# CE – 102 Engineering Mechanics

## Assorted Examples and Tutorial sets



# Department of Civil Engineering Indian Institute of Technology Bombay

January-April 2019

### **General Assessment Scheme**

Exam	Weightage	Tentative Date
Quiz 1	10%	05 <sup>th</sup> February, 2019; 7 pm – 8 pm
Mid Semester	30%	22 <sup>nd</sup> – 28 <sup>th</sup> February 2019; As per Institute Time Table
Quiz 2	10%	02 <sup>nd</sup> April, 2019; 7 pm – 8 pm
End Semester	50%	22 <sup>nd</sup> April – 05 <sup>th</sup> May 2019; As per Institute Time Table

## Tentative Syllabus for Exams<sup>\*</sup>

Topics	Q1	MS	Q2	ES
Introduction, Fundamentals of Mechanics, Equivalent Force- couple systems, simplest resultant	Y	Y		Y
Equilibrium of 2D and 3D systems		Y		Y
Truss		Y		Y
Friction, Belt Friction			Y	Y
Methods of Virtual work and Potential energy			Y	Y
Dynamics				Y

\*Exact syllabus for the individual exams will be announced in class.

#### **Course content**

Topics	No. of Lectures	Tutorials
	(approx.)	
Introduction, Fundamentals of	6	2
Mechanics, Equivalent Force-couple		
systems, simplest resultant		
Equilibrium of 2D and 3D systems	6	2
Truss	4	1
Friction Belt Friction	4	2
	·	-
Methods of Virtual work and Potential	4	2
energy		
Dynamics	3	1

#### **References:**

- Shames, I. H., *Engineering Mechanics-Statics and Dynamics*, Prentice Hall India Ltd., New Delhi, 4<sup>th</sup> edition (1996).
- Beer, F. P. and E. R. Johnston, D. F. Mazurek, P. J. Cornwell, E. R. Eisenberg, Vector Mechanics for Engineers – Statics and Dynamics, Tata-McGraw-Hill, New Delhi, 9<sup>th</sup> edition (2010).
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- Meriam, J. L. and L. G. Kraige, *Engineering Mechanics Vol-1 & 2*, Wiley India Ltd., New Delhi, 6<sup>th</sup> edition (2011).
- Hibbler, R. C., Engineering Mechanics (Statics), Prentice hall Publications, 13<sup>th</sup> Edition (2012).

NOTE: Bring this hand-out and calculator to all lectures and tutorials without fail.

Shape		x	<u>y</u>	Area
Triangular area	$\frac{1}{ \overline{y} } \xrightarrow{\mu \in C} \stackrel{h}{ \overline{y} }$		<u>h</u> 3	<u>bh</u> 2
Quarter-circular area	Cr C	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area	$c \rightarrow c$	<u>3a</u> 8	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area	$O$ $\overline{x}$ $x$	0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel	$y = kx^{2}$ $h$ $h$ $\overline{x}$	<u>3a</u> 4	3 <u>h</u> 10	<u>ah</u> 3
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	ar <sup>2</sup>
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle	$\frac{1}{\alpha}$	$\frac{r \sin \alpha}{\alpha}$	0	2ar

#### Centroids of Common Shapes of Areas and Lines

[Reference : Beer and Johnston (2010)]



#### Mass Moments of Inertia of Common Geometric Shapes



[Reference : Beer and Johnston (2010)]

Support or Connection	Reaction	Number of Unknowns
Rollers Rocker Frictionless surface	Force with known line of action	1
Short cable Short link	Force with known line of action	1
Collar on frictionless pin in slot	90° Force with known line of action	1
Frictionless pin or hinge	or α Force of unknown direction	2
Fixed support	Force and couple	3

#### Reactions at Supports and Connections for a Two-Dimensional Structure

[Reference : Beer and Johnston (2010)]



#### Reactions at Supports and Connections for a Three-Dimensional Structure

[Reference : Beer and Johnston (2010)]

1. Determine the resultant of the forces given in Fig. 1.



2. The hydraulic cylinder *BD* exerts on member ABC a force *P* directed along line *BD*. Knowing that *P* must have a 750 N component perpendicular to member *ABC*, determine (a) the magnitude of force *P* and (b) its component parallel to *ABC*.



3. A collar that can slide on a vertical rod is subjected to the three forces shown. Determine (a) the value of the angle for which the resultant of the three forces is horizontal, (b) the corresponding magnitude of the resultant.



4. Three cables are used to support the container weighing 10 kN. Find the tension force in each of the cables.



5. Find the moment of 300 N force about C by using (a) definition, (b) vector algebra, (c) horizontal and vertical components of 300 N force, (d) the components of 300 N force along AC and perpendicular to AC. Also determine the smallest force applied at B which creates the same moment about C.



6. The rectangular plate is supported by the brackets at A and B and by a wire CD as shown in Fig.3. Knowing that the tension in the wire is 200 N, determine the moment about A of the force exerted by the wire at C.



7. A 36-N force is applied to a wrench (spanner) to tighten a showerhead. Knowing that the centerline of the wrench is parallel to the x axis. Determine the moment of the force about A.



8. The frame ACD is hinged at A and D and is supported by a cable that passes through a ring at B and is attached to hooks at G and H. Knowing that the tension in the cable is 1125 N, determine the moment about the diagonal AD of the force exerted on the frame by portion BH of the cable.



9. Four tugboats are used to bring an ocean liner to its pier. Each tugboat exerts 100 kN push in the direction shown. Determine the point on hull where a single, more powerful tugboat should push to produce the same effect as the original four tugboats. Also determine the total push and its direction to be exerted by the single tugboat.



10. A concrete foundation mat in the shape of a regular hexagon with 3-m sides supports four column loads as shown. Determine the magnitude and the point of application of the resultant four loads.



11. Compute the resultant force system of the applied loads at position *B*. Also find the simplest resultant in the plane *BCDE*.



12. Find the simplest resultant for the forces acting on the simply supported beam



13. An automatic valve consists of a  $225 \times 225$  mm square plate of uniform thickness weighing 200 N (total). The valve is pivoted about a horizontal axis through *A* located at a distance 100 mm above the lower edge. Determine the depth of water *d* for which the valve will open.



15. The quarter circular uniform gate AB has a width of 6 m. The gate controls the flow of water over the edge B. The gate has total weight of 6800 kg and is hinged about its upper edge A. Find P required to keep the gate closed.



14. Find the simplest resultant of the hydrostatic pressure on the arch dam.



16. Compute reactions at A and B.



17. Compute reactions at A and B.



18. Compute reactions at the fixed end.



19. Determine the tension in cable ABD and reaction at support C. Assume the pulley to be frictionless.



20. A 70 kg (W) overhead garage door consists of a uniform rectangular panel AC 2100 mm high (h), supported by the cable AE attached at the middle of the upper edge of the door and by two sets of frictionless rollers at A and B. Each set consists of two rollers one either side of the door. The rollers A are free to move in horizontal channels, while rollers B are guided by vertical channels. If the door is held in the position for which BD=1050 mm, determine (a) the tension in the cable AE, (2) the reaction at each of the four rollers. Assume a= 1050 mm, b= 700mm



21. Knowing that each pulley has a radius of 250 mm, determine the components of reactions at D and E.



22. The shear shown is used to trim electronic-circuit-board laminates. Knowing that P = 400 N, determine (*a*) the vertical component of the force exerted on the shearing blade at D, (*b*) the reaction at *C*.



23. Determine the force in member BD and the components of reaction at C.



24. The small crane is mounted on one side of the bed of a pickup truck as shown. The weight of the barrel is 120 kg. For the piston,  $\theta = 40^{\circ}$ , determine the magnitude of the force supported by the pin at O and the oil pressure P against the 50 mm diameter piston of the hydraulic cylinder BC.



25. A uniform pipe of radius cover of radius r = 240 mm and mass 30 kg is held in a horizontal position by the cable CD. Determine the tension in the cable.



26. A window is temporarily held open in the 50° position shown by a wooden prop CD until a crank-type opening mechanism can be installed. If a = 0.8m and b = 1.2 m and the mass of the window is 50 kg with mass center at its geometric center, determine the compressive force  $F_{CD}$  in the prop.



27. The rigid L-shaped member ABC is supported by a ball-and-socket joint at A and by three cables. Determine the tension in each cable and the reaction at A caused by the 1 kN load applied at G.



28. Three identical steel balls, each of mass m, are placed in the cylindrical ring which rests on a horizontal surface and whose height is slightly greater than the radius of the balls. The diameter of the ring is such that the balls are virtually touching one another. A fourth identical ball is then placed on top of the three balls. Determine the force P exerted by the ring on each of the three lower balls.



29. Using the method of joints determine the forces in the members of the truss shown. Given AD = 1m, DB = 2m, CD = 1m and P = 3kN.



30 Using method of joints, determine the forces in the members of the truss shown.



31 and 32. For the given loading, determine the zero force member in the trusses shown.



34. Determine the forces in members marked 1, 2 and 3 of the truss shown below.



35. Determine the forces in members GJ and IK of the truss shown.







36. Determine the forces in the members AB, BC, AD and AE.



38. A 100 N force acts as shown on a 300 N block placed on an inclined plane. The coefficients of friction between the block and plane are  $\mu_s = 0.25$  and  $\mu_k = 0.20$ . Determine whether the block is in equilibrium and find the value of the friction force.



37. The diagonal members in the central panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the force in member DE and in the counters which are acting under the given loading.



39. The coefficient of friction between the block and the rail are  $\mu_s=0.30$  and  $\mu_k=0.25$ . Knowing that  $\theta = 30$  deg, determine the smallest value of P required (a) to start the block up the rail (b) to keep it from moving down



40. The uniform pole has a weight of 30 kN and length of 26 m. If it is placed against the smooth wall and on the rough floor in the position d = 10 m. Will it remain in this position when it is released?. The  $\mu_s$  (Coefficient of static friction) = 0.3.



41. A 60 kg cabinet is mounted on casters, which can be locked to prevent their rotation. The coefficient of static friction is 0.35. If h = 600 mm, determine the magnitude of force P required to move the cabinet to the right (a) if all casters are locked. (b) if the casters at B are locked and casters at A are free to rotate, (c) if the casters at B are free to rotate.



42. A 60 kg cabinet is mounted on casters, which can be locked to prevent their rotation. The coefficient of static friction is 0.35. Assuming that the casters at both A and B are locked, determine (a) the force P required to move the cabinet to the right. (b) the largest allowable value of h if the cabinet is not to tip over.



43. The coefficients of friction are  $\mu_s = 0.4$ and  $\mu_k = 0.3$  between all surfaces of contact. Determine the force **P** for which motion of the 30 kg block is impending if cable *AB* (*a*) is attached as shown, (*b*) is removed.



44. The moveable bracket shown may be placed at any height on the 30 mm diameter pipe. If the coefficient of friction between the pipe and bracket is 0.25, determine the minimum distance x at which the load can be supported. Neglect the weight of the bracket.



45. A light metal panel is welded to two short sleeves of 25-mm inside diameter which may slide on a horizontal rod. The coefficient of friction between the sleeves and the rod is 0.40. A cord attached to the corner C is used to move the panel along the rod. Determine the range of values  $\theta$ for which the panel will start moving to the right.



46. The friction tongs shown are used to lift a 350 kg casting. Knowing that h = 864 mm. determine the smallest allowable value of the coefficient of static friction between the casting and blocks *D* and *D*'.



47. Two large cylinders, each of radius r = 500 mm, rotate in the opposite direction and form the main elements of a crusher for stone aggregate. The distance d is set equal to the maximum desired size of crushed aggregate. If d = 20 mm and  $\mu_s = 0.30$ , determine the size s of the largest stones which will be pulled by the crusher by friction alone.



48. What is the force *F* required to hold the two cylinders, each having a mass of 500 N? Take coefficient of static friction = 0.2 for all surfaces of contact. Diameter of each cylinder = 2m



49. What is the force F required to hold the three cylinders, each having a mass of 500 N? Take coefficient of static friction = 0.2 for all surfaces of contact. Diameter of each cylinder = 2m



50. A pulley requires 200 Nm torque to get it rotating in the direction as shown. The angle of wrap is  $\pi$  radians, and  $\mu_s = 0.25$ . What is the minimum horizontal force F required to create enough tension in the belt so that it can rotate the pulley?



51. A flat belt connects pulley *A* to the driving pulley *B*. The coefficients of friction are  $\mu_s = 0.25$  and  $\mu_k = 0.20$  between both pulleys and the belt. Knowing that the maximum allowable tension in the belt is 600 N, determine the largest torque which can be exerted by the belt on pulley *A*.



52. Knowing that the coefficient of static friction is 0.25 between the rope and the horizontal pipe and 0.20 between the rope and the vertical pipe, determine the range of values of P for which equilibrium is maintained.



53. A cord is wiped twice around a pole A and three times around second pole B. Finally cord goes over a half barrel section and supports a mass M of 500 kg. What is tension T required to maintain this load? Take coefficient of friction 0.1 for all surfaces of contact.



54. Assuming frictionless contacts, determine the magnitude of P for equilibrium



55. The pressure p driving a piston of diameter 100 mm is 1 N/mm<sup>2</sup>. At the configuration shown, what weight W will the system hold if friction is neglected



56. Determine the magnitude of the couple M required to maintain the equilibrium of the mechanism.



57. What is the relation among P,Q and  $\theta$  for equilibrium?



58. Find the horizontal and vertical components of the reaction at hinge E of the system shown below



59. A hydraulic lift table consisting of two identical linkages and hydraulic cylinders is used to raise a 1000-kg crate. Members *EDB* and *CG* are each of length 2*a* and member *AD* is pinned to the midpoint of *EDB*. Determine the force exerted by each cylinder in raising the crate for  $\theta = 60^{\circ}$ , a = 0.70 m, and L = 3.20 m.



60. Determine the moment 'M' required to keep the system in equilibrium. All connections are hinged. Points E and F are mid-points of members BC and CD respectively.



61. A force **P** is applied to slider *C* as shown. The constant of the spring is 1.6 kN/m, and the spring is un-stretched when member *BD* is horizontal. Neglecting friction between the slider and the guide rod and knowing that BC = BD = 150 mm, determine the magnitude of **P** so that when the system is in equilibrium.



62. A 10 kg block is attached to the rim of a 300 mm radius disk as shown. Knowing that the spring *BC* is unstretched when  $\theta =$ 0, determine the position or positions of equilibrium and state whether the equilibrium is stable, unstable, or neutral.



63. The spring is un-stretched when  $\theta = 30^{\circ}$ . At any position of the pendulum, the spring remains horizontal. If the spring constant is k = 50 N/m, at what position will the system be in equilibrium.



64. If the springs are un-stretched when  $\theta = \theta_0$ , find the angle  $\theta$  when the weight *W* is applied on the system. Use the method of minimum potential energy.



65. Two bars are attached to a single spring of constant k that is un-stretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.



66. A 10 kg rectangular plate shown in the fig, is suspended at its centre from a rod having a torsional stiffness k = 1.5 Nm/rad. Develop the equation of motion for small angular rotation  $\theta$  (in the plane of the plate) and determine the natural period of vibration of the plate.



67. The uniform beam is supported at its ends by two springs A and B, each having the same stiffness k. When nothing is supported on a beam, it has a period of vibration of 0.83 s. If a 50 kg mass is placed at its centre, the period of vertical vibration is 1.52 s. Compute the stiffness of each spring and the mass of the beam.



68. What is the natural frequency of motion for block A for small oscillation? Consider BC to have negligible mass and body A to be a particle. When body A is attached to the rod, the static deflection is 25 mm. The spring constant  $K_1 = 1.75$  N/mm. Body A weighs 110 N. What is  $K_2$ ?



69. The bent rod shown in the Fig. has negligible mass and supports a 5 kg collar at its end. Develop the equation of a motion and determine the natural period of vibration for the system.



70. Consider a rod ABC to have a uniformly distributed mass of 3 kg (as shown in Fig. of Problem 69). Develop the equation of motion and determine the natural period of vibration for the system.

1. A bracket is subjected is subjected to the system of forces as shown in Fig. 1. (a) Replace the given force-couple system by an equivalent force couple system at B. (b) Locate the points where the line of action of the resultant intersects lines AB, BC and CD.



Fig.1

Fig. 2

2. A transmission tower is held by three wires attached to a pin at A and anchored by bolts B, C, and D. Knowing the tension in cable AC is 3.6 kN (see Fig. 2), determine the required value of tension in each of the cables AB and AD so that the resultant of the three forces applied by the cables at A is vertical.



3. Three children are standing on a 15 x 15 m raft (See Fig.3). The weights of the children at points A, B, and C are 85 N, 60 N, and 90 N, respectively. If a fourth child of weight 95 N climbs onto the raft, determine where she should stand if the other children remain in the positions shown and the line of action of the resultant of the four weights is to pass through the center of the raft.

4. A machine component is subjected to the forces shown in Fig. 4, each of which is parallel to one of the coordinate axes. Replace these forces with an equivalent force-couple system at *A*.

1. Gate AB shown in the figure below is 6 m wide and weighs 50,000 kg when submerged. It is hinged at B and rests against a smooth wall at A. Determine the water level h which will just cause the gate to open (see Fig. 1).



2. A freshwater marsh is drained to the ocean through an automatic tide gate that is 4 m wide and 3 m high. The gate is held by hinges located along its top edge at *A* and bears on a sill at *B*. If the water level in the marsh is h = 6 m, determine the ocean level *d* for which the gate will open. (Specific weight of salt water = 10.05 kN/m<sup>3</sup>.)





3. Find the resultant of the wind forces on the advertising board shown in Fig. 3. The intensity of the wind force normal to the board is 10(1+z) N/m<sup>2</sup>, where z is the vertical distance from ground level.



**Fig. 3** 18

#### **CE102** Engineering Mechanics

#### Tutorial – 3

1. A 500-kg concrete slab is supported by a chain and sling attached to the bucket of the front-end loader shown in Fig. 1. The action of the bucket is controlled by two identical mechanisms, only one of which is shown. Knowing that the mechanism supports half of the 500-kg slab, determine the force (a) in the cylinder CD, (b) in cylinder FH.



Fig. 1

Fig. 2

2. A light rod (see Fig. 2) AD supports a 150N vertical load and is attached to collars B and C, which may slide freely on the rods shown. Knowing that the wire attached at A forms an angle  $\alpha = 30^{\circ}$  with the horizontal, determine (a) the tension in the wire, (b) the reactions at B and C.

3. The elevation of a platform is controlled by two identical mechanisms only one of which is shown in Fig. 3. A load of 5 kN is applied to the mechanism shown. Knowing that the pin at C can transmit only a horizontal force, determine (a) the force in link BE, (b) the components of the force exerted by the hydraulic cylinder on pin H.



4. A vertical load P is applied at end B of rod BC as shown in Fig. 3. (a) Neglecting the weight of the rod, express the angle  $\theta$  corresponding to the equilibrium position in terms of P, I and the counterweight W. (b) Determine the value of  $\theta$  corresponding to equilibrium if P = 2W.

#### **CE102** Engineering Mechanics

#### Tutorial – 4

1. Two rods are welded together to form a T-shaped lever which leans against a frictionless wall at D and is supported by bearings at A and B as shown in Fig. 1. A vertical force P of magnitude 600 N is applied at the mid-point E of rod DC. Determine the reaction at D.



2. A 100 kg uniform rectangular platform is supported in the position shown in Fig. 2 by hinges A and B, and by a cable DCE that passes over a frictionless hook at C. Assuming that the tension is the same in both parts of the cable, determine (a) the tension in the cable, (b) the reactions at A and B. Assume that the hinge at B does not exert any axial thrust.



3. A 2000 N load hangs from the corner C of a rigid pipe ABCDE which has been bent as shown. The pipe is supported by the ball and socket joints A and D which are fastened respectively to the floor and a vertical wall, and by a cable attached at the mid-point E of the portion BC of the pipe and at a point G on the wall. Determine (a) where G should be located if the tension in the cable is to be minimum and (b) the corresponding minimum value of the tension in the cable.

#### **CE 102 Engineering Mechanics**

#### **Tutorial 5**

1. Determine the forces in the members FH, EH, EG, LM, MK and LK.





2. Find the forces in member BD and DE.



3. The truss shown in Fig. 3 has six joints and nine members. The truss is resting on hinge support at A and roller support at D. If the allowable force in members AB as well as CD is 4710 N, determine the magnitude of the largest load P that can be applied at joints E and F.





#### **CE 102 Engineering Mechanics**

#### **Tutorial 6**

1. What is the height h of the step shown in the figure so that the force P will roll the 25 kg cylinder over the step at the same time that there is impending slippage at A? Take coefficient of static friction to be equal to 0.3.



- 2. What is the minimum coefficient of friction required just to maintain the bracket (Fig. 2) and its 250 kg load? The center of gravity is 1.8m from the center line.
- 3. Two slender rods of negligible weight are pin-connected at *A* and attached to the 18-N block *B* and the 80-N block *C* as shown in Fig. 3. The coefficient of static friction is 0.55 between all surfaces of contact. Determine the range of values of *P* for which equilibrium is maintained.



4. A 200 N block rests as shown in Fig. 4 on a wedge of negligible weight. The coefficient of static friction  $\mu_s$  is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For P = 100 N, determine the value of  $\mu_s$  for which motion is impending.

1. The setup shown is used to measure the output of a small turbine. When the flywheel is at rest, the reading of each spring scale is 70 N. If a 12.60 Nm couple must be applied to the flywheel to keep it rotating clockwise at a constant speed, determine (*a*) the reading of each scale at that time, (*b*) the coefficient of kinetic friction. Assume that the length of the belt does not change. (see fig.1)



- 2. A differential band brake is used to control the speed of a drum which rotates at a constant speed. Knowing that the coefficient of kinetic friction between the belt and the drum is 0.30 and that a couple of magnitude 125 N. m is applied to the drum, determine the corresponding magnitude of the force **P** that is exerted on end *D* of the lever when the drum is rotating (*a*) clockwise, (*b*) counter-clockwise. (see Fig. 2)
- 3. In the pivoted motor mount shown, the weight **W** of the 175-N motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums A and B is 0.40, and neglecting the weight of platform CD, determine the largest couple which can be transmitted to drum B when the drive drum A is rotating clockwise. (See Fig. 3)



Fig. 3

Fig. 4

4. An idler pulley that is free to rotate is used to increase angle of wrap for the pulleys as shown in the figure. If the tension in the slack side is 1000 N, Find the maximum torque that can be transmitted by the pulleys for a coefficient of friction 0.3. (See Fig. 4)

1. Knowing that the line of action of the force Q passes through point *C*, derive an expression for the magnitude of Q required to maintain equilibrium.





2. The mechanism shown is acted upon by the force **P** as shown in Fig. 2. Derive an expression for the magnitude of the force Q required to maintain equilibrium.







- 3. Determine the couple *M* which must be applied at *O* in order to support the mechanism in the position  $\theta = 30^{\circ}$ as shown in Fig. 3. The masses of the disk at *C*, bar *OA*, and bar *BC* are  $m_o$ , m, and 2m respectively.
- 4. A power-operated loading platform designed for the back of a truck is shown as shown in Fig. 4. The position of the platform is controlled by the hydraulic cylinder, which applies force at *C*. The links are pivoted to the truck frame at *A*, *B*, and *F*. Determine the force *P* supplied by the cylinder in order to support the platform in the position shown. The mass of the platform and the links may be neglected compared with that of the 250 kg crate with center of mass at *G*.



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1. A vertical force P of magnitude 150 N is applied to the linkage at B as shown in Fig. 1. The constant of the spring is 2000 N/m., and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage, determine the value of  $\theta$  corresponding to equilibrium.



- 2. Rod ABC is connected through a pin and slot to a sleeve which slides on a vertical rod as shown in Fig. 2. Before the weight W of 100 N is applied at C, the rod is inclined at an angle of 45°. If K of the spring is 8000 N/m, what is the angle  $\theta$  for equilibrium? The length of AB is 300 mm and the length of BC is 200 mm when  $\theta = 45^{\circ}$ . Neglect friction and all weights other than W.
- 3. A slender rod *AB* of mass *m* is attached to two blocks *A* and *B* which can move freely in the guides shown. Knowing that the spring is unstretched when y = 0 determine the value of *y* corresponding to equilibrium when m = 12 kg, l = 750 mm, and k = 900 N/m.



4. Two identical rods are pinned together at B. At B, there is a torsional spring with rotational stiffness 500 Nm/rad. What is the max weight W that each rod can have for a case of stable equilibrium when the rods are collinear?

1. What are the differential equation of motion about the static equilibrium configuration and the natural frequency of motion of body for small motion of BC? Neglect inertial effects from BC. Assume  $k_1$ = 15 N/m,  $K_2$  = 20 N/m,  $K_3$  = 30 N/m and  $W_A$ = 30 N.



Fig. 1

2. What is the equivalent torsional spring constant on the disc from the shafts? The modulus of rigidity for the shafts is  $10 \times 10^{10}$  N/m<sup>2</sup>. What is the natural frequency of the system? If the disc is twisted  $10^{0}$  and then released, what will its angular position be in 1 s? Neglect the mass of the shaft. The disc weighs 143 N.



Fig. 2

3. Derive the equation of motion for small rotation the rigid uniform beam of mass 12 kg. Take K = 2500N/m, C = 8 kg/s. What is the damping ratio? If the initial rotation of 0.1 rad at A is applied and the bar is released, compute the displacement at D at the end of 3 s.



**Fig. 3** 26

4. For the rigid uniform plate shown in the diagram, derive the equation of motion for small oscillation.



Fig. 4

5. (a) If the bar *ABCDE* is of negligible mass, what is the natural frequency of free oscillation of the system for small amplitude of motion? Take K = 25 N/m, m = 2 kg, l = 1 m. (b) If mass of bar *ABCDE* is 0.4 kg/m, what is the natural frequency of free oscillation of the system for small amplitude of motion? Take K = 25 N/m, m = 2 kg, l = 1 m.



Fig. 5