Stresses in Rigid Pavements

Stresses in CC Pavement

- Temperature Stresses
  - Due to the temperature differential between the top and bottom of the slab, curling stresses (similar to bending stresses) are induced at the bottom or top of the slab
- Frictional stresses
  - Due to the contraction of slab due to shrinkage or due to drop in temperature tensile stresses are induced at the middle portion of the slab
- Wheel Load Stresses
  - CC slab is subjected to flexural stresses due to the wheel loads

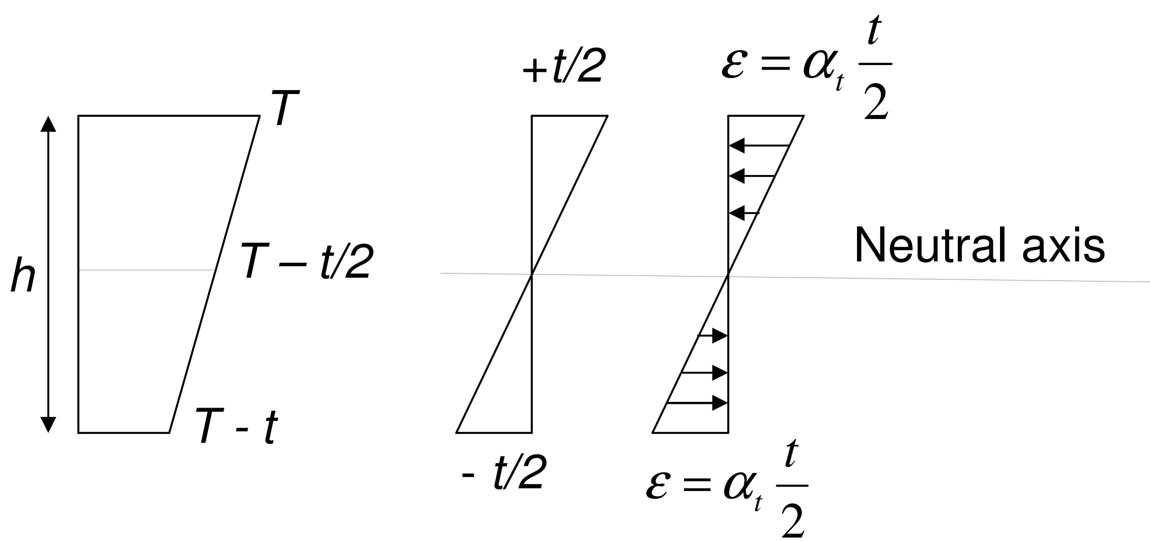
Temperature Stresses

- Temperature differential between the top and bottom of the slab causes curling (warping) stress in the pavement
- If the temperature of the upper surface of the slab is higher than the bottom surface then top surface tends to expand and the bottom surface tends to contract resulting in compressive stress at the top, tensile stress at bottom and vice versa

Notation

- E = modulus of elasticity
- C_x and C_y = Bradbury's coefficients
- T = temperature
- a = radius of contact
- h = thickness of cement concrete slab
- k = modulus of subgrade reaction
- l = raduis of relative stiffness
- t = temperature differential
- α_t = coefficient of thermal expansion
- ε = strain
- μ = Poisson's ratio
- σ = stress

Temperature Differential



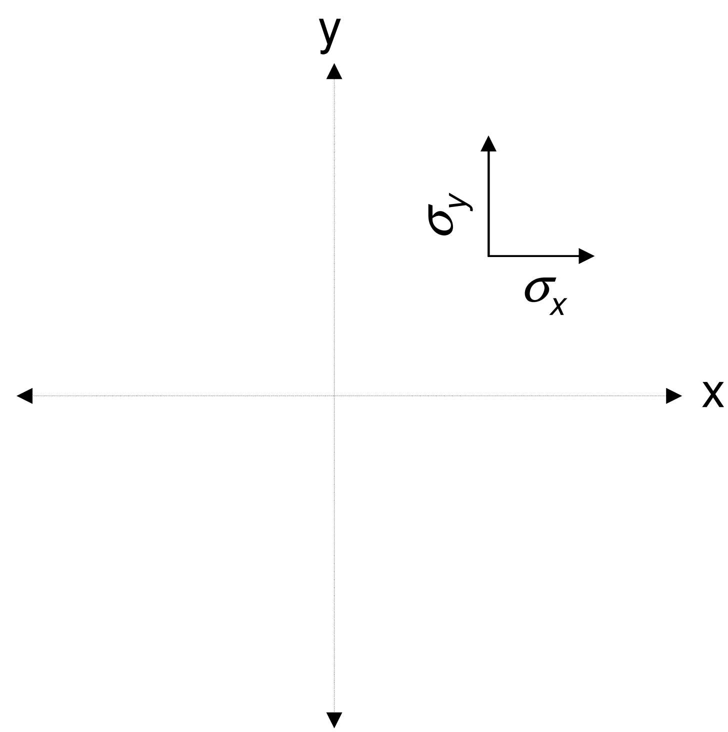
- Temperature at top = T
- Temperature differential = t
- Temperature at bottom = $T - t$
- Average Temperature (at mid height) = $(T + T - t)/2 = T - t/2$
- Increase in temperature of top fibre above average temperature = $t/2$
- Decrease in temperature of bottom fibre below average temperature = $t/2$

Curling Stresses in Infinite Slab

The equations of strain in an infinite slab, that bends in both x and y directions, are

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} \qquad \text{..................(A)}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} \qquad \text{..................(B)}$$



Assuming that the slab bends only in x direction,

$\varepsilon_y = 0$; i.e., from (B) $\quad \sigma_y = \mu\ \sigma_x$

Substituting in (A)

$$\sigma_x = \frac{E\varepsilon_x}{1-\mu^2}\,; \qquad as\ \varepsilon_x = \varepsilon_y = \alpha_t\, t$$

$$\sigma_x = \frac{E\alpha_t t}{2\left(1-\mu^2\right)} \qquad \text{..................(C)}$$

Curling Stresses in infinite Slab

Assuming that the slab bends only in y direction,

$\varepsilon_x = 0$; i.e., from (A) $\quad \sigma_x = \mu\, \sigma_y$

Substituting in (B)

$$\sigma_x = \frac{\mu E \varepsilon_y}{1-\mu^2}; \qquad as\ \varepsilon_x = \varepsilon_y = \alpha_t\, t$$

$$\sigma_x = \frac{\mu E \alpha_t t}{2\left(1-\mu^2\right)} \qquad \text{..................(D)}$$

When bending occurs in both x and y directions, as is the case for temperature curling, the stress due to bending in both the directions must be superimposed.

$\therefore\ \sigma_x = \sigma_x$ due to bending in x direction + σ_x due to bending in y direction

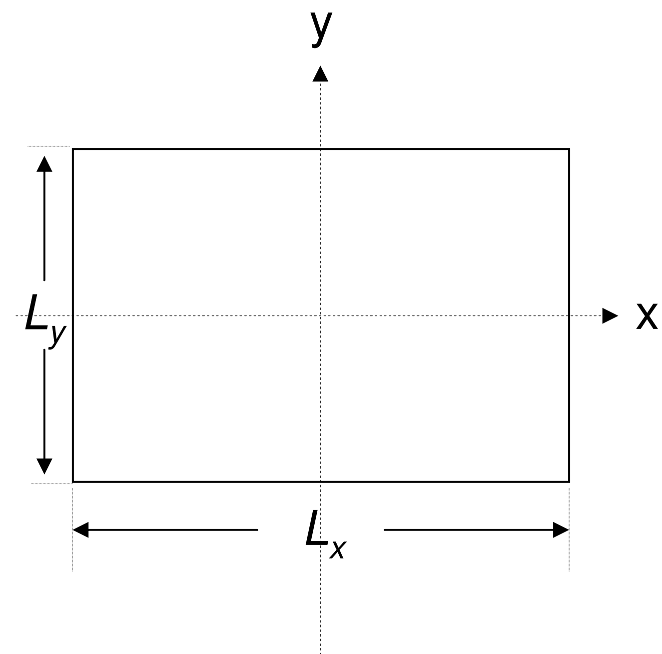
$$\text{i.e., } \sigma_x = \frac{E \alpha_t t}{2\left(1-\mu^2\right)} + \mu \frac{E \alpha_t t}{2\left(1-\mu^2\right)}$$

Curling Stresses in Finite Slab

Total temperature stress in x direction

$$\sigma_x = C_x \frac{E\alpha_t t}{2(1-\mu^2)} + C_y \mu \frac{E\alpha_t t}{2(1-\mu^2)}$$

$$\text{i.e., } \sigma_x = \frac{E\alpha_t t}{2(1-\mu^2)}\left(C_x + \mu C_y\right) \quad \text{..........(E)}$$



C_x and C_y are the correction factors for a finite slab (unconstrained edges)

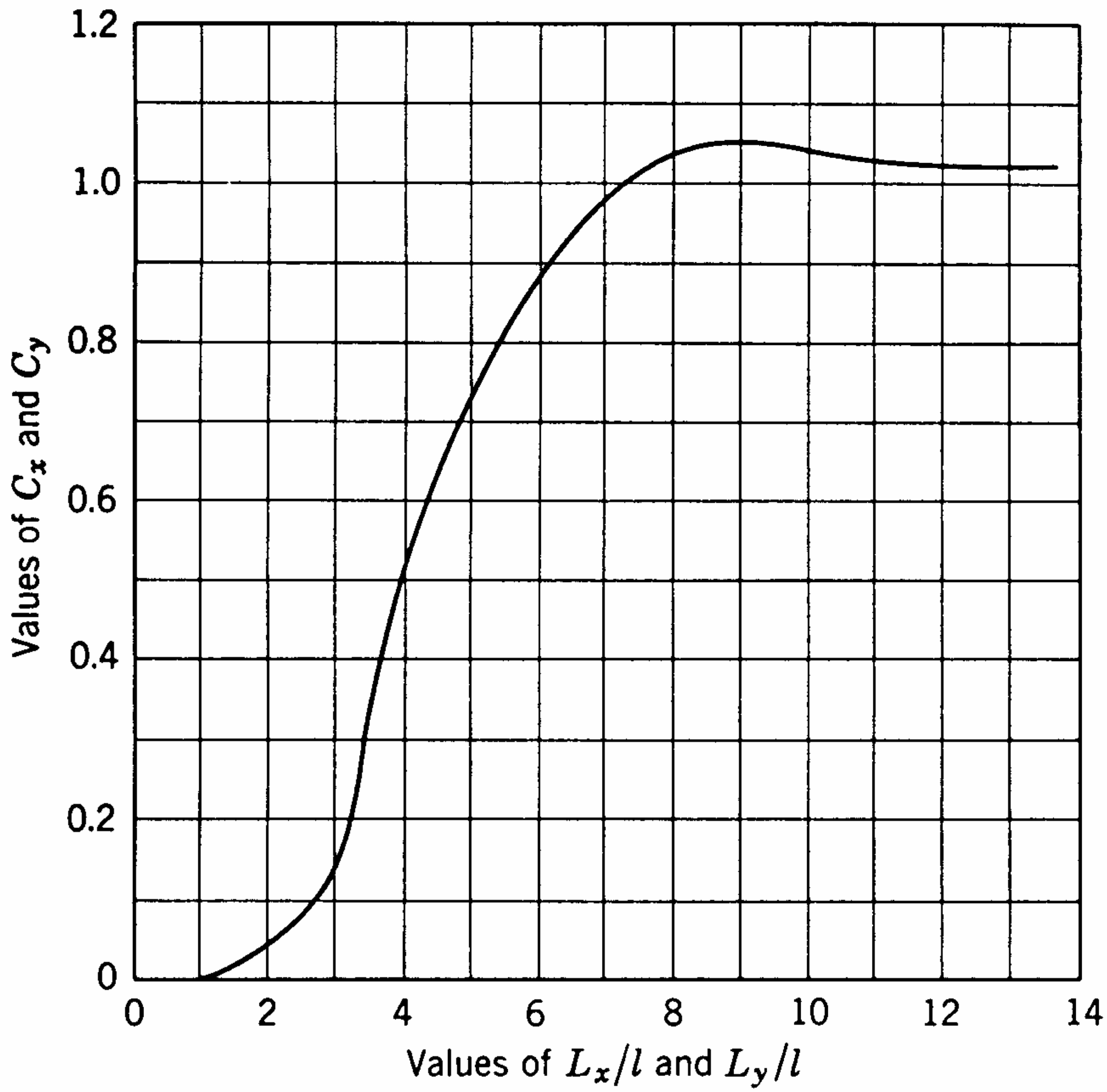
Equation (E) gives the stress at interior due to temperature differential.

The stress at edge is obtained by substituting $\mu = 0$ in Eq. (E)

$$\text{i.e., } \sigma = \frac{C E\alpha_t t}{2}$$

$$\text{corner stress} = \frac{E\alpha_t t}{\sqrt[3]{1-\mu^2}}\sqrt{\frac{a}{l}}$$

Bradbury’s Warping Stress Coefficients



Bradbury's Warping Stress Coefficients (IRC-58, 2002)

L/l	***C***	***L/l***	***C***
1	0.000	7	1.030
2	0.040	8	1.077
3	0.175	9	1.080
4	0.440	10	1.075
5	0.720	11	1.050
6	0.920	12	1.000

Radius of Relative Stiffness

$$l = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$$

Where,

E = Modulus of Elasticity of concrete, MPa

h = thickness of slab, m

μ = Poisson's ratio

k = modulus of subgrade reaction, MN/m^3

Example Problem

- A concrete slab 7.62 m long, 3.66 m wide and 203 mm thick, is subjected to a temperature differential of 11.1 ^oC. Assuming that k = 54.2 MN/m3 and $\alpha_t = 9\times10^{-6}$ /^oC. Determine the maximum curling stress in the interior, edge and corner of the slab. Take the radius of contact as a = 152 mm.

Temperature Differentials

- Maximum temperature differentials occur during the day in the spring and summer months.
- During midday of summer, the surface of the slab, which is exposed to the sun, warms faster than the subgrade which is relatively cool.
- During night time the surface of the slab becomes cool when compared to the subgrade.
- Usually, night time temperature differentials are one half the day time temperature differentials.
- The actual temperature differentials depend on the location..
- Temperature differential is expressed as temperature gradient per mm of slab thickness.
- The temperature gradients vary between 0.067 to 0.1 $^{o}C/mm$.

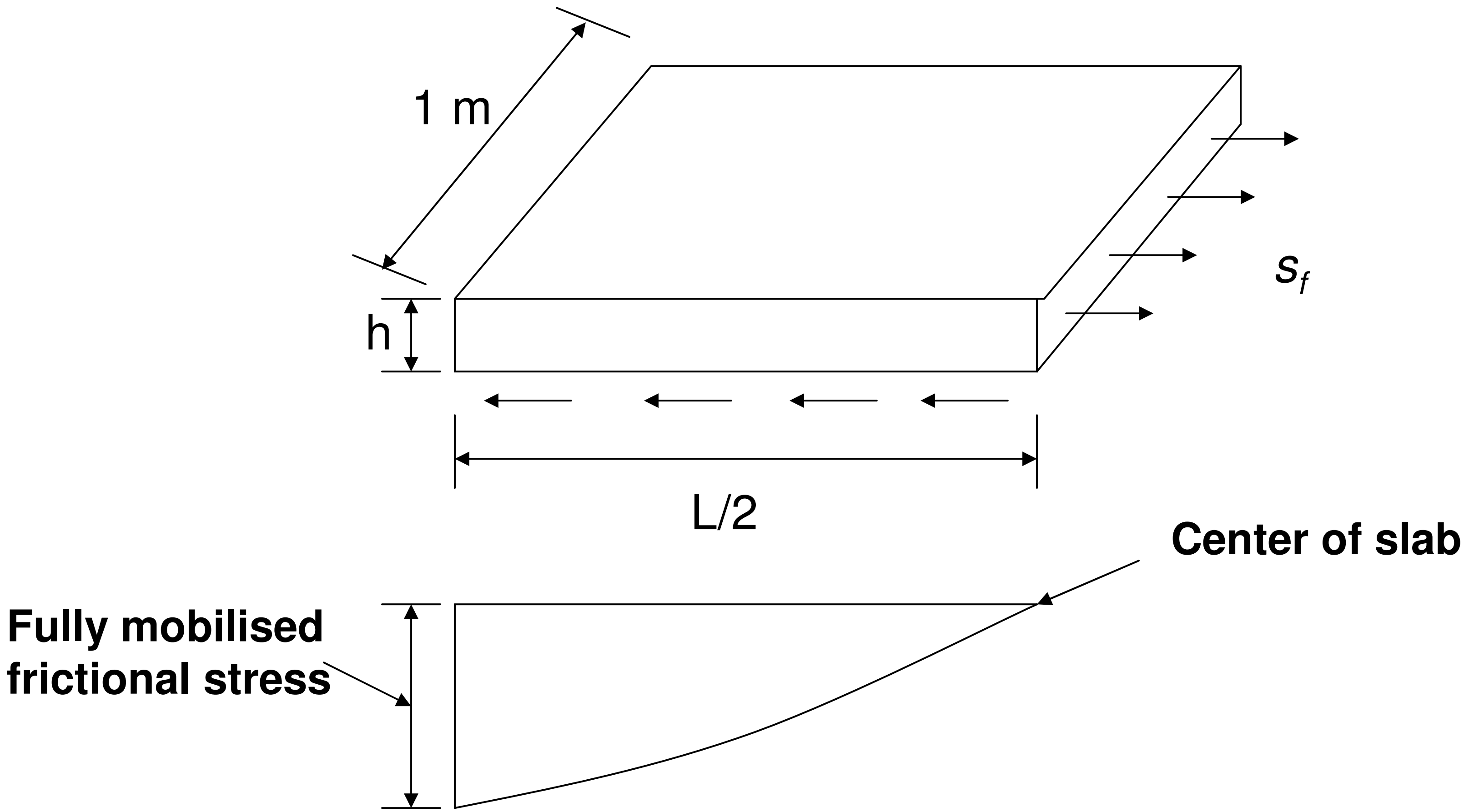
Temperature Differentials Recommended by IRC

Zone	States	Temperature Differential, ^{o}C in slab of thickness			
		15 cm	20 cm	25 cm	30 cm
I	Punjab, U.P., Uttaranchal, Gujarat, Rajasthan, Haryana and North M.P. Excluding hilly regions.	12.5	13.1	14.3	15.8
II	Bihar, Jharkhand, West Bengal, Assam and Eastern Orissa excluding hilly regions and coastal areas	15.6	16.4	16.6	16.8
III	Maharashtra, Karnataka, South M.P., Chattisgarh, Andhra Pradesh, Western Orissa and North Tamil Nadu, excluding hilly regions and coastal areas	17.3	19.0	20.3	21.0
IV	Kerala and South Tamilnadu excluding hilly regions and coastal areas	15.0	16.4	17.6	18.1
V	Coastal areas bounded by hills	14.6	15.8	16.2	17.0
VI	Coastal areas unbounded by hills	15.5	17.0	19.0	19.2

Frictional Stresses

- The friction between a concrete slab and its foundation causes tensile stress
  - in the concrete,
  - In the steel reinforcements and
  - In tie bars
- For plain concrete pavements, the spacing between contraction joints is so chosen that the stresses due to friction will not cause the concrete to crack.
- Longer joint spacing than that above requires the provision of temperature steel to take care of the stresses caused by friction.
- The number of tie bars are also determined by frictional stresses.

Stresses Due to Friction



Variation of frictional stress

Stresses Due to Friction

- Frictional force per unit width of slab

  $\gamma_c \times h \times 1 \times (L/2) \times f_a = (\gamma\, h\, L\, f_a)/2$

  Where,

  γ_c = unit weight of concrete, kN/m^3

  h = thickness of slab, m

  L = length of slab, m

- *Tensile force in the slab at the middle*
  - $S_f = s_f \times h \times 1 = s_f\, h$
  - Where, S_f = tensile force, kN; s_f = tensile stress, kN/m^2

- Equating the two
  - $s_f = (\gamma_c\, L\, f_a)/2$

Spacing of Contraction Joints

- The contraction joints are spaced to limit the tensile stress induced in the slab to the value that can be born by the slab during curing period
- Spacing is found out by taking the allowable tensile stress as 80 kPa during curing period of concrete
- $L = (2s_f)/(\gamma_c\ f_a)$
- *For* $s_f = 80$ *kPa,* $\gamma = 23.6$ kN/m3 and $f_a = 1.5$

  L = 4.52 m

  Therefore, the spacing of contraction joints is kept as 4.5 m.

Contraction Joint Spacing Specified by IRC

Slab Thickness (cm)	Maximum Contraction joint spacing (m)
15	4.5
20	4.5
25	4.5
30	5.0
35	5.0

A Fully Developed Crack at a Contraction Joint

Spacing of Contraction Joints Based on Allowable Joint Opening

- The spacing of joints in plain concrete pavements depends more on shrinkage characteristics of concrete rather than stress in the concrete
- The spacing of joints can be computed by limiting the joint opening to maintain the load transfer
- $\Delta L = CL(\alpha_t \Delta T + \varepsilon)$
- Where, L = joint spacing, ΔL = joint opening (1.3 mm and 6.4 mm for un-dowelled and dowelled joints respectively), α_t *= coefficient of thermal expansion of concrete,* ε = drying shrinkage coefficient (0.5 to 2.5×10^{-4}), ΔT = temperature at placement minus the lowest mean monthly temperature, C is the adjustment factor due to slab-subbase friction, 0.65 for stabilized base and 0.8 for granular base

Example Problem

- Find the allowable joint spacing of dowelled and undowelled contraction joints for the following data based on joint opening criteria:

  Allowable joint opening for dowelled joints: 6.4 mm

  Allowable joint opening for undowelled joints: 1.3 mm

  $\Delta T = 33\ ^{o}C$; $\alpha_t = 9.9 \times 10^{-6}\ /^{o}C$; $\varepsilon = 1.0 \times 10^{-4}$; $C = 0.65$

Temperature Steel

- Temperature steel is provided in the form of wire fabric or bar mats at mid depth and is discontinued at joints.
- This temperature steel does not increase the structural capacity of the slab
- Temperature steel is used to increase the spacing of contraction joints
- Temperature steel ties the cracked concrete together and maintains load transfer through aggregate interlock

Steel Stresses

Equating the force in steel to frictional force

$A_s\ f_s = (\gamma_c\ h\ L\ f_a)/2$

Where,

A_s = area of temperature steel per unit width of slab

f_s = *stress in steel*

The area of steel required per unit width can be computed as

$A_s = (\gamma_c\ h\ L\ f_a)/(2\ f_s)$

Example Problem on Temperature Steel

- Determine the wire fabric required for a two lane concrete pavement 203 mm thick, 18.3 m long and 7.3 m wide with a longitudinal joint at the centre.

  f_s = 297 MPa; γ_c = 25 kN/m^3; f_a = 1.5

Tie Bars

- Tie bars are placed across the longitudinal joint to tie the two slabs together so that the joint will be tightly closed and the load transfer across the joint can be ensured
- The amount of tie bar steel is worked out as

$$A_s = (\gamma_c\ h\ L'\ f_a)/(f_s)$$

Where, L' = distance from the longitudinal joint to the free edge where no tie bars exist

Length of Tie Bar

- The length of tie bars is governed by the allowable bond stress
- For deformed bars, an allowable bond stress of 2.4 MPa may be assumed
- $l = (f_s d)/(2\tau)$
- Where, f_s = allowable tensile stress in tie bar steel, d = diameter of the tie bar, τ = allowable bond stress
- The length l should be increased by 76 mm for misalignment

Example Problem on Tie bar Design

- Determine the diameter, spacing, and length of tie bars required for a two lane concrete pavement, 203 mm thick, 18.3 m long and 7.3 m wide with a longitudinal joint at the centre.

  f_s = 200 MPa; τ = 2.4 MPa γ_c = 25 kN/m^3; f_a = 1.5

Details of Tie Bars for Two Lane CC Pavement (IRC: 58-2002)

Slab Thickness (cm)	Diameter (mm)	Maximum Spacing (cm)		Minimum Length (cm)	
		Plain Bars	Deformed Bars	Plain Bars	Deformed Bars
25	12	45	72	58	64
	16	80	128	72	80
30	12	37	60	58	64
	16	66	106	72	80
35	12	32	51	58	64
	16	57	91	72	80

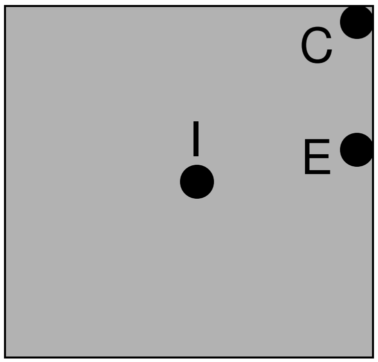
f_s = 1200 kg/cm^2 for plain bars, 2000 kg/cm^2 for deformed bars; τ=17.5 kg/cm^2 for plain bars, 24.6 kg/cm^2 for deformed bars.

Methods of Analyses for Wheel Load Stresses in a Rigid Pavement

- The following three methods were used for computing the wheel load stresses in a rigid pavement.
  - Closed form formulae – Westergaard
  - Influence Charts – Pickett and Ray (1951)
  - Finite Element Computer Programmes
- The first two methods assume the slab as an elastic plate resting on liquid foundation
- The liquid foundation assumes the subgrade to be a set of springs.
- Deflections at any given point is proportional to the force at that point and independent of the forces at all other points.

Westergaard Equations

- Westergaard considered three cases of loading:
  - Corner loading
  - Edge loading
  - Interior loading



Corner Loading

The maximum stress (bending tension) σ_c and deflection δ_c due to the load at corner of a rigid slab is given by Westergaard as

$$\sigma_c = \frac{3P}{h^2}\left[1-\left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

$$\delta_c = \frac{P}{k\ell^2}\left[1.1-0.88\left(\frac{a\sqrt{2}}{\ell}\right)\right]$$

Where:
k = modulus of subgrade reaction
ℓ = radius of relative stiffness
a = load contact radius
P = load

The above equation for σ_c as modified by Kelly is being used by IRC for computing the wheel load stress at corner

$$\sigma_c = \frac{3P}{h^2}\left[1-\left(\frac{a\sqrt{2}}{l}\right)^{1.2}\right]$$

Interior Loading

The maximum stress (bending tension) σ_i and deflection δ_i due to the load at interior of a rigid slab is given by Westergaard as

$$\sigma_i = \frac{0.316P}{h^2}\left[4\log\left(\frac{\ell}{b}\right)+1.069\right]$$

$$b = a \quad \text{when } a \geq 1.724h$$

$$b = \sqrt{1.6a^2+h^2} - 0.675h \quad \text{when } a < 1.724h$$

$$\delta_i = \frac{P}{8k\ell^2}\left\{1+\frac{1}{2\pi}\left[\ln\left(\frac{a}{2\ell}\right)-0.673\right]\left(\frac{a}{\ell}\right)^2\right\}$$

Edge Loading

The maximum stress (bending tension) σ_e and deflection δ_e due to the load at edge of a rigid slab is given by Westergaard as

$$\sigma_e = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.666\left(\frac{a}{\ell}\right) - 0.034\right]$$

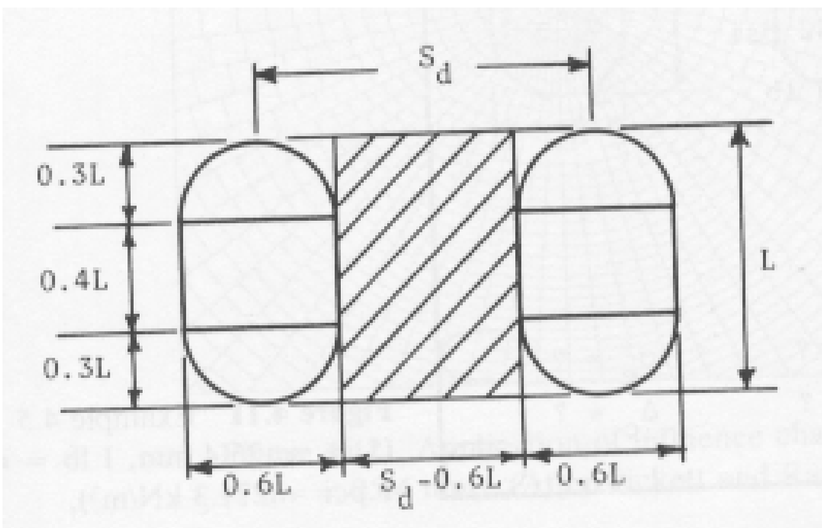
$$\delta_e = \frac{0.431P}{k\ell^2}\left[1 - 0.82\left(\frac{a}{\ell}\right)\right]$$

The above equation for σ_e as modified by Teller and Sutherland was used by IRC in its old specification for computing the wheel load stress at edge

$$\sigma_e = 0.529\,\frac{P}{h^2}(1+0.54\,\mu)\,\left(4\log_{10}\frac{l}{b} + \log_{10} b - 0.4048\right)$$

Dual Tyres

For computing the stresses due to a dual wheel using Westergaard equations, the dual wheel is converted into an equivalent single wheel load by computing the equivalent circular contact area as follows:



$$L \approx \sqrt{\frac{P_d}{0.5227q}}$$

Then, area of the equivalent circle:

$$\pi a^2 = 2 \times 0.5227L^2 + (S_d - 0.6L)L$$

$$a = \sqrt{\frac{0.8521 \times P_d}{q\pi} + \frac{S_d}{\pi}\left(\frac{P_d}{0.5227q}\right)^{1/2}}$$

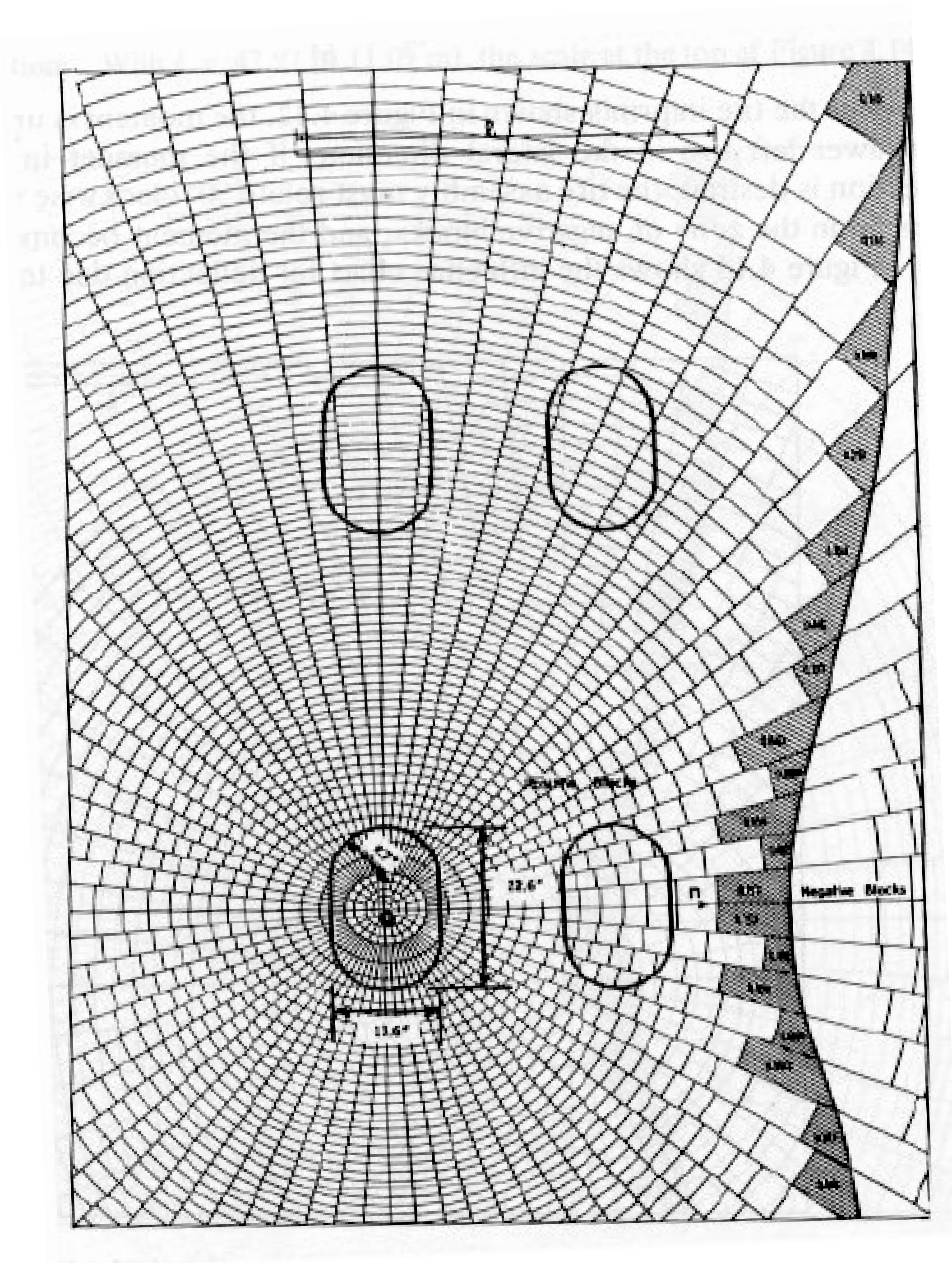
Example Problem on Dual Tyres

- Using Westergaard equations determine the maximum stress at interior, edge and corner if a 44.5 kN load is applied on a set of dual tyres spaced at 356 mm on centres. Use the following data:

  q=610 kPa; k = 27.2 MN/m^3; h = 254 mm; E = 27.6 GPa; μ = 0.15.

Limitations of Closed Form Solutions

- Applicable to only single wheel load with circular contact area
- Load locations are fixed (i.e., interior, edge and corner).
- Applicable to only large slabs
- Assumes full subgrade support
- No consideration is given to load transfer across joints



Influence Charts by Pickett and Ray (1951)

- These influence charts were developed using Westergaard theory for liquid foundation by taking poisson's ratio of 0.15 for concrete slab.
- These were previously used by PCA for rigid pavement design
- Influence charts were prepared for finding stresses and deflections at interior and edge locations.
- These were extensively used for the design of airfield rigid pavements

Computations using Influence Charts

These charts can be used for any shape of contact area and any wheel/axle/gear configuration

The bending moment is obtained about point O in the direction n.

The contact areas are to be drawn to the scale shown and the number of blocks (N) with in the contact areas are counted from the respective charts.

Bending moment, M is obtained as

$$M = \frac{pl^2 N}{10000}$$ and the stress can be computed as $$\text{Stress} = \frac{6M}{h^2}$$

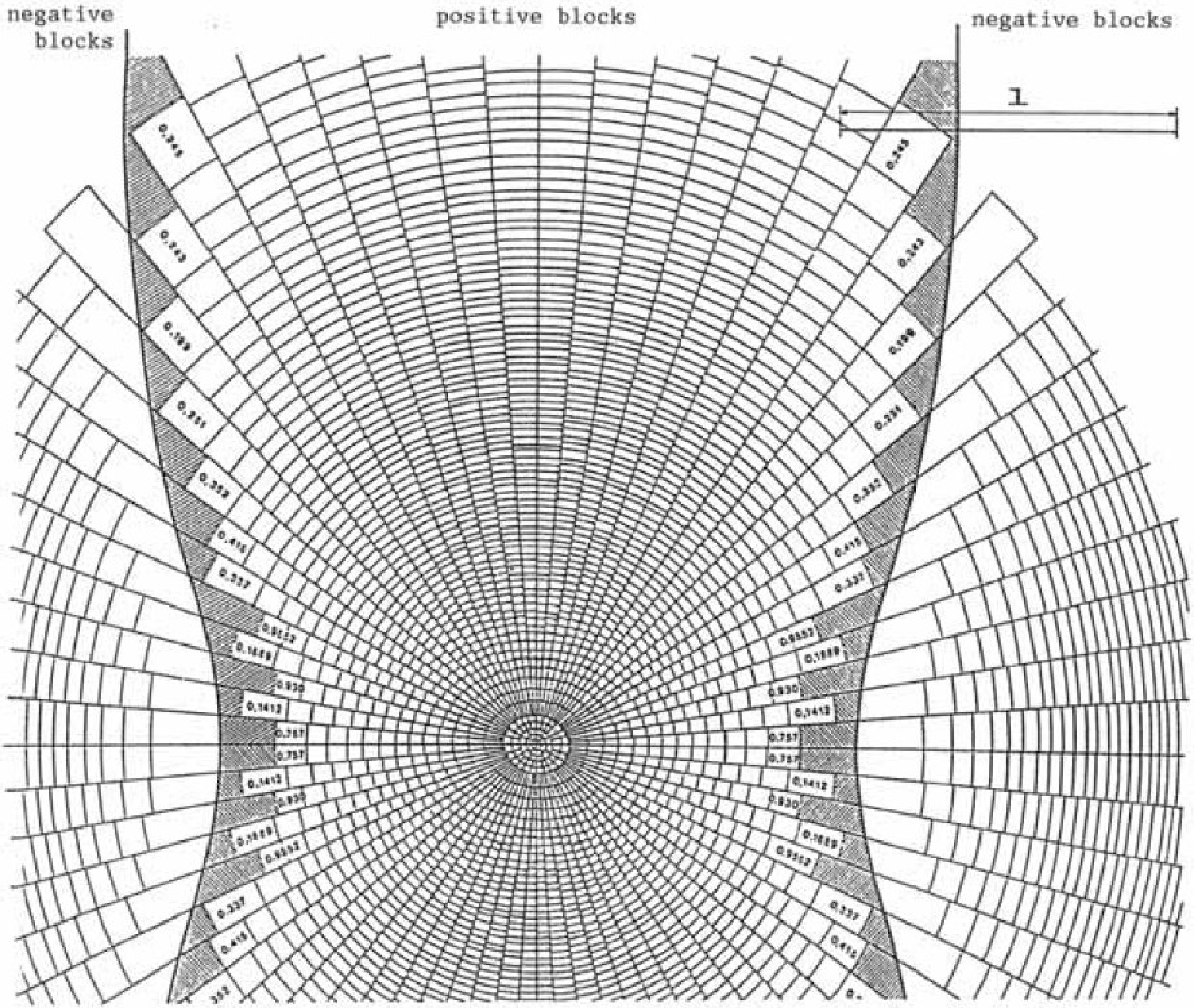
Deflection is computed as

$$\Delta = \frac{0.0005\, pl^4 N}{D}$$ Where, D is modulus of rigidity and is given as

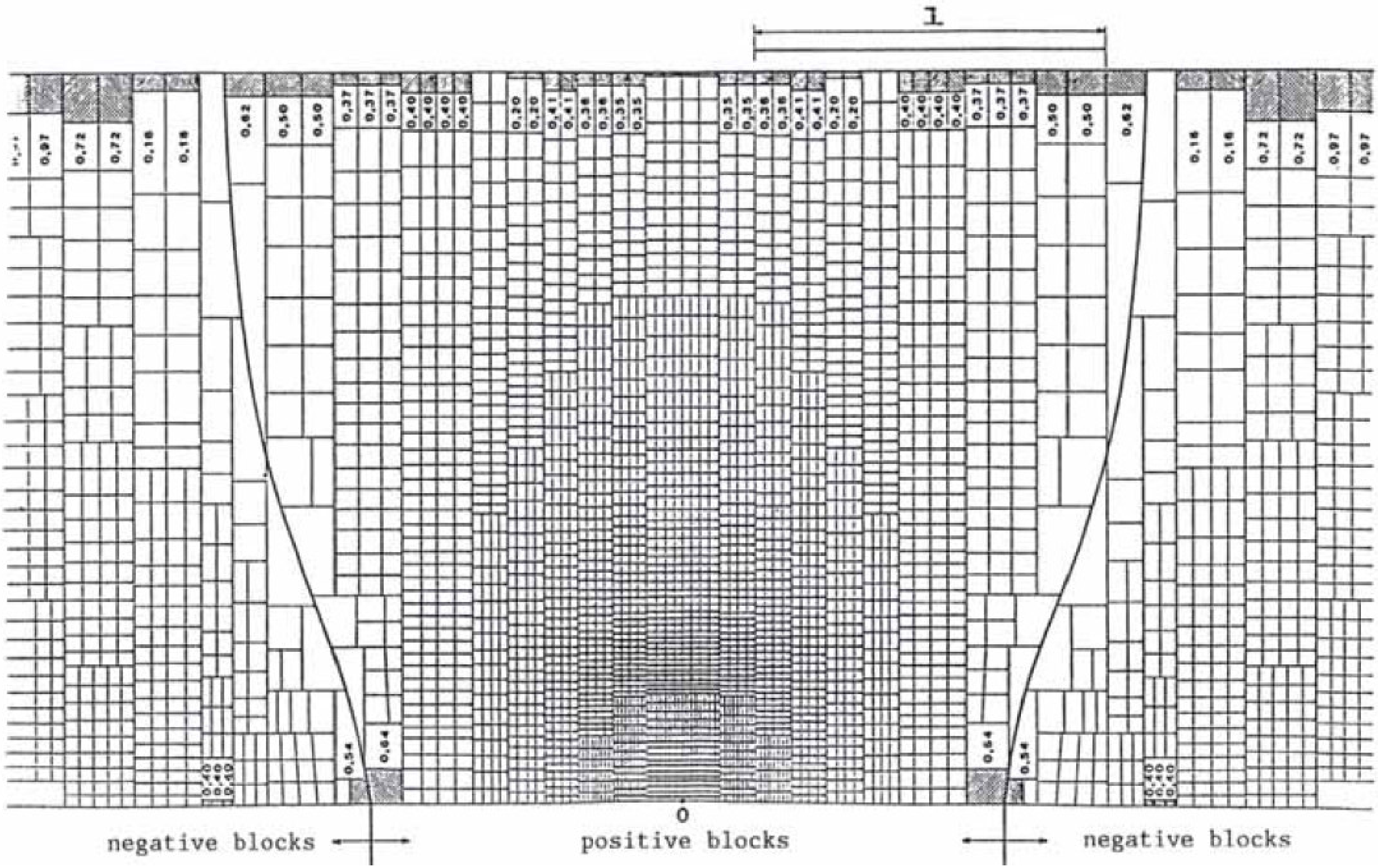
$$D = \frac{Eh^3}{12(1-\mu^2)}$$

Influence chart for moment due to the load at interior

Pickett and Ray (1951)

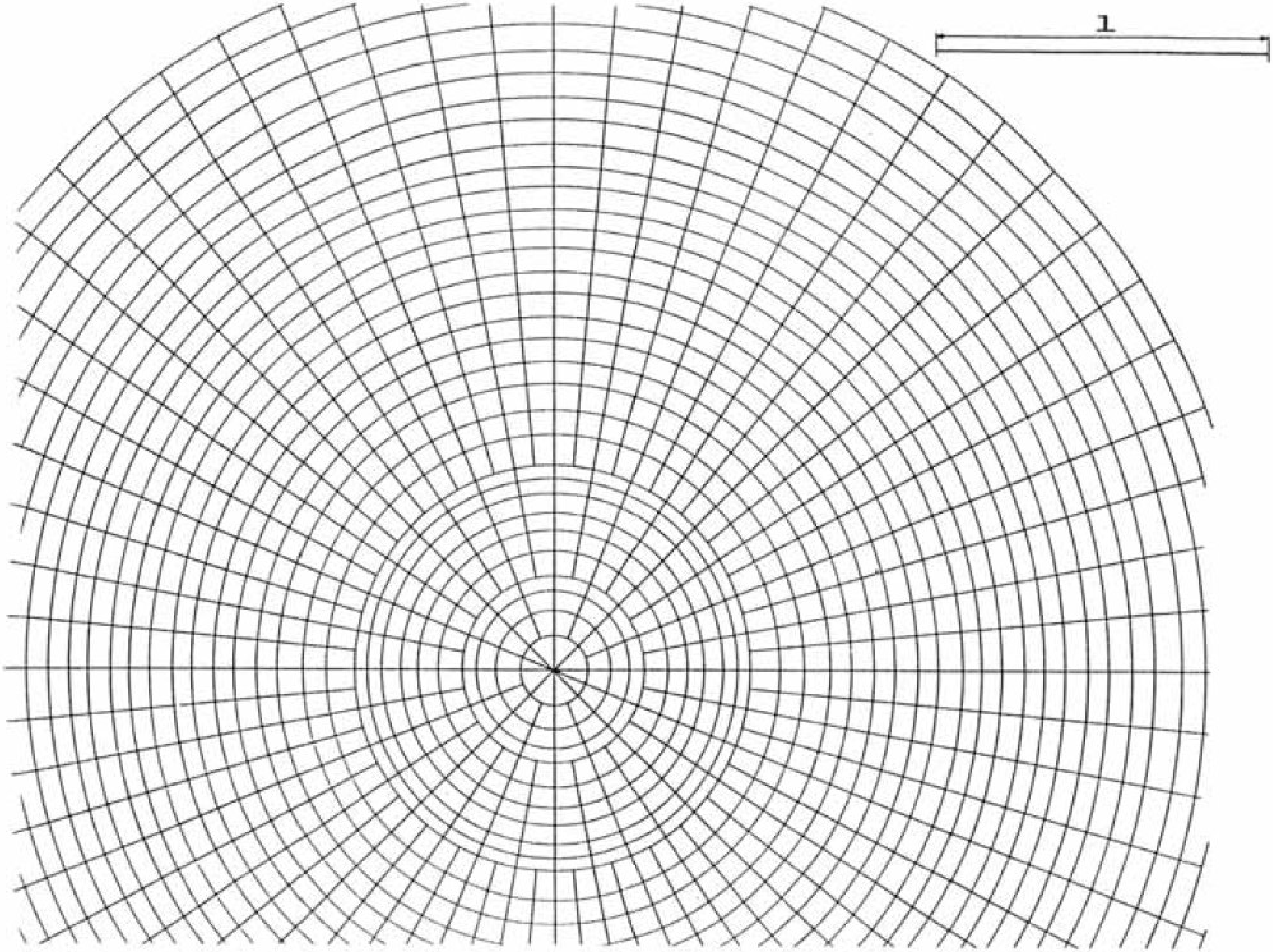

negative blocks
positive blocks
negative blocks
1

Influence Chart for Moment due to Load at Edge
Pickett and Ray (1951)



Influence chart for deflection due to the load at interior

Pickett and Ray (1951)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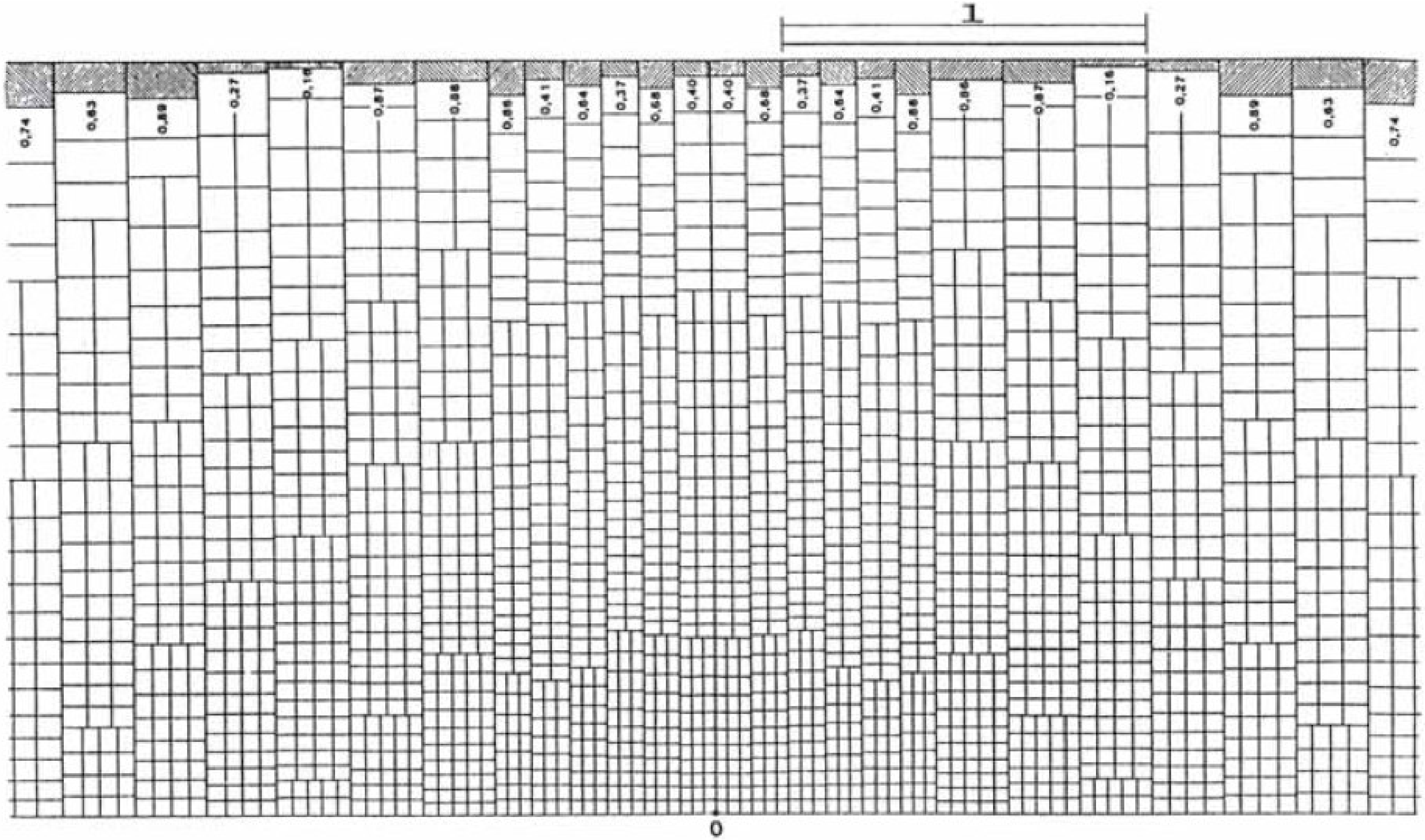
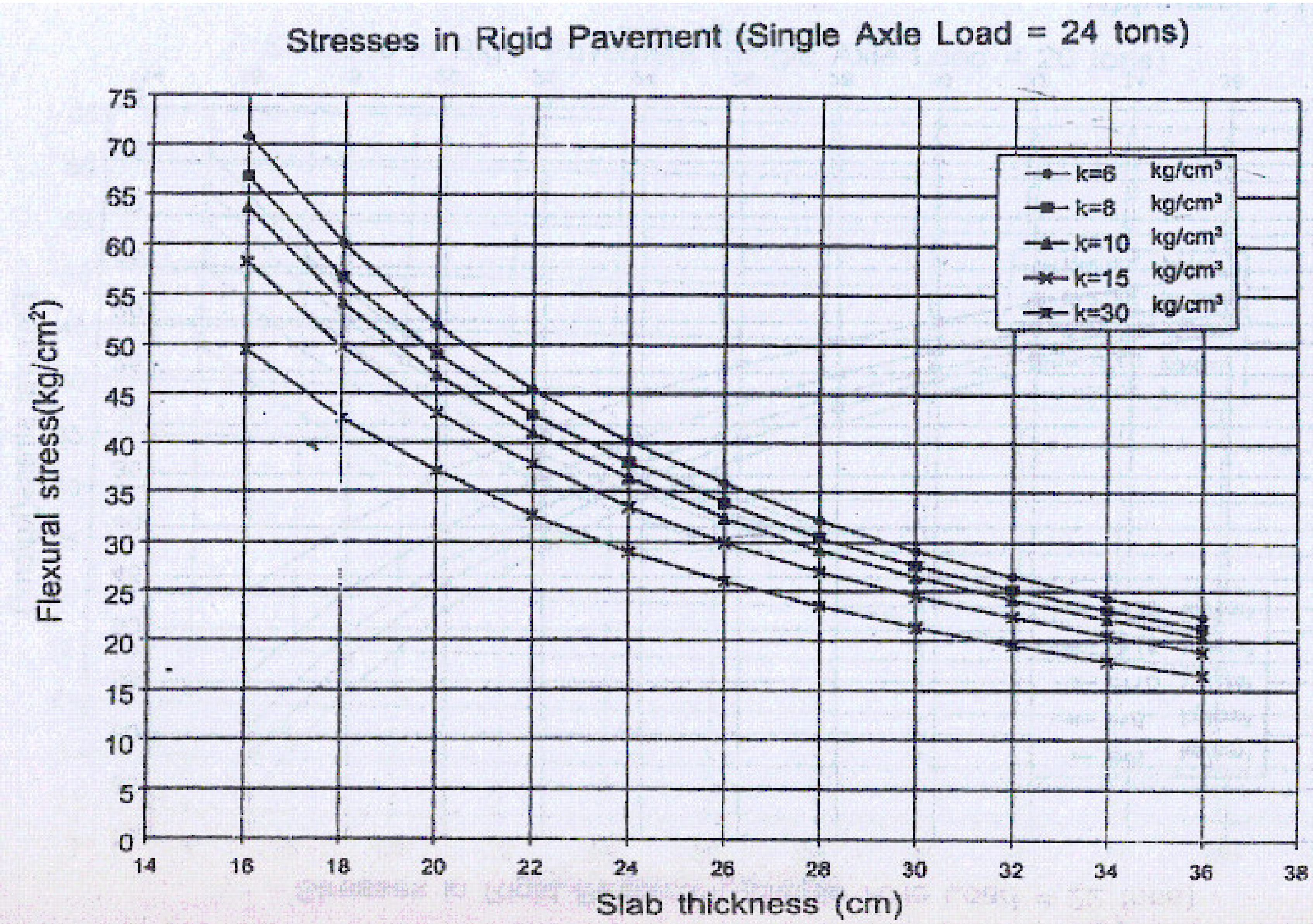
Influence Chart for Deflection due to Load at Edge
Pickett and Ray (1951)



IRC Recommendations on Wheel Load Stresses

- The loads causing failure of pavements are mostly applied by single and tandem axles, stress must be determined for the condition shown in chart's given by Picket &Ray for stress computation in the interior as well as edge region

- Using fundamental concept of Westergaard and Picket &Ray's pioneering work a computer program IITRIGID developed at IIT, Kharagpur was used for edge load condition



Stresses in Rigid Pavement (Single Axle Load = 24 tons)
Flexural stress(kg/cm²)
Slab thickness (cm)
k=6 kg/cm³
k=8 kg/cm³
k=10 kg/cm³
k=15 kg/cm³
k=30 kg/cm³
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
14
16
18
20
22
24
26
28
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32
34
36
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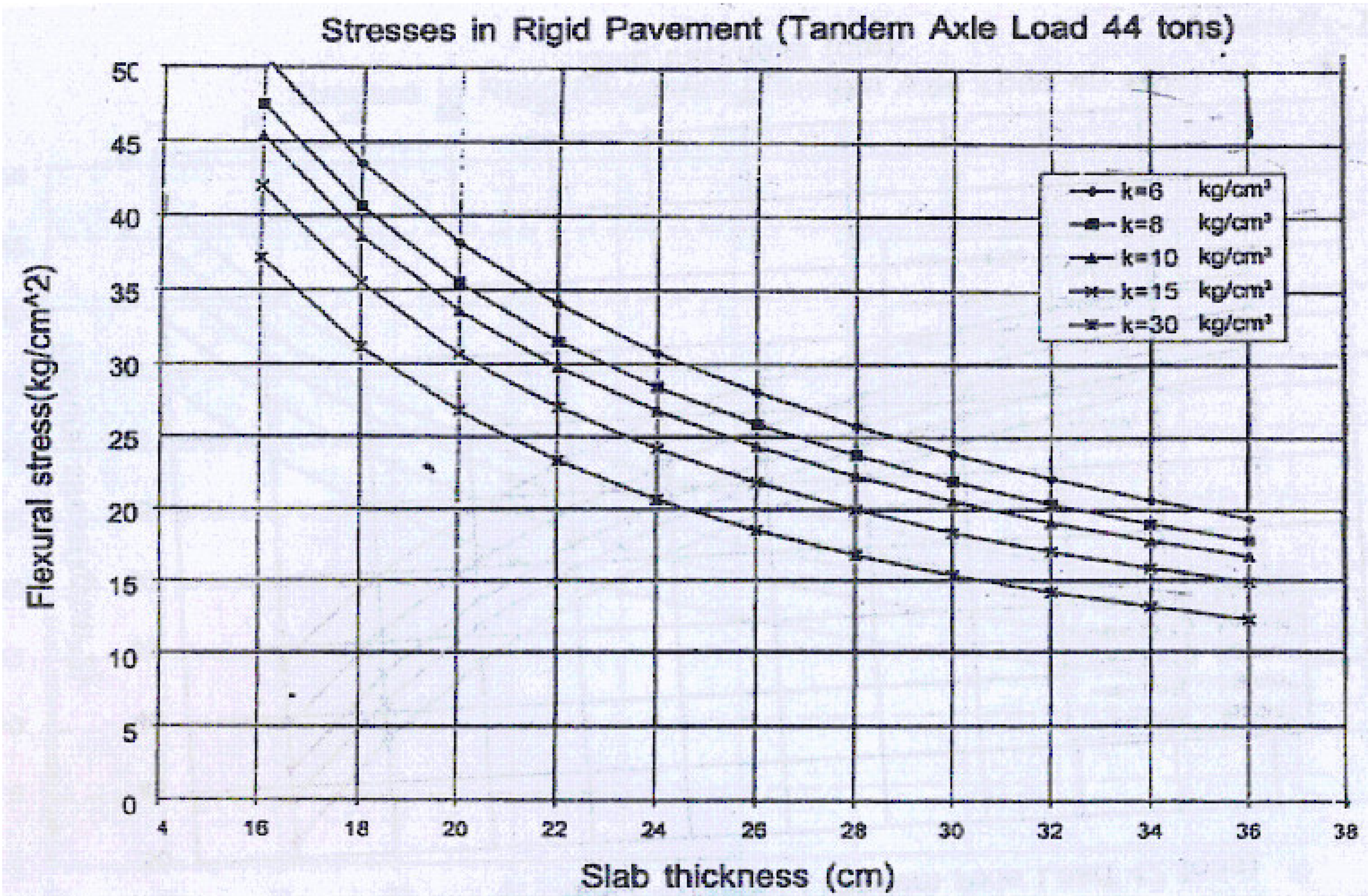
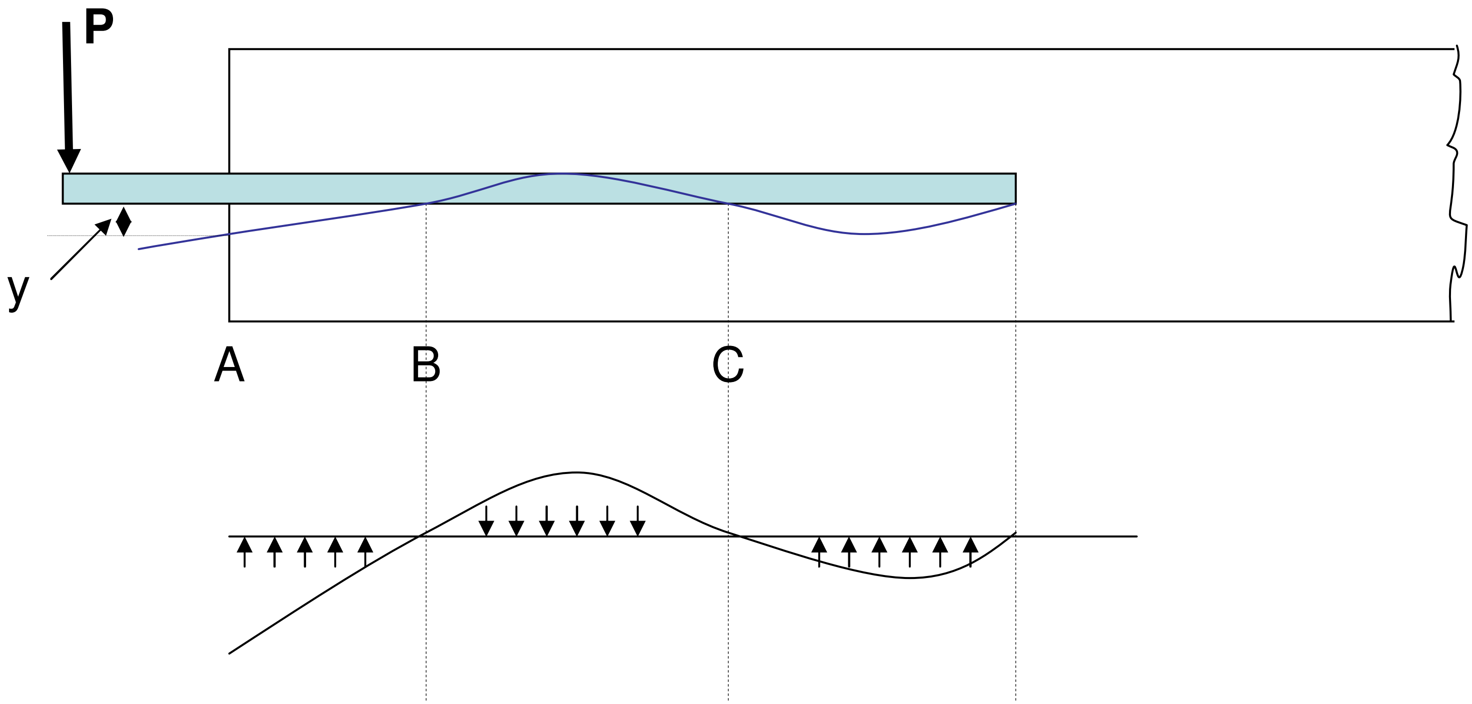


Stresses in Rigid Pavement (Tandem Axle Load 44 tons)
Flexural stress(kg/cm^2)
Slab thickness (cm)
k=6 kg/cm³
k=8 kg/cm³
k=10 kg/cm³
k=15 kg/cm³
k=30 kg/cm³
50
45
40
35
30
25
20
15
10
5
0
4
16
18
20
22
24
26
28
30
32
34
36
38

Pressure Exerted on a Loaded Dowel



Assumptions:

- Dowel is infinite in length
- Extends into an elastic body

Deflection of Dowel

- Dowel bar encased in concrete deflect as shown in the figure
- A – B: deflect downward exerting pressure at the lower face of the dowel
- B – C: B is point of contra flexure, the pressure is on the top
- Beyond C: again at C the bearing is on the bottom of the dowel bar
- Based on the equation of deflection curve by Thimoshenko, Friberg gave the deflection at the joint face (maximum deflection) as

$$y_0 = \frac{P(2+\beta z)}{4\beta^3 E_d I_d}$$

Deflection of Dowel

- P = load on one dowel
- z = joint width
- β = relative stiffness of dowel embedded in concrete
- E_d = Modulus of elasticity of dowel steel
- I_d = Moment of inertia of the dowel bar = $\frac{1}{64}\pi d^4$

$$\beta = \sqrt[4]{\frac{Kd}{4E_d I_d}}$$

- K = modulus of dowel support (81.5 to 409 GN/m^3)

Maximum Bearing Stress

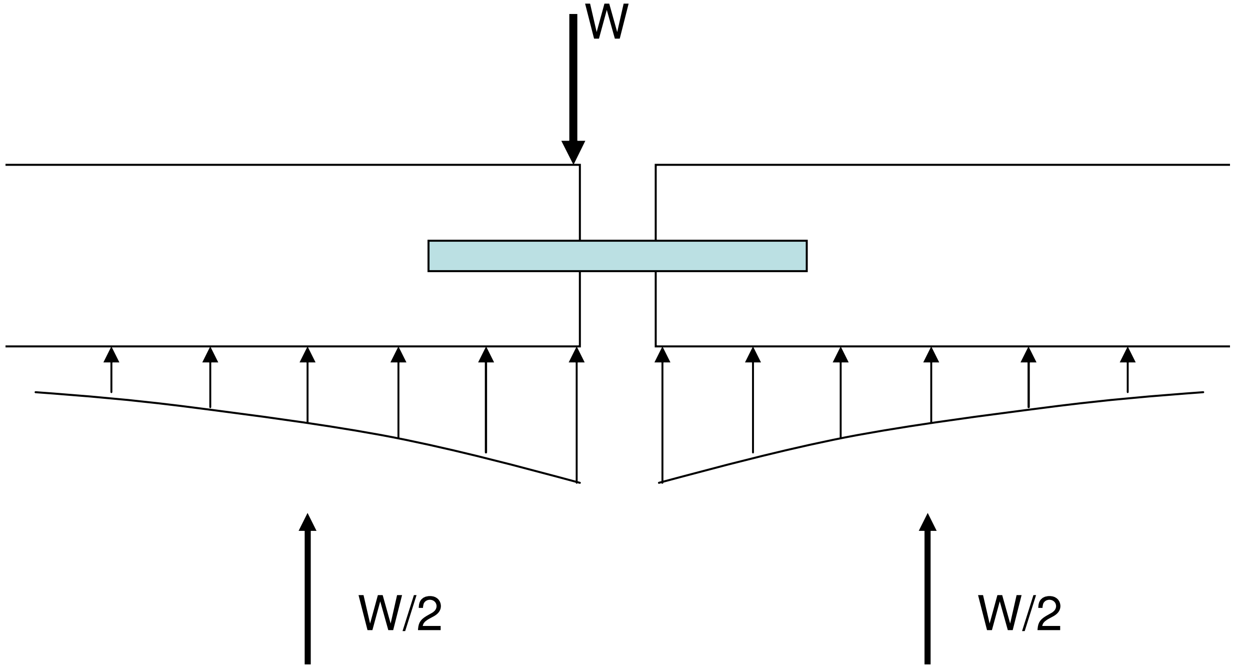
- Bearing stress is proportional to deflection

$$\sigma_b = K\, y_0 = \frac{KP(2 + \beta z)}{4\beta^3 E_d\, I_d}$$

Governing Stress in Dowel Bar

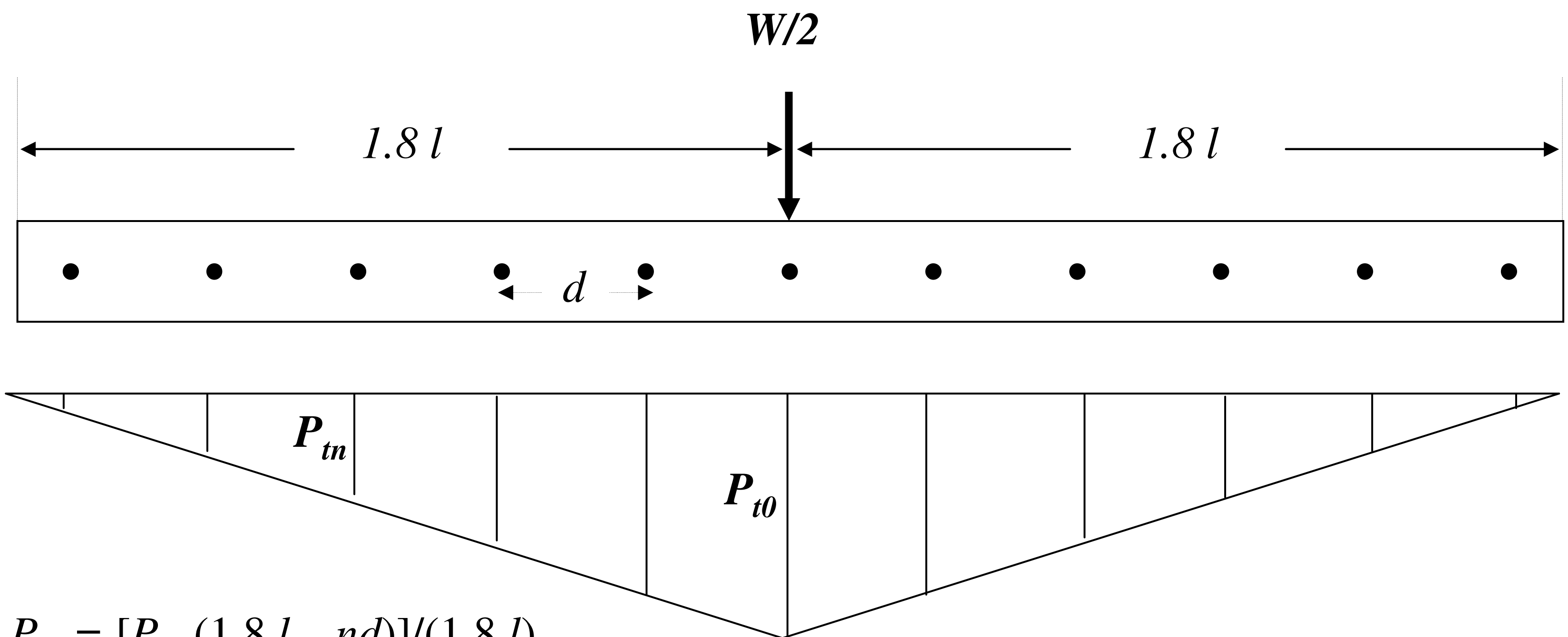
- Dowel is subjected to the following stresses
  - Shear
  - Bending
  - Bearing
- Because concrete is much weaker in steel, the size and spacing of dowels required are governed by the bearing stress between dowel and concrete.

Dowel Group Action



- Part of W is transferred to the adjacent slab through the dowel group.
- If the dowels are 100% efficient, 50% of W is transferred to the other slab
- If the pavement is old, less than 50% of W is transferred to the other slab

Maximum Load on Dowel

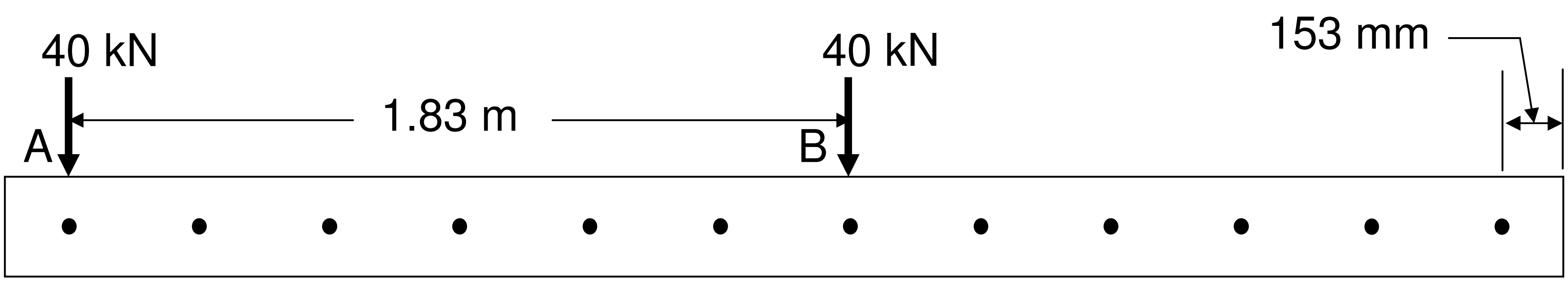


$P_{tn} = [P_{t0}\,(1.8\,l - nd)]/(1.8\,l)$

- Maximum negative moment for both interior and edge loadings occur at a distance of 1.8l from the load.
- The shear in each dowel decreases inversely with the distance of the dowel from the point of loading, being maximum for the dowel under or nearest to the point of loading and zero at 1.8l.

Example Problems

- A concrete pavement 203 mm thick is having a joint width of 5.1 mm, a modulus of subgrade reaction of 27 kN/m^3, and a modulus of dowel support of 407 GN/m^3. A load of 40 kN is applied over the outermost dowel at a distance of 152 mm from the edge. The dowels are 19 mm in diameter and 305 mm on centres. Determine the maximum bearing stress between dowel and concrete. (E = 27.6 GPa, E_d = 200 GPa)
- A concrete slab resting on a foundation with k = 13.6 Mn/m^3. Twelve dowels at 305 mm centres are placed at the joint on the 3.66 m lane. Two 40 kN wheel loads are applied at points A and B. Determine the maximum load on one dowel.



Design Parameters of Dowels

- IRC specifies that the efficiency of load transfer may be taken as 40%
- As per IRC recommendations, the distance at which the shear force becomes zero from the maximum loaded dowel is 1.0 l and not 1.8 l
- The permissible bearing stress (MPa) of concrete is calculated as

$$\sigma_b = [(10.16-\phi)\times f_{ck}]/(9.525)$$

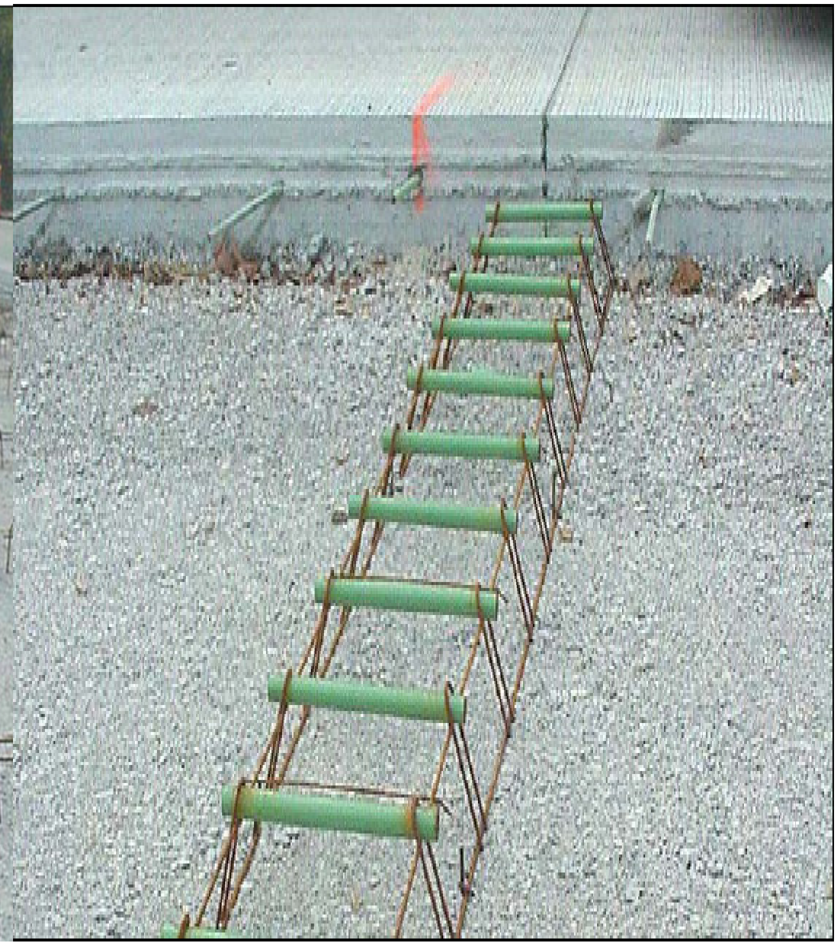
Where, ϕ = diameter of dowel in cm

f_{ck} = characteristic strength of concrete in MPa

Recommended Dimensions of Dowel Bars (IRC:58-2002)

Slab thickness, cm	Dowel Bar Dimensions (mm)		
	Diameter	Length	Spacing
20	25	500	250
25	25	500	300
30	32	500	300
35	32	500	300

Dowel Bars

Analysis of Traffic Loading for Pavement Design

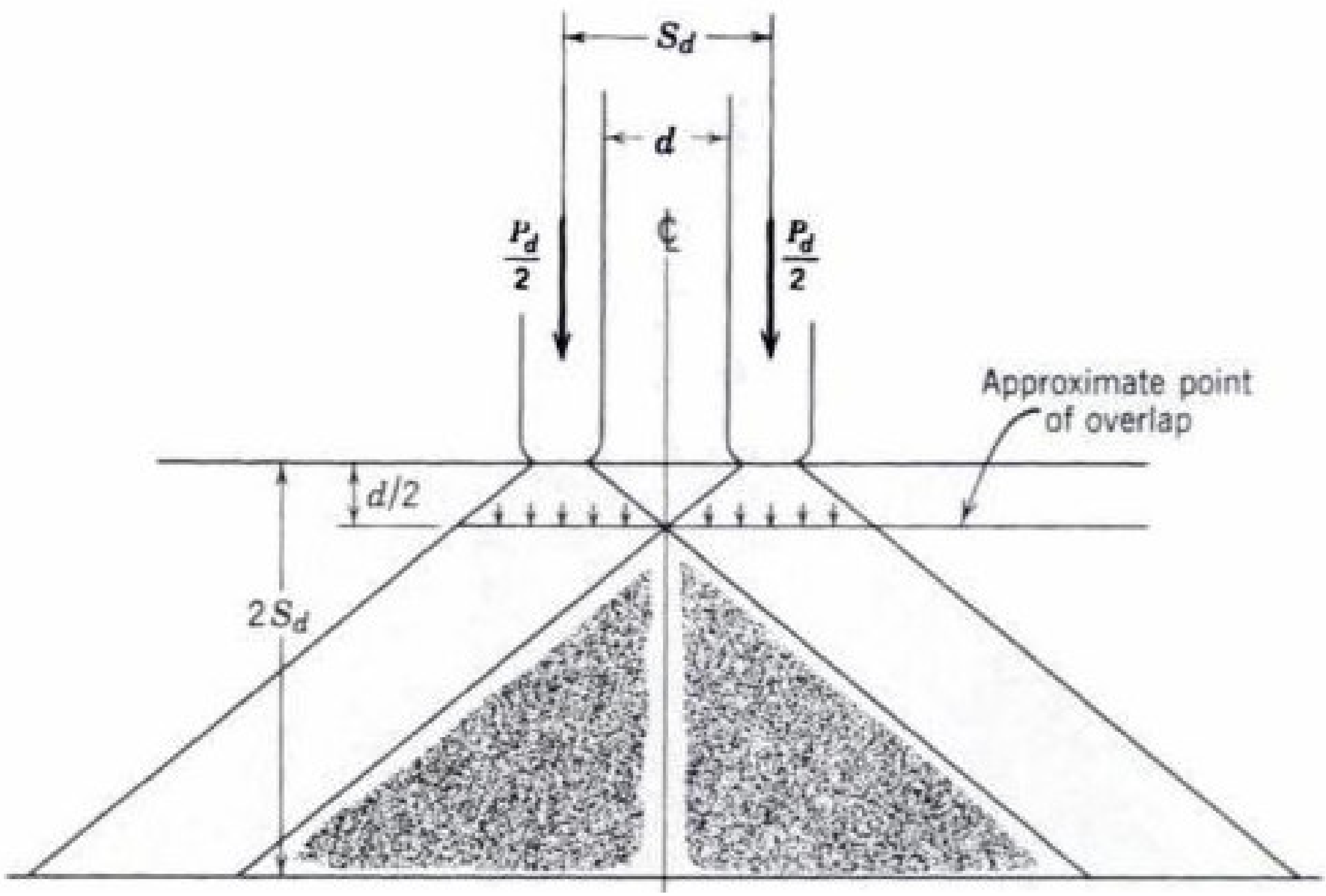
Three Different Approaches

- Fixed traffic
  - Design is governed by single wheel load
  - Load repetitions is not considered as a variable
  - Multiple wheels are converted into single wheel
  - Heaviest wheel load anticipated is used in design
- Fixed vehicle
  - Design is governed by the number of repetitions of standard vehicle or axle load, usually 80 kN single axle load
  - Repetitions of non-standard axles are converted into equivalent repetitions of standard axle using equivalent axle load factors
  - The cumulative number of repetitions of standard axle during the design life is termed as Equivalent Single Wheel Load (ESAL) and is the single traffic parameter for design purpose.
- Variable traffic and variable vehicle (Spectrum of Axles Approach)
  - Both vehicle and traffic are considered independently. i.e., treat all axles separately and use spectrum of axles in the design

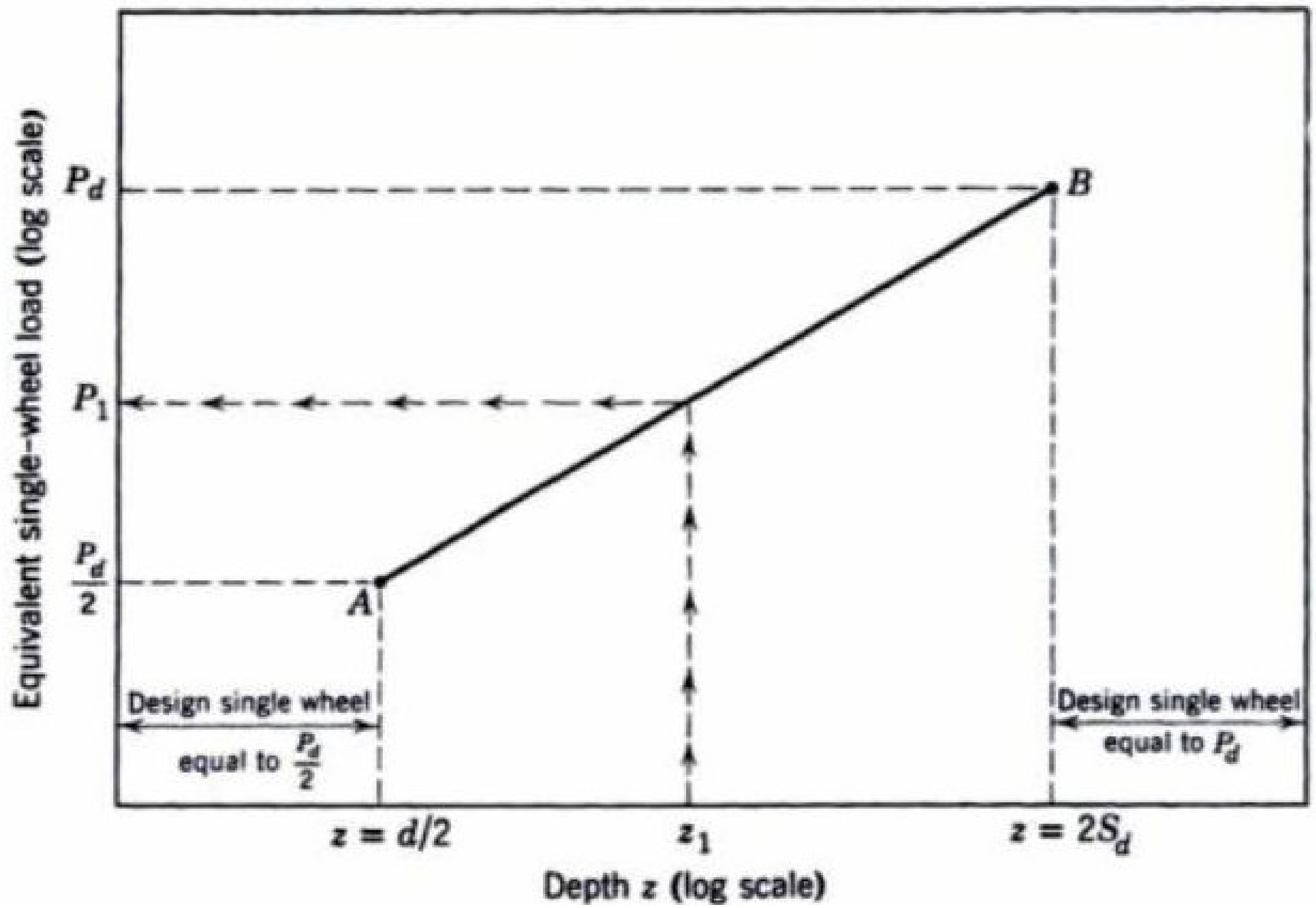
Equivalent Single Wheel Load

- In fixed traffic approach, multiple wheels are converted into equivalent single wheel load.
- ESWL can be determined based on the following approaches
  - Equal stress criteria
    - EWSL is the wheel load that causes the same stress at the top of subgrade as that of the multiple wheels
  - Equal deflection criteria
    - EWSL is the wheel load that causes the same deflection at the top of subgrade as that of the multiple wheels
- EWSL depends on the thickness of the pavement

Empirical Method (Boyd and Foster, 1951)



Empirical Method (Boyd and Foster, 1951)



Empirical Method (Boyd and Foster, 1951)

The equation in the previous plot for ESWL can be written as

$$\log(ESWL) = \log(P_d / 2) + \frac{0.301 \log(2z / d)}{\log(4S_d / d)}$$

Example Problem

- A set of dual tyres has a total load of 40 kN, a contact radius of 114 mm, and a centre to centre spacing of 343 mm. Determine ESWL by Boyd and Foster method for a 343 mm thick pavement.

Theoretical Method

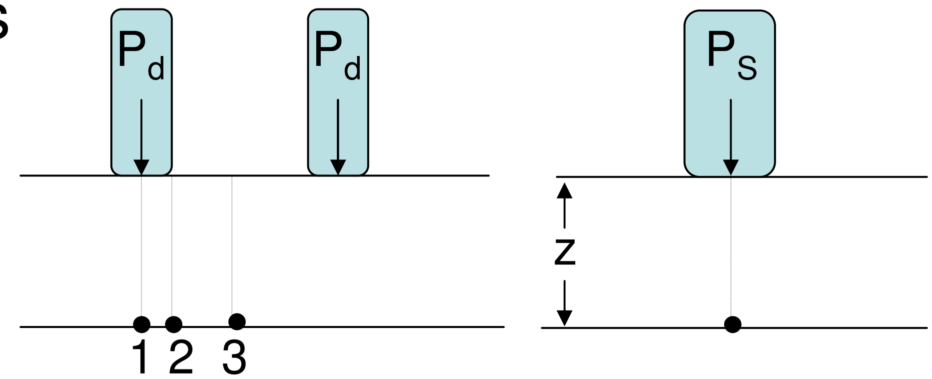
In one layer system, the vertical stress σ_z at the axis of symmetry at a depth z is given by

$$\sigma_z = p\left[1 - \frac{z^3}{\left(a^2 + z^2\right)^{3/2}}\right]$$

i.e., $$\sigma_z = \frac{P}{\pi a^2}\left[1 - \frac{z^3}{\left(a^2 + z^2\right)^{3/2}}\right]$$

$\therefore\ \sigma_z \propto P$ for a constant a

i.e., $$\frac{P_d}{P_S} = \frac{\sigma_{zd}}{\sigma_{zs}}$$



- σ_{zs} is the maximum of stresses at 1, 2 and 3
- σ_{zd} is the maximum stress due to P_d

Equivalent Deflection Criteria (Foster & Ahlvin)

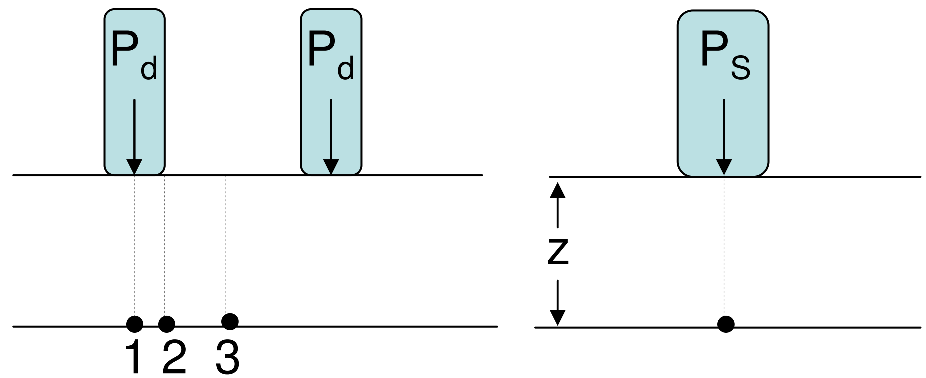
The general equation for deflection is given by

$$\Delta = \frac{pa}{E} F$$

$$\therefore \Delta \, \alpha \, P$$

i.e., $\dfrac{P_S}{P_d} = \dfrac{\Delta_S}{\Delta_d}$

$$\therefore \quad P_S = P_d \frac{\Delta_S}{\Delta_d}$$



Δ_S = max (Δ_1, Δ_2, Δ_3)

Δ_d = maximum deflection due to P_d

ESAL Approach

Equivalent Single Axle Load

- Equivalent Single Axle Load is the equivalent repetitions of standard axle during the design life of the pavement.
- IRC terms this ESAL as cumulative number of standard axles during the design life
- The number of repetitions of different types of axles are converted into equivalent repetitions of standard axle by using Equivalent Axle Load Factors (EALF)

Equivalent Axle Load Factor

- Equivalent Axle Load Factor (EALF) defines the damage per pass to a pavement by an axle relative to the damage per pass of a standard axle
- EALF for X- Type Axle = No. of repetitions of standard axle for causing a specified damage / No. of repetitions of X-type axle for causing the same damage
- For example, if 100,000 repetitions of a X-type axle causes a rut depth of 12 mm compared to 200,000 repetitions of a standard axle for causing the same rut depth, then EALF of X-type axle = 2. Here, the damage is measured in terms of rut depth.

Fourth Power Rule

- Approximate EALF can be worked out using the fourth power rule
- Single Axle Load

  EALF = (Axle Load in Kg/8160)^4
- Tandem Axle Load

  EALF = (Axle load in Kg/14968)^4
- However, as the EALF depends on axle load as wheel as the pavement configuration, the exact EALF can be worked out only by using distress models

Vehicle Damage Factor (VDF)

- Instead of converting each axle pass into equivalent standard axle passes, It will be convenient to convert one truck pass into equivalent standard axle passes.
- The factor that converts the number of trucks into equivalent standard axle repetitions is termed as vehicle damage factor or truck factor
- Therefore, Vehicle damage factor is the number of standard axles per truck.

Determining VDF

- For all important projects, the VDF need to be worked out from axle load survey
- In axle load survey the axles of a sample of about 10% of randomly chosen trucks are weighed using axle load pads
- Different configurations of trucks should be proportionately represented in the sample
- A stratified sample would be ideal for this purpose
- Annual Daily Traffic (ADT) of trucks need to be obtained for the road from the recent volume surveys or if not available should be estimated by conducting traffic volume survey

Summary Table of Axle Load Survey

Single Axles

Axle Load	No. of Axles	EALF	No of Std Axles
2	0	0.004	0.00
4	1	0.058	0.06
6	6	0.292	1.75
8	144	0.924	133.03
10	20	2.255	45.11
12	14	4.677	65.48
14	8	8.665	69.32
16	4	14.782	59.13
18	2	23.877	47.35
Total			**421.23**

Summary Table of Axle Load Survey

Tandem Axles

Axle Load	No. of Axles	EALF	No of Std Axles
4	0	0.005	0.00
8	14	0.082	1.14
12	21	0.413	8.68
16	101	1.306	131.87
20	44	3.188	140.25
24	42	6.610	277.61
28	44	12.246	538.80
Total			**1098.36**

Computation of VDF

- Total number of standard axles of the sampled trucks = 421.23+1098.36

  = 1519.59
- No. of Trucks sampled = 250
- VDF = No. of Std. Axles/No. of Trucks

  = 1519.59/250 = 6.08
- Therefore VDF = 6.08

Traffic on Design Lane

- The lane that carries maximum truck volume is the design lane
- The distribution of truck traffic across the width of the carriageway is to be considered in working out the traffic on design lane
- On any given road, one direction may carry more loads than the other
- within high traffic direction, each lane may carry a different portion of the loading
- The outer most lane often carries the most trucks and therefore is usually subjected to the heaviest loading
- Hence inputs for traffic distribution factors are important in the pavement analysis

Need for Distribution Factors

Traffic on Design Lane

- It is worked out by finding the directional distribution and lane distribution factors
- Directional Distribution Factor (D)
  - The ADT of trucks is the sum of daily truck traffic volume in both directions
  - D factor is the proportion of ADT of trucks occurring in the maximum direction
  - The D factor normally varies between 0.5 to 0.6

Traffic on Design Lane

- Lane Distribution Factor (L)
  - Is the proportion of truck traffic occurring on the design lane
  - Lane Distribution Factor depends on
    - Number of lanes
    - Traffic volume
- Daily Truck Traffic on Design Lane

$$= (\text{ADT of Trucks}) \times (D) \times (L)$$

Factors Suggested by IRC

- Undivided Roads (Single Carriageway)

No. of Traffic Lanes in Two Directions	Percentage of Trucks in Design Lane (D×L)
1	100
2	75
4	40

Factors Suggested by IRC

- Divided Roads (Dual Carriageway)

No. of Traffic Lanes in each Direction	Percentage of Trucks in Design Lane (L)
1	100
2	75
3	60
4	45

Computation of ESAL

- ESAL = (ADT of Trucks) $\times$ (365) $\times$ (D) $\times$ (L) $\times$ (VDF) $\times$ GF

- GF = $\dfrac{(1+r)^n - 1}{r}$

r = Growth rate in decimal

n = Design life in years