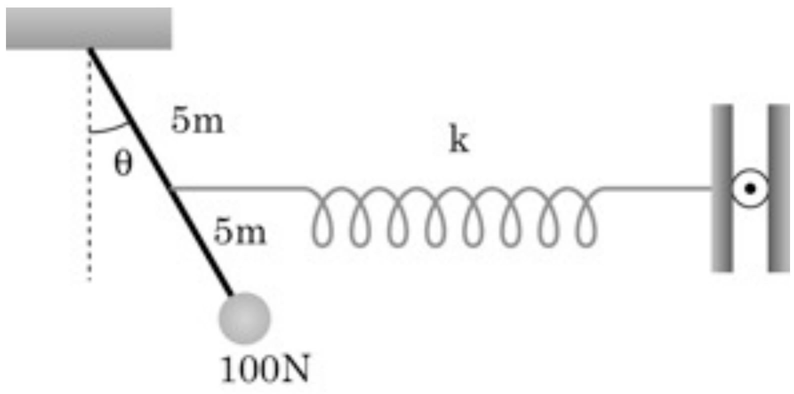
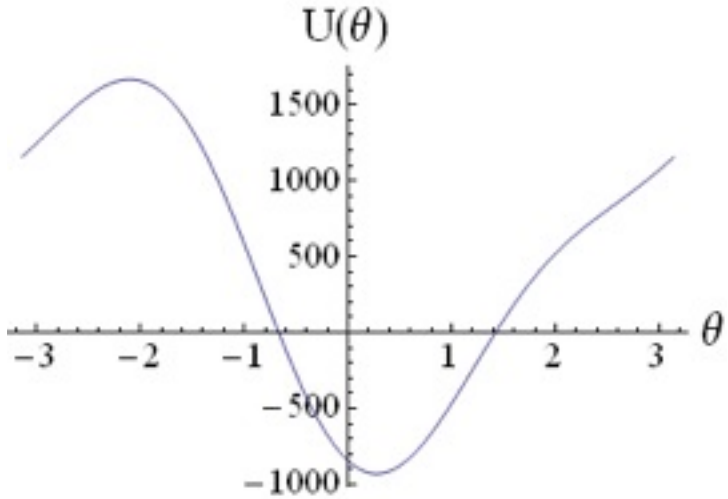
Problem 1

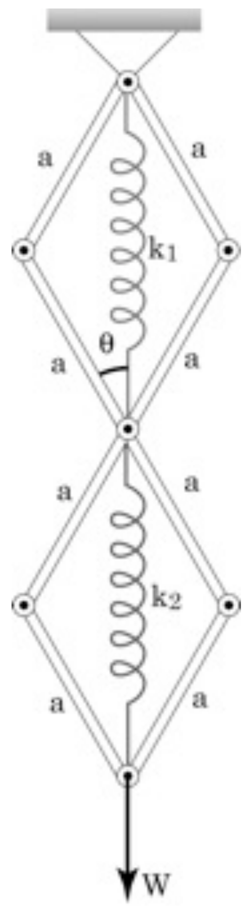
- The spring is un-stretched when $\theta = 30^o$. At any position of the pendulum, the spring remains horizontal. If the spring constant is k = 50 N/m, at what position will the system be in equilibrium. **(Soln: No need to solve this problem. It involves Newton-Raphson like method for actual solution)**





Problem 2

- If the springs are un-stretched when $\theta = \theta_o$, find the angle θ when the weight W is applied on the system. Use the method of minimum potential energy.

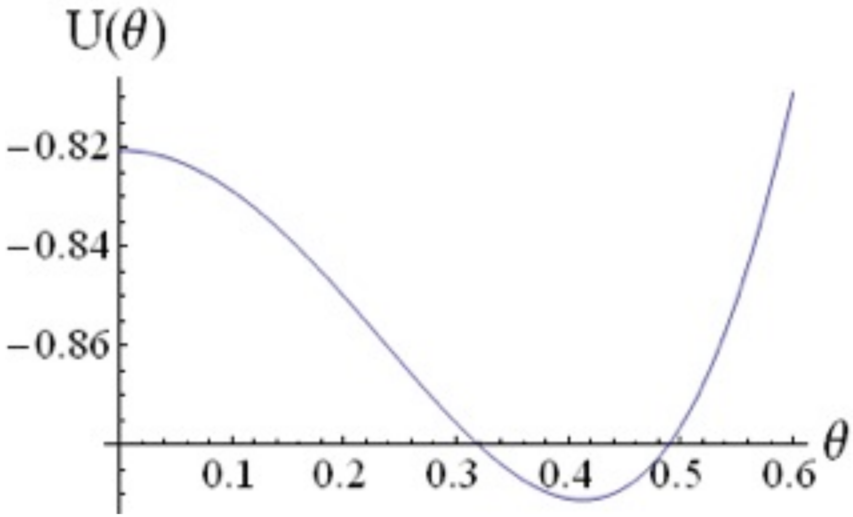


$$\theta = 0 \; ; cos(\theta) = cos(\theta_0) + \frac{W}{(k_1 + k_2)a}$$

Energy in non-dimensional form

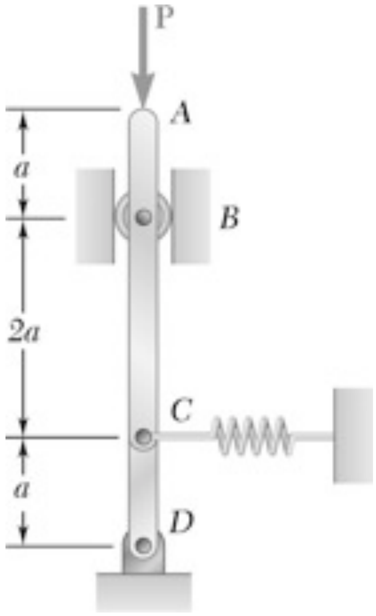
$$\frac{r}{2}\left(\left(\mathrm{Cos}[\theta] - \mathrm{Cos}[\theta o]\right)\right)^2 - \mathrm{Cos}[\theta] \quad (* \; r = \frac{a\,k}{W} \; *)$$

$$r_0 = 20; \theta_0 = \pi/6$$



Problem 3

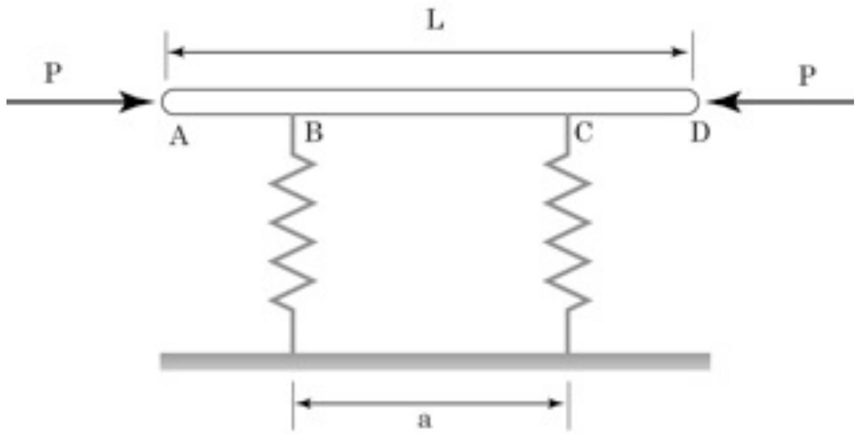
- Two bars are attached to a single spring of constant k that is un-stretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.



$$P \leq \frac{4}{7}ka$$

Problem 4

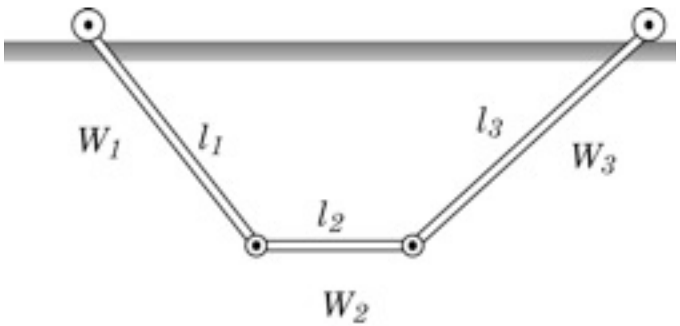
- The horizontal bar AD is attached to two springs of constant k and is in equilibrium in the position shown. Determine the range of values of the magnitude P of the two equal and opposite horizontal forces P and -P for which the equilibrium position is stable if (a) AB = CD, (b) if AB = 2CD.



$$P \leq \frac{ka^2}{4L} (\text{ stability both for parts (a) and (b)})$$

Problem 5

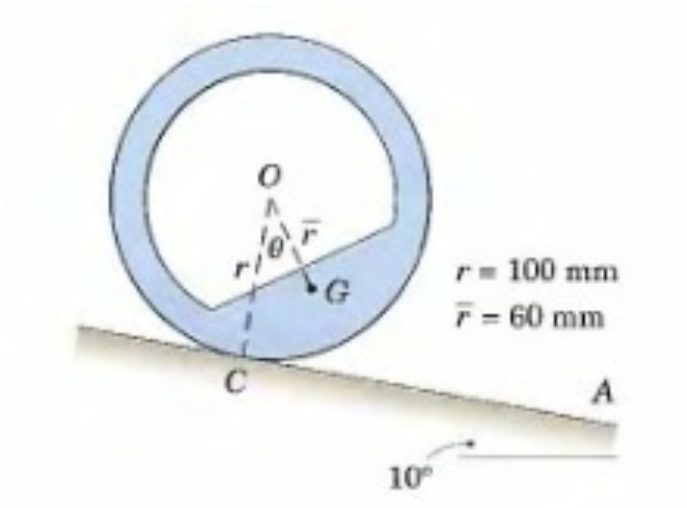
- Determine the angle of inclination of each linkage in the figure shown. The rollers move without friction on the support.



Solution: l_1 and l_2 vertical

Problem 6

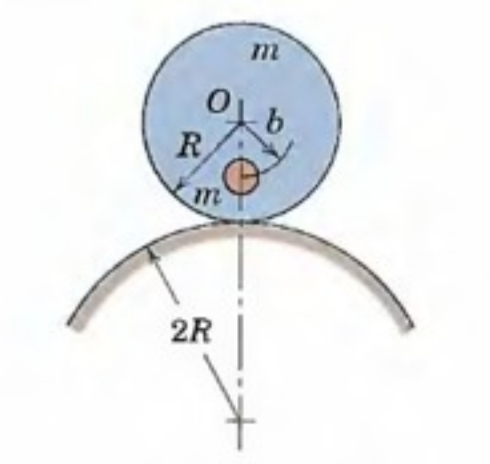
- Determine the equilibrium values of θ and the stability of equilibrium at each point for the unbalanced wheel on the 10^o incline. Static friction is sufficient to prevent slipping. The mass center is at G.



$$\theta = -6.82^o,\ \text{stable;}\ \theta = 207^o\ \text{unstable}$$

Extra Problem 1

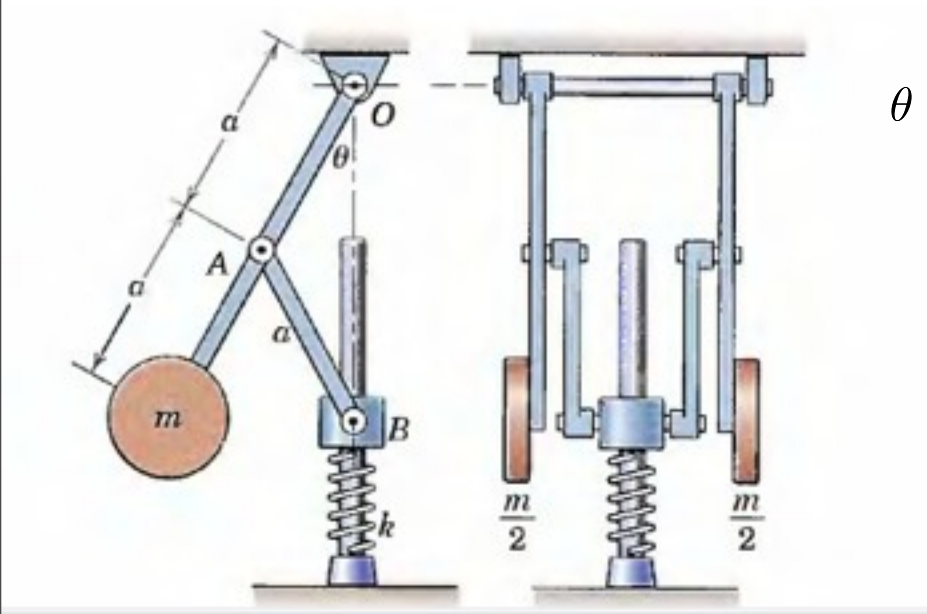
- The uniform disk of radius R and mass m rolls without slipping on the fixed cylinder surface of radius 2R. Fastened to the disk is a lead cylinder also of mass m with its center located a distance b from the center O of the disk. Determine the minimum value of b for which the disk will remain in stable equilibrium on the cylindrical surface.



$$b_{\min} = \frac{2}{3}R$$

Extra Problem 2

- In the mechanism shown the spring of stiffness k is uncompressed when θ = 60º. Also the masses of the parts are small compared with the sum m of the masses of the two cylinders. The mechanism is constructed so that the arms may swing past the vertical, as seen in the right-hand side view. Determine the values of θ for equilibrium and investigate the stability of the mechanism in each position. Neglect friction.



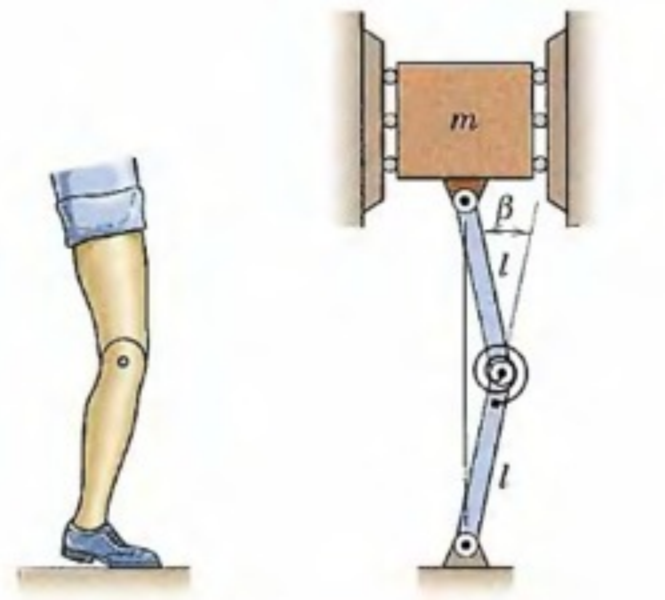
$$\theta = 0, \text{stable if k} < \text{mg/a; unstable if k} > \text{mg/a}$$

$$\theta = \cos^{-1}\frac{1}{2}(1+\frac{mg}{ka})(\text{only if } k > mg/a); \text{stable}$$

Extra Problem 3

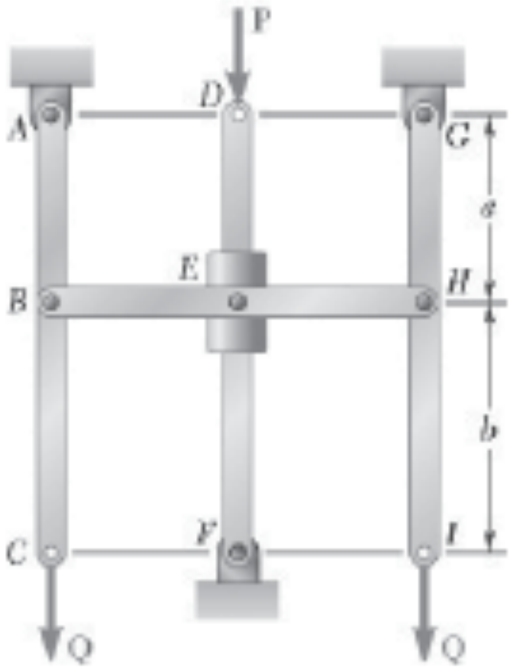
- One of the critical requirements in the design of an artificial leg for an amputee is to prevent the knee joint from buckling under load when the leg is straight. As a first approximation, simulate the artificial leg by the two light links with a torsion spring at their common joint. The spring develops a torque M = K β, which Is proportional to the angle of the bend β at the joint. Determine the minimum value of K which will ensure the stability of the knee joint for β = 0.

$$K_{\min} = \frac{1}{2} mgl$$



Extra Problem 4

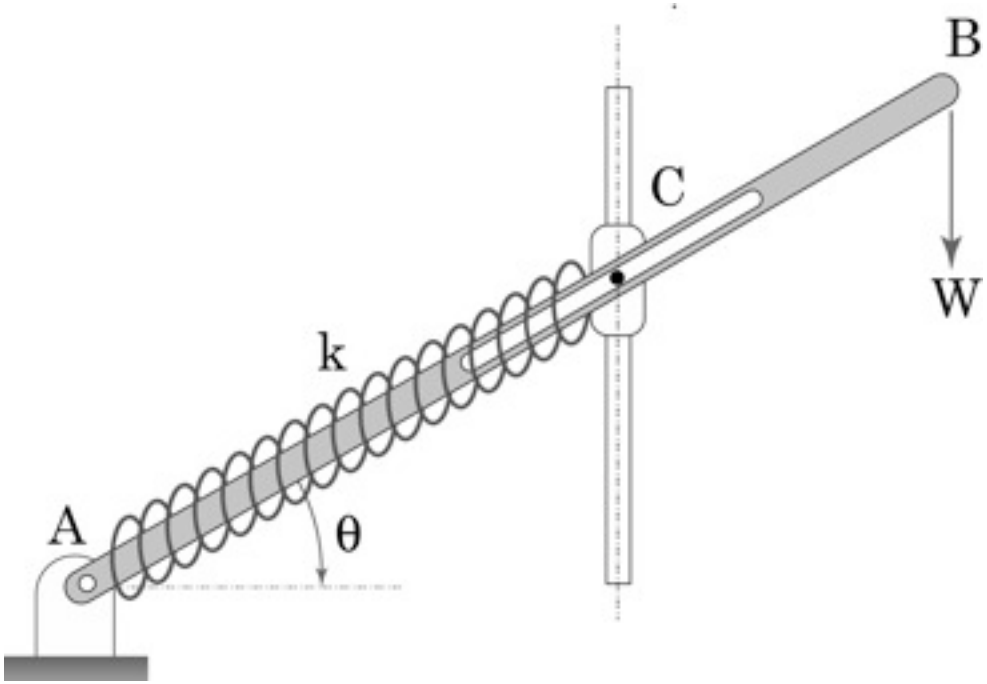
- The horizontal bar BEH is pinned to collar E and to vertical bars AC and GI. The collar can slide freely on bar DF. Determine the range of values of Q for which the equilibrium of the system is stable in the position shown when a = 480mm, b = 400mm, and P = 600N.



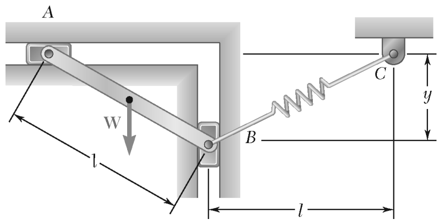
$$Q > 432N$$

Extra Problem 5

- Rod ABC is connected through a pin and slot to a sleeve which slides on a vertical rod. Before the weight W of 100N is applied at C, the rod is inclined at an angle of 45º. If K of the spring is 8000N/m, what is the angle θ for equilibrium? The length of AC is 300mm and the length of CB is 200mm when θ=45º. Neglect friction and all weights other than W.



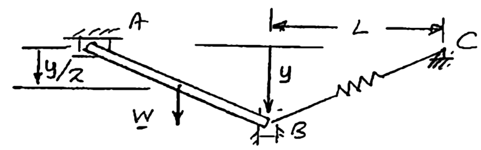
$$\theta = 19.22^o \text{ unstable}$$



PROBLEM 10.77

A slender rod AB, of weight W, is attached to two blocks A and B that can move freely in the guides shown. Knowing that the spring is unstretched when $y = 0$, determine the value of y corresponding to equilibrium when $W = 80$ N, $l = 500$ mm, and $k = 600$ N/m.

SOLUTION



Deflection of spring = s, where $$s = \sqrt{l^2 + y^2} - l$$

$$\frac{ds}{dy} = \frac{y}{\sqrt{l^2 - y^2}}$$

Potential energy: $$V = \frac{1}{2}ks^2 - W\frac{y}{2}$$

$$\frac{dV}{dy} = ks\frac{ds}{dy} - \frac{1}{2}W$$

$$\frac{dV}{dy} = k\left(\sqrt{l^2 + y^2} - l\right)\frac{y}{\sqrt{l^2 + y^2}} - \frac{1}{2}W$$

$$= k\left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y - \frac{1}{2}W$$

Equilibrium $$\frac{dV}{dy} = 0: \quad \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y = \frac{1}{2}\frac{W}{k}$$

Now $$W = 80 \text{ N}, \quad l = 0.500 \text{ m}, \quad \text{and} \quad k = 600 \text{ N/m}$$

Then $$\left(1 - \frac{0.500 \text{ m}}{\sqrt{(0.500)^2 + y^2}}\right)y = \frac{1}{2}\frac{(80 \text{ N})}{(600 \text{ N/m})}$$

or $$\left(1 - \frac{0.500}{\sqrt{0.25 + y^2}}\right)y = 0.066667$$

Solving numerically, $$y = 0.357 \text{ m}$$

$y = 357$ mm ◀