# **Engineering Mechanics**

3D Equilibrium

# Support reactions in 3D structures



# Support reactions in 3D Struct.







Hinge and bearing supporting radial load only



Two force components (and two couples)

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 $(\mathbf{M}_n)$ 



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#### Categories of Equilibrium in 3D

CAT	EGORIES OF EQUILIBRIUM IN THREE	DIMENSIONS
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point	$F_{1}$ $F_{2}$ $F_{2}$ $F_{3}$ $F_{5}$ $F_{4}$ $F_{4}$ $F_{5}$ $F_{4}$	$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line	$F_{1}$ $F_{2}$ $F_{2}$ $F_{2}$ $F_{3}$ $F_{5}$ $F_{4}$	$\begin{split} \Sigma F_x &= 0 & \Sigma M_y = 0 \\ \Sigma F_y &= 0 & \Sigma M_z = 0 \\ \Sigma F_z &= 0 \end{split}$
3. Parallel	$F_{5}$ $F_{4}$ $F_{1}$ $F_{2}$ $x$ $z$	$\begin{split} \Sigma F_x &= 0 \qquad \Sigma M_y = 0 \\ \Sigma M_z &= 0 \end{split}$
4. General	F <sub>1</sub> F <sub>2</sub> M Z F <sub>4</sub> F <sub>3</sub>	$\begin{split} \Sigma F_x &= 0 & \Sigma M_x = 0 \\ \Sigma F_y &= 0 & \Sigma M_y = 0 \\ \Sigma F_z &= 0 & \Sigma M_z = 0 \end{split}$

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#### **Reactions at Supports and Connections** for a Three-Dimensional Structure







Hinge and bearing supporting radial load only

Two force components (and two couples)

 $(M_{-})$ 









Universal Joint

Ball Support (or ball caster)

Ball and socket joint

#### Categories of Equilibrium in 2D

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS			
Force System	Free-Body Diagram	Independent Equations	
1. Collinear	$F_1$ $F_2$ $F_3$ $-x$	$\Sigma F_x = 0$	
2. Concurrent at a point	$F_1$ $F_2$ $F_4$ $F_3$ $F_2$ $F_3$ $F_2$ $F_3$	$\Sigma F_x = 0$ $\Sigma F_y = 0$	
3. Parallel	$ \begin{array}{c} & y \\ \hline F_2 \\ \hline F_3 \\ \hline F_4 \end{array} \begin{array}{c} y \\ \hline \\$	$\Sigma F_x = 0$ $\Sigma M_z = 0$	
4. General	$F_1$ $F_2$ $F_3$ $y$ $f_4$ $F_4$ $F_3$ $y$ $f_4$ $F_$	$\Sigma F_x = 0 \qquad \Sigma M_z = 0$ $\Sigma F_y = 0$	

# Conditions of Equilibrium

CAT	EGORIES OF EQUILIBRIUM IN THRE	EE DIMENSIONS
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point	F <sub>1</sub> F <sub>5</sub> F <sub>4</sub> F <sub>1</sub> F <sub>2</sub> F <sub>2</sub> F <sub>2</sub> F <sub>2</sub> F <sub>2</sub> F <sub>2</sub> F <sub>2</sub> F <sub>2</sub>	$\begin{split} \Sigma F_x &= 0\\ \Sigma F_y &= 0\\ \Sigma F_z &= 0 \end{split}$
2. Concurrent with a line	$F_1$ $F_2$ $F_1$ $F_2$ $F_2$ $F_3$ $F_5$ $F_4$	$\begin{split} \Sigma F_x &= 0 & \Sigma M_y = 0 \\ \Sigma F_y &= 0 & \Sigma M_z = 0 \\ \Sigma F_z &= 0 \end{split}$
3. Parallel	$F_1$ $F_2$ $F_3$ $F_3$ $F_2$ $F_3$ $F_2$ $F_3$ $F_3$ $F_2$ $F_3$ $F_3$ $F_3$ $F_4$ $F_4$ $F_4$ $F_4$ $F_5$	$\Sigma F_x = 0 \qquad \Sigma M_y = 0$ $\Sigma M_z = 0$
4. General	F <sub>1</sub> F <sub>4</sub> F <sub>2</sub> M y z z	$\begin{split} \Sigma F_x &= 0 & \Sigma M_x = 0 \\ \Sigma F_y &= 0 & \Sigma M_y = 0 \\ \Sigma F_z &= 0 & \Sigma M_z = 0 \end{split}$



(a) Complete fixity Adequate constraints



(b) Incomplete fixity Partial constraints



(c) Incomplete fixity Partial constraints



(d) Excessive fixity Redundant constraint

#### Adequacy of Constraints

#### Adequacy of Constraints



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(b) Incomplete fixity Partial constraints



(c) Incomplete fixity Partial constraints



(d) Excessive fixity Redundant constraints

A uniform pipe cover of radius r = 240 mm and mass 30 kg is held in a horizontal position by the cable CD. Determine the tension in the cable.



#### Problem 23 - Solution

Weight of pipe cover,  $W = 30 \text{ kg x } 9.81 \text{ m/s}^2 = 294 \text{ N}$ 

The reactions acting at points A and B would also consist of couples about the x and y axes. However these couples are not shown in the FBD as they are not significant in this problem and the tension in the cable can be obtained without taking these couples into consideration.

The self weight and cable tension acting on the pipe cover can be represented as follows:



$$\vec{W} = (-294N)\vec{j}$$
$$\vec{T} = T\hat{r}_{DC}$$

$$B = (0,0,0), C = (0,240mm,80mm), D = (480mm,0,240mm),$$
  

$$\vec{r}_{DC} = \vec{r}_{BC} - \vec{r}_{BD} = (-480mm)\vec{i} + (240mm)\vec{j} + (-160mm)\vec{k}$$
  

$$\left|\vec{r}_{DC}\right| = \sqrt{\left(\!\left(480mm\right)^2 + \left(240mm\right)^2 + \left(160mm\right)^2\right)}\!= 560mm$$
  

$$\hat{r}_{DC} = \left(-\frac{6}{7}\right)\vec{i} + \left(\frac{3}{7}\right)\vec{j} + \left(-\frac{2}{7}\right)\vec{k}$$
  
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#### Problem 23 - Solution



The rigid L-shaped member ABC is supported by a ball-and-socket joint at A and by three cables. Determine the tension in each cable and the reaction at A caused by the 1 kN load applied at G.





$$\begin{array}{c|c} \text{Problem 24 -} \\ \sum \vec{M}_{A} = 0 \Rightarrow \vec{r}_{AB} \underbrace{ \begin{array}{c} \vec{F}_{11} + \vec{r}_{AB} \times \vec{f}_{2} + \vec{r}_{AG} \times \vec{F} + \vec{r}_{AC} \times \vec{T}_{3} = 0 \\ \Rightarrow T_{1} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 152cm & 0 & 0 \\ -0.923 & 0 & 0.385 \end{vmatrix} + T_{2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 152cm & 0 & 0 \\ -0.923 & 0.385 \end{vmatrix}} + T_{2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 152cm & 0 & 0 \\ -0.923 & 0.385 \end{vmatrix} + \left. \begin{array}{c} \vec{i} & \vec{j} & \vec{k} \\ 152cm & 0 & 0 \\ -0.923 & 0.385 \end{vmatrix} + \left. \begin{array}{c} \vec{i} & \vec{j} & \vec{k} \\ 152cm & 0 & 0 \\ -0.923 & 0.385 \end{vmatrix} \right| + \left. \begin{array}{c} \vec{i} & \vec{j} & \vec{k} \\ 152cm & 0 & -76cm \\ 0 & -1 & 0 \end{vmatrix} \right| + \left. \begin{array}{c} \vec{i} & \vec{j} & \vec{k} \\ 152cm & 0 & -76cm \\ -0.923 & 0.385 & 0 \end{vmatrix} = 0 \\ \Rightarrow T_{1}(-58.52\vec{j}) + T_{2}(58.52\vec{k}) + (-38\vec{i} - 152\vec{k}) \\ + T_{3}(29.26\vec{i} + 70.148\vec{j} + 58.52\vec{k}) = 0 \\ \Rightarrow (-38 + 29.26T_{3})\vec{i} + (-58.52T_{1} + 70.148T_{3})\vec{j} & A_{2} \\ + (58.52T_{2} - 152 + 58.52T_{3})\vec{k} = 0 \end{array} \right)$$



$$\sum M_{A} = (-38 + 29.26T_{3})\vec{i} + (-58.52T_{1} + 70.148T_{3})\vec{j}$$
$$+ (58.52T_{2} - 152 + 58.52T_{3})\vec{k} = 0$$

Equating the individual moment components of  $M_A$  to zero we get three equations of equilibrium as shown below:

$$-38 + 29.26T_3 = 0$$

$$-58.52T_1 + 70.148T_3 = 0$$

$$T_1 = 1.56kN$$

$$58.52T_2 - 152 + 58.52T_3 = 0$$

$$T_2 = 1.3kN$$

#### Problem 24 - Solution



#### Problem 24 - Solution



Equating the individual force components to zero we get three equations of equilibrium as shown below:

$$-0.923T_{1} - 0.923T_{2} - 0.923T_{3} + A_{x} = 0 \quad A_{x} = 3.84kN$$
  

$$0.385T_{2} + 0.385T_{3} + A_{y} - 1 = 0 \qquad A_{y} = 0$$
  

$$0.385T_{1} + A_{z} = 0 \qquad A_{z} = -0.6kN$$

# Problem 24 - Solution



#### Check Stability



#### Problem A

• The uniform 10-lb rod AB is supported by a ball-and-socket joint at A and leans against both the rod *CD* and the vertical wall. Neglecting the effects of friction, determine (a) the force which rod (a)CD exerts on AB, (b)the reactions at A and В.



 $\mathbf{B} = (3.00 \text{ lb})\mathbf{k}$ 

 $\mathbf{A} = -(3.20 \text{ lb})\mathbf{i} + (5.73 \text{ lb})\mathbf{j} - (3.00 \text{ lb})\mathbf{k}$ 

# Problem B

In the planetary gear system shown, the radius of the central gear A is a = 24 mm, the radius of the planetary gears is b, and the radius of the outer gear E is

(a+2b). A clockwise couple of magnitude 15N.m is applied to the central gear A, and a counterclockwise couple of magnitude 75N.m is applied to the spider *BCD*. If the system is to be in equilibrium, determine (*a*) the required radius *b* of the planetary gears, (*b*) the couple  $M_E$  that must be applied to the outer  $\neg \neg \neg \neg F$ 

 $r_B = 36.0 \text{ mm}$ 



 $\mathbf{M}_E = 60.0 \,\,\mathrm{N} \cdot \mathrm{m}$ 

#### Planetary Gear



http://machinedesign.com/mechanicaldrives/planetary-gears-review-basicdesign-criteria-and-new-options-sizing

Three identical steel balls, each of mass m, are placed in the cylindrical ring which rests on a horizontal surface and whose height is slightly greater than the radius of the balls. The diameter of the ring is such that the balls are virtually touching one another. A fourth identical ball is then placed on top of the three balls. Determine the force P exerted by the ring on each of the three lower balls.



 $P = \frac{mg}{3\sqrt{2}}$ 

A window is temporarily held open in the 50° position shown by a wooden prop CD until a crank-type opening mechanism can be installed. If a=0.8m and b=1.2 m and the mass of the window is 50 kg with mass center at its geometric center, determine the compressive force  $F_{CD}$  in the prop.



#### Problem 26 - Solution



 $\vec{r}_A = a\vec{j}; \vec{r}_B = a\vec{j} - b\vec{k}; \vec{r}_C = -\frac{b}{4}\vec{k};$ 

 $\vec{r}_D = a\sin\theta \vec{i} + a(1 - \cos\theta)\vec{j};$ 

Reactions at hinges A and B: (i) No M<sub>z</sub> Reactions (ii)Other Reactions are not required in this problem

B

$$\vec{r}_{E} = \frac{a}{2}\sin\theta \vec{i} + a\left(1 - \frac{\cos\theta}{2}\right)\vec{j} - \frac{b}{2}\vec{k}. \quad \vec{r}_{CD} = a\sin\theta \vec{i} + a(1 - \cos\theta)\vec{j} + \frac{b}{4}\vec{k}$$
$$\vec{r}_{AE} = \frac{a}{2}\sin\theta \vec{i} - \frac{a}{2}\cos\theta \vec{j} - \frac{b}{2}\vec{k} \qquad |\vec{r}_{CD}| = \sqrt{(a\sin\theta)^{2} + (a(1 - \cos\theta))^{2} + \left(\frac{b}{4}\right)^{2}}$$
$$\vec{r}_{AD} = a\sin\theta \vec{i} - a\cos\theta \vec{j}$$

# Problem 26 - Solution B

$$\vec{r}_{AE} = \frac{a}{2}\sin\theta \vec{i} - \frac{a}{2}\cos\theta \vec{j} - \frac{b}{2}\vec{k}$$
  

$$\vec{r}_{AD} = a\sin\theta \vec{i} - a\cos\theta \vec{j}$$
  

$$\vec{r}_{CD} = a\sin\theta \vec{i} + a(1-\cos\theta)\vec{j} + \frac{b}{4}\vec{k}$$
  

$$\left|\vec{r}_{CD}\right| = \sqrt{(a\sin\theta)^2 + (a(1-\cos\theta))^2 + \left(\frac{b}{4}\right)^2}$$
  

$$\vec{W} = -W\vec{j}$$
  

$$\vec{F}_{CD} = F_{CD}\hat{r}_{CD} = \frac{F_{CD}}{\left|\vec{r}_{CD}\right|} \left(a\sin\theta \vec{i} + a(1-\cos\theta)\vec{j} + \frac{b}{4}\vec{k}\right)$$
  

$$Let \ \vec{F}_{CD} = F\left(a\sin\theta \vec{i} + a(1-\cos\theta)\vec{j} + \frac{b}{4}\vec{k}\right)$$
  

$$F = \frac{F_{CD}}{\left|\vec{r}_{CD}\right|} = \frac{F_{CD}}{\sqrt{(a\sin\theta)^2 + (a(1+\cos\theta))^2 + (\frac{b}{4})^2}}$$

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E

W

С

D

# Problem 26 - Solution B. $\vec{r}_{AE} = \frac{a}{2}\sin\theta \hat{i} - \frac{a}{2}\cos\theta \hat{j} - \frac{b}{2}\hat{k}$ $\vec{r}_{AD} = a\sin\theta \hat{i} - a\cos\theta \hat{j}$ $\vec{r}_{CD} = a\sin\theta \vec{i} + a(1 - \cos\theta)\vec{j} + \frac{b}{4}\vec{k}$ $\left|\vec{r}_{CD}\right| = \sqrt{\left(a\sin\theta\right)^2 + \left(a(1-\cos\theta)\right)^2 + \left(\frac{b}{4}\right)^2}$ $\vec{W} = -W\vec{j}$ $\vec{F}_{CD} = F\left(a\sin\theta\vec{i} + a(1-\cos\theta)\vec{j} + \frac{b}{4}\vec{k}\right) \qquad F = \frac{F_{CD}}{|\vec{r}_{CD}|} = \frac{F_{CD}}{\sqrt{\left(a\sin\theta\right)^2 + \left(a(1+\cos\theta)\right)^2 + \left(\frac{b}{4}\right)}}$ $\sum M_{AB} = -\vec{k}.\vec{r}_{AE} \times W - \vec{k}.\vec{r}_{AD} \times \vec{F}_{CD} = 0$ $\Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ \frac{a}{2}\sin\theta & -\frac{a}{2}\sin\theta & -\frac{b}{2} \\ 0 & -W & 0 \end{vmatrix} + F \begin{vmatrix} 0 & 0 & -1 \\ a\sin\theta & -a\cos\theta & 0 \\ a\sin\theta & a(1-\cos\theta) & \frac{b}{4} \end{vmatrix} = 0$ 29

# Problem 26 - Solution

$$\vec{W} = -W\vec{j}$$

$$\vec{F}_{CD} = F\left(a\sin\theta\vec{i} + a(1-\cos\theta)\vec{j} + \frac{b}{4}\vec{k}\right)$$

$$F = \frac{F_{CD}}{|\vec{r}_{CD}|} = \frac{F_{CD}}{\sqrt{(a\sin\theta)^2 + (a(1+\cos\theta))^2 + (\frac{b}{4})}}$$

$$\sum M_{AB} = \vec{k}.\vec{r}_{AE} \times \vec{W} + \vec{k}.\vec{r}_{AD} \times \vec{F}_{CD} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ \frac{a}{2}\sin\theta & -\frac{a}{2}\sin\theta & -\frac{b}{2} \\ 0 & -W & 0 \end{vmatrix} + F \begin{vmatrix} 0 & 0 & -1 \\ a\sin\theta & -a\cos\theta & 0 \\ a\sin\theta & a(1-\cos\theta) & \frac{b}{4} \end{vmatrix} = 0$$

$$W \frac{a}{2}\sin\theta - Fa^2(\sin\theta(1-\cos\theta) + \sin\theta\cos\theta) = 0$$

$$W \frac{a}{2}\sin\theta - Fa^2\sin\theta = 0$$

$$F = \frac{W}{2a}$$
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# Problem 26 - Solution B, $\vec{W} = -\vec{Wj}$ $\vec{F}_{CD} = F\left(a\sin\theta\vec{i} + a(1-\cos\theta)\vec{j} + \frac{b}{4}\vec{k}\right)$ $F = \frac{F_{CD}}{|\vec{r}_{CD}|} = \frac{F_{CD}}{\sqrt{(a\sin\theta)^2 + (a(1+\cos\theta))^2 + (\frac{b}{4})^2}}$ $F = \frac{W}{2a}$ $F_{CD} = F \left| \vec{r}_{CD} \right| = \frac{W}{2a} \sqrt{\left( a \sin \theta \right)^2 + \left( a \left( 1 - \cos \theta \right) \right)^2 + \left( \frac{b}{4} \right)^2}$ W

Putting W = mg; m = 50 kg; g = 9.81 m/s<sup>2</sup>; a = 0.8 m; b = 1.2 m;  $\theta = 50^{\circ}$ 

$$F_{CD} = 226.9N$$



• Two shafts AC and CF, which lie in the vertical xy plane, are connected by a universal joint at C. The bearings at B and Ddo not exert any axial force. A couple of magnitude (clockwise when viewed from the positive x axis) is applied to shaft CF at F. At a time when the arm of the crosspiece attached to shaft CF is horizontal, determine (a) the magnitude of the couple which must be applied to shaft ACat A to maintain equilibrium, (b) the reactions at B, D, and E.

## **Universal Joint**



Image courtesy of ClearMechanic.com

The frame ACD is supported by ball-and-socket joints at A and D and by a cable that passes through a ring at B and is attached to hooks at G and H. Knowing that the frame supports at point C a load of magnitude determine the tension in the cable.



T = 100.7 lb

• The rigid L-shaped member *ABC* is supported by a ball and socket at *A* and by three cables. Determine the tension in each cable and the reaction at *A* caused by the 500-lb load applied at *G*.



• A uniform 0.5 x 0.75m steel plate *ABCD* has a mass of 40 kg and is attached to ball-and-socket joints at *A* and *B*. Knowing that the plate leans against a frictionless vertical wall at *D*, determine (*a*) the location of *D*, (*b*) the reaction at *D*.



The two bars *AB* and *OD*, pinned together at C, form the diagonals of a horizontal square *AOBD*. The ends *A* and *O* are attached to a vertical wall by ball and socket joint, point *B* is supported by a cable *BE*, and a vertical load *P* is applied at *D*. Find the components of the reactions at *A* and *O*.



Three identical steel balls, each of mass m, are placed in the cylindrical ring which rests on a horitontal surface and whose height is slightly greater than the radius of the balls. The diameter of the ring is such that the balls are virtually touching one another. A fourth identical ball is then placed on top of the three balls. Determine the force P exerted by the ring on each of the three lower balls.



• A rectangular sign over a store has a mass of 100kg, with the center of mass in the center of the rectangle. The support against the wall at point C may be treated as a ball-and-socket joint. At corner D support is provided in the y-direction only. Calculate the tensions T1 and T2 in the supporting wires, the total force supported at C, and the lateral force R supported at D



• A window is temporarily held open in the 50o position shown by a wooden prop CD until a crank-type opening mechanism can be installed. If a=0.8m and b=1.2 m and the mass of the window is 50 kg with mass center at its geometric center, determine the compressive force FCD in the prop.



• Two rods are welded together to form a T-shaped lever which leans against a frictionless wall at *D* and is supported by bearings at *A* and *B*. A vertical force *P* of magnitude 600 N is applied at the midpoint *E* of rod *DC*. Determine the reaction at *D*.  $[F_D = 375 N]$ 





A camera weighing 0.53 lb is mounted on a small tripod weighing 0.44 lb on a smooth surface. Assuming that the weight of the camera is uniformly distributed and that the line of action of the weight of the tripod passes through D, determine (a) the vertical components of the reactions at A, B, and C when  $\theta = 0$  (b) the maximum value of  $\theta$  if the tripod is not to tip over. [(a) Na= 0.656lb, NB= NC= 0.157lb, (b)  $\min = 62 \text{deg.}$ ]

• A 450 N load *P* is applied at the corner *C* of rigid pipe *ABCD* which has been bent as shown. The pipe is supported by the ball and socket joints *A* and *D*, fastened respectively to the floor and to a vertical wall, and by a cable attached at the midpoint *E* of the portion *BC* of the pipe and at a point *G* on the wall. Determine (a) where *G* should be located if the tension in the cable is to be minimum, (b) the corresponding minimum value of the tension. [(a) height of G = 5m from A, same horizontal component as A, on the wall (b) Tmin = 300N]



• Three identical steel balls, each of mass m, are placed in the cylindrical ring which rests on a horitontal surface and whose height is slightly greater than the radius of the balls. The diameter of the ring is such that the balls are virtually touching one another. A fourth identical ball is then placed on top of the three balls. Determine the force P exerted by the ring on each of the three lower balls. [N = W/3 $\sqrt{2}$ ]



3) The power unit of the post-hole digger supplies a torque of 4000 N.m to the auger. The arm *B* is free to slide in the supporting sleeve *C* but is not free to rotate about the horizontal axis of *C*. If the unit if free to swivel about the vertical axis of the mount *D*, determine the force exerted against the right rear wheel by the block *A* (or *A'*), which prevents the unbraced truck from rolling. (*Hint* : view the system from above.)

