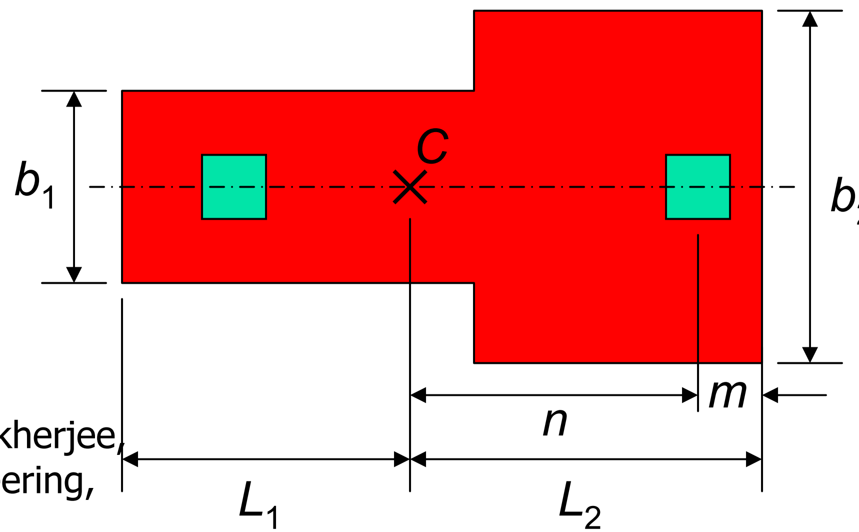


$$\frac{b_2}{b_1} = \frac{3(n+m)-L}{2L-3(n+m)}$$

$$(b_1+b_2) = \frac{2R}{q_e L}$$

$$c_1 = \frac{L(b_1+2b_2)}{3(b_1+b_2)}$$

$$c_2 = \frac{L(2b_1+b_2)}{3(b_1+b_2)}$$

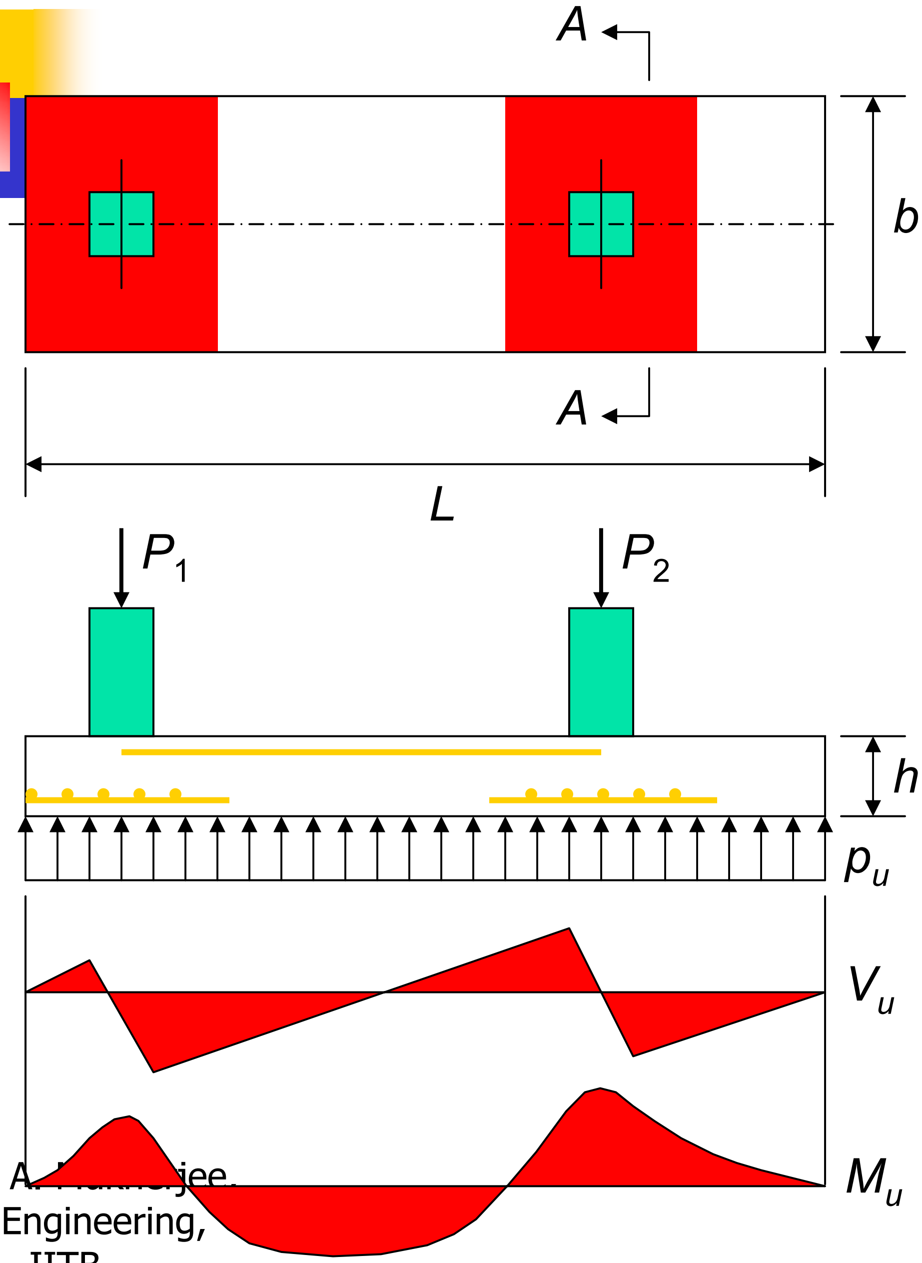


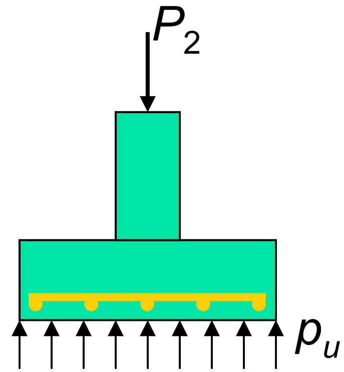
$$b_1 = \frac{2(n+m)-L_2}{L_1(L_1+L_2)}$$

$$b_2 = \frac{R}{q_e L_2} - \frac{L_1 b_1}{L_2}$$

$$L_1 b_1 + L_2 b_2 = \frac{R}{q_e}$$

Reinforcement in Combined Footings

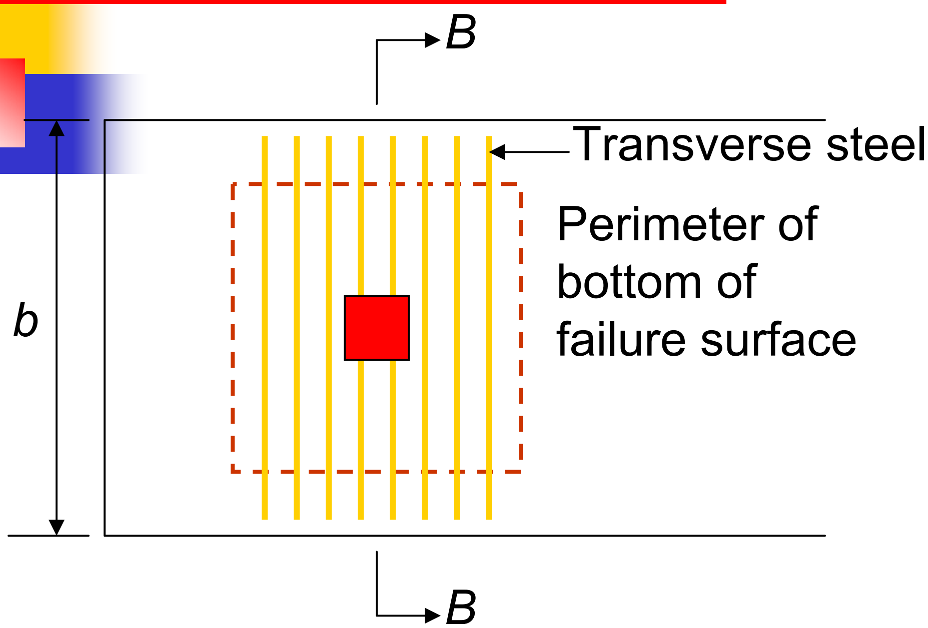


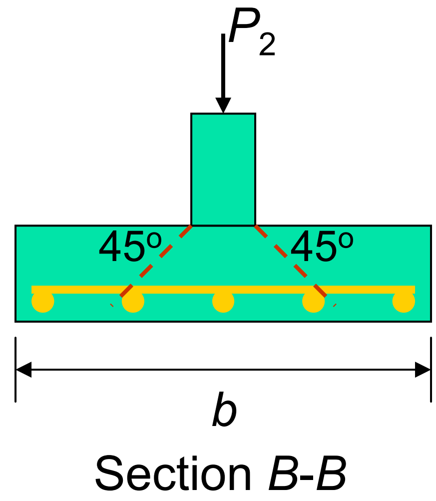


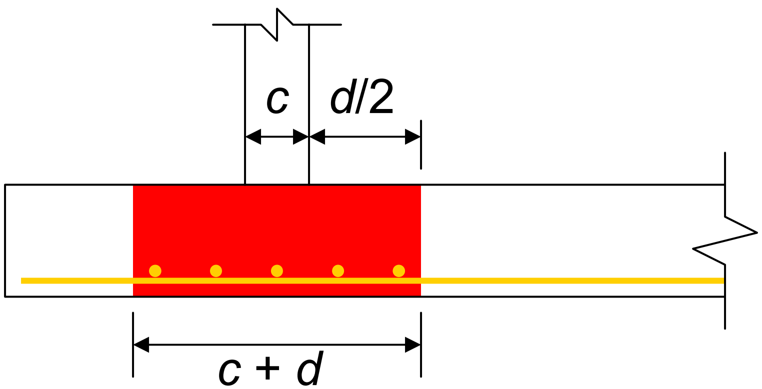
Section A-A

Transverse reinforcement

Transverse Reinforcement





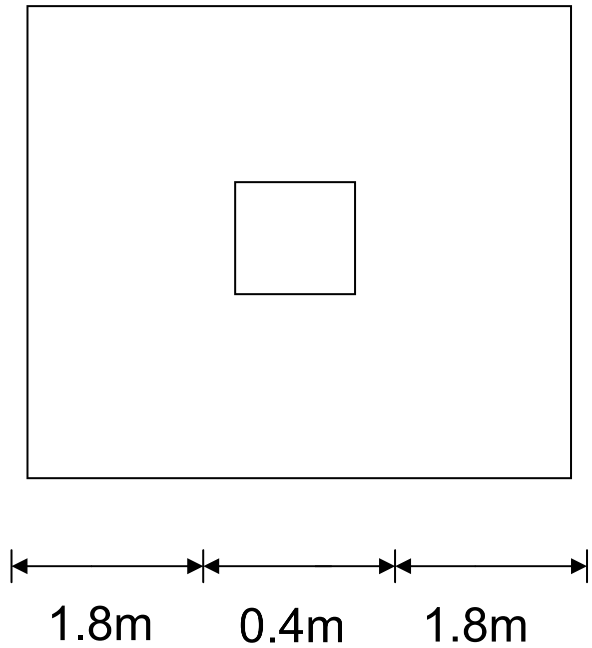


PROBLEM

Design and detail a square isolated R.C. footing of uniform thickness for 400mm square R.C. column carrying an axial load of 2100 KN including self weight of column.
Take safe bearing capacity of soil 150 KN/m2 at 1.5m below the existing ground level.
Use M20 grade concrete and Fe 415 steel for design.

1. CALCULATION OF FOOTING PLAN DIMENSION

Assuming self weight of footing @10% of column load,
Total load transferred to the soil = 1.1 X 2100 = 3310 KN.
Safe Bearing Capacity of soil = 150 KN/m2.
Area of footing required = (3310/ 150) = 15.4 m2.
Size of square footing required = √ 15.4 = 3.9243 m.
Let us provide a Square footing of size 4m X 4m .



Net upward soil pressure on the base of footing = 2100/16 =131.25 KN/m2 < 150 KN/m2 (Hence O.K.)

For 1m width of the footing the net upward loading intensity of the footing base (w) = 131.25 KN/m = 131.25 N/mm.

2. CALCULATION OF DEPTH FROM FLEXURAL CONSIDERATION.

Maximum moment at face of column = M = (131.25 X 1.82)/2
= 212.625 KN-m.

Ultimate moment = Mu = 1.5 X 212.625 = 318.9375 KN-m.

Required depth (dreq) = √ (318.9375 X 106)/(2.76 X 1000) = 339.94mm
~ 340mm.

3. CALCULATION OF DEPTH FROM TWO WAY SHEAR CONSIDERATION.

Clause 36.6.3.1 (pg-58) of IS 456:2000

Allowable Shear Stress = ta = Ks tc , where

Ks = (0.5 + βc) or 1 whichever is less.

βc = (short side of column)/(long side of column)

= 400/400 =1.

Ks = 0.5+1 = 1.5>1. Therefore Ks = 1.

Гc = 0.25√ fck = 0.25√20 = 1.118 Mpa.

Therefore ta = Ks tc = 1 X 1.118 = 1.118 Mpa.

Critical Perimeter = 4(400 + 2 X d/2) = 4(400 + d) mm.
Critical Area = 4(400 + d)d mm2.
Allowable load in Punching or two way Shear = 4(400 + d)dta
= 4.472(400 + d)d.

Load causing punching = [P-(400+d)2w]
= [40002-(400+d)2]w
= (4400+d)(3600-d)w
= (4400+d)(3600-d) X 0.13125

Equating Allowable load in punching with Load causing punching we get the value of “d” = 497.114 mm ~ 500mm.

Since depth required from Two way shear consideration > depth required from flexure consideration, hence Two way shear governs the design.
Overall depth required = Dreq. = 500 + 50 + 20 +10 = 580mm.
[Assuming 50mm clear cover and 20f tor bars for reinforcement in both top and bottom layer of reinforcement.]

To be on safer side so that footing does not fail in One way Shear check let us increase the depth of the footing by 10%.

Dreq. = 580 + 58 = 638mm.

Let us provide D = 650 mm.

Effective depth provided for top layer of reinforcement

= dt = 650-50-20-10=570mm.

Effective depth provided for bottom layer of reinforcement

= db = 650-50-10=590mm.

4. <u>CALCULATION OF REINFORCEMENT</u>

A. <u>For top layer</u>

Mu /(bdt2) = (318.9375 X 106) / (1000 X 5702) = 0.98.

From Table-2, page 48 of SP16:1980 we can get,

Percentage of steel required = Pt = 0.289.

Area of Steel required Ast = (0.289 X 1000 X 570)/ 100 = 1647.3 mm2.

Spacing required for 20f tor bars = (1000 X 314)/ 1647.3 = 190.6 mm c/c.

Therefore let us provide 20f tor bars @ 175mm c/c in top layer of reinforcement.

Provided Pt = (100 X 314) / (175 X 570) =0.315%.

B For bottom layer

Mu /(bdb2) = (318.9375 X 106) / (1000 X 5902) = 0.92.
From Table-2, page 48 of SP16:1980 we can get,
Percentage of steel required = Pt = 0.2704.
Area of Steel required Ast = (0.2704 X 1000 X 590)/ 100
= 1595.36 mm2.
Spacing required for 20f tor bars = (1000 X 314)/ 1595.36
= 196 mm c/c.
Therefore let us provide 20f tor bars @ 175mm c/c in bottom layer of reinforcement.
Provided Pt = (100 X 314) / (175 X 590) =0.304%.

5. CHECK FOR ONE WAY SHEAR

The critical section for one way shear check is 570mm (dt) away from the column face.

Pt provided as reinforcement in the top layer = 0.315%.
From table 19 of IS 456:2000, page-73, by interpolation we get,
Permissible Shear Stress = tperm = 0.3912 Mpa.

Shear at critical section = (1800 – 570) X 131.25 = 161437.5 N.
Therefore Shear stress developed = tdev = (161437.5) / (1000 X 570)
= 0.283 Mpa.
Therefore tdev < tperm. Hence O.K.

6. CHECK FOR BEARING

From Clause-34.4, page-65 of IS 456:2000 we get,

Allowable Bearing pressure = $0.45f_{ck}\sqrt{(A1/A2)}$ or $0.45f_{ck} \times 2$ whichever is lower.

A1 = (400+db)(400+dt)
= (400+590)(400+570)
= 960300mm2

A2 = 400 X 400 = 160000mm2.

$\sqrt{(A1/A2)} = 6.002 > 2$.

Therefore Allowable Bearing pressure = 0.45 X 20 X 2 = 18 N/mm2.

Actual Bearing Pressure developed at the junction of Column and Footing = (2100 X 1000) / (400 X 400) = 13.125 N/mm2 < 18 N/mm2.

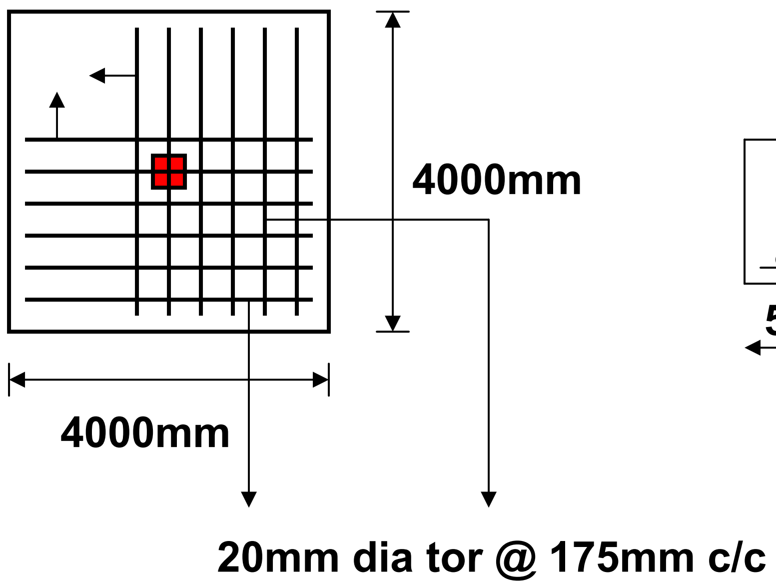
(Hence Safe).

FINAL DESIGN

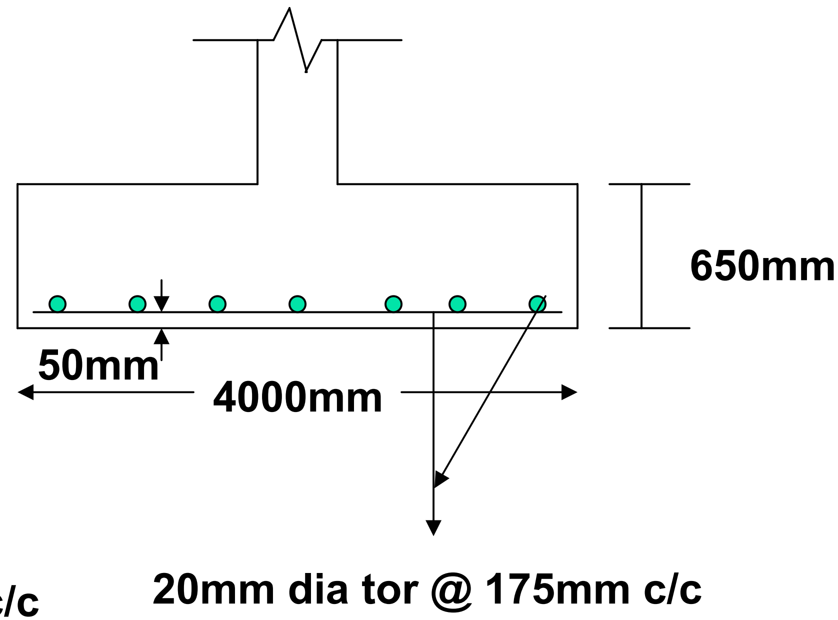
PLAN DIMENSION:	4m X 4m.
OVERALL DEPTH:	650mm.
TOP LAYER REINFORCEMENT:	20 F TOR STEEL BARS @ 175mm C/C.
BOTTOM LAYER REINFORCEMENT:	20 F TOR STEEL BARS @ 175mm C/C.

CLEAR COVER TO REINFORCEMENT: 50mm.

REINFORCEMENT DETAILING



PLAN



ELEVATION