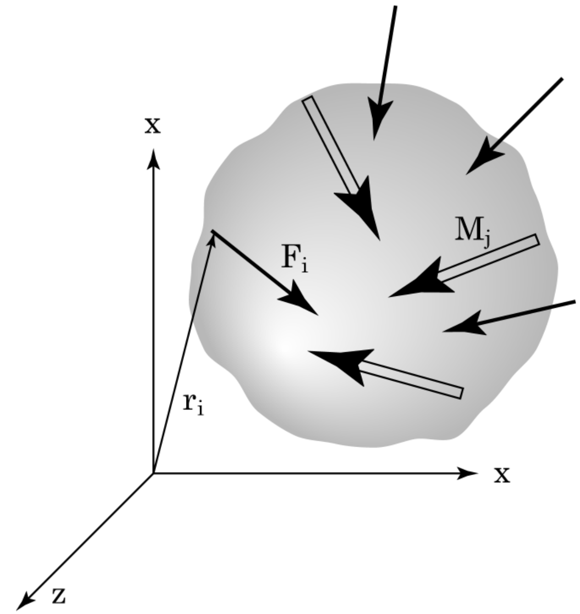
Engineering Mechanics

Equilibrium of Rigid Bodies

Equilibrium

- System is in equilibrium if and only if the *sum* of all the *forces* and *moment* (*about any point*) equals *zero.*



$$\vec{F} = \sum_i \vec{F_i} = \vec{0}$$
$$\vec{M_o} = \sum_j \vec{M_j} + \sum_i \vec{r_i} \times \vec{F_i} = \vec{0}$$

Supports and Equilibrium

- Any structure is made of many components.
- The components are the be connected by linkages.
- Other wise the structure will lose its integrity.
- Different component of structure *talk* to each other via linkages.
- The structure should be *globally* supported to prevent it from falling over.

Different Structural Supports

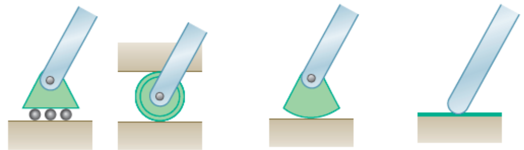
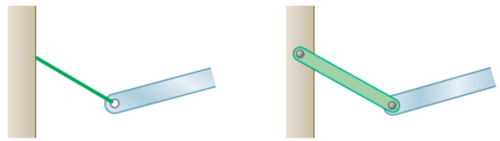
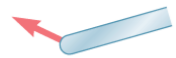
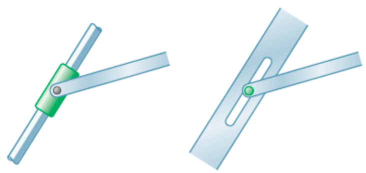
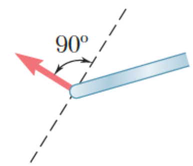
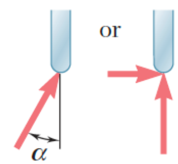
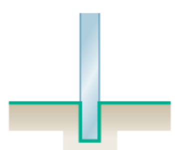
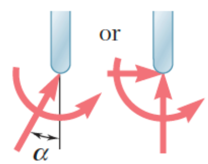
- Supports are required to maintain system in equilibrium.
- Too few supports makes system *unstable* general loading
- Too many supports make the system *over-rigid*.

Constraints and Reactions

- There is an intricate relationship between *kinematics (motion)* and *reactions (forces)*.
- Always note that in the case of supports *displacement (rotation)* and *force (torque)* in any given *direction* are complementary.
- If a support *rigidly* constrains a given *degree of freedom (DOF)* for a rigid body then it gives rise to a reaction corresponding to that *DOF*.
- Similarly if a support *freely* allows motion of particular *DOF* then there is *no reaction* from the support in that direction.

Support reactions in 2D structures

Reactions at Supports and Connections for a Two-Dimensional Structure

Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of action	1
 Frictionless pin or hinge, Rough surface	 Force of unknown direction	2
 Fixed support	 Force and couple	3

Roller/Rocker

pin/hinge

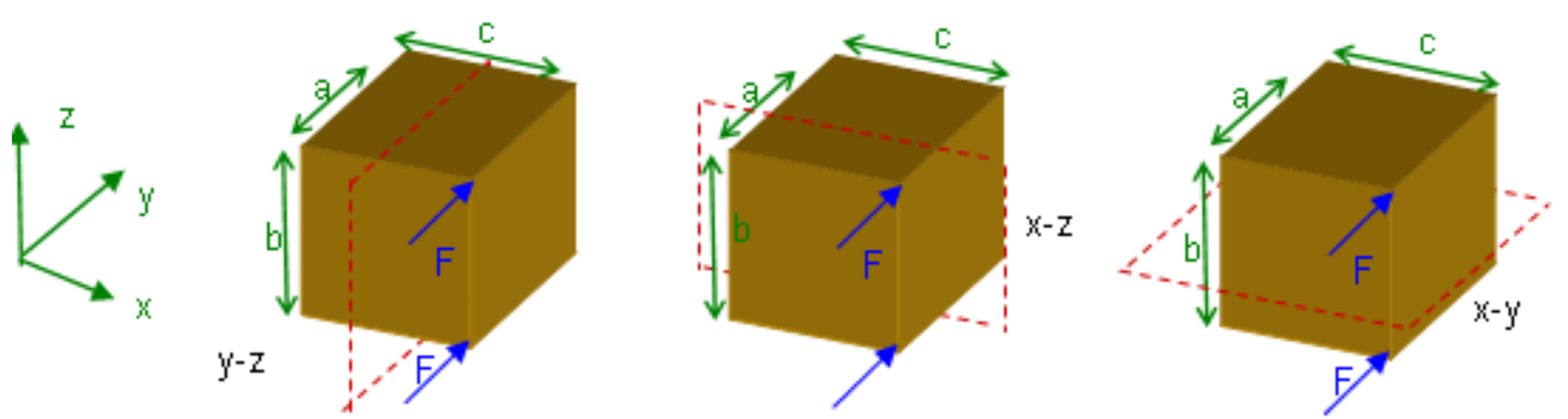
Simple Examples

Roller Support

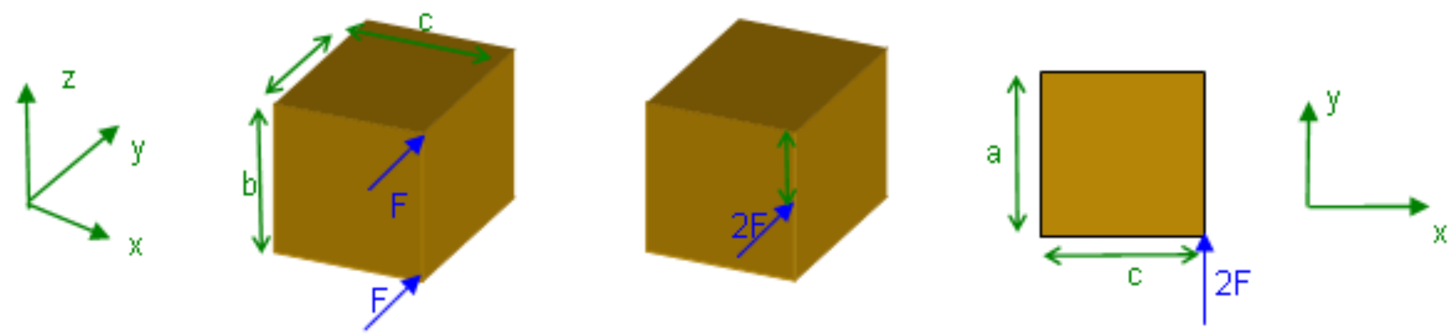
Fixed Support

Using Symmetry to convert 3D problems to 2D

(adapted from http://oli.web.cmu.edu)

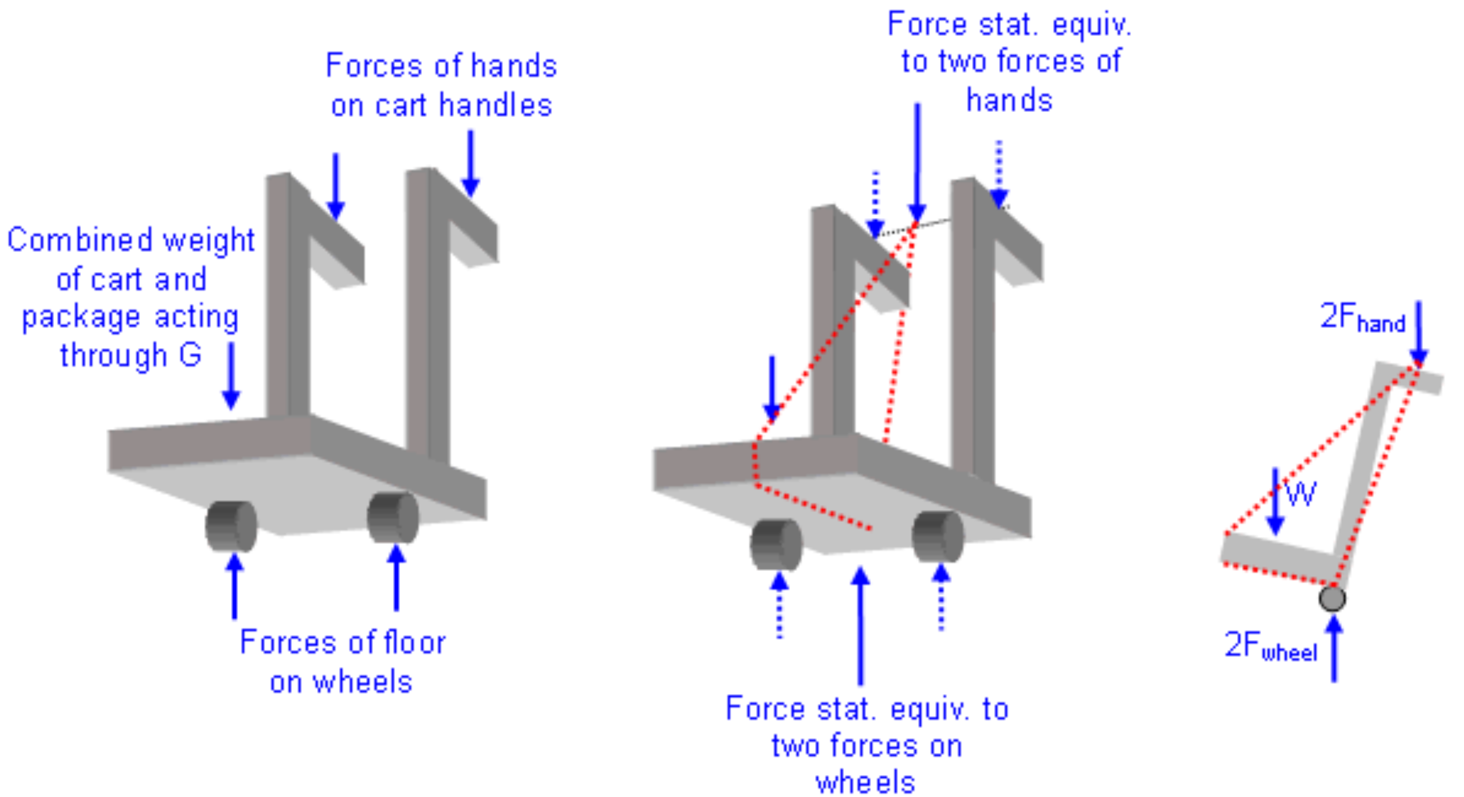


- Box has 3-planes of symmetry.
- Loading had only one plane of symmetry
- Using symmetry and static equivalence, the problem can be converted into a 2D problem



Simple Example

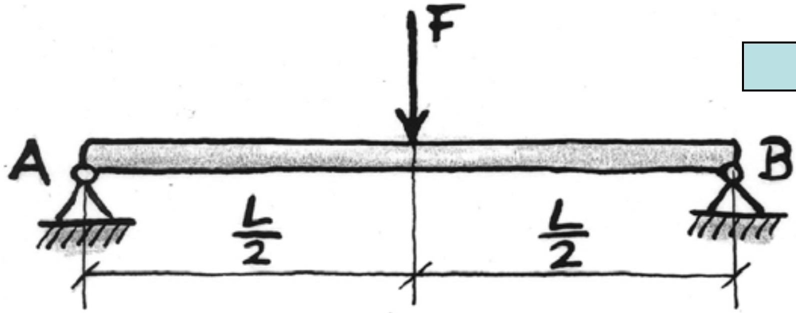
(http://oli.web.cmu.edu)



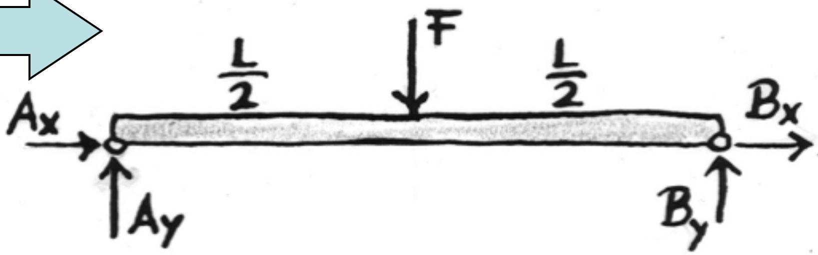
Free Body Diagram (FBD)

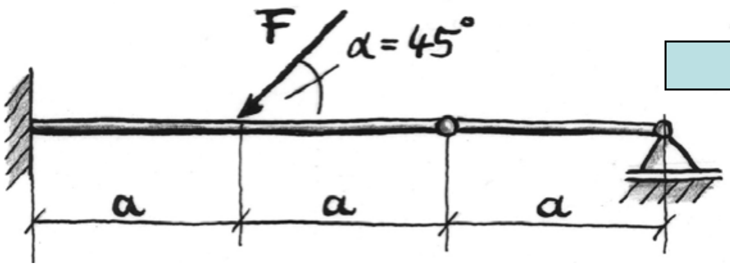
- **Single most important concept in engineering mechanics**.
- Zoom in on a given component of a structure.
- Means replace supports (connections) with the corresponding reactions.
- **Replace *kinematic* constraints with corresponding *reactions*.**
- Concepts will get more clear as we proceed further.

Simple examples

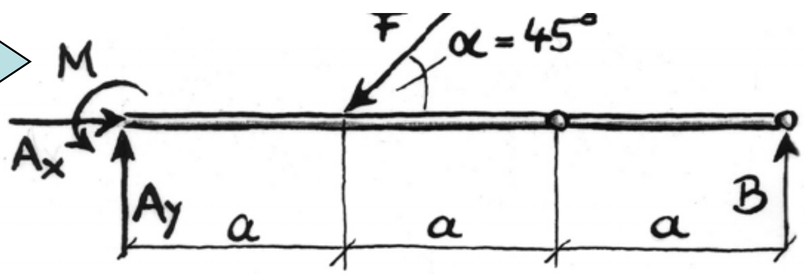


FBD

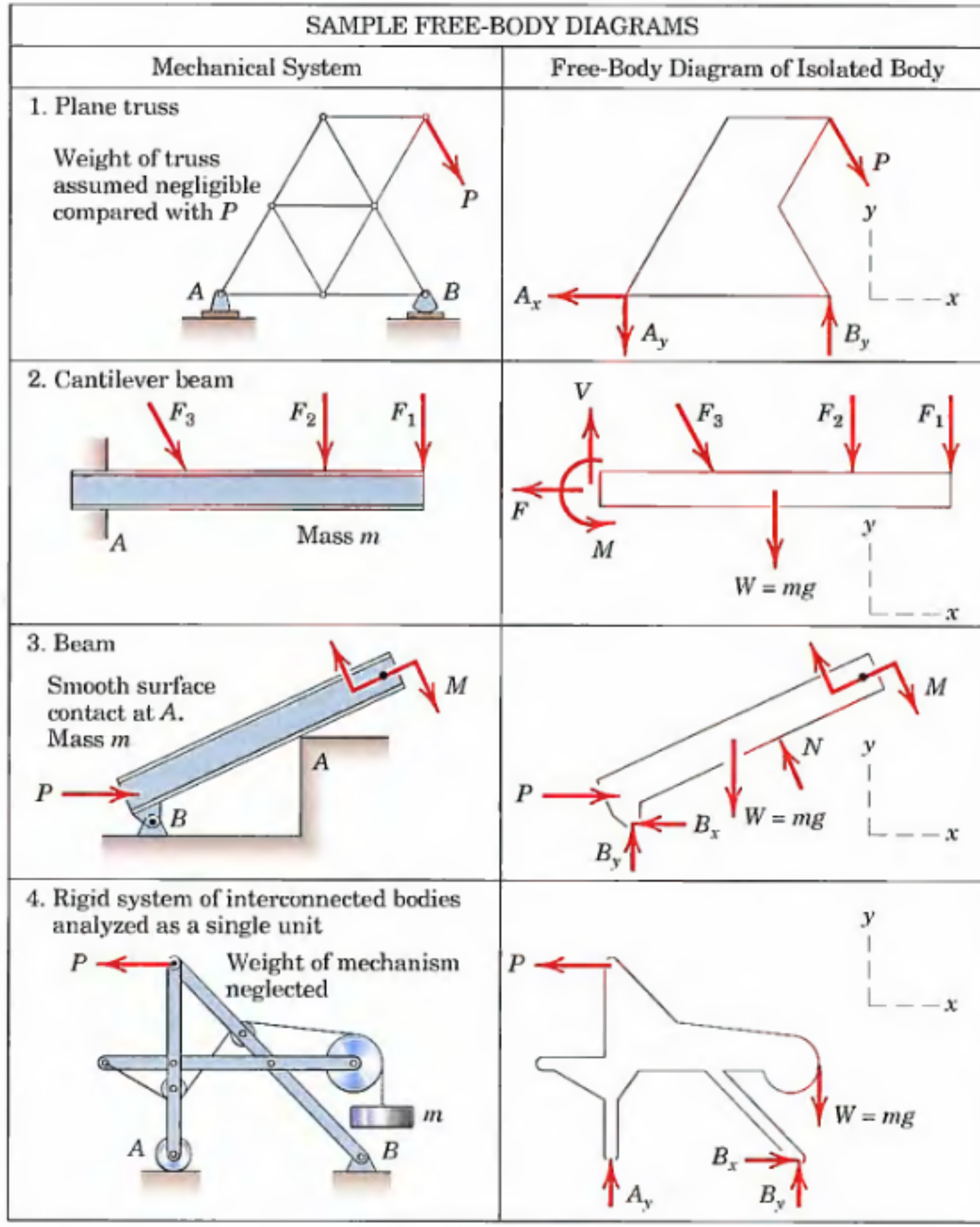




FBD



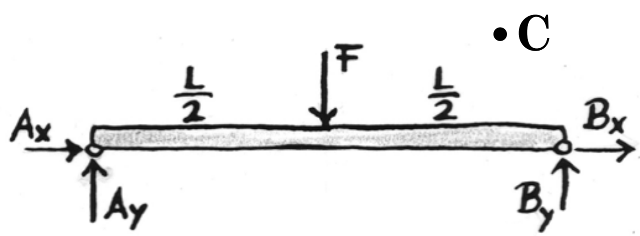
More Examples of FBD



Equations of equilibrium in 2D

- ***Three*** equations per free body.
- More than ***three*** equations per free body is illegal.

$$\sum F_x = 0 \qquad (1)$$
$$\sum F_y = 0 \qquad (2)$$
$$\curvearrowright \sum M_A = 0 \qquad (3)$$



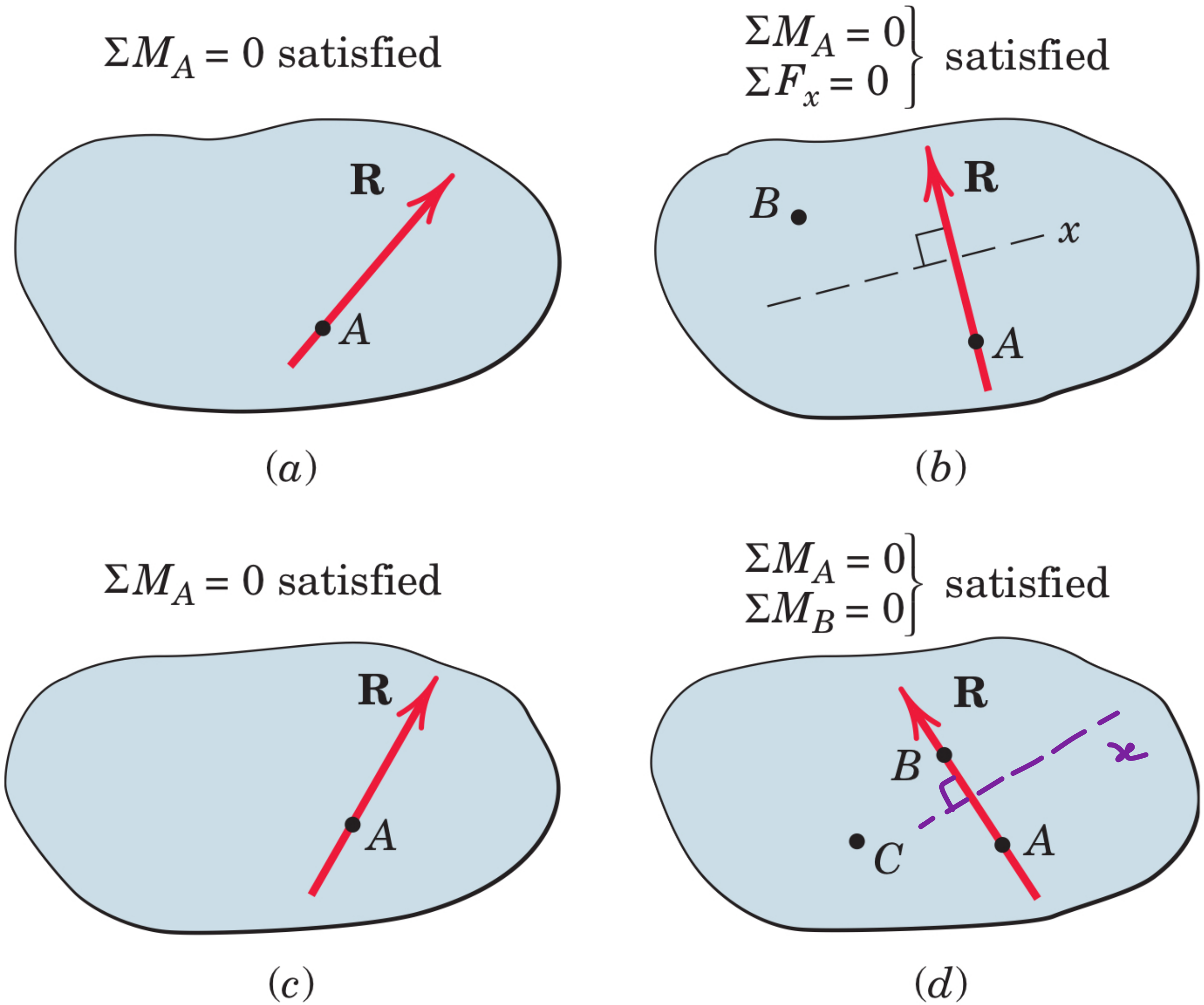
We can also use equations like this

$$\sum F_x = 0 \qquad (1)$$
$$\sum M_B = 0 \qquad (2)$$
$$\sum M_A = 0 \qquad (3)$$

or like this where A, B, C are not in a straight line

$$\sum M_A = 0 \qquad (1)$$
$$\sum M_B = 0 \qquad (2)$$
$$\sum M_C = 0 \qquad (3)$$

Another way to understand equations of equilibrium



only way to satisfy

$\Sigma M_A = 0$

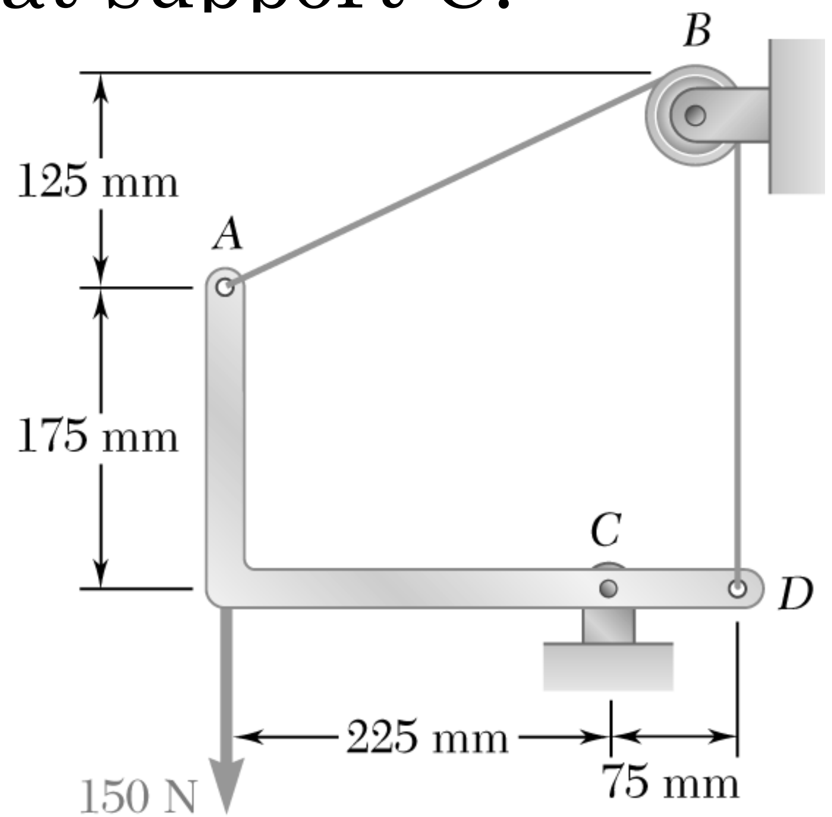
$\Sigma M_B = 0$

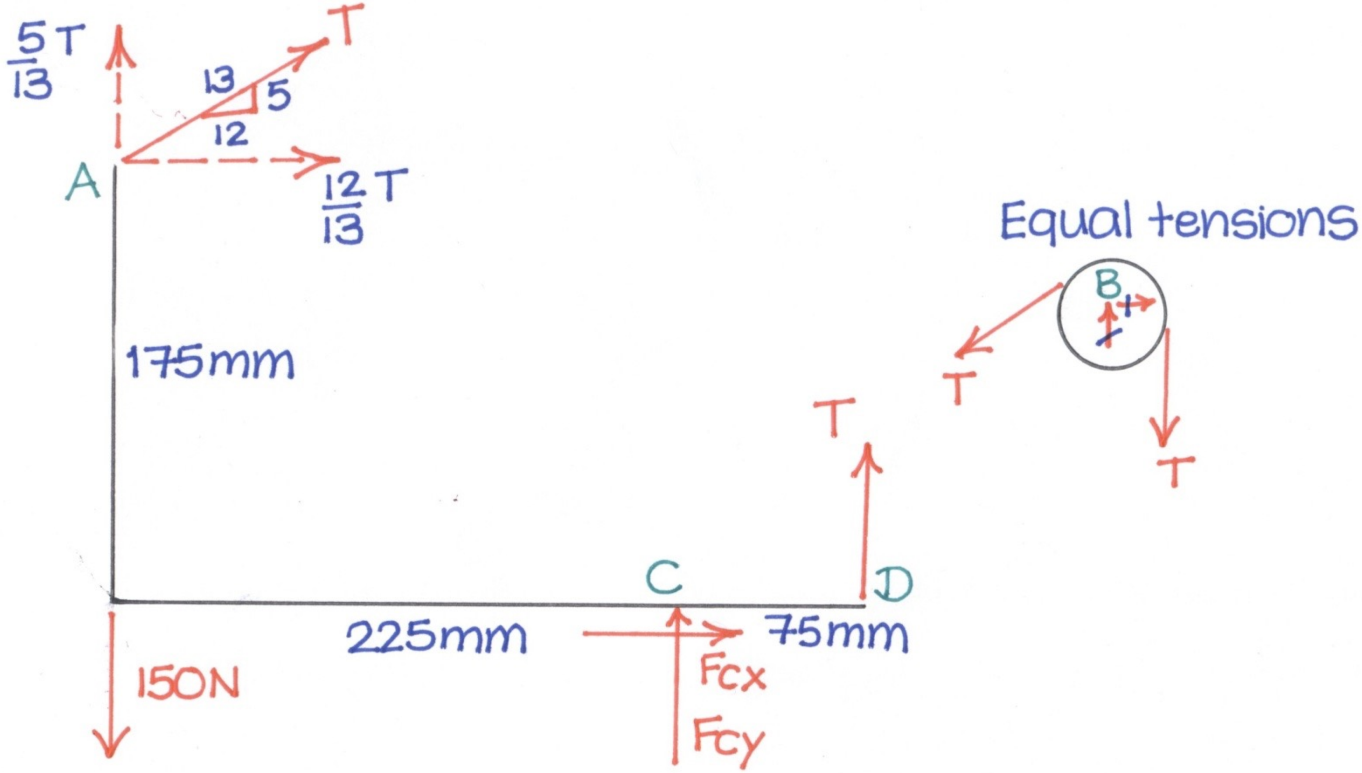
$\Sigma M_C = 0$

if $\vec{F} = 0$

Problem 1

- Determine the tension in cable *ABD* and reaction at support *C*.





Without string ACD rotates about C

$$\circlearrowright \Sigma M_C = \frac{12}{13} T * 175 + \frac{5}{13} T * 225 - 150 * 225 - T * 75$$

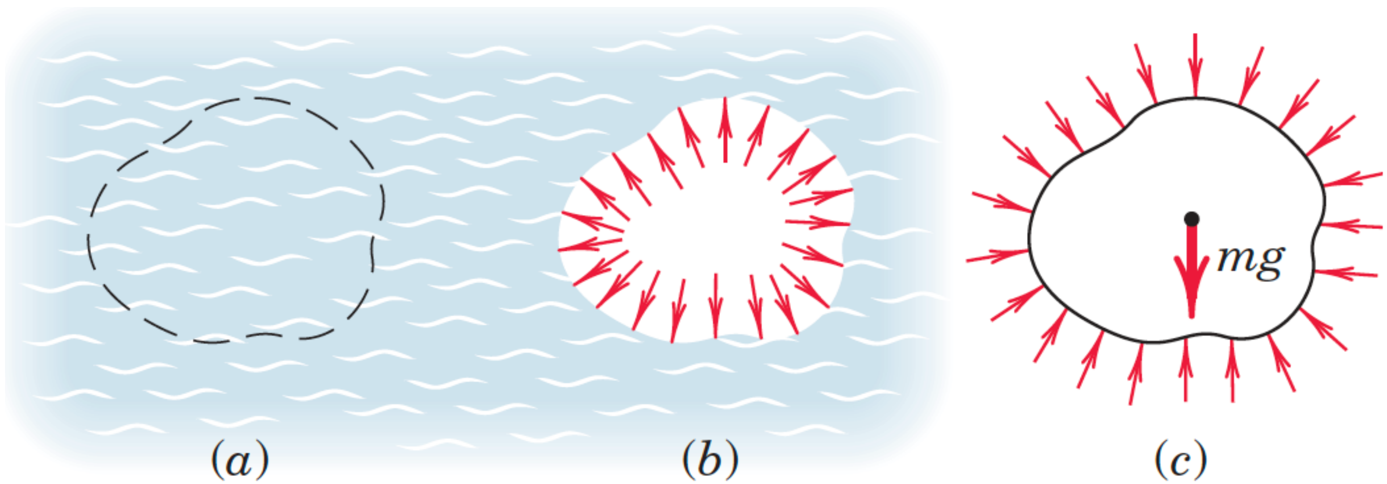
$$= 0$$

$$\Rightarrow T = 195\,N$$

$$\Sigma F_x \rightarrow = 0 \Rightarrow F_{Cx} = -180N; \; 180N \leftarrow$$

$$\Sigma F_y \uparrow = 0 \Rightarrow F_{Cy} = -120N \quad 120N \downarrow$$

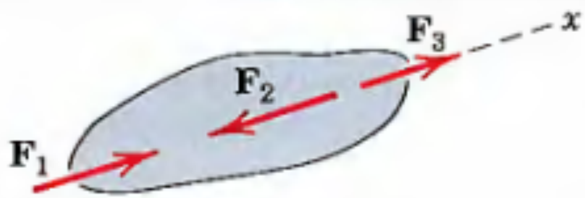
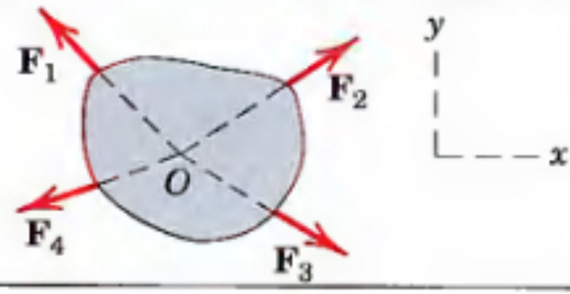
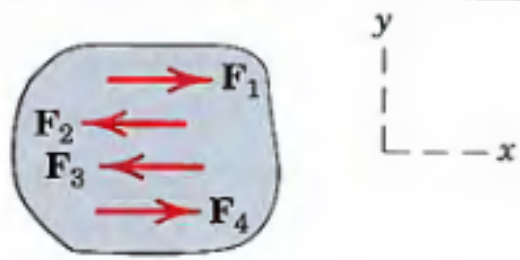
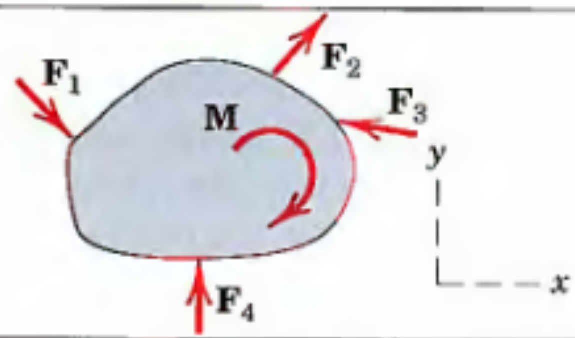
Hydrostatic force: FBD



$F = \rho_w g V$

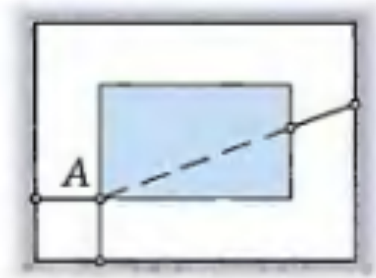
V = volume of mass

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS

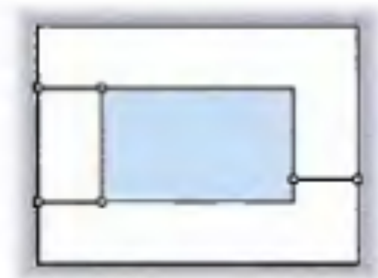
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\quad \Sigma M_z = 0$ $\Sigma F_y = 0$

Adequacy of Constraints

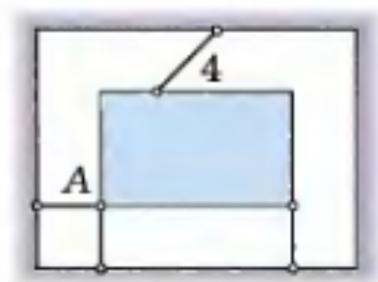
(*a*) Complete fixity
Adequate constraints



(*b*) Incomplete fixity
Partial constraints



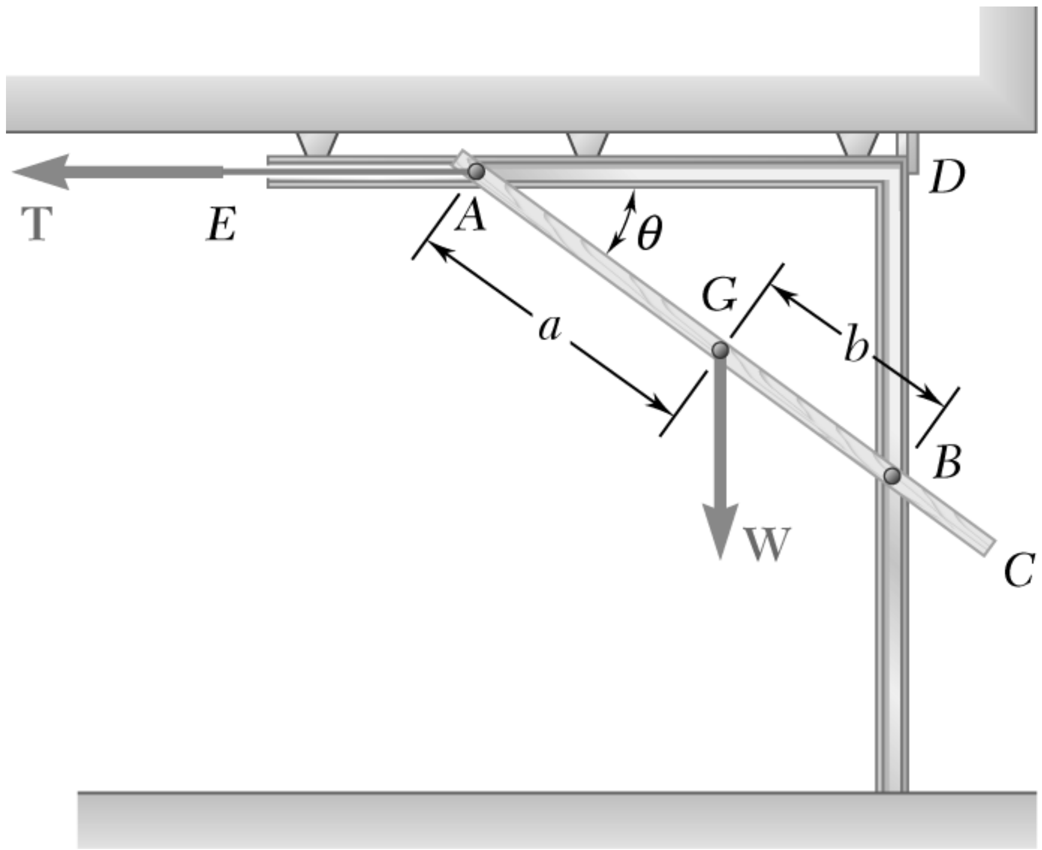
(*c*) Incomplete fixity
Partial constraints

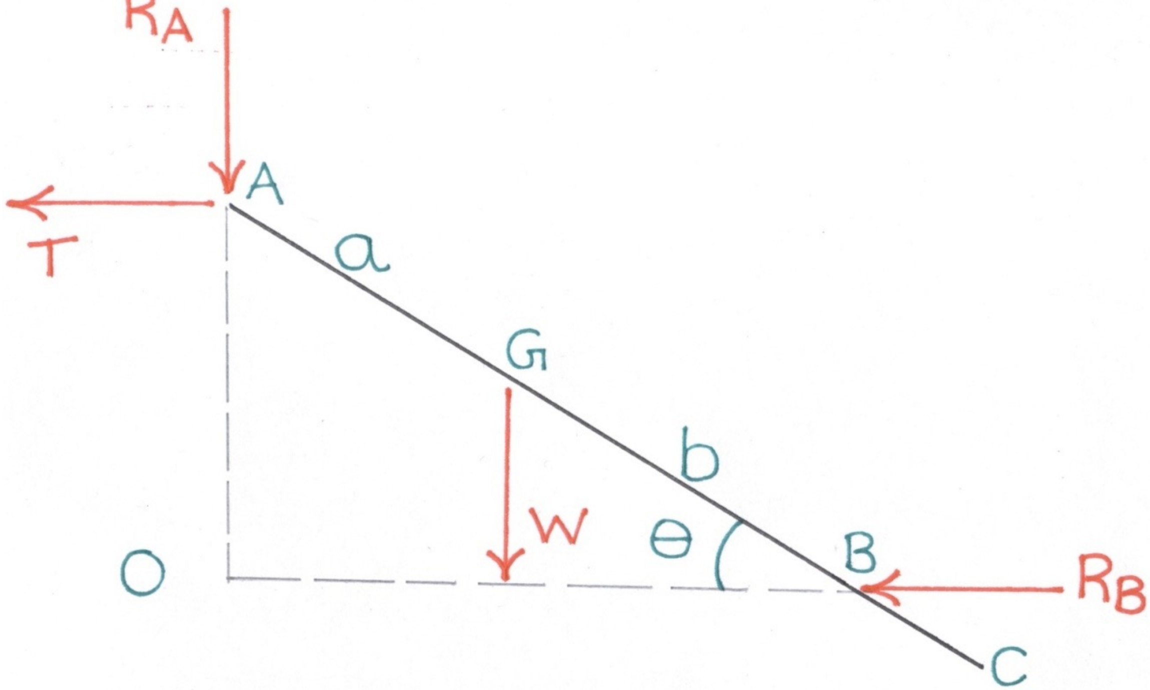


Single Rigid Body Supported Globally

Problem 2

- A 70 kg (W) overhead garage door consists of a uniform rectangular panel AC 2100 mm high (h), supported by the cable AE attached at the middle of the upper edge of the door and by two sets of frictionless rollers at A and B. Each set consists of two rollers one either side of the door. The rollers A are free to move in horizontal channels, while rollers B are guided by vertical channels. If the door is held in the position for which BD=1050 mm, determine (a) the tension in the cable AE, (2) the reaction at each of the four rollers. Assume a = 1050 mm, b = 700mm





R_A, R_B intersect @ O

T balances torque of W about O

$$\circlearrowleft \Sigma M_O = -T(a+b)\sin\theta + Wa\cos\theta = 0$$

$$\Rightarrow T = \frac{Wa\cos\theta}{(a+b)\sin\theta} = 549.4\text{ N}$$

(÷ 2)

$$\rightarrow \Sigma F_x = 0 \Rightarrow F_{Bx} = 549.4\text{ N}$$

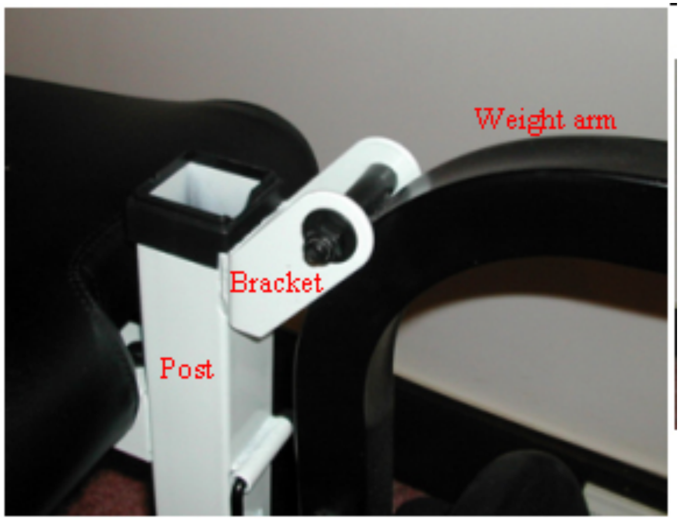
(÷ 2)

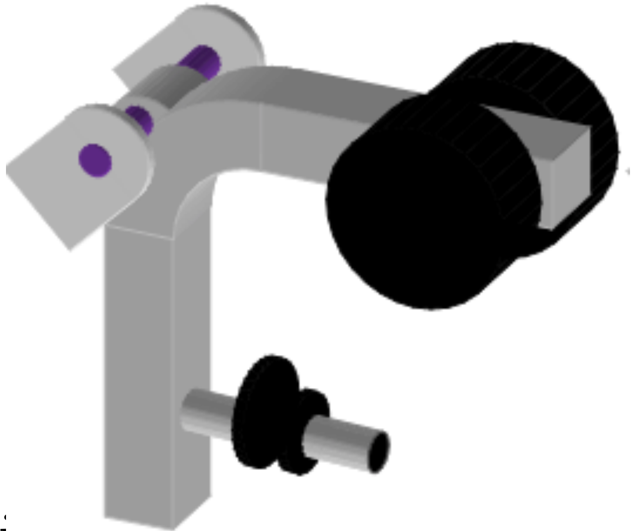
$$\uparrow \Sigma F_y = 0 \Rightarrow R_A = 686.7\text{ N}$$

(÷ 2)

Multiple Rigid Bodies Connected To Each Other

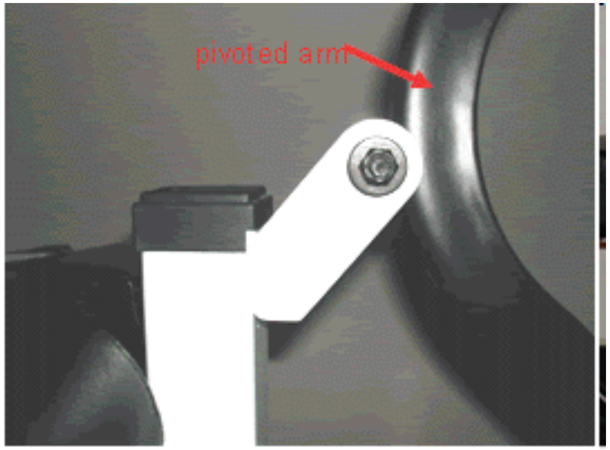
Pin Connections

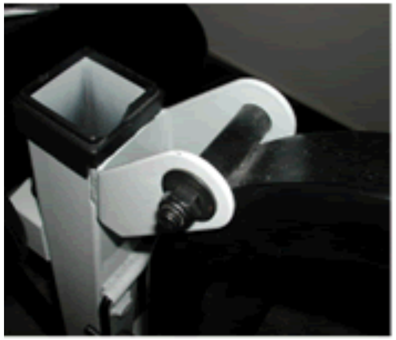




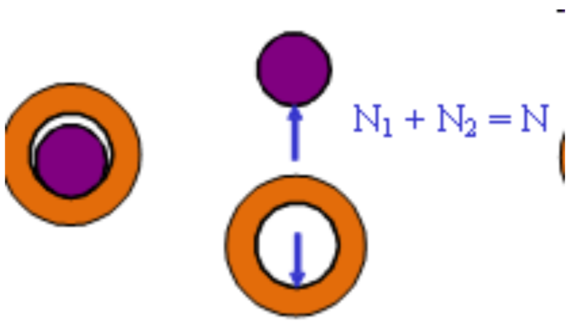
- All figures from http://oli.web.cmu.edu

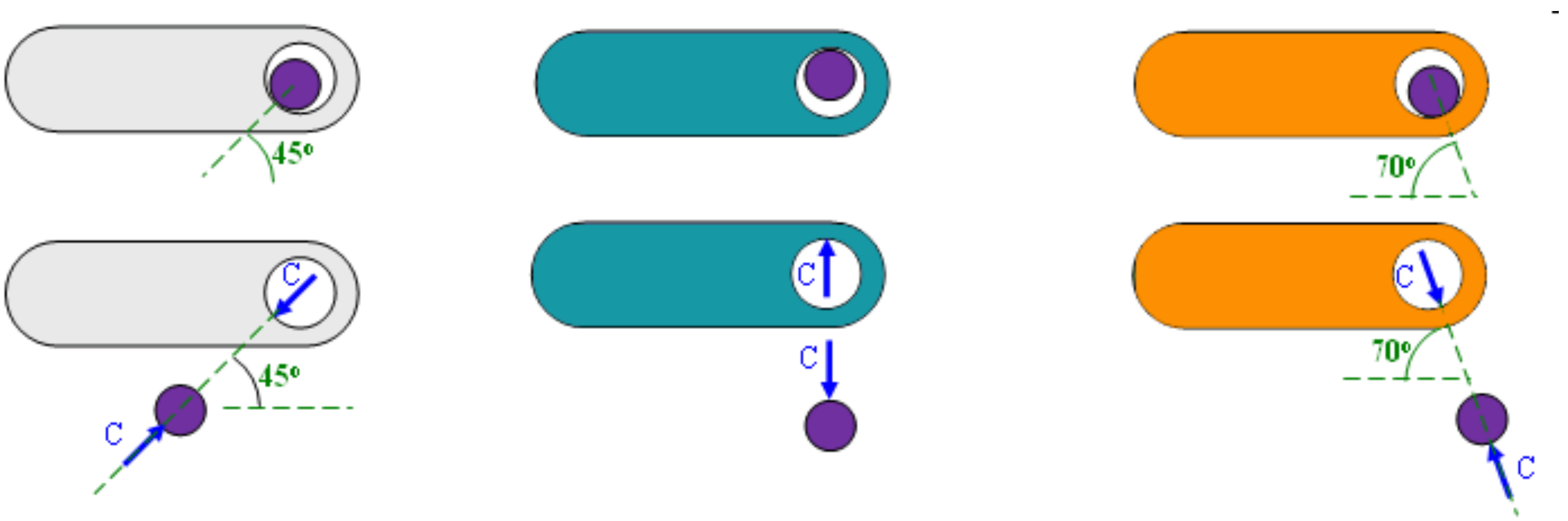
Modeling 3D Problem as 2D



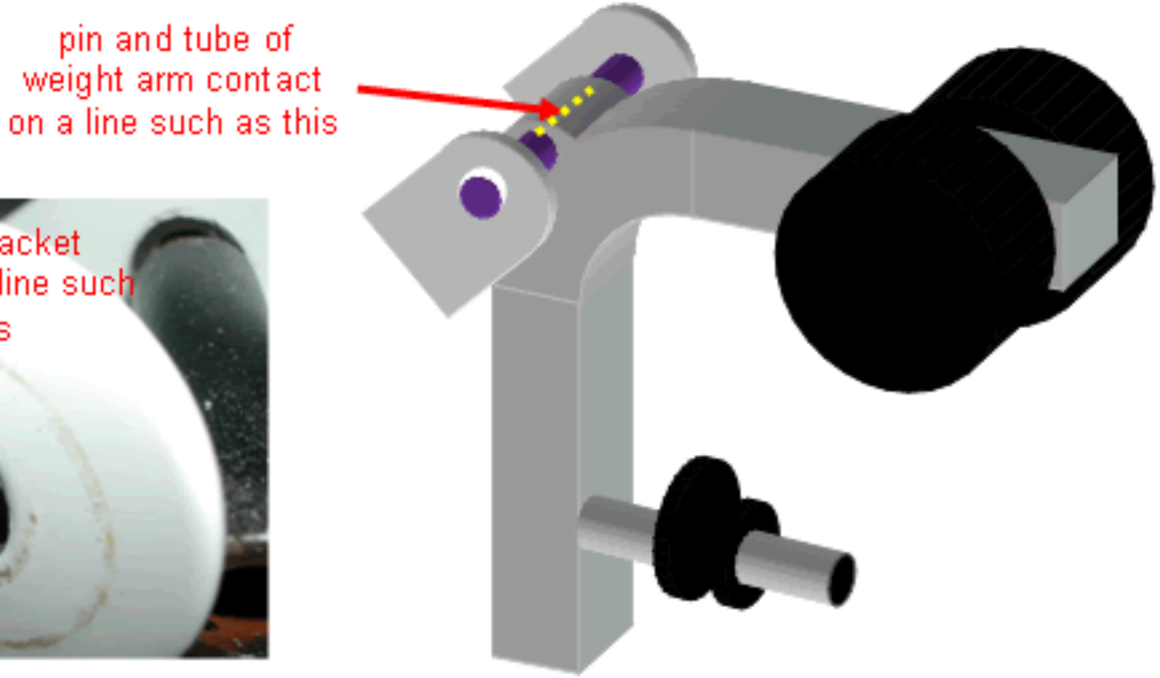


Point Connections

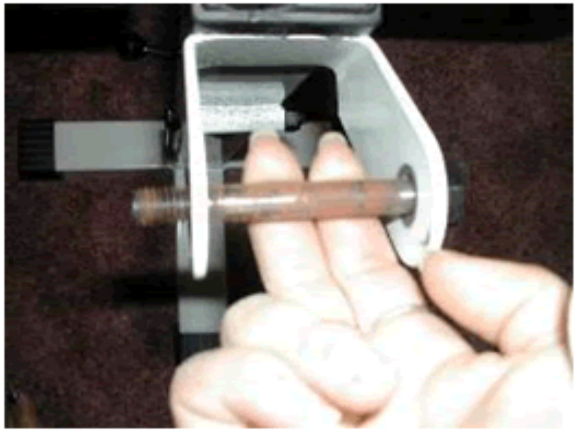




Contd..


pin and tube of
weight arm contact
on a line such as this


pin and bracket
contact on a line such
as this



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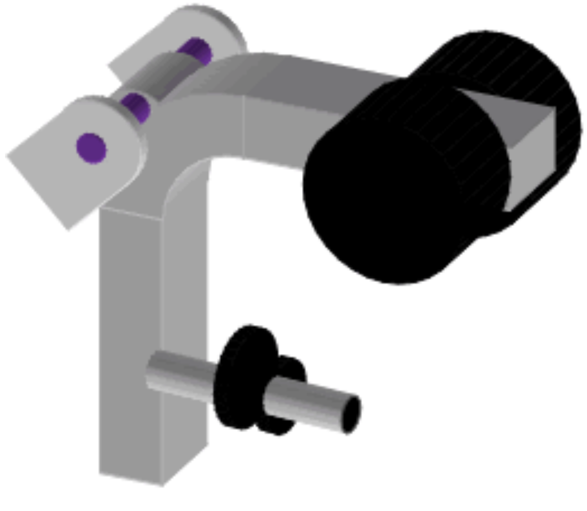
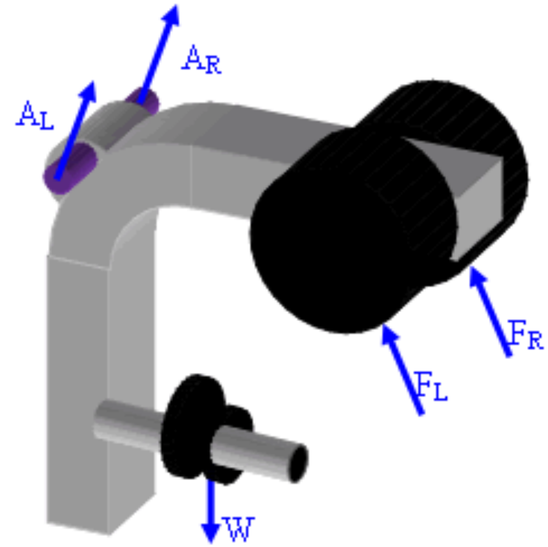
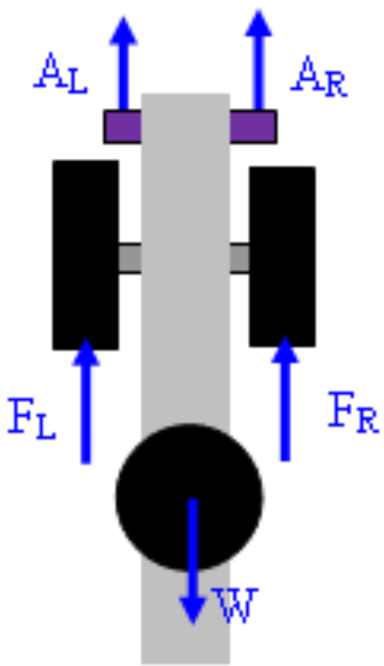
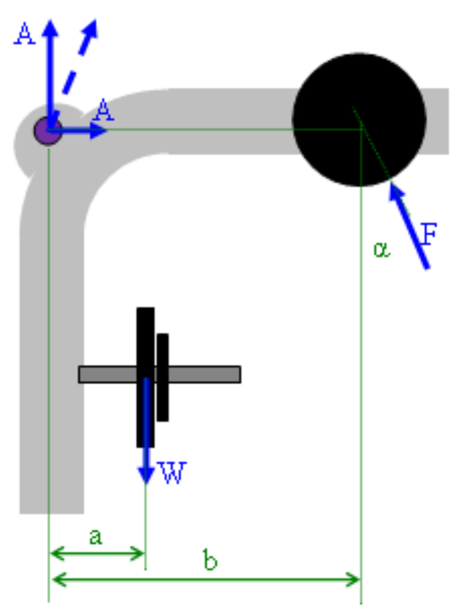
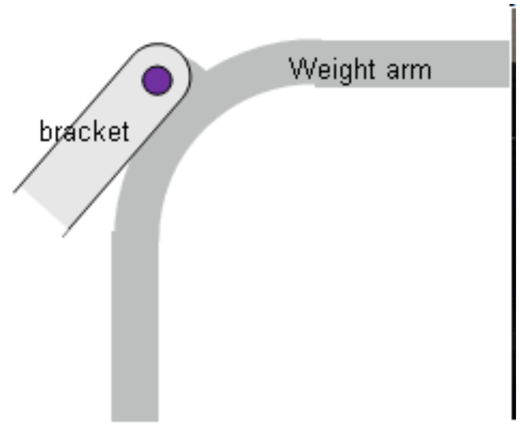


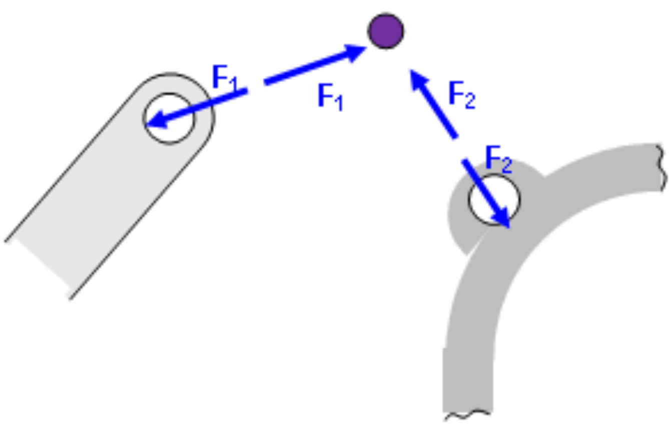


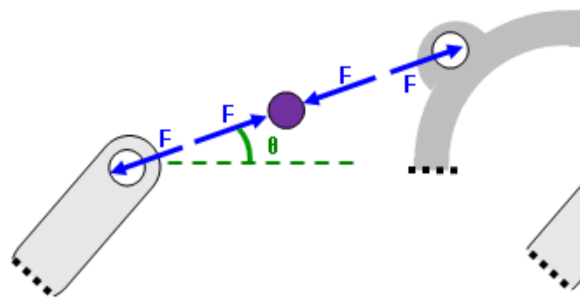


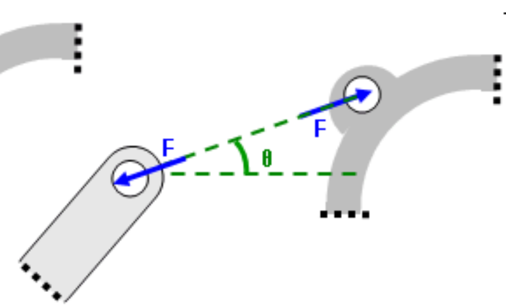


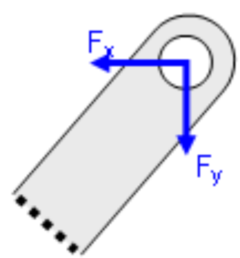
Free Body Diagram at Pin-Connection

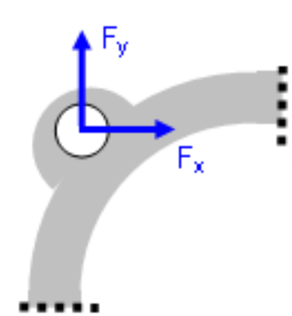




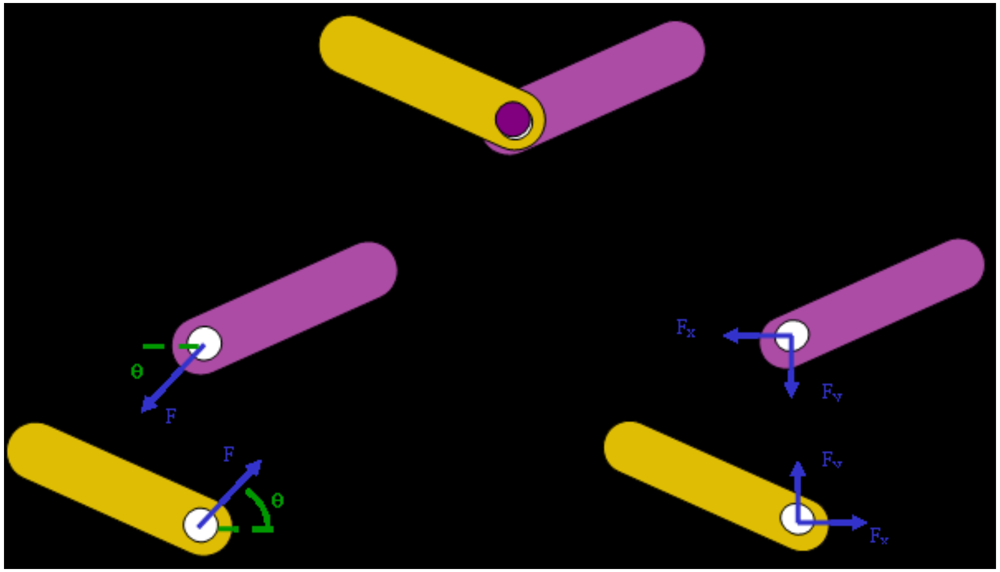






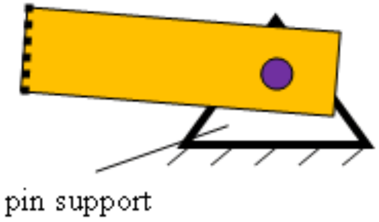


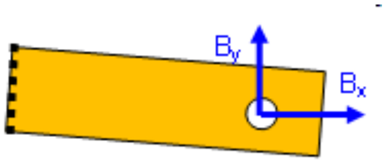
Summary



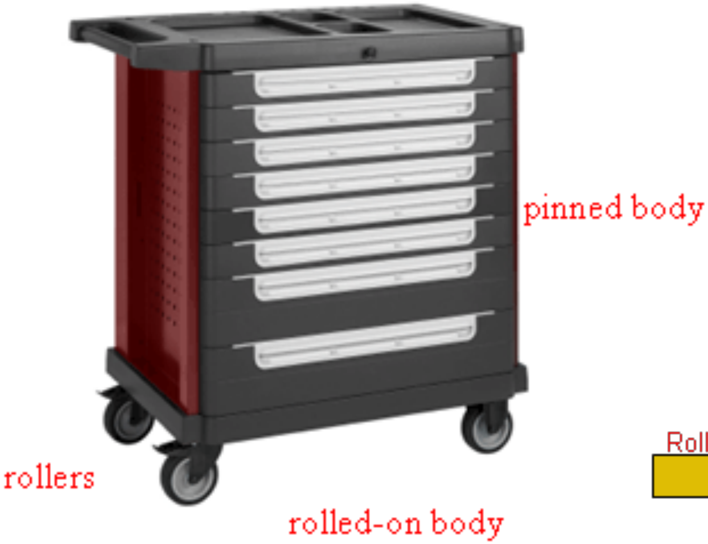
• Pin connection

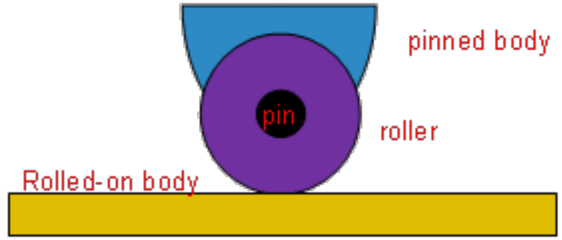
• Pin support

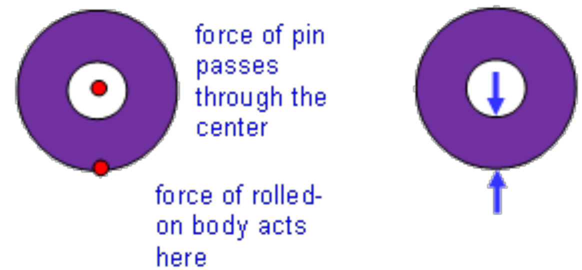


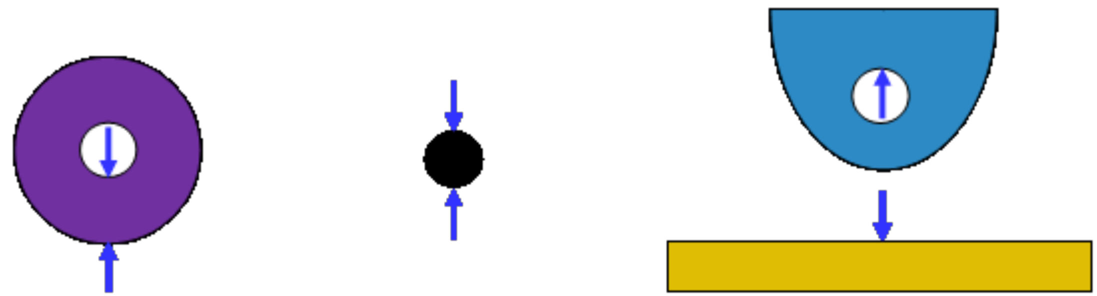


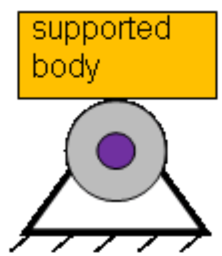
Roller



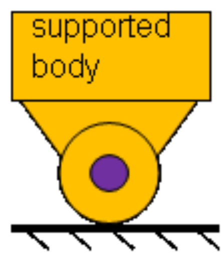






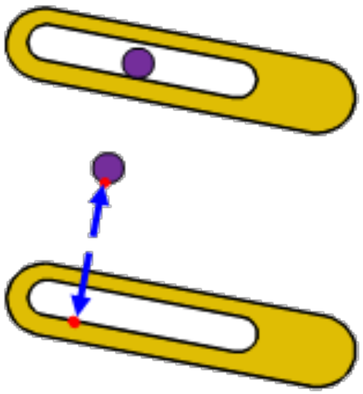


roller pinned to ground (stationary)

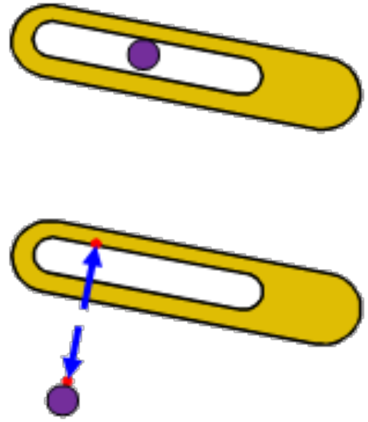


roller and pinned body move with respect to ground

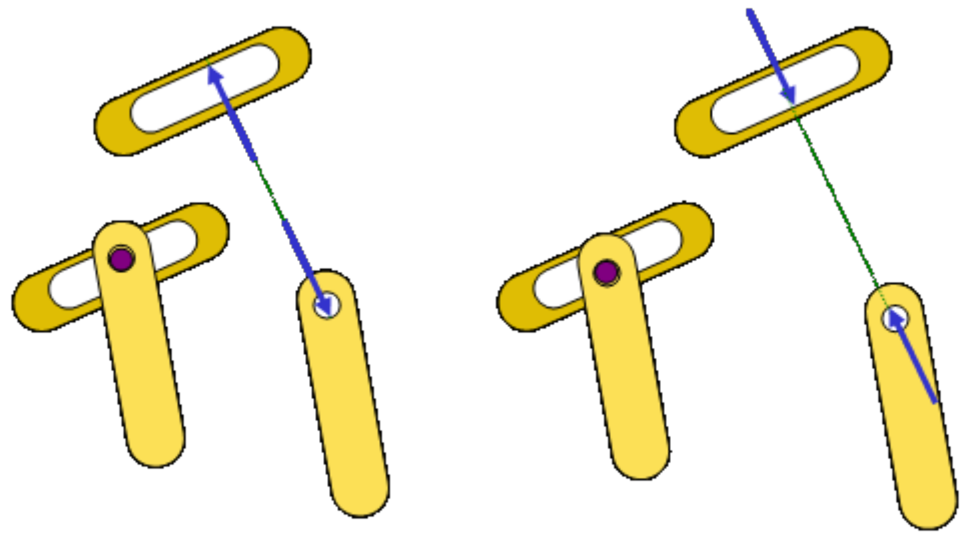
Slot Connection



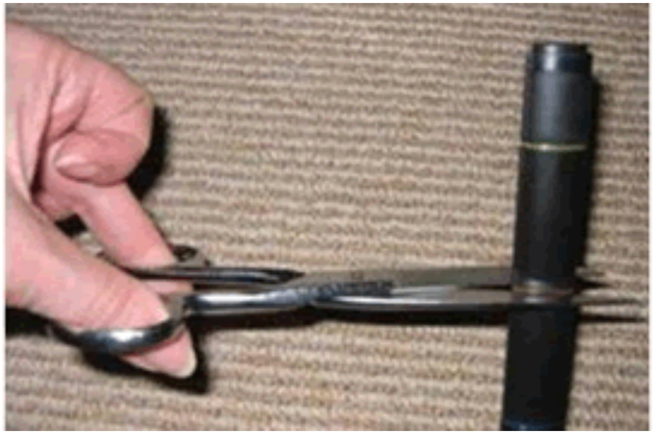
pin (or roller) contacts
lower part of slot



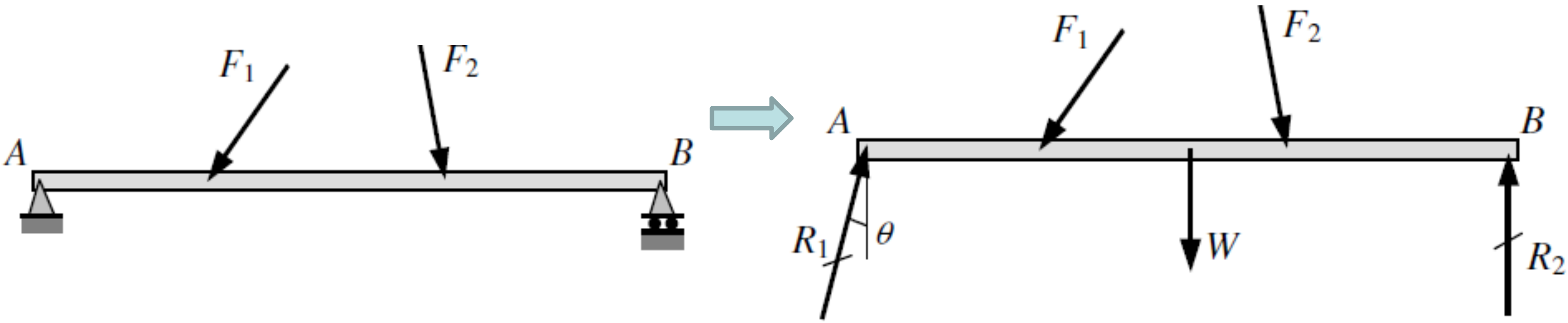
pin (or roller) contacts
upper part of slot

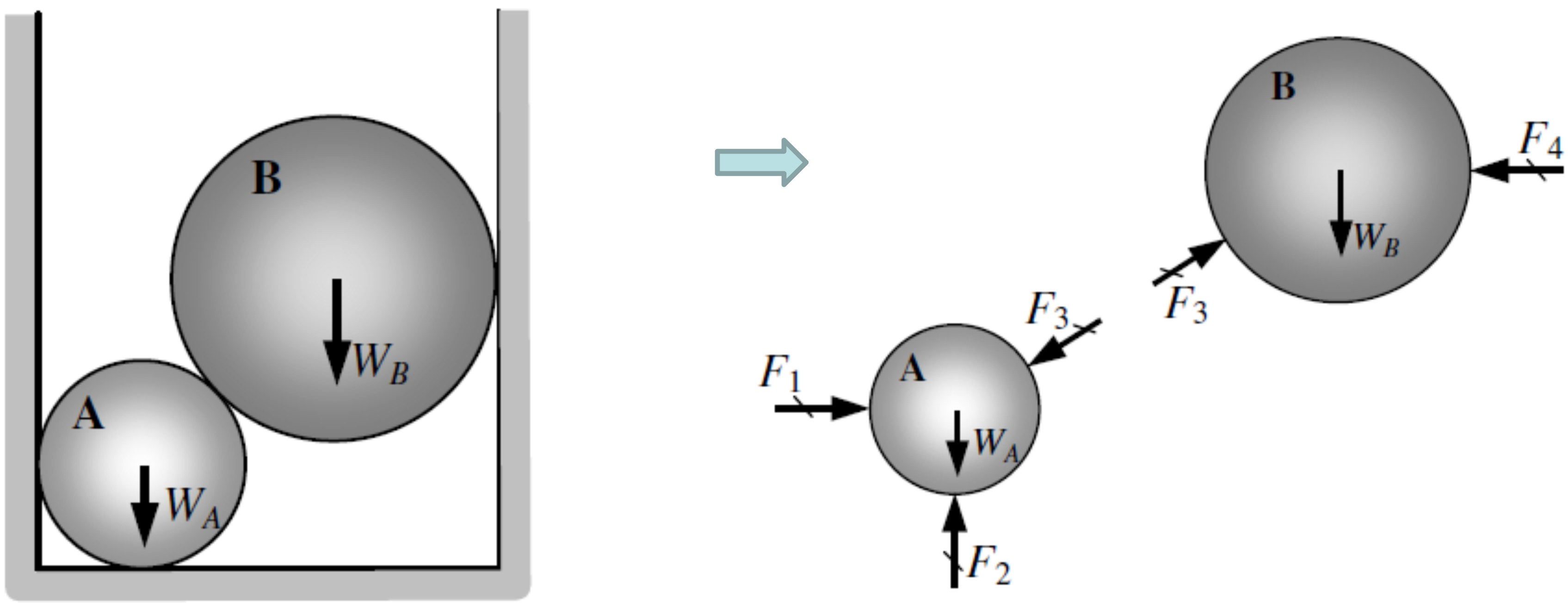


Non-Symmetrical but bodies connected by pin are very close to each other



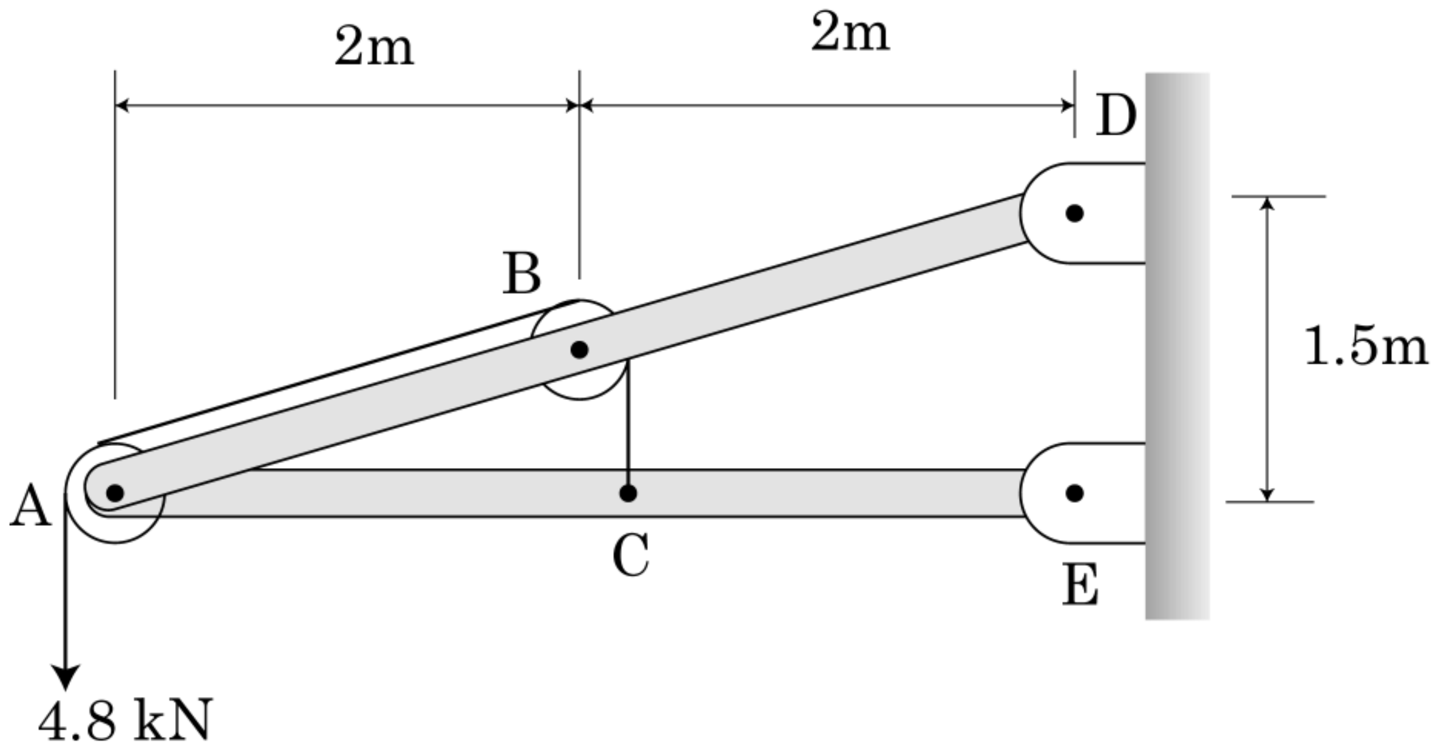
Free body diagrams – Examples

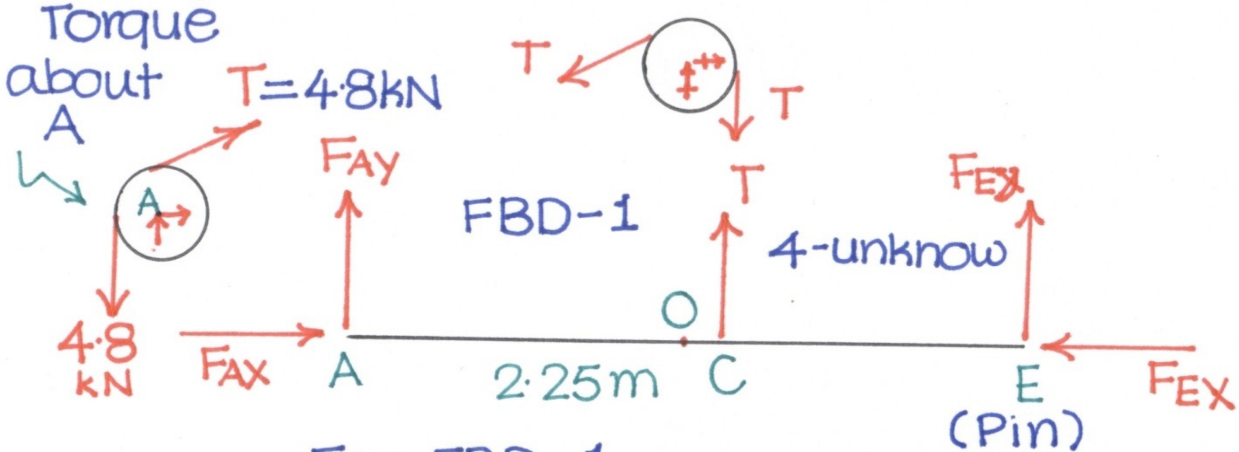




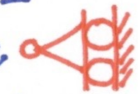
Problem 3

- Knowing that each pulley has a radius of 250mm, determine the components of reactions at *D* and *E*.





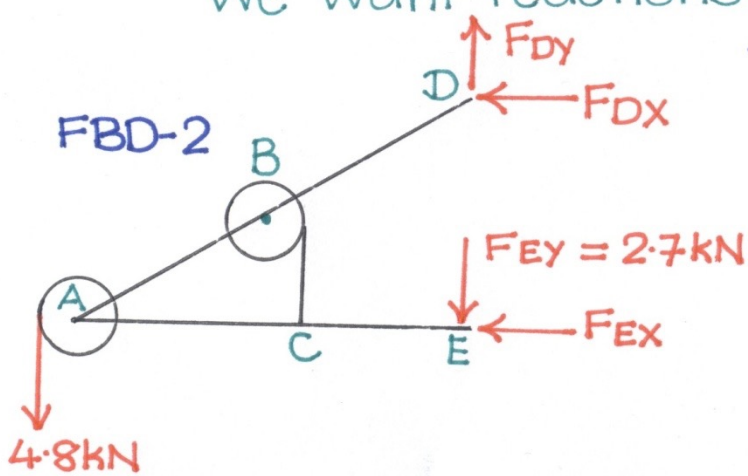
For FBD-1

If 'E' were  AE would be imbalanced w.r.t rotation about A

$\circlearrowright \sum M_A = 0 \Rightarrow F_{Ey} = -2.7\,kN$

OR $2.7\,kN \downarrow$

We want reactions at D, E



$\circlearrowright \sum M_D = 0$

$\Rightarrow F_{Ex} = 13.6\,kN \leftarrow$

$\rightarrow \sum F_x = 0$

$\Rightarrow F_{Dx} = -13.6\,kN \rightarrow$

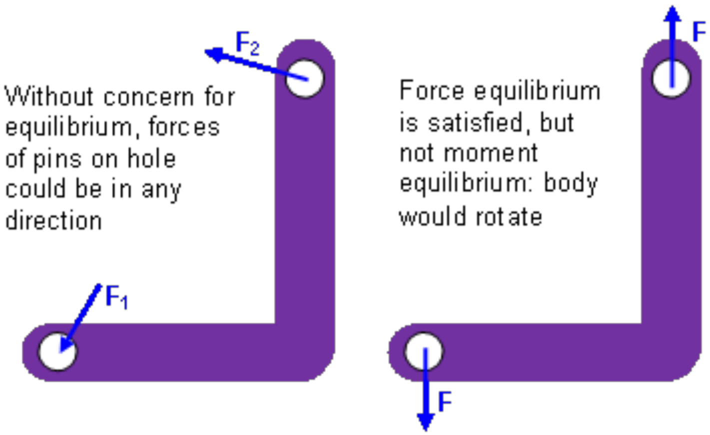
$\uparrow \sum F_y = 0$

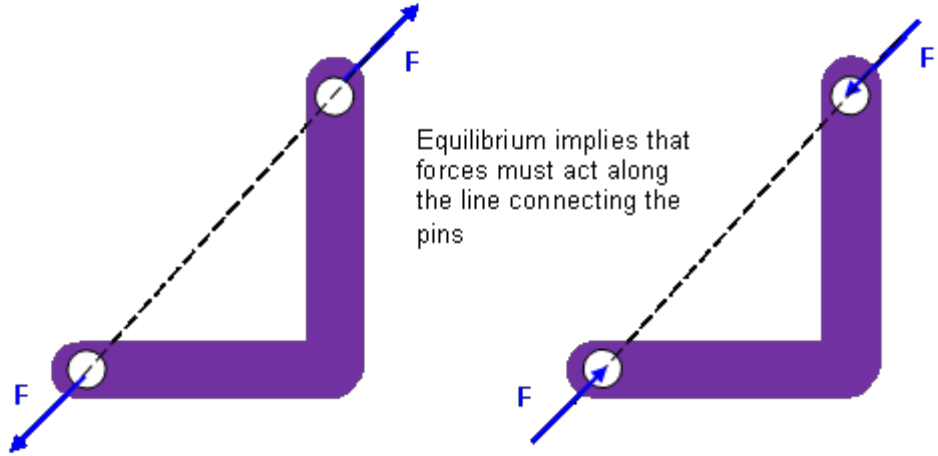
$\Rightarrow F_{Dy} = 7.5\,kN \uparrow$

Link: Two-Force Member

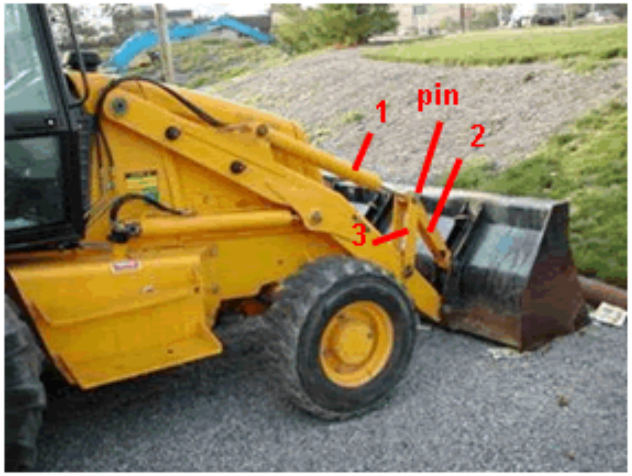
- Member with negligible weight and arbitrary shape connected to other members by pins

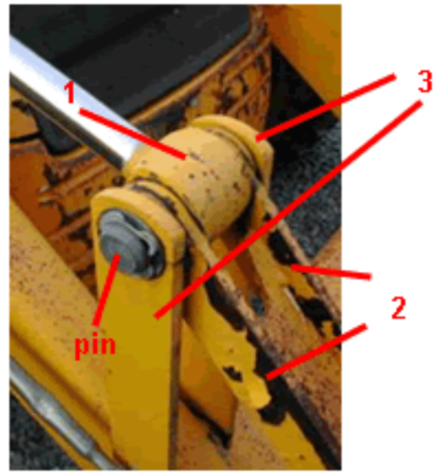
Two Force member





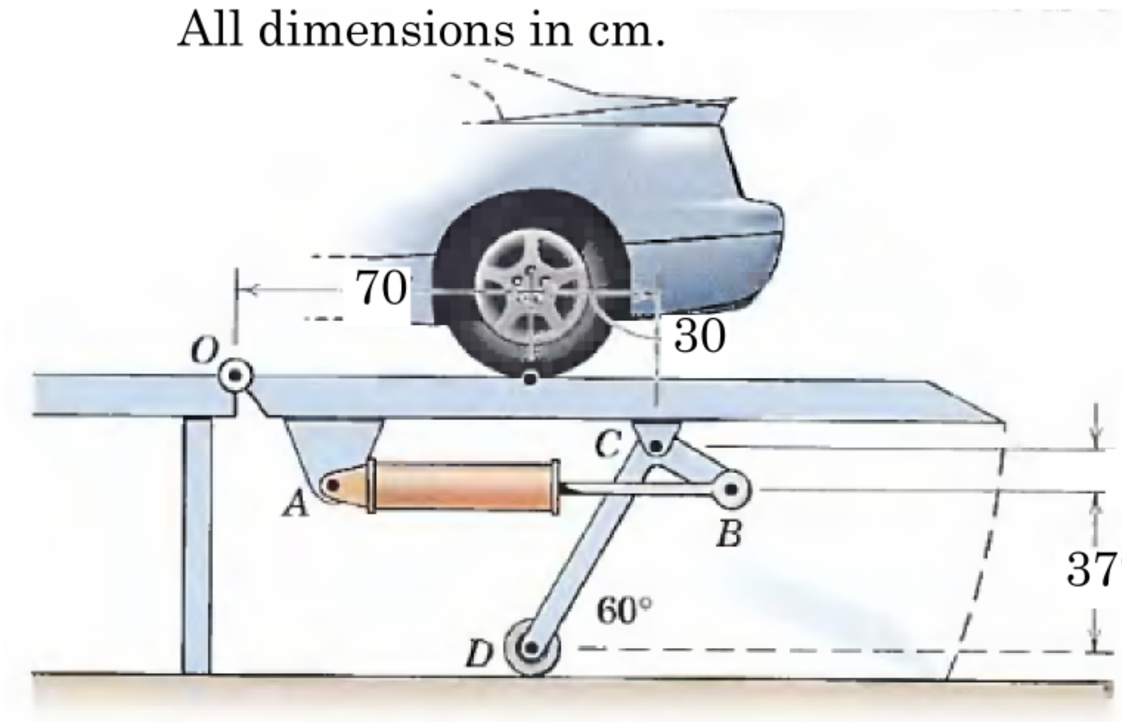
Hydraulic Cylinder

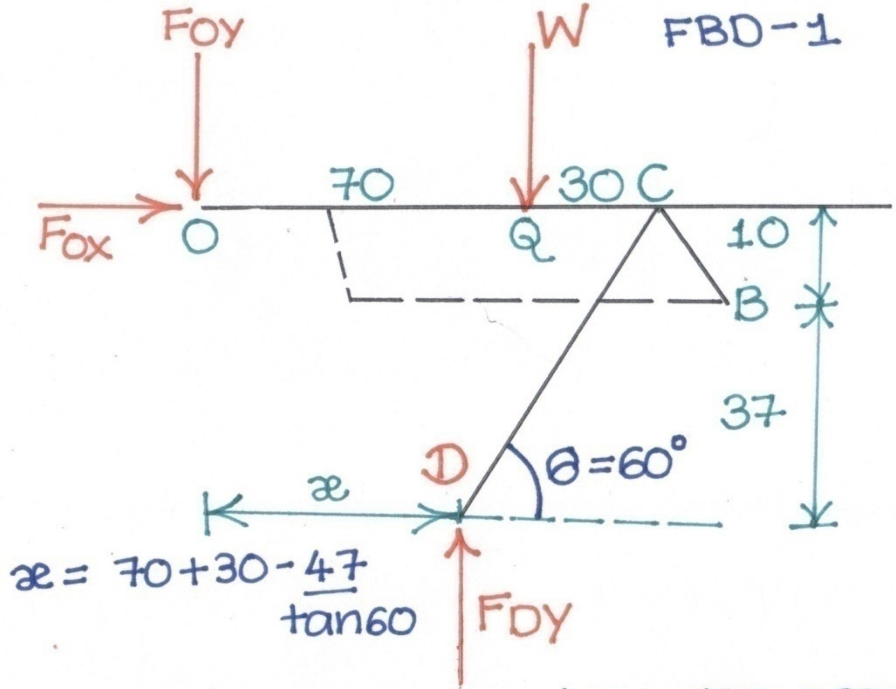




Problem 4

- The car hoist allows the car to be driven on to the platform, after which the rear wheel is raised. If the loading from the rear wheel is 3300kg, determine the force in the hydraulic cylinder AB. Neglect the weight of the platform itself. Member BCD is a right angle bell crank pinned to the ramp at C

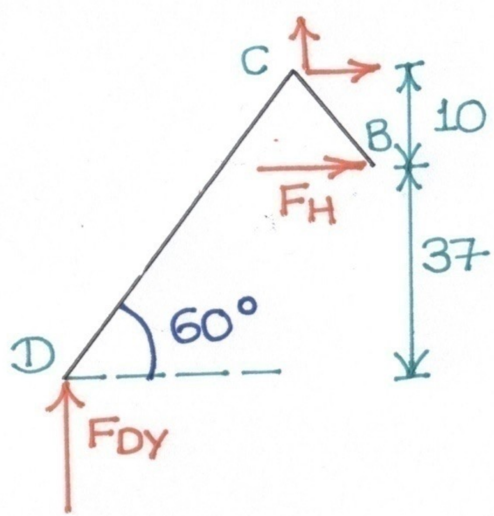




$$x = 70 + 30 - \frac{47}{\tan 60}$$

Without reaction at D entire assembly rotates about O; For FBD-1

$$\circlearrowleft \sum M_O = 0 \Rightarrow F_{DY} = 0.961W$$



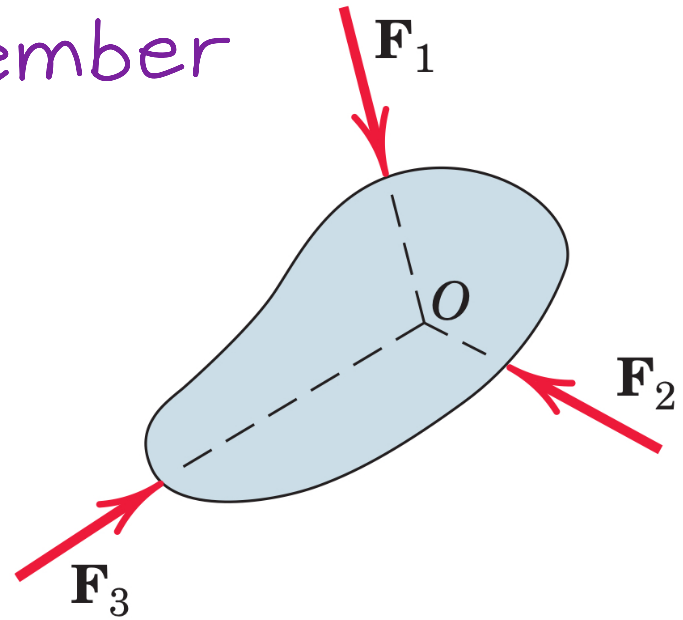
$F_H \rightarrow$ 2-force

without F_H : DCB rotates about 'C'

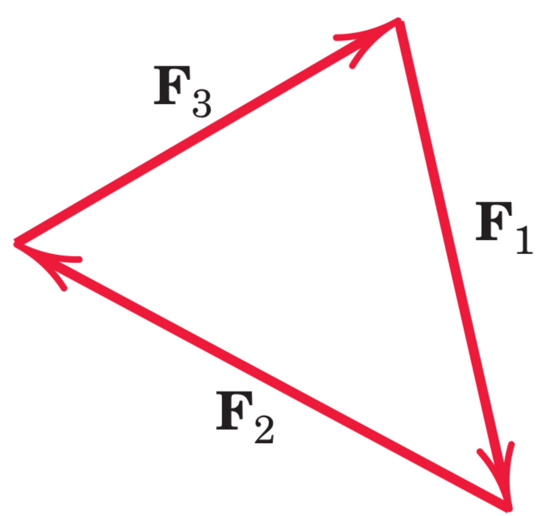
$$\circlearrowleft \sum M_C = 0 \Rightarrow F_H = 84.4\,kN \text{ (C)}$$

$$F_H = 2.61W$$

3-force member

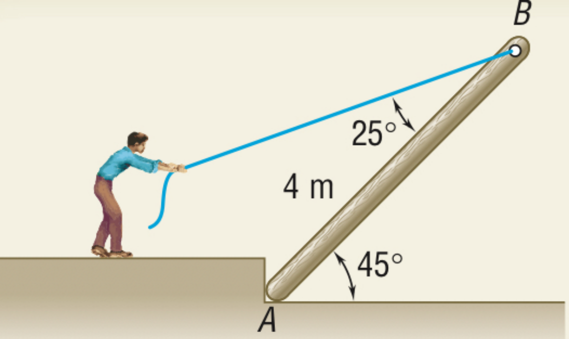


(*a*) Three-force member

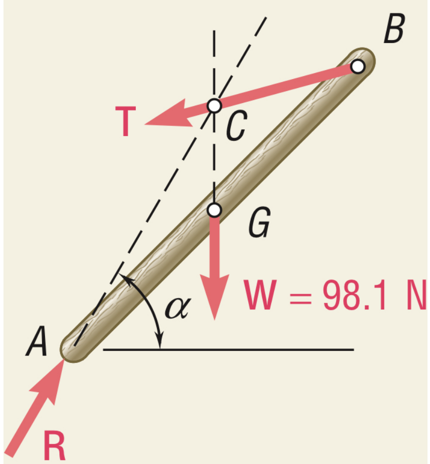


(*b*) Closed polygon satisfies $\Sigma \mathbf{F} = \mathbf{0}$

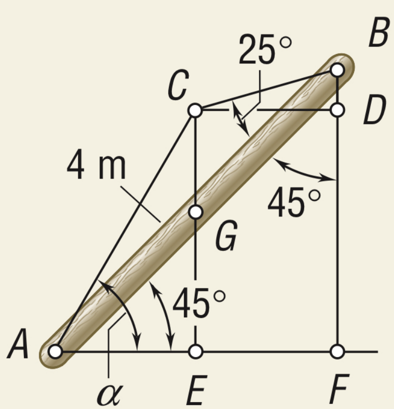
SAMPLE PROBLEM 4.6



A man raises a 10-kg joist, of length 4 m, by pulling on a rope. Find the tension T in the rope and the reaction at A.

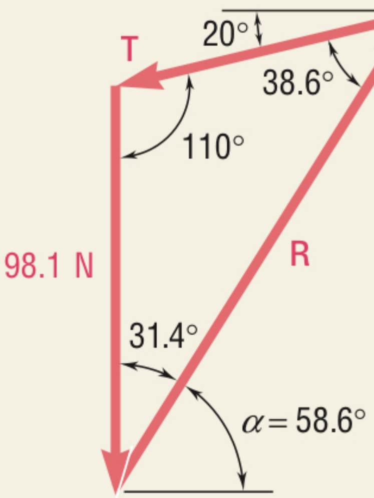


$$AF = BF = (AB)\cos 45° = (4\text{ m})\cos 45° = 2.828\text{ m}$$
$$CD = EF = AE = \tfrac{1}{2}(AF) = 1.414\text{ m}$$
$$BD = (CD)\cot(45° - 25°) = (1.414\text{ m})\tan 20° = 0.515\text{ m}$$
$$CE = DF = BF - BD = 2.828\text{ m} - 0.515\text{ m} = 2.313\text{ m}$$



$$\tan a = \frac{CE}{AE} = \frac{2.313\text{ m}}{1.414\text{ m}} = 1.636$$

$$\alpha = 58.6°$$ ◀



Force Triangle. A force triangle is drawn as shown, and its interior angles are computed from the known directions of the forces. Using the law of sines, we write

$$\frac{T}{\sin 31.4°} = \frac{R}{\sin 110°} = \frac{98.1\ \text{N}}{\sin 38.6°}$$

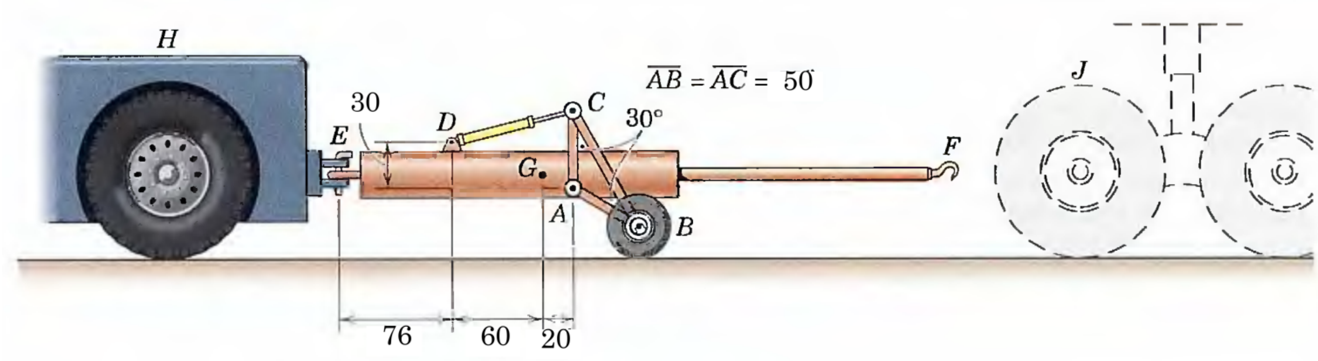
$T = 81.9\ \text{N}$ ◀

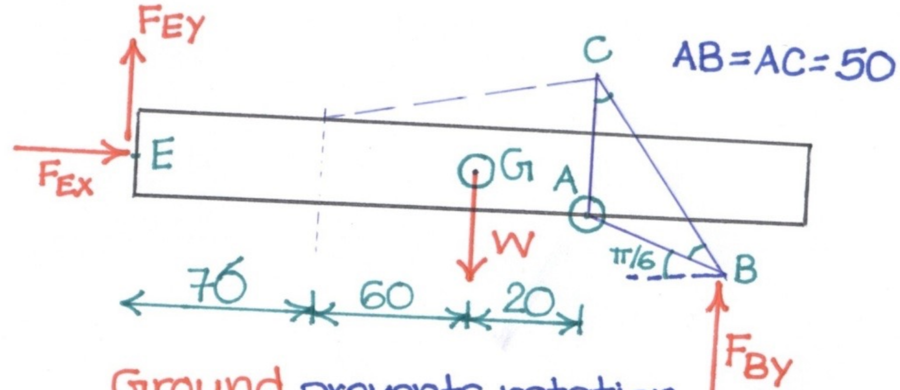
$\text{R} = 147.8\ \text{N}\ \mathbf{a}\,58.6°$ ◀

Problem 5

- An adjustable tow bar connecting the tractor unit *H* with the landing gear *J* of a large aircraft is shown in the figure. Adjusting the height of the hook *F* at the end of the tow bar is accomplished by the hydraulic cylinder *CD* is activated by a small hand-pump (not shown). For the nominal position shown of the triangular linkage *ABC*, calculate the force *P* supplied by the cylinder to the pin *C* to position the tow bar. The rig has a total weight of 220kg and is supported by the tractor hitch *E*.

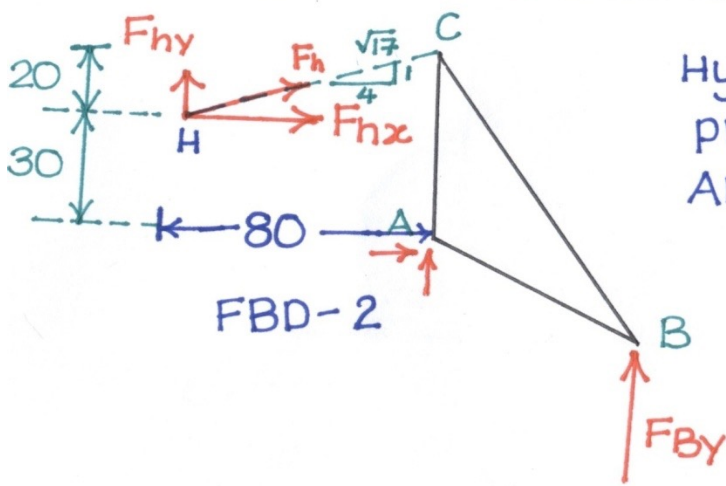
All dimensions in cm.





Ground prevents rotation about E

$\circlearrowleft \Sigma M_E = 0 \Rightarrow F_{BY} = 0.682W$

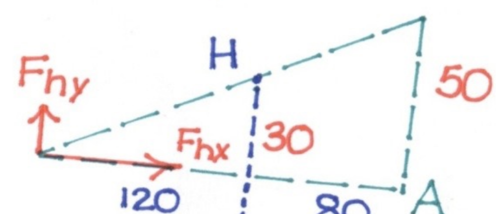


FBD-2

Hydraulic cylinder prevents rotation of ABC about A

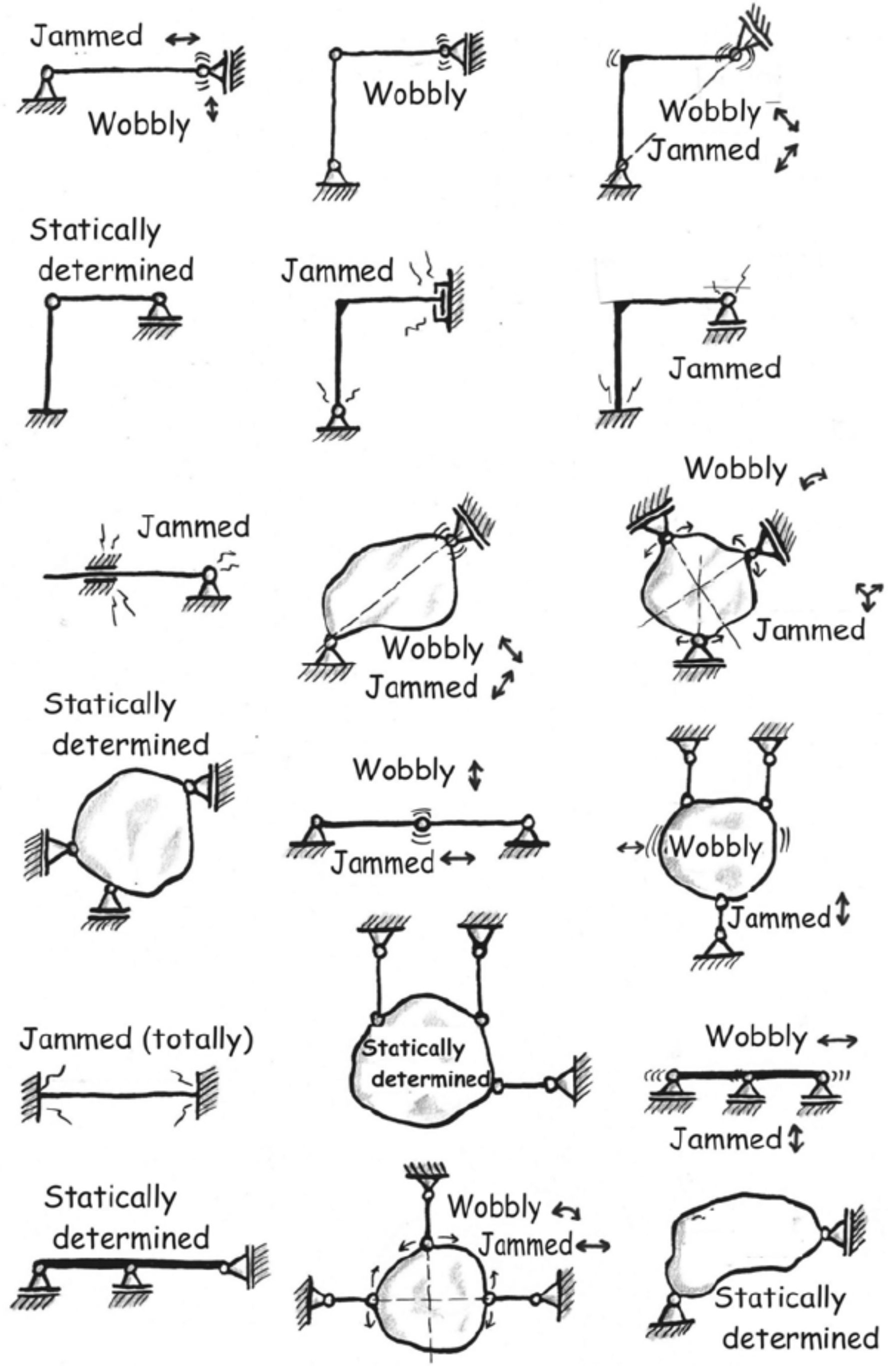
$\circlearrowleft \Sigma M_A = 0 \Rightarrow F_{hy} = 0.148W$

$\Rightarrow F_h \approx 0.61W$



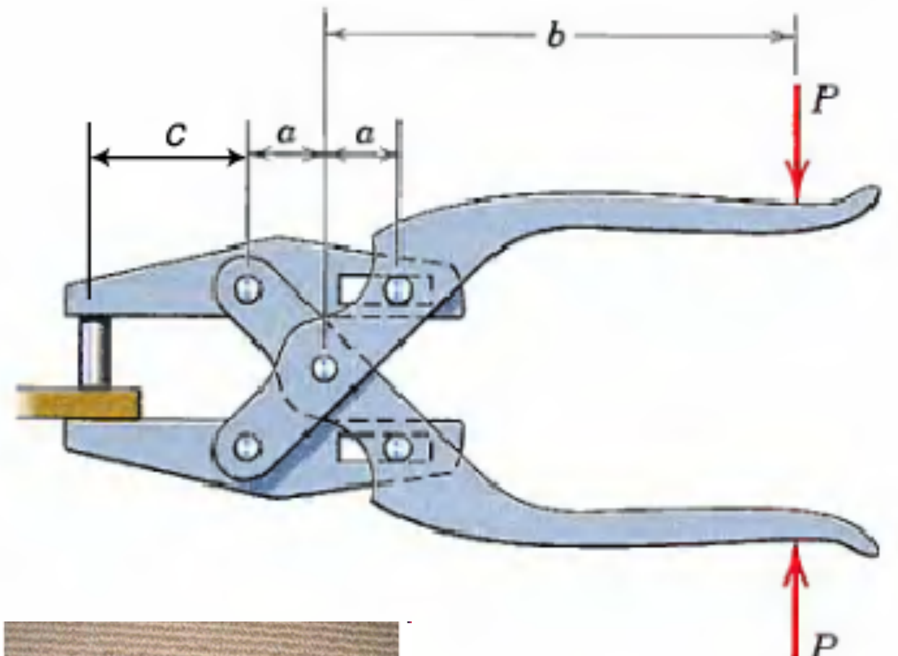
Torque about A only from $F_{hy}\uparrow$

System constrained to various degrees

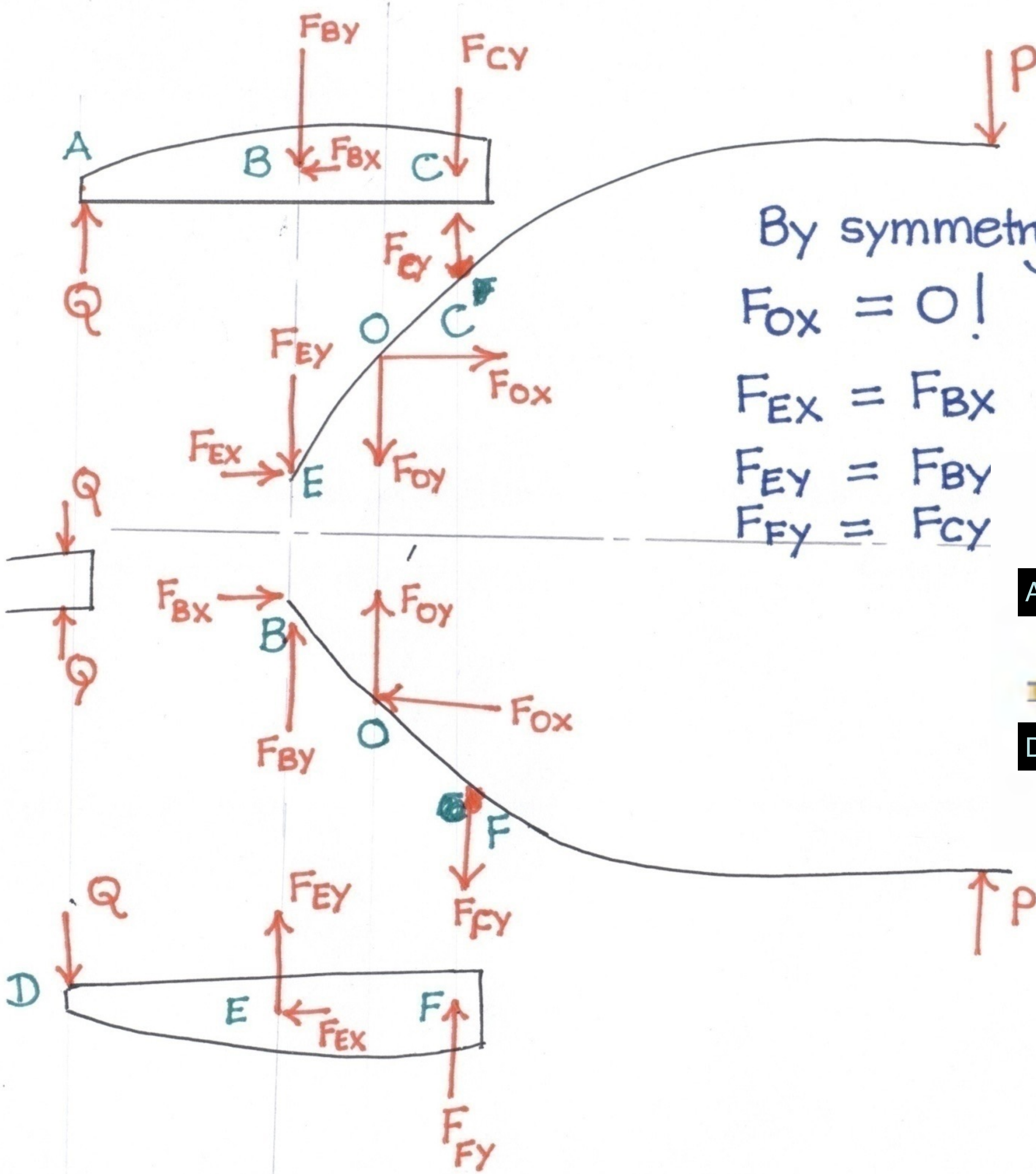


Problem 6

- For the paper punch shown in the figure find the punching force Q corresponding to a hand grip P.



compare with



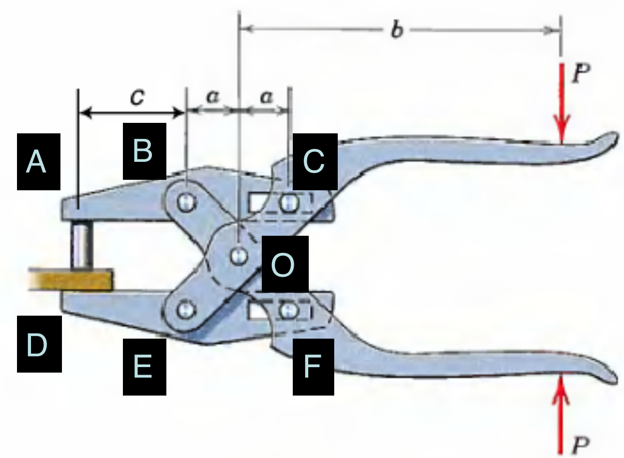
By symmetry

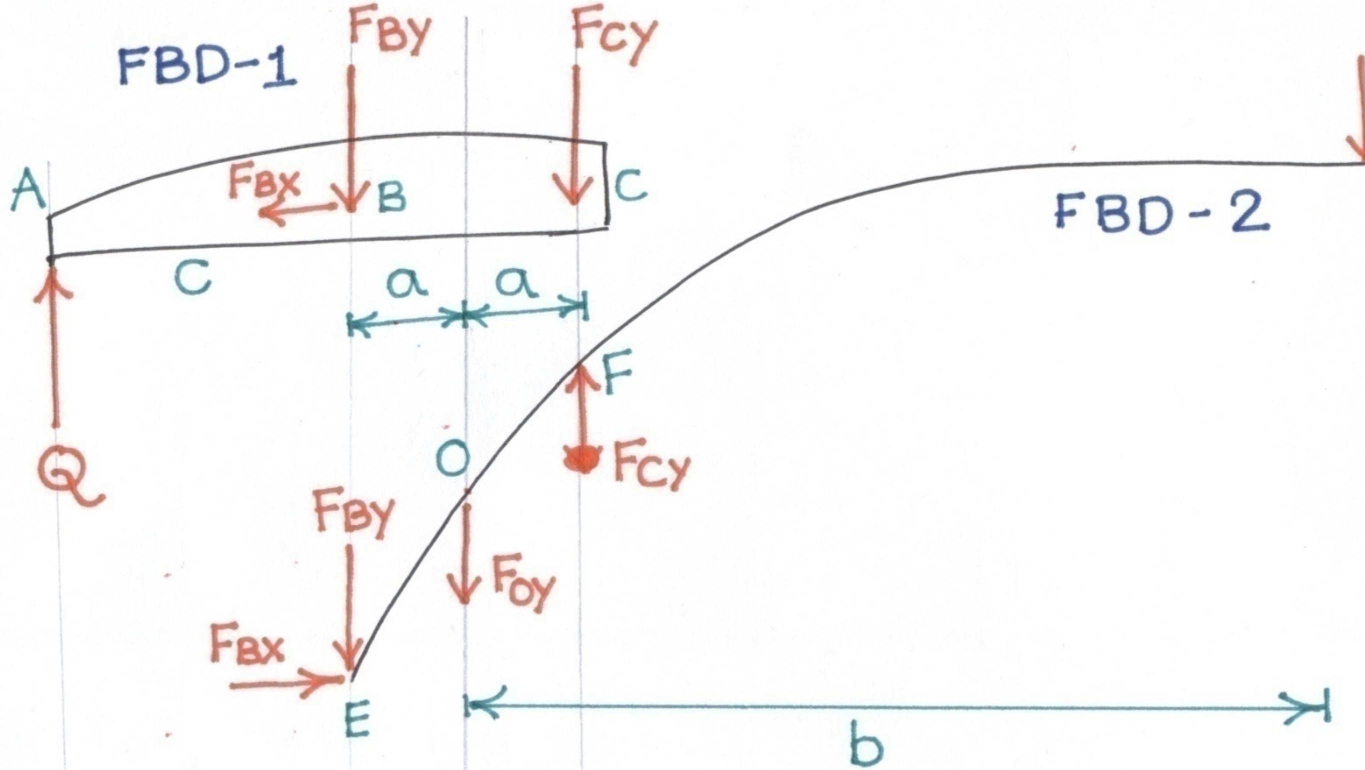
$$F_{OX} = 0!$$

$$F_{EX} = F_{BX}$$

$$F_{EY} = F_{BY}$$

$$F_{FY} = F_{CY}$$





FBD-1; $\rightarrow \sum F_x = 0 \Rightarrow F_{Bx} = 0$

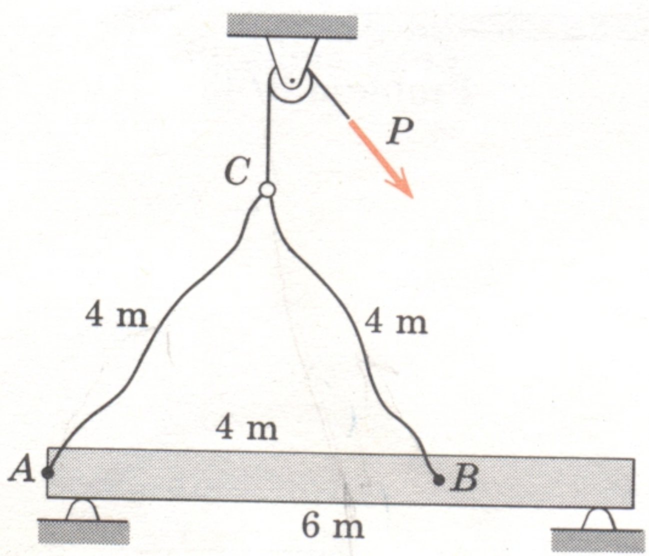
$\uparrow \sum F_y = 0 \Rightarrow F_{By} + F_{Cy} = Q$

FBD-2; $\circlearrowleft \sum M_O = 0 \Rightarrow Pb = (F_{By} + F_{Cy}) a$

$\Rightarrow Pb = Qa$

$$Q = \frac{Pb}{a}$$

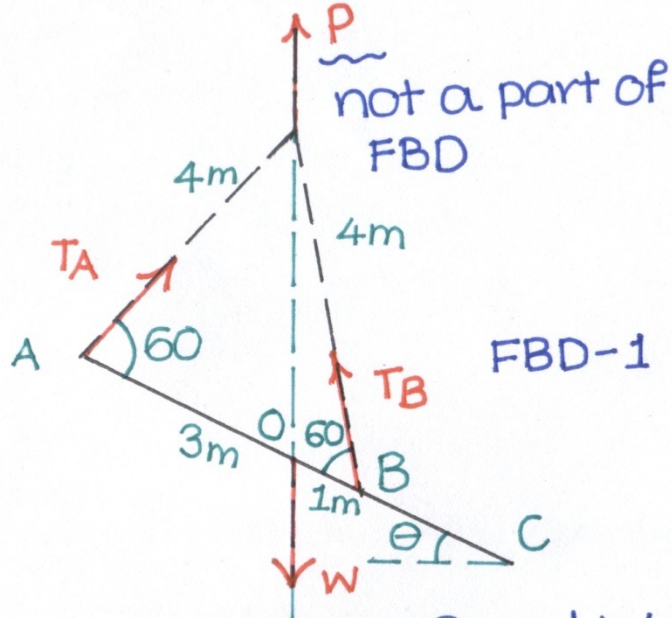
Problem 7



Problem 3/52

Meriam,, *Statics*

The uniform beam has an overall length of 6m and a mass of 300kg. The force P applied to the hoisting cable is slowly increased to raise the ring C, the two 4-m ropes AC and BC, and the beam. Compute the tensions in the ropes at A and B when the beam is clear of its supports and the force P is equal to the weight of the beam



For equilibrium P and W are in line; θ is unknown

$$T_A \cos(60-\theta) = T_B \cos(60+\theta)$$

$$T_A \frac{\sqrt{3}}{2} * 4 = W\cos\theta * 1$$

$$T_B \frac{\sqrt{3}}{2} * 4 = W\cos\theta * 3$$

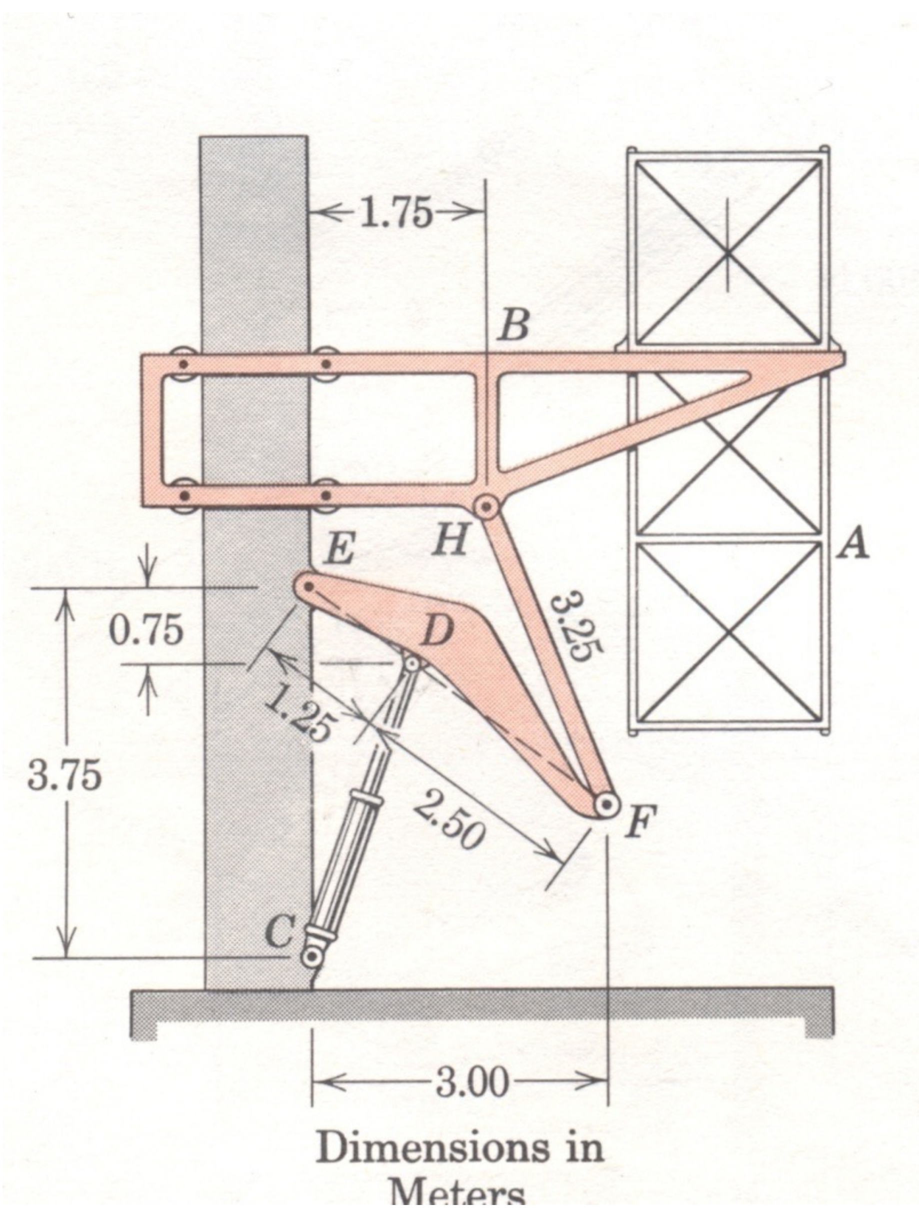
$$T_B = 3T_A$$

$$\frac{\cos(60-\theta)}{\cos(60+\theta)} = 3 \Rightarrow \theta = \cos^{-1}\sqrt{\frac{12}{13}}$$

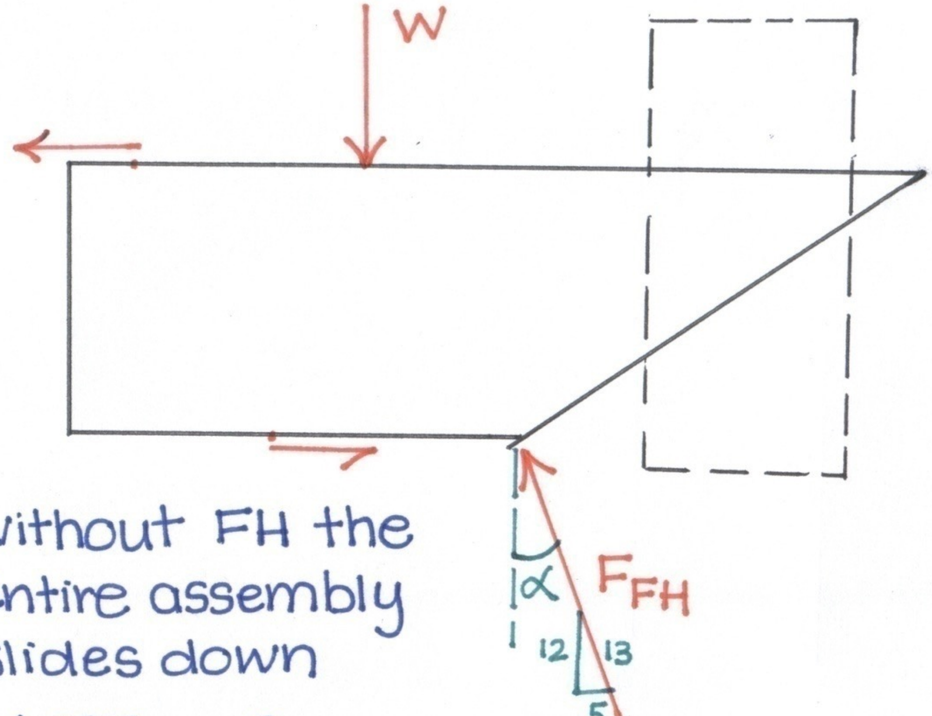
$$T_A = W/\sqrt{13}$$

$$T_B = 3W/\sqrt{13}$$

Problem 8



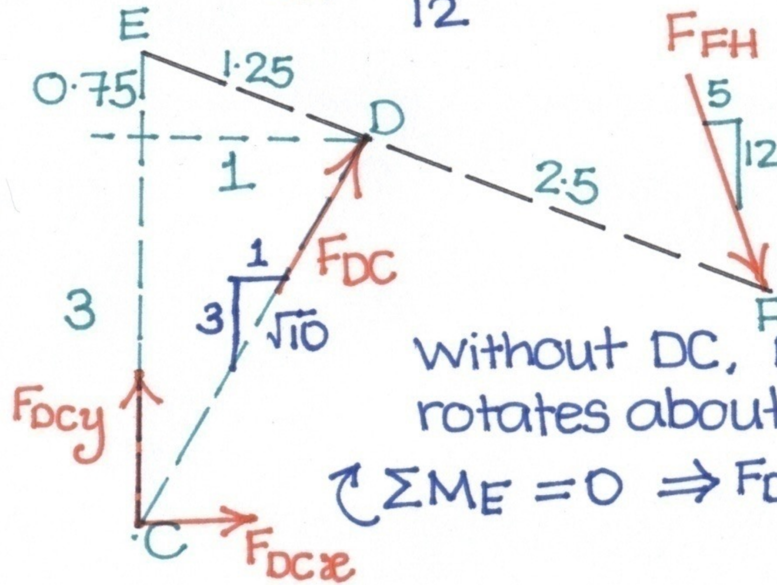
The figure shows a special rig for erecting vertical sections of a construction tower. The assembly A has a mass of 1.5Mg and is elevated by the platform B which itself has a mass of 2Mg. The platform is guided up the fixed vertical columns by rollers and is activated by the hydraulic cylinder *CD* and links *EDF* and *FH*. For the particular position shown calculate the force *R* exerted by the hydraulic cylinder at *D*. Neglect mass of cylinder and links.



Without FH the entire assembly slides down

$\downarrow \Sigma F_y = 0$

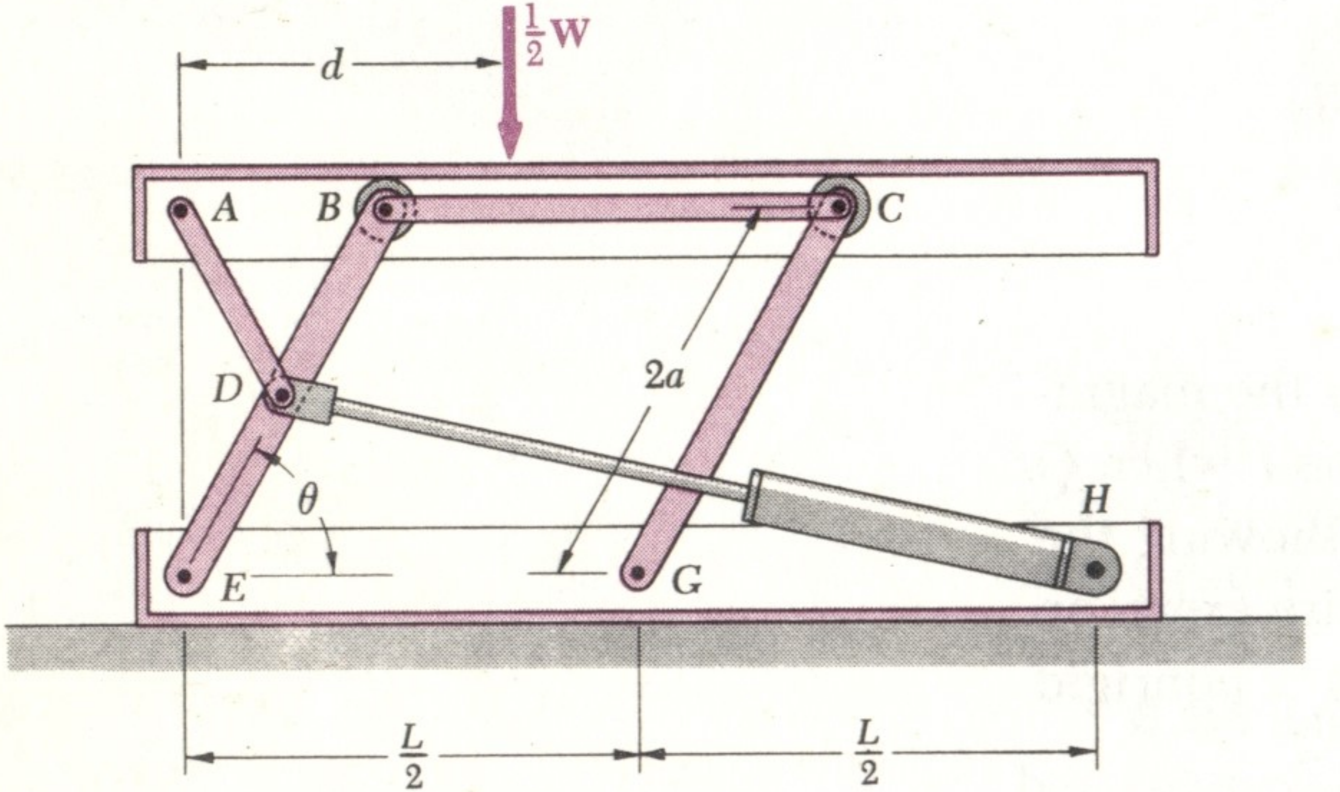
$\Rightarrow F_{FH} = \frac{13}{12} W$



Without DC, EDF rotates about C

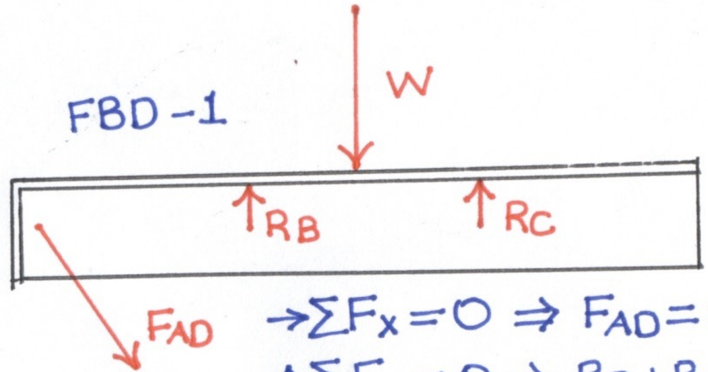
$\curvearrowleft \Sigma M_E = 0 \Rightarrow F_{DC} = 59.7$ kN (C)

Pr



A hydraulic-lift table is used to raise a 1000 kg crate. It consists of two identical linkages on which hydraulic cylinders exert equal forces. Members *EDB* and *CG* are each of length *2a*, and member *AD* is pinned to the midpoint of *EDB*. If the crate is placed on the table, so that half of its weight is supported by the system shown, determine the force exerted by each cylinder in raising the crate for θ = 60 deg, a = 0.70m, and L = 3.20m. Show that the result obtained is independent of distance d.

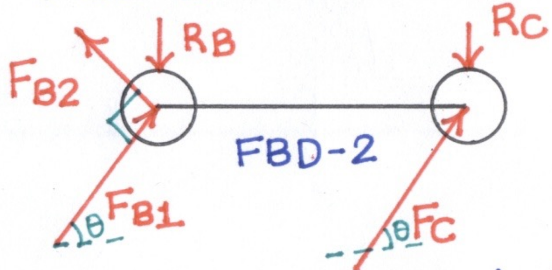
FBD-1



$\rightarrow \Sigma F_x = 0 \Rightarrow F_{AD} = 0$

$\uparrow \Sigma F_y = 0 \Rightarrow R_B + R_C = W$

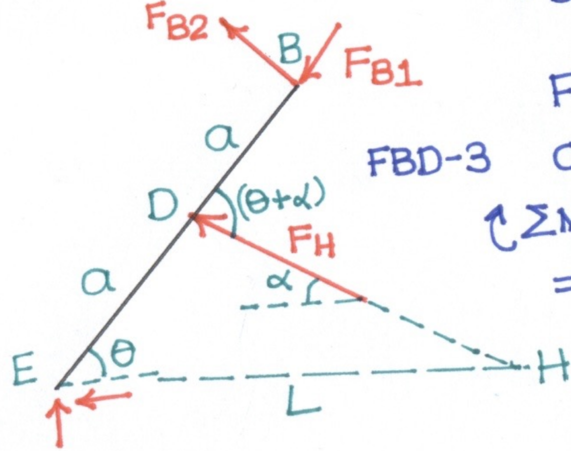
FBD-2



Cylinder DH prevents EB from being 2-force. adds F_{B2} in ⊥ dir^n.

$\nwarrow \Sigma F = 0 \Rightarrow F_{B2} = (R_B + R_C)\cos\theta$

$F_{B2} = W\cos\theta$



FBD-3

F_H prevents rotation of EDB about E

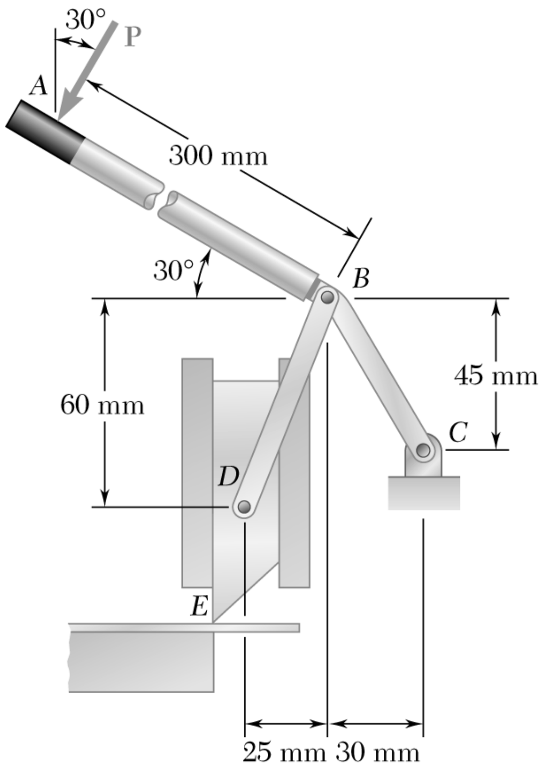
$\circlearrowleft \Sigma M_E = 0$

$\Rightarrow F_{B2} * 2a = F_H a \sin(\theta+\alpha)$

$F_H = \dfrac{W\cos\theta}{\sin(\theta+\alpha)}$

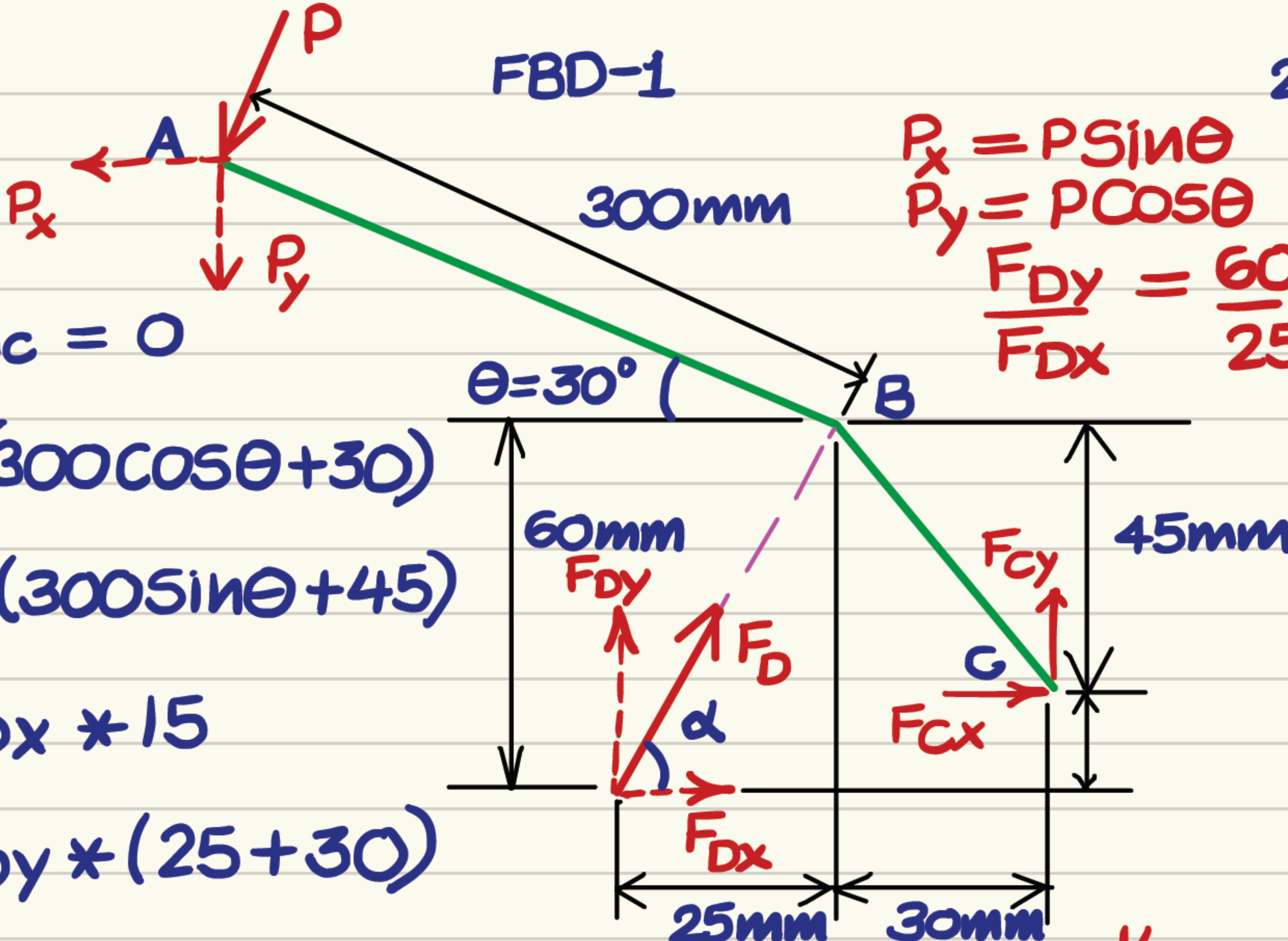
Problem 1

- The shear shown is used to cut and trim electronic-circuit board laminates. For the position shown, determine (a) the vertical component of force exerted on the shearing blade at *D*, and (b) the reaction at *C*. The value of P = 400N



FBD-1

29/1/17



$P_x = P\sin\theta$

$P_y = P\cos\theta$

$$\frac{F_{Dy}}{F_{Dx}} = \frac{60}{25} = \frac{12}{5}$$

$\circlearrowright \Sigma M_C = 0$

$$P_y(300\cos\theta + 30) + P_x(300\sin\theta + 45) + F_{Dx} * 15 - F_{Dy} * (25+30) = 0$$

$$F_D = (F_{Dx}^2 + F_{Dy}^2)^{1/2}$$

$$F_{Dx} = \frac{5}{13} F_D \; ; \; F_{Dy} = \frac{12}{13} F_D$$

$\rightarrow \sum F_x = -P_x + F_{Dx} + F_{Cx} = 0$

$\Rightarrow F_{Cx} = -991.4\,N$

$\uparrow \sum F_y = -P_y + F_{Dy} + F_{Cy} = 0$

$\Rightarrow F_{Cy} = -2512.9\,N$

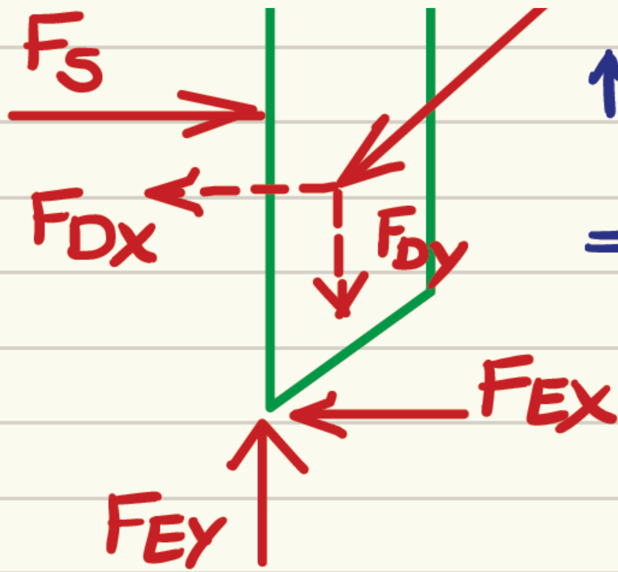
$F_C = (F_{Cx}^2 + F_{Cy}^2)^{1/2}$

$= 2701.4\,N$

$\alpha = \tan^{-1}\left(\frac{F_{Cy}}{F_{Cx}}\right)$

$F_C = 2.7\,kN, \alpha = 68.5^\circ$

FBD-2



$\uparrow \sum F_y = 0$

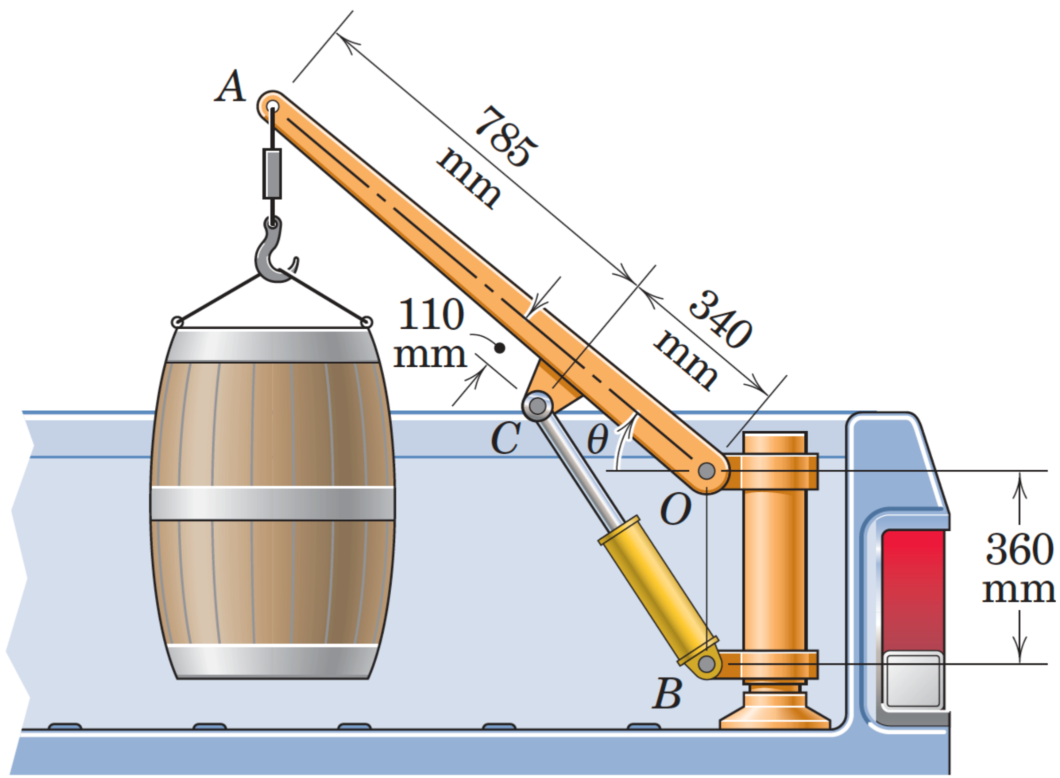
$\Rightarrow -F_{Dy} + F_{Ey} = 0$

$\Rightarrow F_{Dy} = 2859.3\,N$

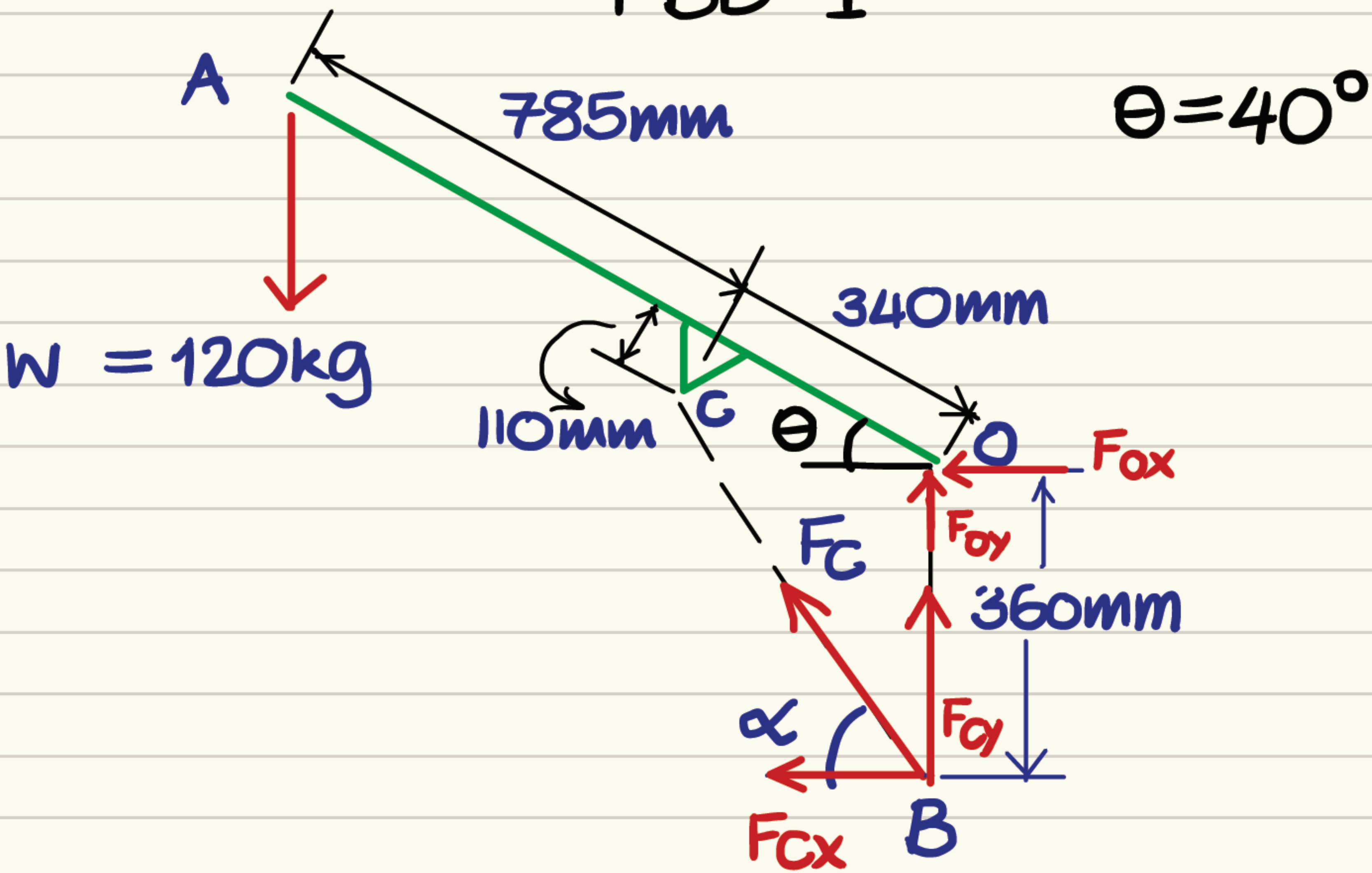
$F_{Dy} = 2.86\,kN$

Problem

The small crane is mounted on one side of the bed of a pickup truck as shown. The weight of the barrel is 120 kg. For the piston, θ = 40º, determine the magnitude of the force supported by the pin at O and the oil pressure P against the 50 mm diameter piston of the hydraulic cylinder BC.



FBD-1



$$\circlearrowleft \Sigma M_O = W * (785+340)\cos\theta - F_{cx} * 360 = 0$$

$$\Rightarrow F_{cx} = 287.26 kg$$

$$\frac{F_{cy}}{F_{cx}} = \tan\alpha = \frac{340\sin\theta - 110\cos\theta + 360}{340\cos\theta + 110\sin\theta}$$

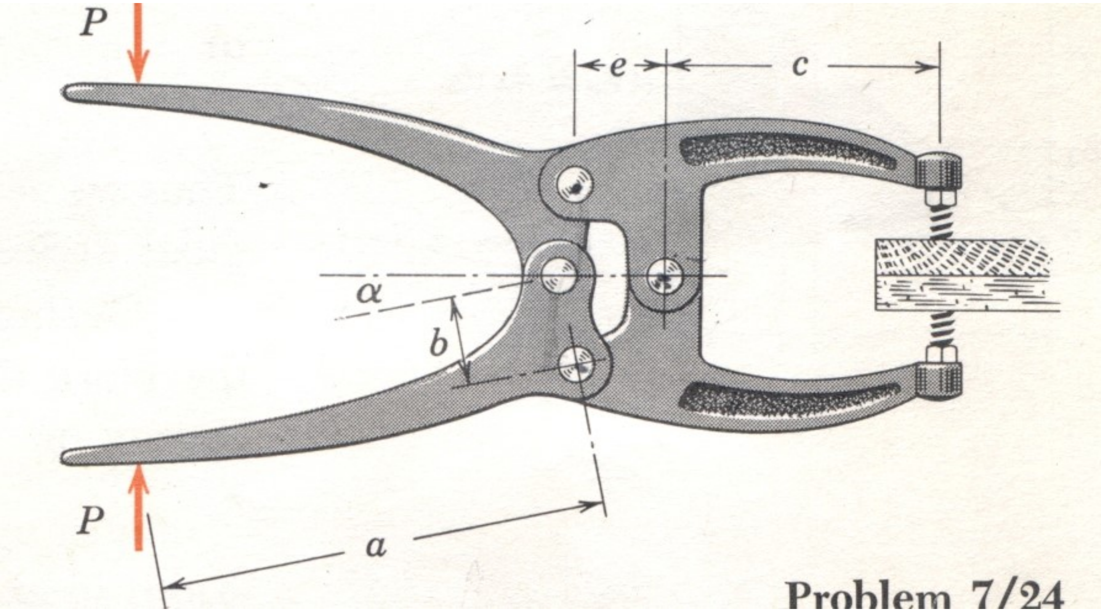
$$= 1.4926$$

$$\Rightarrow F_{cy} = F_{cx}\tan\alpha =$$

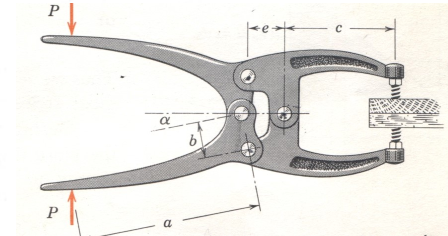
$$F_c = (F_{cx}^2 + F_{cy}^2)^{1/2} = 516.1 \text{ kg}$$

$$F_c = 516.1 \times 9.81 \text{ N} = 506.3 \text{ N}$$

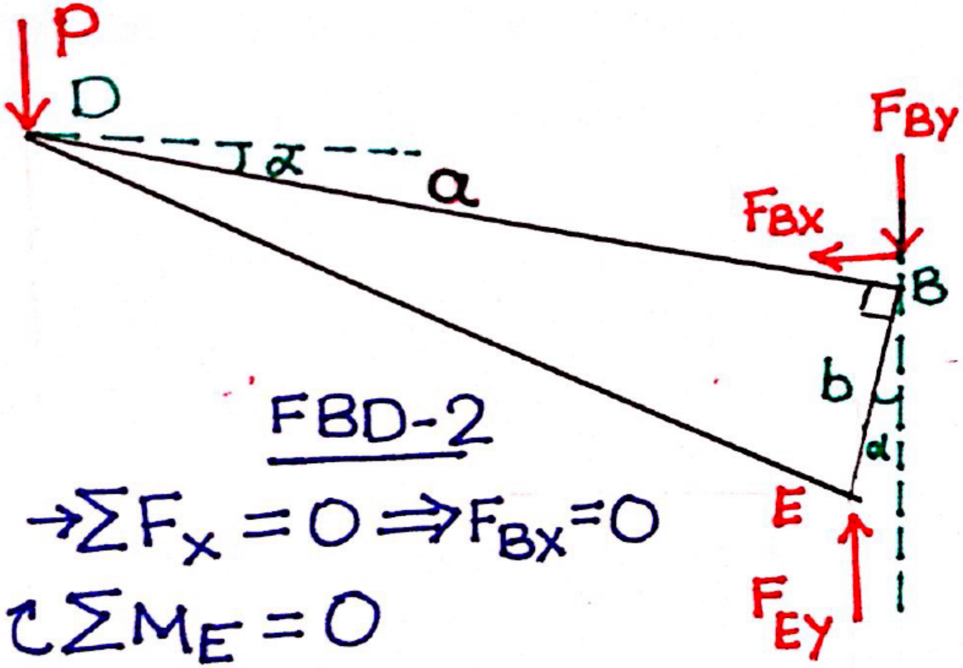
Problem 2

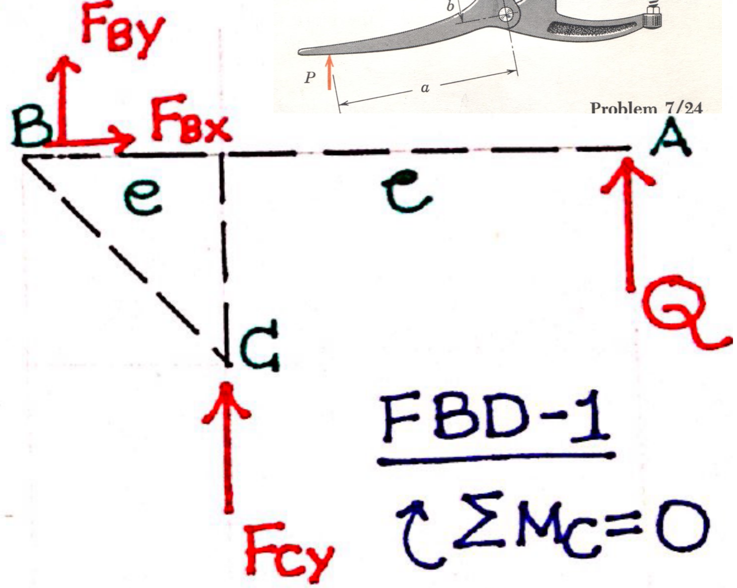


- Obtain the clamping force Q developed for the pliers when the handle force is P.



Problem 7/24





FBD-2

$$\rightarrow \sum F_x = 0 \Rightarrow F_{Bx} = 0$$

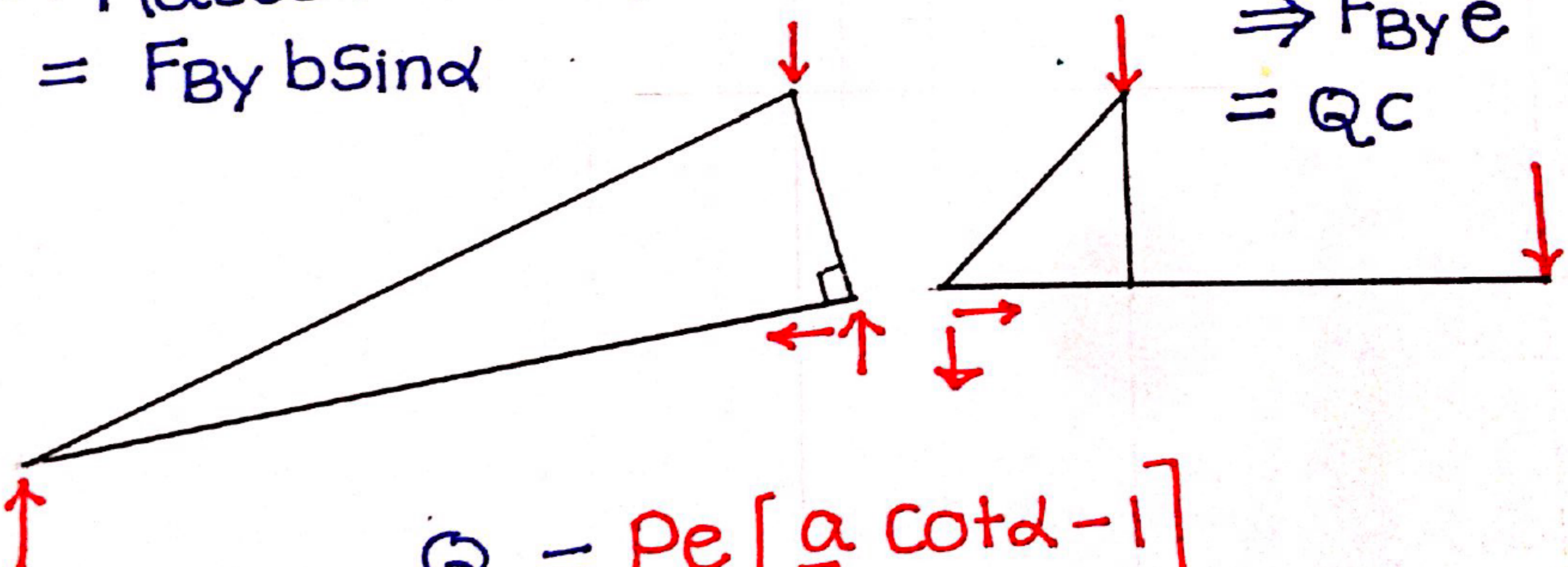
$$\circlearrowright \sum M_E = 0$$

$$\Rightarrow P(a\cos\alpha - b\sin\alpha) = F_{By}\, b\sin\alpha$$

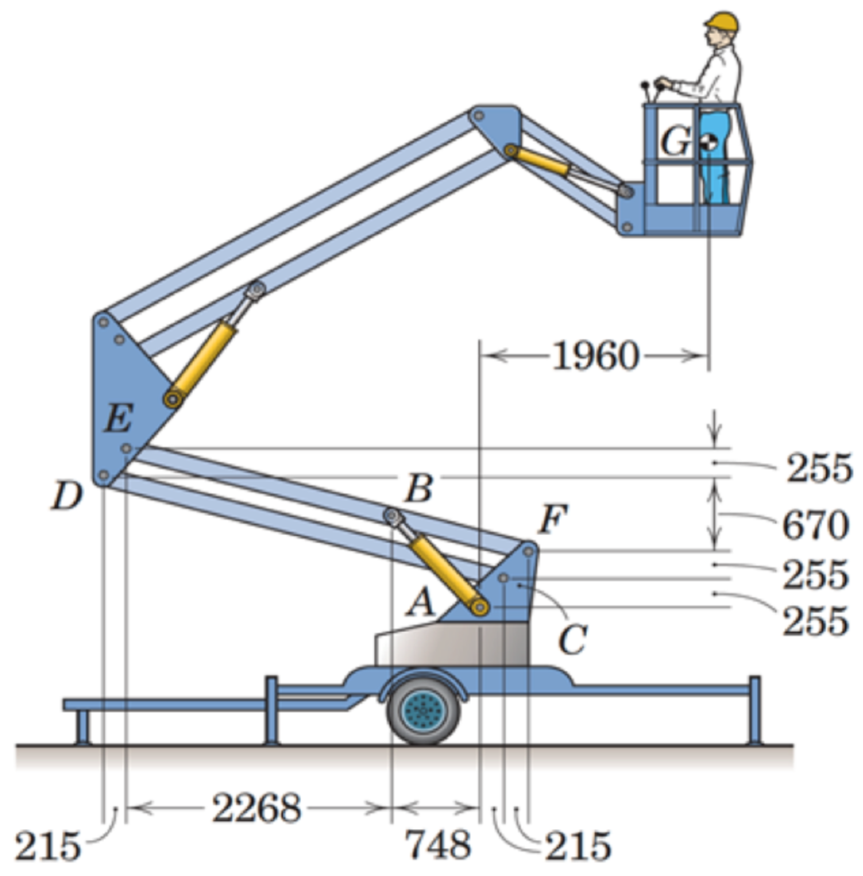
FBD-1

$$\circlearrowright \sum M_C = 0$$

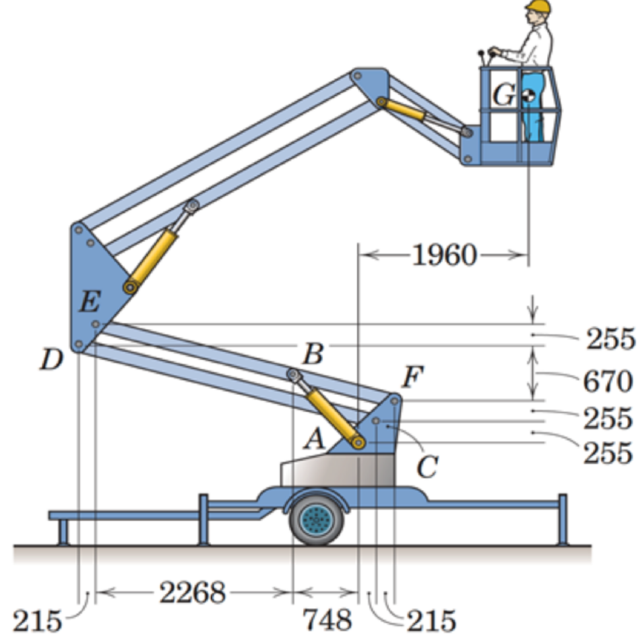
$$\Rightarrow F_{By}\, e = Qc$$



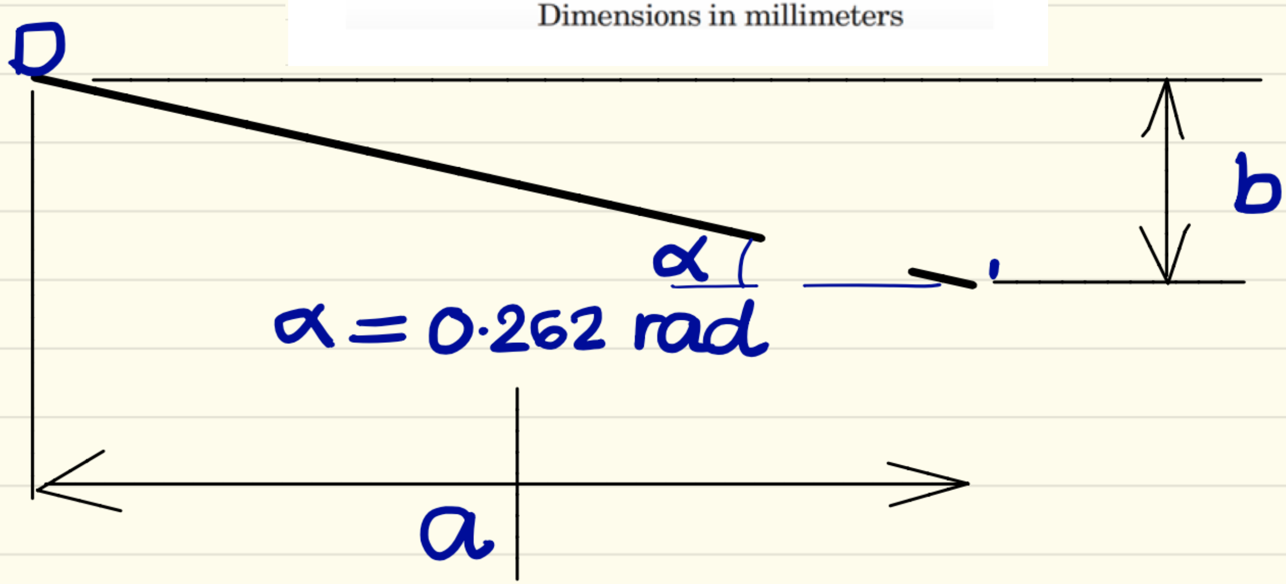
$$Q = \frac{Pe}{c}\left[\frac{a}{b}\cot\alpha - 1\right]$$



Determine the force in cylinder *AB* due to the combined weight of the bucket and operator. The combined mass is 180 kg with mass centre at *G*.



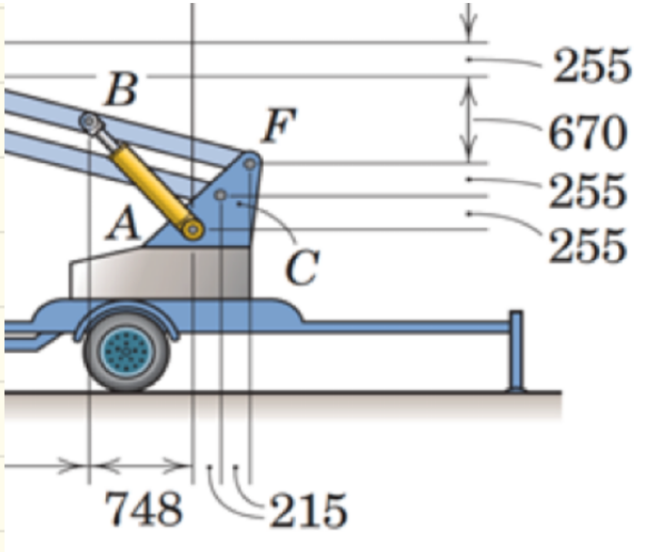
Dimensions in millimeters

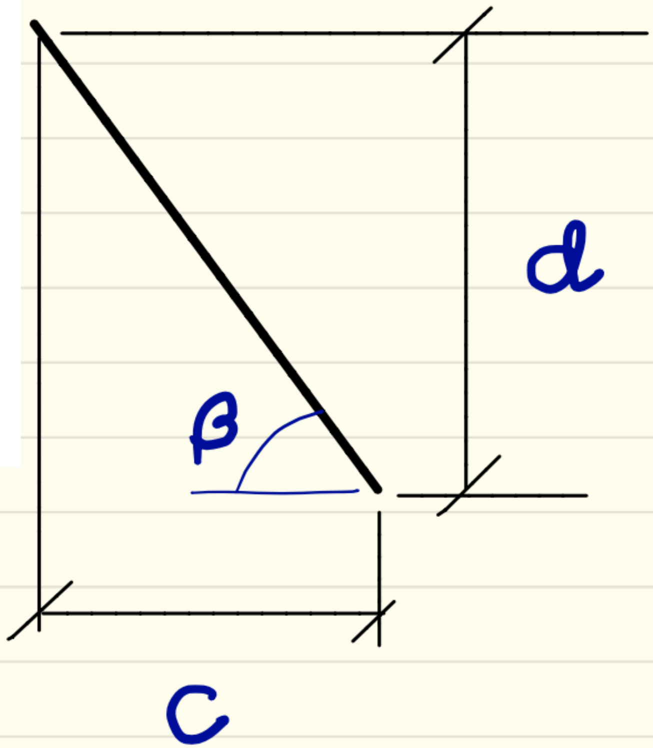


$a = 2268 + 748 + 215 + 215 = 3346$

$b = 670 + 255 = 925$

Angle of Hydraulic cylinder AB

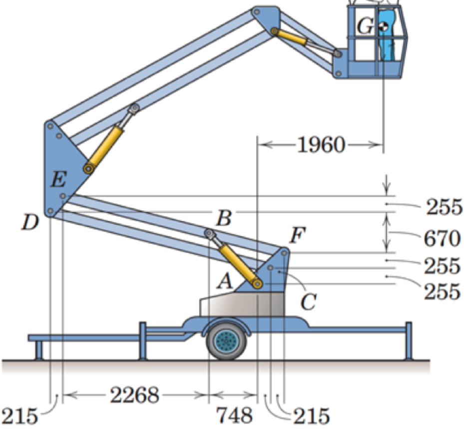




$\beta = 0.829 \text{ rad}$

$c = 748$

$d = 255 + 255 + (748 + 2 \times 215)\dfrac{b}{a} = 826.21$

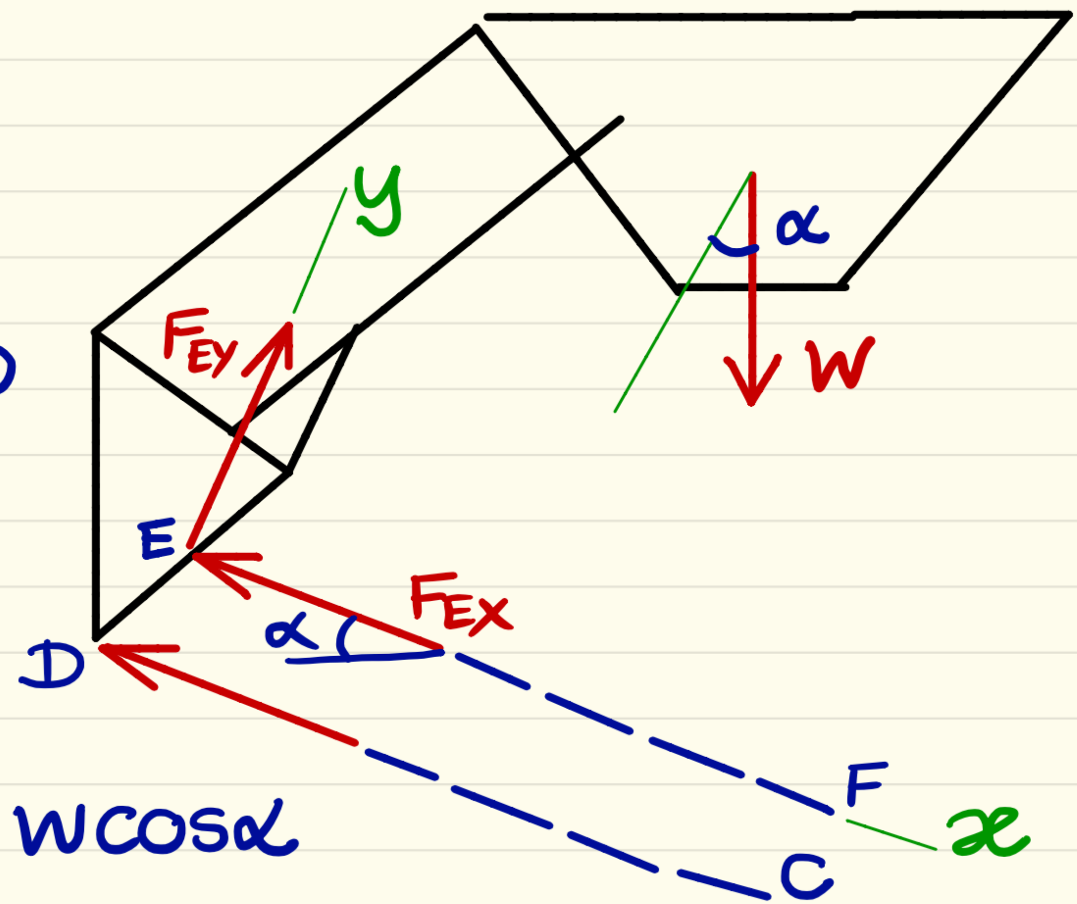


Dimensions in millimeters

FBD-1

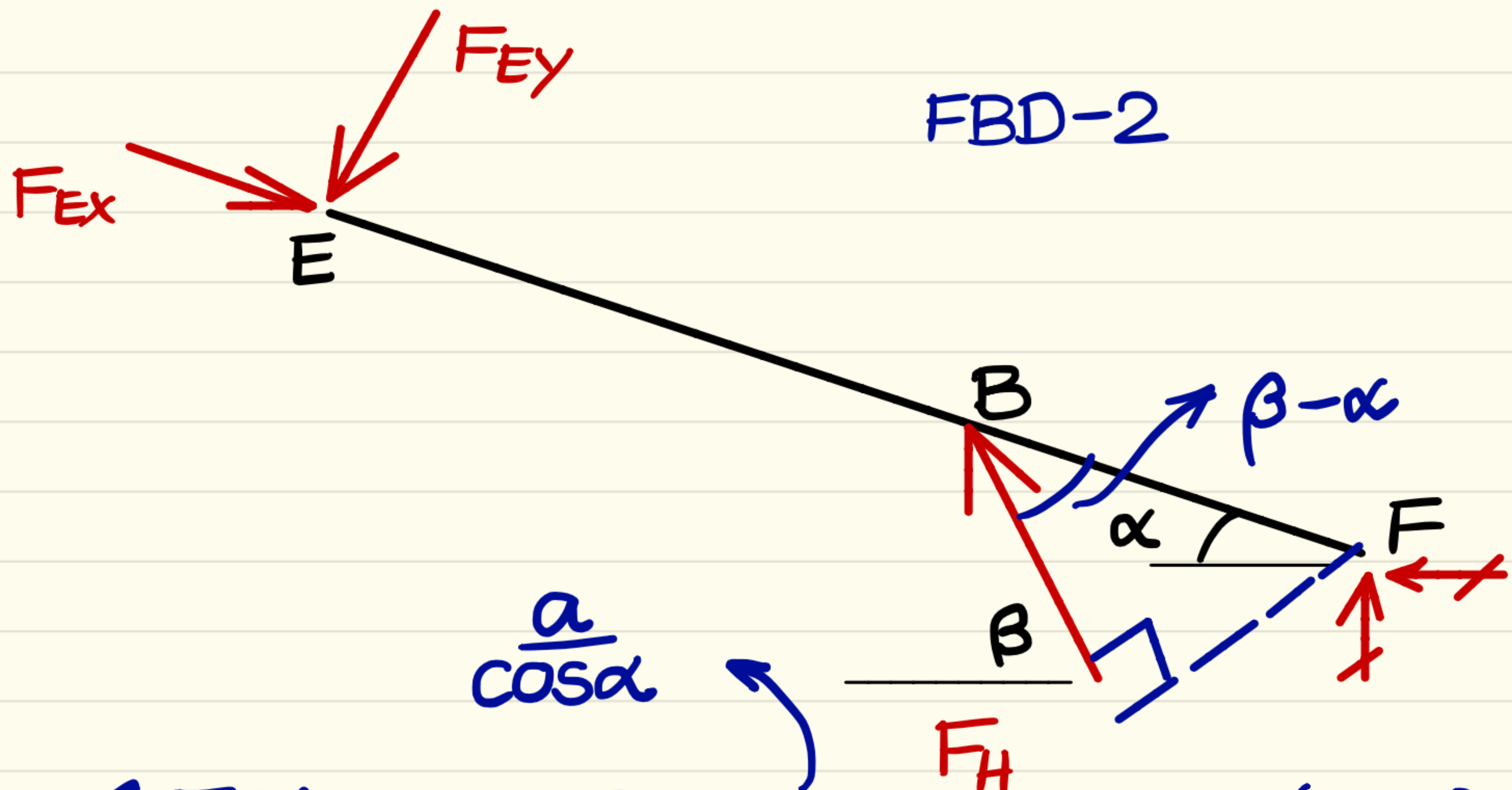
$\Sigma F_y =$

$W\cos\alpha - F_{EY} = 0$



$F_{EY} = W\cos\alpha$

Force F_h Hydraulic cylinder AB

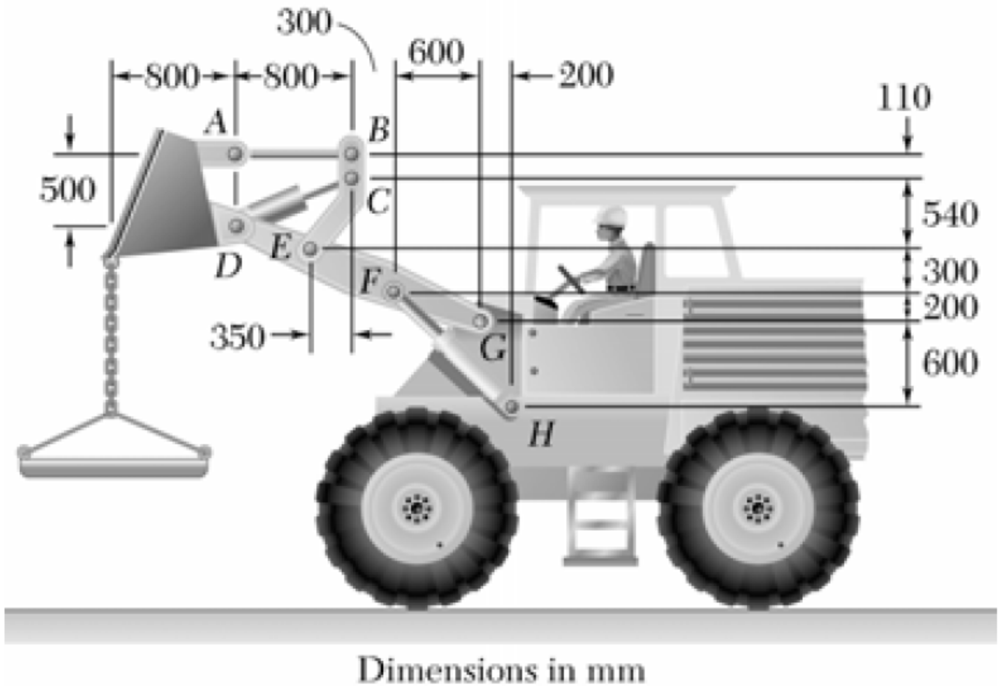


$$\circlearrowright \Sigma M_F = F_{Ey} * EF + F_H \sin(\beta-\alpha)\, BF = 0$$

$$EF = \frac{a}{\cos\alpha}, \quad BF = \frac{748+2\times 215}{\cos\alpha}$$

$g = 9.81$

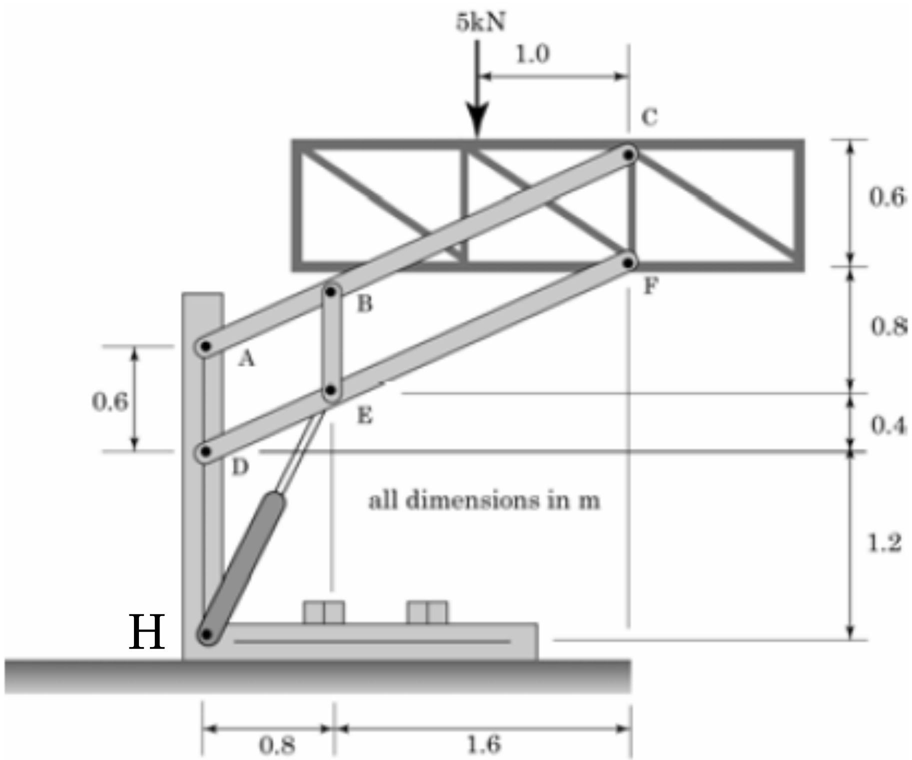
$$\Rightarrow \boxed{F_h = 5.21\,W \cong 9200\,N\ (C)}$$



Problem 3

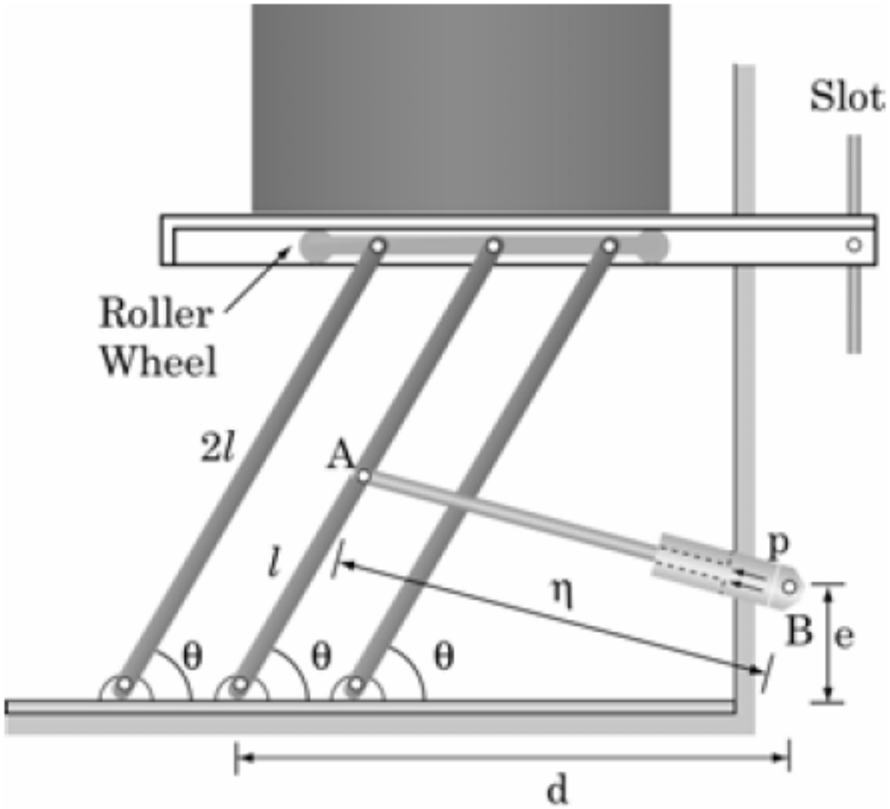
- A 500-kg concrete slab is supported by a chain and sling attached to the bucket of the front-end loader shown. The action of the bucket is controlled by two identical mechanisms, only one of which is shown. Knowing that the mechanism supports half of the 500-kg slab, determine the force (a) in the cylinder *CD, (b) in cylinder FH*.

Problem 4



- The elevation of a platform is controlled by two identical mechanisms only one of which is shown. A load of 5 kN is applied to the mechanism shown. Knowing that the pin at *C* can transmit only a horizontal force, determine (a) the force in link *BE, (b) the components of the force exerted by the* hydraulic cylinder on pin *H*.

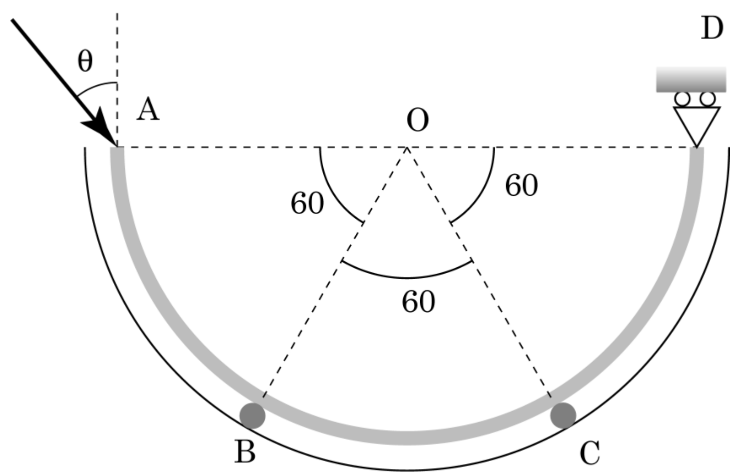
Problem 5



- A hydraulic lift platform for loading trucks supports a weight *W of 5000N. Only one side of the system has been* shown; the other side is identical. If the diameter of the piston in the cylinder (two) is 40 mm, what pressure *p is needed to support W when θ* = 60°. Assume *l = 240 mm, d = 600 mm, and e = 100 mm. Neglect friction* everywhere.

Problem 6

- A semicircular rod ABCD is supported by a roller at D and rests on two frictionless cylinders at B and C. Find the maximum angle force P can make from the vertical if applied at point A and the rod remains in equilibrium.



Problem 11

- In the toy folding chair shown, members ***ABEH*** and ***CFK*** are parallel. Determine the components of all forces acting on member ***ABEH*** when a 160 N weight is placed on the chair. Draw completely all free body diagrams required. It may be assumed that the floor is frictionless and that half the weight is carried by each side of the chair and is applied at point *M* as shown.