Two-layer Systems

- The effect of layers above subgrade is to reduce the stress and deflections in the subgrade.
- Burmister (1958) obtained solutions for two-layer problem by using strain continuity equations.
- Vertical stress depends on the modular ratio (i.e., \( E_1/E_2 \)).
- Vertical stress decreases considerably with increase in modular ratio.
- For example,
  
  \[ \sigma_z \] at interface = 65% of contact pressure for \( a/h_1 = 1 \) and \( E_1/E_2 = 1 \).
  
  \[ \sigma_z \] at interface = 8% of contact pressure for \( a/h_1 = 1 \) and \( E_1/E_2 = 100 \).
Variation of Subgrade Stress with Modular Ratio
Vertical Stress in a Two-layer System

Vertical stress influence coefficient $= \sigma_z/p$

Values of parameter, $z/\alpha$

Layer 1

Layer 2

Interface 1-2

Boussinesq $E_1/E_2 = 1.0$

$E_1/E_2 = 100$

$E_1/E_2 = 50$

$E_1/E_2 = 20$

$E_1/E_2 = 10$

$E_1/E_2 = 5$

$E_1/E_2 = 2$

Base course or pavement layer

Subgrade layer

$h$
Vertical Surface Deflection in a Two-layer System

- Burmister (1958) developed a chart for computing vertical surface deflection in a two-layer system.
- The deflection factor, $F_2$, is obtained from the chart based on the values of $a/h_1$ and $E_1/E_2$.
- Then the deflection is computed from the following equations:
  - Deflection under a flexible Plate $= \Delta_T = \frac{1.5pa}{E_2} F_2$
  - Deflection under a rigid Plate $= \Delta_T = \frac{1.18pa}{E_2} F_2$
Vertical Surface Deflections for Two Layer Systems (Burmister, 1958)
Interface Deflection in a Two-layer System

- Huang (1969) developed charts for interface deflection in a two-layer system.
- These charts are prepared for varying $E_1/E_2$ values.
- The interface deflection factor, $F$, is obtained from the chart based on the values of $E_1/E_2$, $h_1/a$ and $r/a$ values.
- The interface deflection ($\Delta_S$) is then found from
  \[ \Delta_s = \frac{pa}{E_2} F \]
- The deflection that takes place within the pavement ($\Delta_p$) is given by
  \[ \Delta_p = \Delta_T - \Delta_S \]
Interface Deflection in Two-layer Systems

Source: Huang (1969)
Interface Deflection in Two-layer Systems

Source: Huang (1969)
Interface Deflection in Two-layer Systems

Source: Huang (1969)
Example Problems

• Calculate the surface deflection under the centre of a tyre \((a = 152 \text{ mm}, p = 552 \text{ kPa})\) for a 305 mm pavement having a 345 MPa modulus and subgrade modulus of 69 MPa from two-layer theory. Also calculate the interface deflection and the deflection that takes place within the pavement layer.

• A circular load with a radius of 152 mm and a uniform pressure of 552 kPa is applied on a two-layer system. The subgrade has an elastic modulus of 35 kPa and can support a maximum vertical stress of 55 kPa. What is the required thickness of full depth AC pavement, if AC has an elastic modulus of 3.45 GPa.

Instead of a full depth AC pavement, if a thin surface treatment is applied on a granular base (with elastic modulus of 173 MPa), what is the thickness of base course required?

• A plate bearing test using 750 mm diameter rigid plate was made on a subgrade as well as on 254 mm of gravel base course. The unit load required to cause settlement of 5 mm was 69 kPa and 276 kPa, respectively. Determine the required thickness of base course to sustain a 222.5 kN tyre, 690 kPa pressure and maintain a deflection of 5 mm.
Three-layer System

- Fox and Acum produced the first extensive tabular summary of normal and radial stresses in three-layer systems at the intersection of the plate axis with the layer interfaces.
- Jones (1962) and Peattie (1962) subsequently expanded these solutions to a much wider range of solution parameters.

Three-layer system showing location of stresses presented by Jones (1962) and Peattie (1962)
Notation

• \( \sigma_{z1} \) = Vertical stress at interface 1
• \( \sigma_{z2} \) = Vertical stress at interface 2
• \( \sigma_{r1} \) = Horizontal stress at the bottom of layer 1
• \( \sigma'_{r1} \) = Horizontal stress at the top of layer 2
• \( \sigma_{r2} \) = Horizontal stress at the bottom of layer 2
• \( \sigma'_{r2} \) = Horizontal stress at the top of layer 3
• These stress values are along the axis of symmetry of the load. Therefore, \( \sigma_{r1} = \sigma_{t1} \)

For \( \mu = 0.5 \), \( \varepsilon_z = \frac{1}{E_1} (\sigma_{z1} - \sigma_{r1}) \) and \( \varepsilon_r = \frac{1}{2E_1} (\sigma_{r1} - \sigma_{z1}) \)

Therefore, radial strain = one half the vertical strain
i.e., \( \varepsilon_r = -0.5 \varepsilon_z \)
Parameters in Jones Tables

- Stresses in a three layer system depend on the following ratios:

  \[ K_1 = \frac{E_1}{E_2}; \quad K_2 = \frac{E_2}{E_3} \]
  \[ A = \frac{a}{h_2}; \quad H = \frac{h_1}{h_2} \]

- Jones (1962) presented a series of tables for determining \( \sigma_{z1}, \sigma_{z1} - \sigma_{r1}, \sigma_{z2}, \sigma_{z2} - \sigma_{r2} \).

- His tables also include values of \( \sigma_{z1} - \sigma_{r1} \) and \( \sigma_{z2} - \sigma_{r2} \). But these can be readily obtained from those at bottom of layer 1 and 2.

- For continuity:

  \[ \frac{(\sigma_{z1} - \sigma_{r1})}{E_1} = \frac{(\sigma_{z1} - \sigma_{r1})}{E_2} \]
  \[ i.e., \quad \sigma_{z1} - \sigma_{r1} = (\sigma_{z1} - \sigma_{r1})/K_1 \]
  \[ \sigma_{z2} - \sigma_{r2} = (\sigma_{z2} - \sigma_{r2})/K_2 \]
Computing Stresses from Jones Tables

• Tables presented by Jones (1962) consist of four values of $K_1$ and $K_2$ i.e., 0.2, 2, 20 and 200.

• Therefore, interpolation of stress factors is necessary for many problem solutions. No extrapolation is allowed.

• Four sets of stress factors i.e., $ZZ1$, $ZZ2$, $ZZ1$-$RR1$ and $ZZ2$-$RR2$, are shown. The product of contact pressure and the stress factor gives the stress.

  - $\sigma_{z1} = p(ZZ1)$
  - $\sigma_{z2} = p(ZZ2)$
  - $\sigma_{z1} - \sigma_{r1} = p(ZZ1$-$RR1$)
  - $\sigma_{z2} - \sigma_{r2} = p(ZZ2$-$RR2$)
Jones (1962) Tables
Example Problem

• Given the three layer system shown in figure, determine all the stresses and strains at the two interfaces on the axis of symmetry.

\[ \mu_1 = 0.5, \ h_1=152 \ mm, \ E_1= 2.8 \ \text{GPa} \]

\[ \mu_2 = 0.5, \ h_2 = 152 \ mm, \ E_2 = 138 \ \text{MPa} \]

\[ \mu_3 = 0.5, \ h_3=\infty, \ E_3 = 69 \ \text{MPa} \]