§15.2 Approximate Analysis of a Continuous Beam for Gravity Load

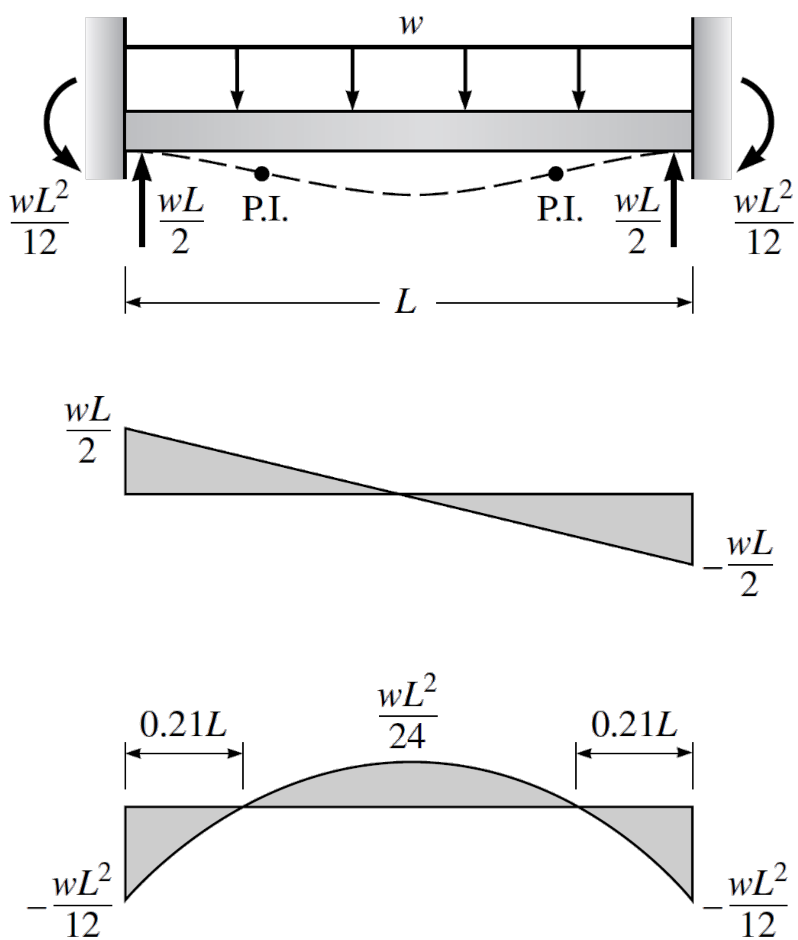

Figure 15.1 Location of points of inflection and shear and moment curves for beams with various idealized end conditions

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§15.2 Approximate Analysis of a Continuous Beam for Gravity Load

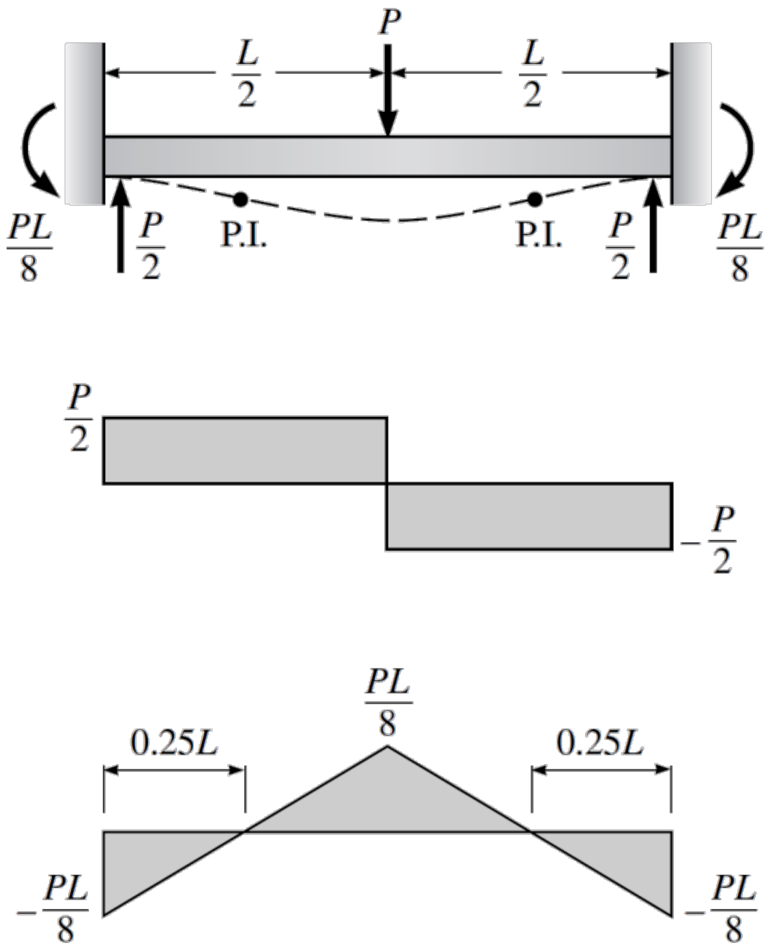


Figure 15.1 Location of points of inflection and shear and moment curves for beams with various idealized end conditions (continued)

§15.2 Approximate Analysis of a Continuous Beam for Gravity Load

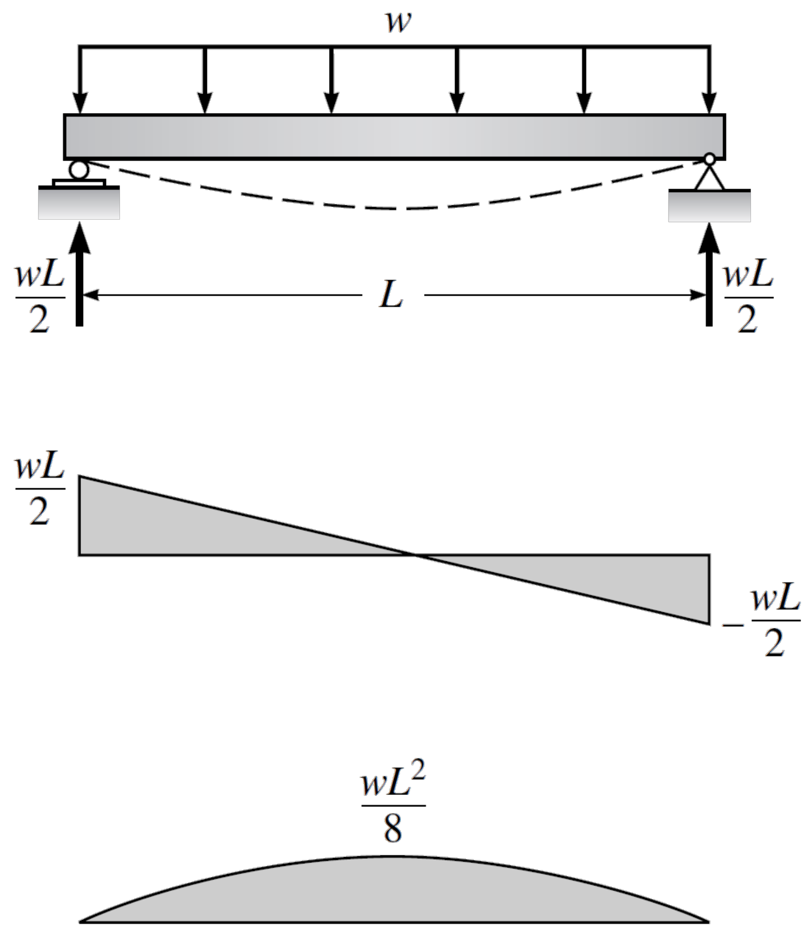


Figure 15.1 Location of points of inflection and shear and moment curves for beams with various idealized end conditions (continued)

§15.2 Approximate Analysis of a Continuous Beam for Gravity Load

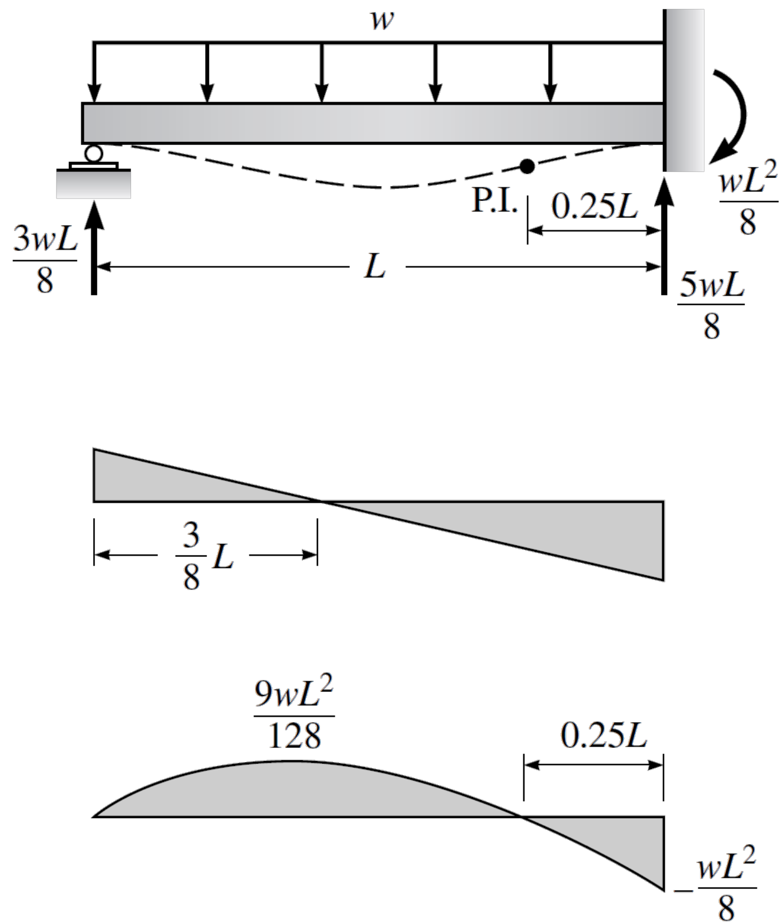
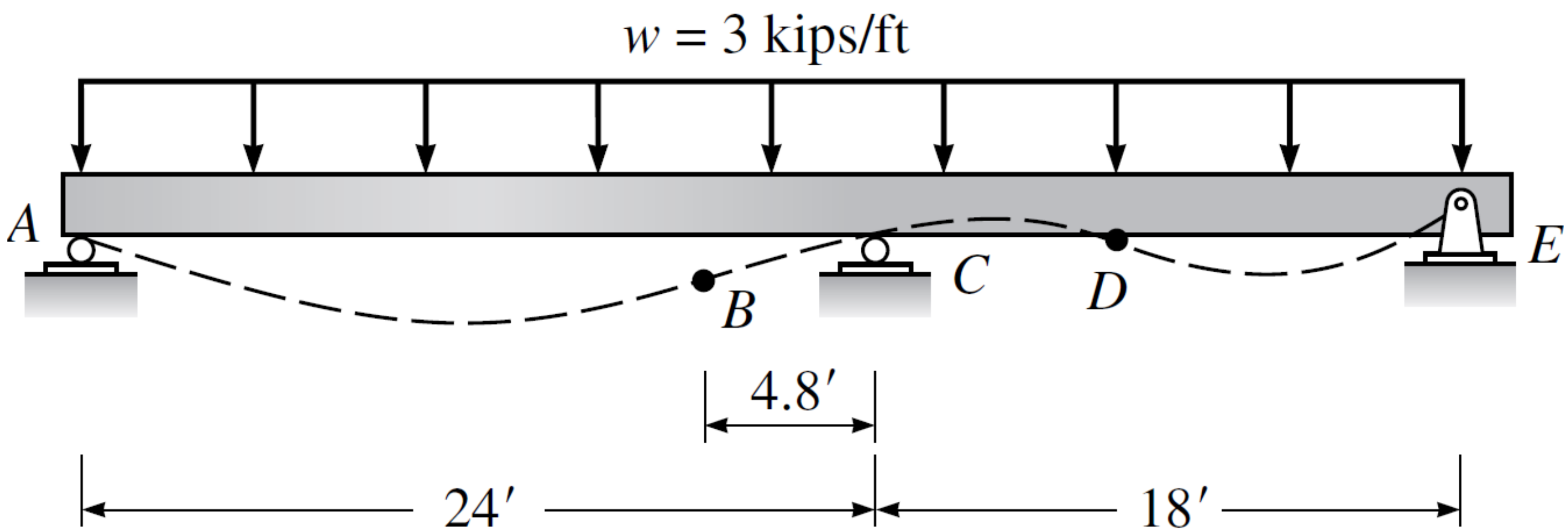


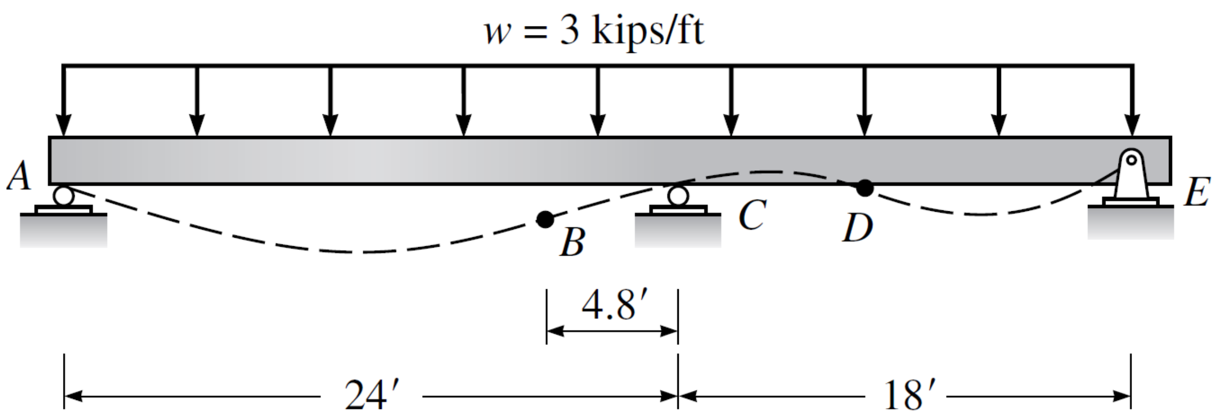
Figure 15.1 Location of points of inflection and shear and moment curves for beams with various idealized end conditions (continued)

Example 15.1

Carry out an approximate analysis of the continuous beam in Figure 15.2*a* by assuming the location of a point of inflection.



Example 15.1 Solution

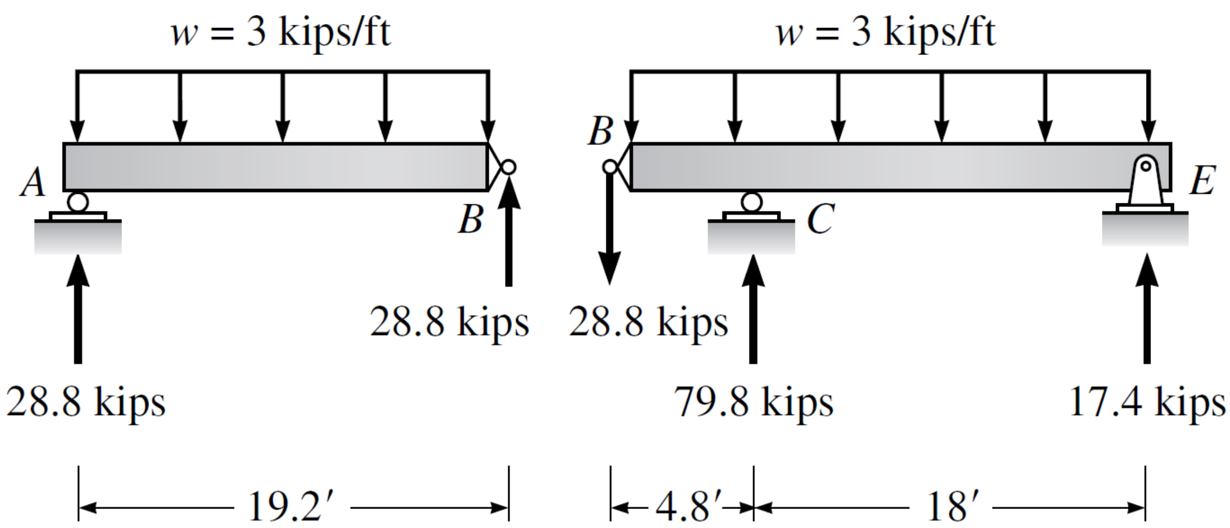


- The approximate location of each point of inflection is indicated by a small black dot on the sketch of the deflection shape.

- Because span *AC* is longer than span *CE*, it applies a greater fixed-end moment to joint *C* than span *CE* does. Therefore, joint *C* rotates counterclockwise.

- The rotation of joint *C* causes the point of inflection at *B* to shift a short distance to the right toward support *C*. Arbitrarily guess that the point of inflection is located $0.2L_{AC}$ = 4.8 ft to the left of support *C*.

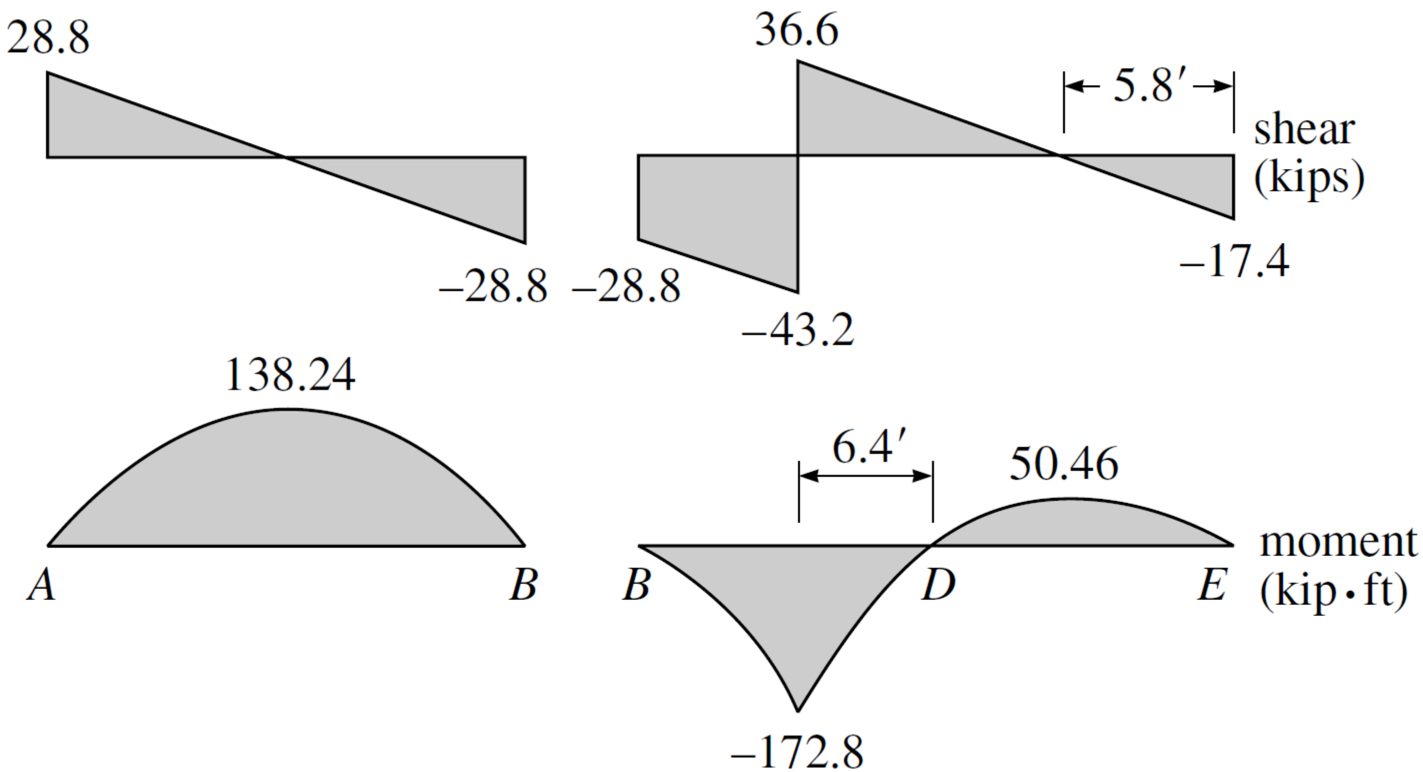
Example 15.1 Solution (continued)



Free bodies of beam on either side of the point of inflection

- Imagine that a hinge is inserted into the beam at the location of the point of inflection, and compute the reactions using the equations of statics.

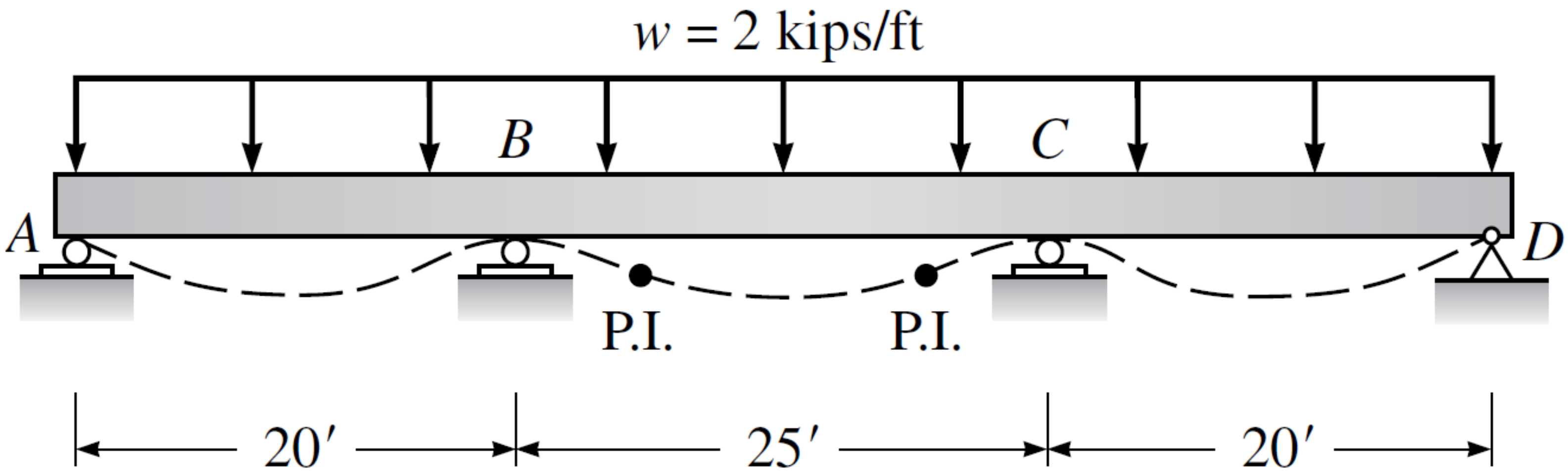
Example 15.1 Solution (continued)



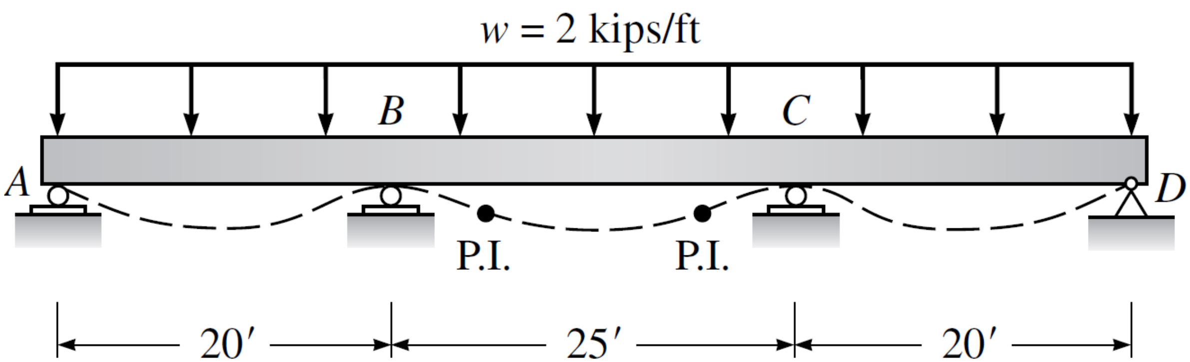
Shear and moment curves based on the approximate analysis

Example 15.2

Estimate the values of moment at midspan of member *BC* as well as at support *B* of the beam in Figure 15.3*a*.



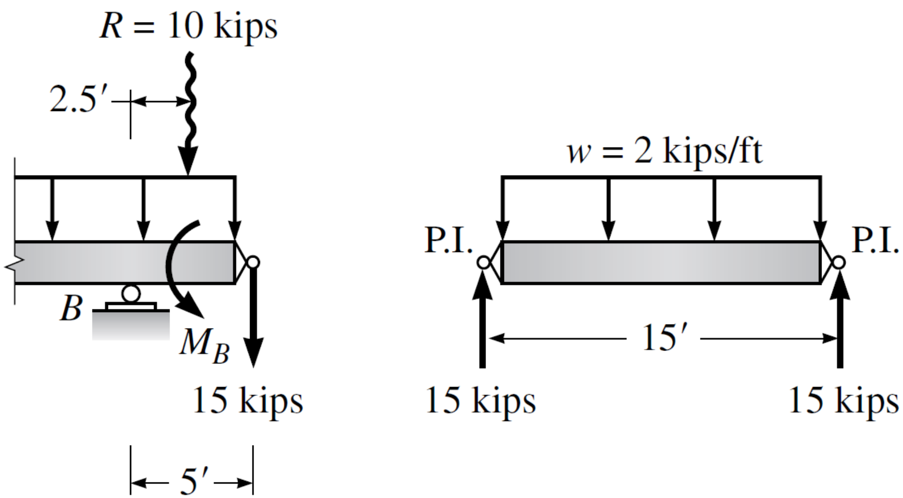
Example 15.2 Solution



- Since the beam is indeterminate to the second degree assume the location of two points of inflection to analyze by the equations of statics.

- Assume points of inflection develop at a distance of 0.2*L* = 5 ft from each support.

Example 15.2 Solution (continued)



Free bodies of the center span

- The moment at midspan equals

$$M \approx \frac{wL^2}{8} = \frac{2(15)^2}{8} = 56.25 \text{ kip} \cdot \text{ft}$$

- Treating the 5-ft segment of beam between the hinge and the support at *B* as a cantilever, compute the moment at *B* as

$$M_B \approx 15(5) + (2)5(2.5) = 100 \text{ kip} \cdot \text{ft}$$

§15.2 Approximate Analysis of a Continuous Beam for Gravity Load

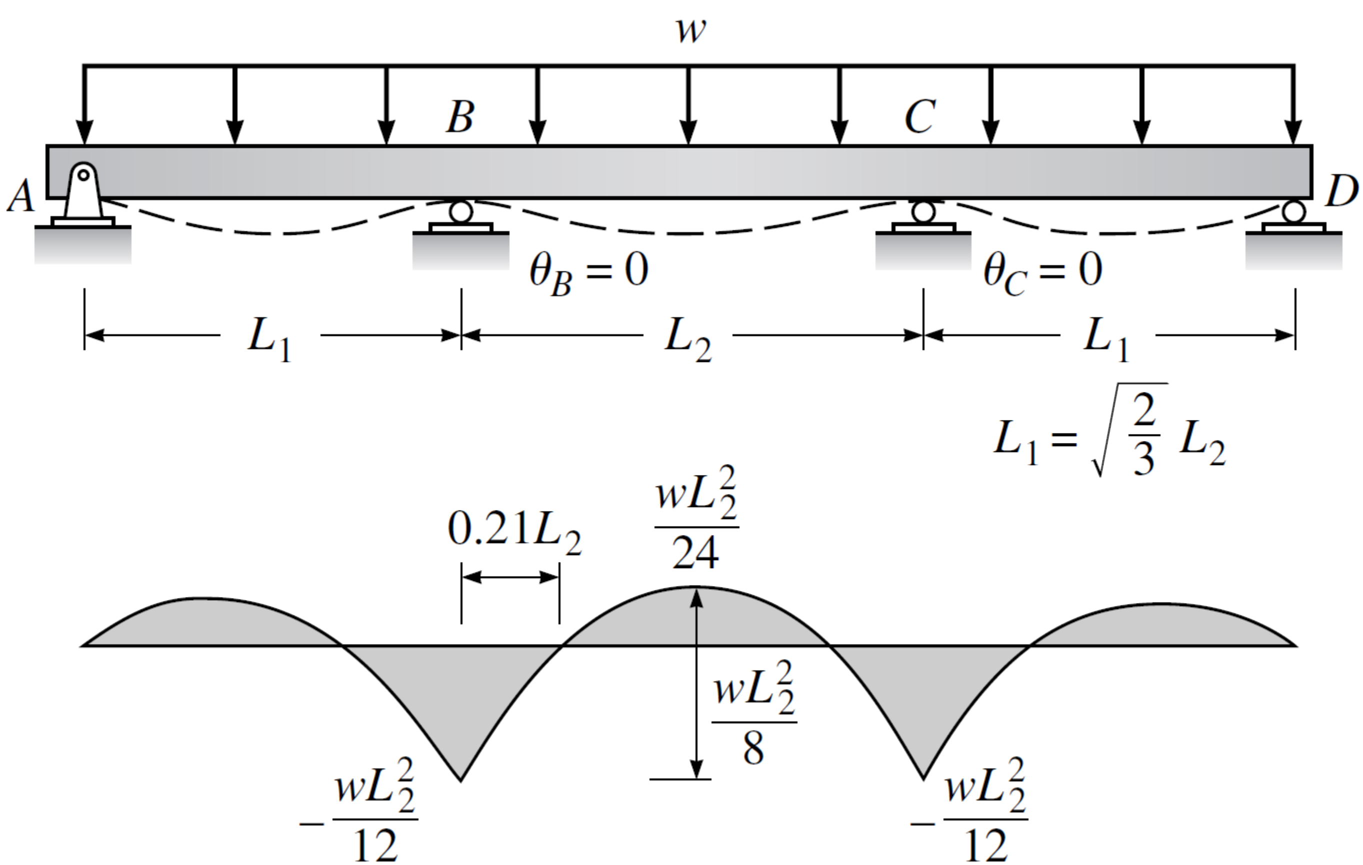


Figure 15.4

§15.2 Approximate Analysis of a Continuous Beam for Gravity Load

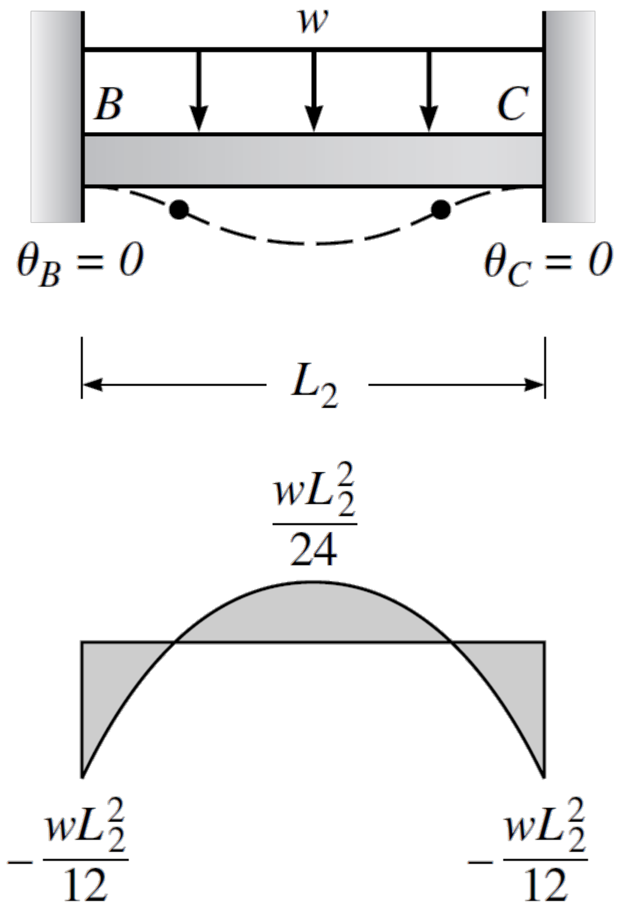


Figure 15.4 (continued)

§15.2 Approximate Analysis of a Continuous Beam for Gravity Load

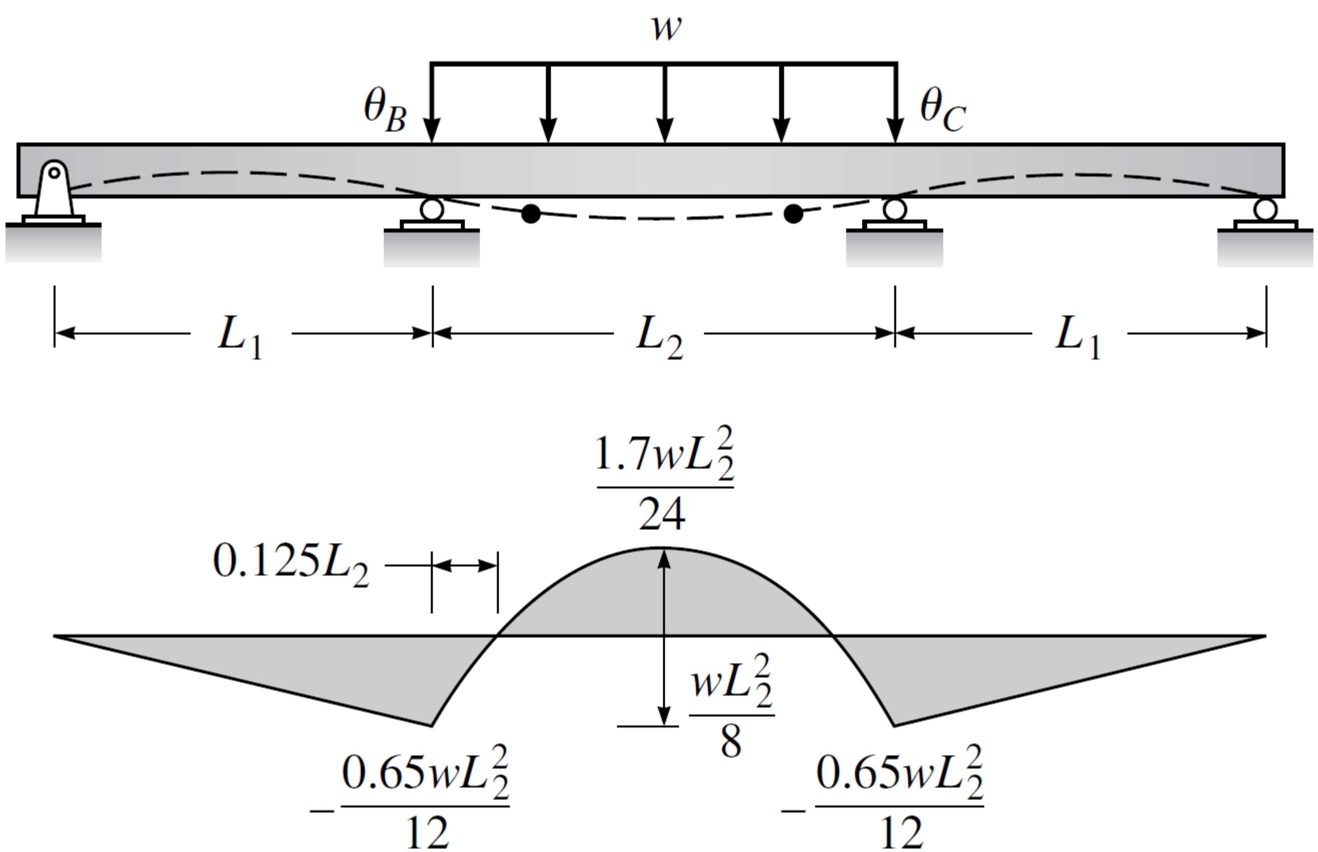


Figure 15.4 (continued)

§15.2 Approximate Analysis of a Continuous Beam for Gravity Load

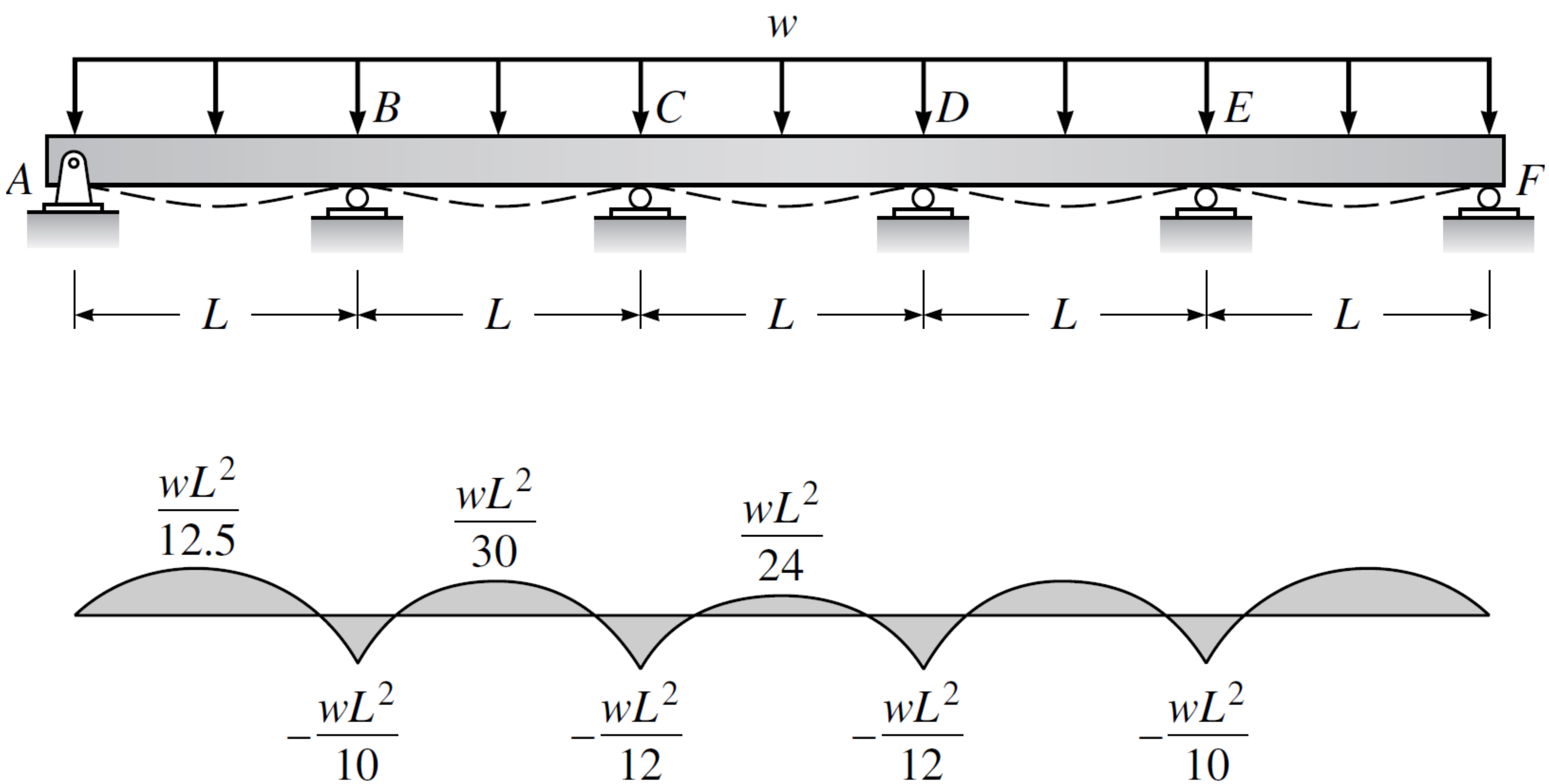
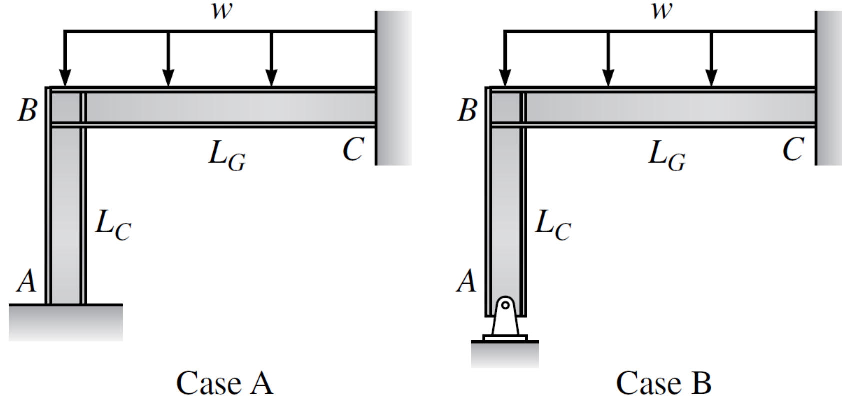


Figure 15.5

§15.3 Approximate Analysis of a Rigid Frame for Vertical Load

Case A: base of column fixed



Case B: base of column pinned

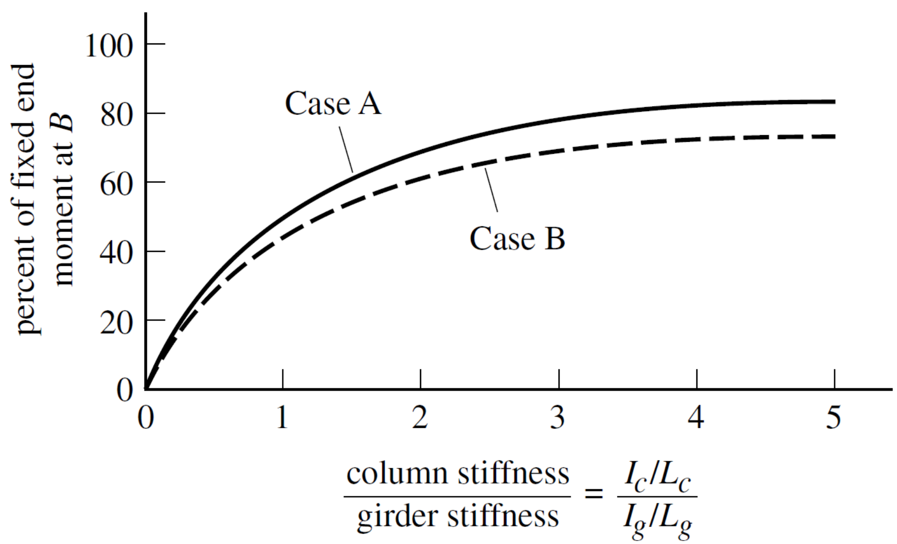
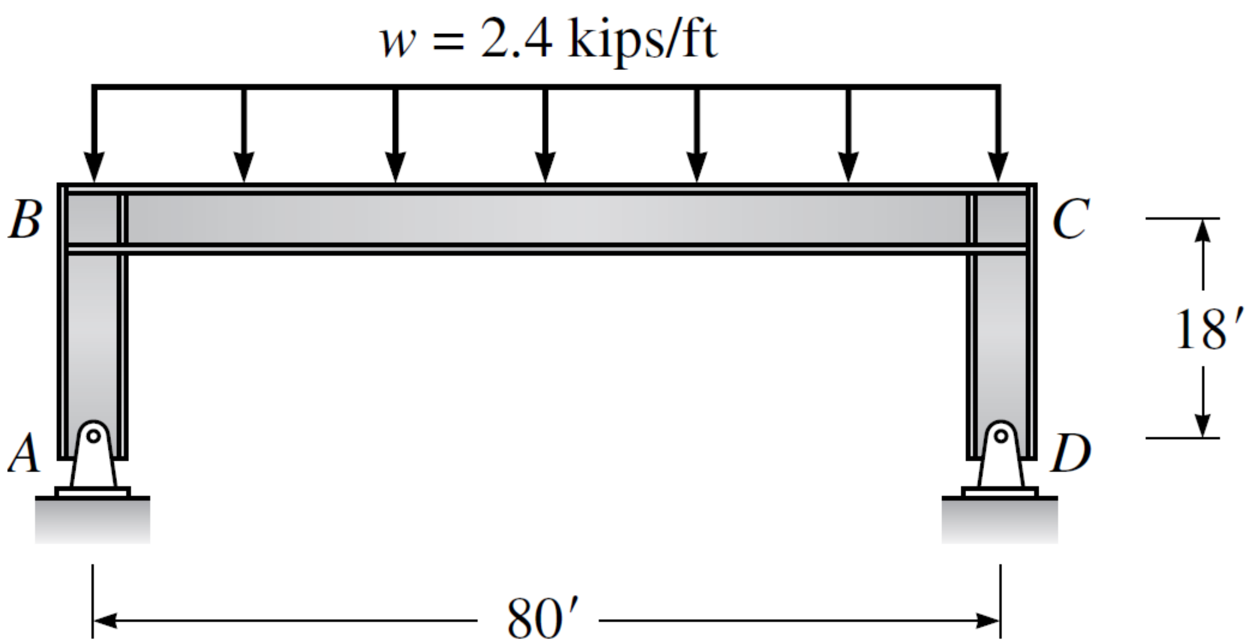


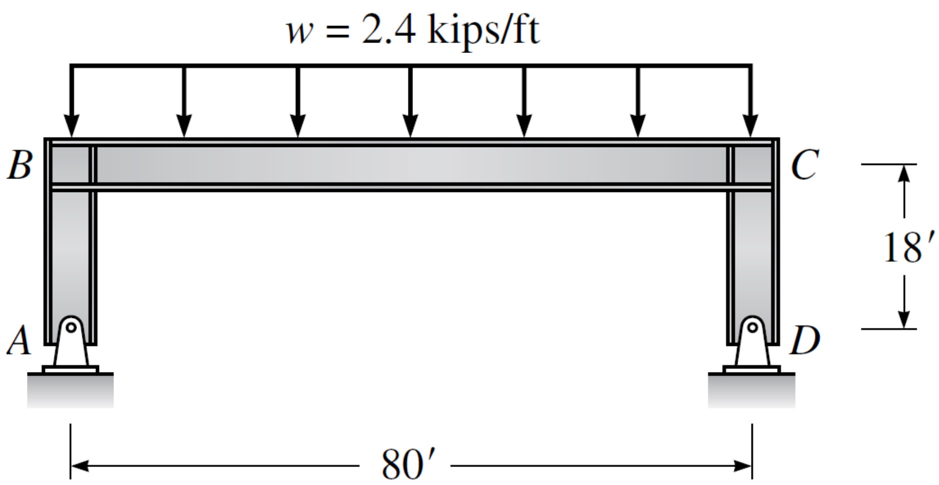
Figure 15.6 Influence of column stiffness on the end moment at joint *B* in a girder whose far end is fixed

Example 15.3

Analyze the symmetric frame in Figure 15.7*a* by estimating the values of negative moments at joints *B* and *C*. Columns and girders are constructed from the same-size members—that is, *EI* is constant.



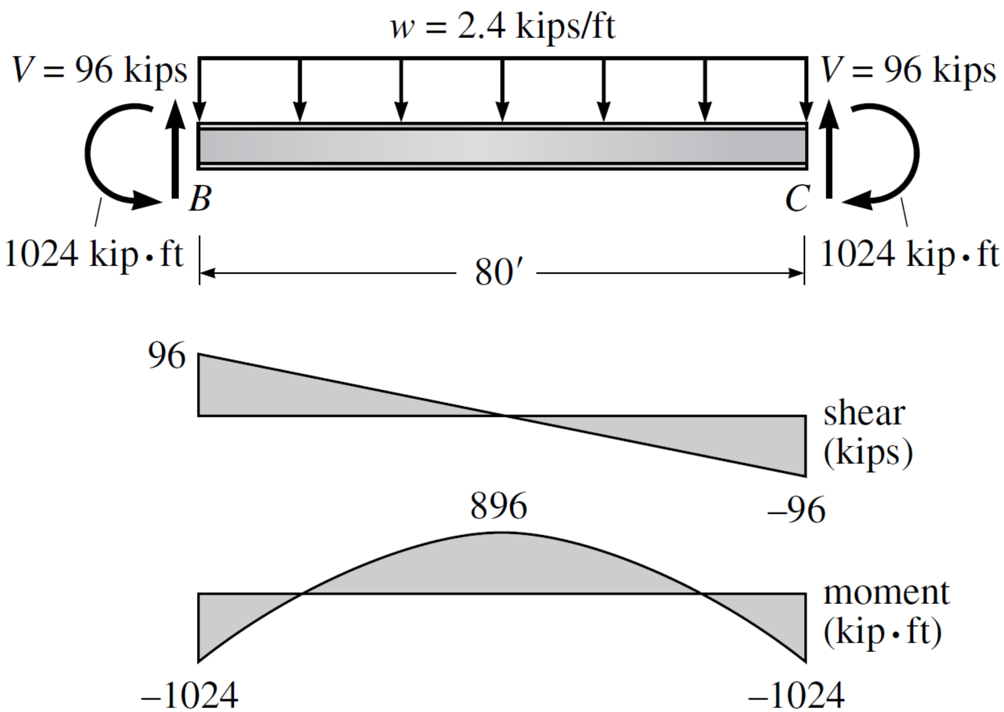
Example 15.3 Solution

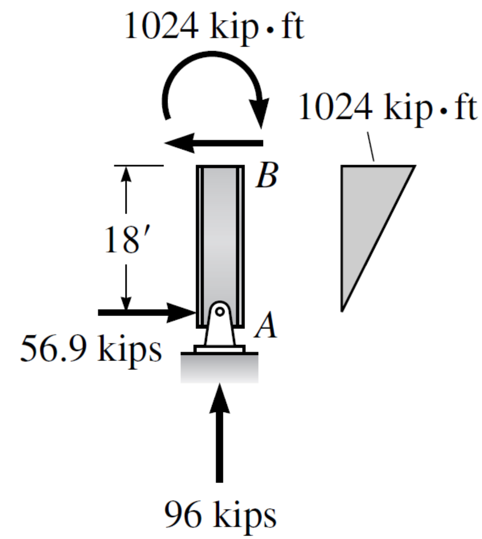


- Assume the negative moments at joints *B* and *C* are equal to 80 percent of the end moments in a fixed-end beam of the same span.

$$M_B = M_C = -0.8\frac{wL^2}{12} = -\frac{0.8(2.4)80^2}{12} = -1024 \text{ kip} \cdot \text{ft}$$

Example 15.3 Solution (continued)





Free body of girder; and approximate shear and moment diagrams

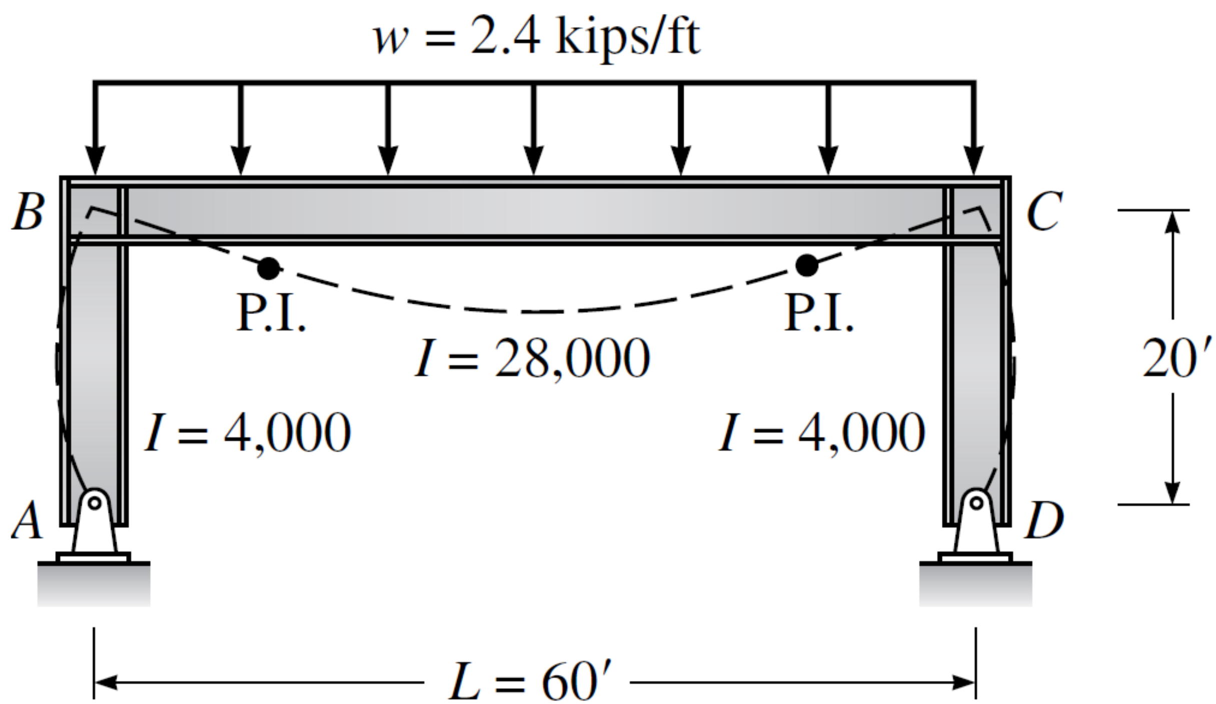
Free body of column with estimated value of the end moment

- Isolate the girder and the column, compute the end shears using the equations of statics, and draw the shear and moment curves.

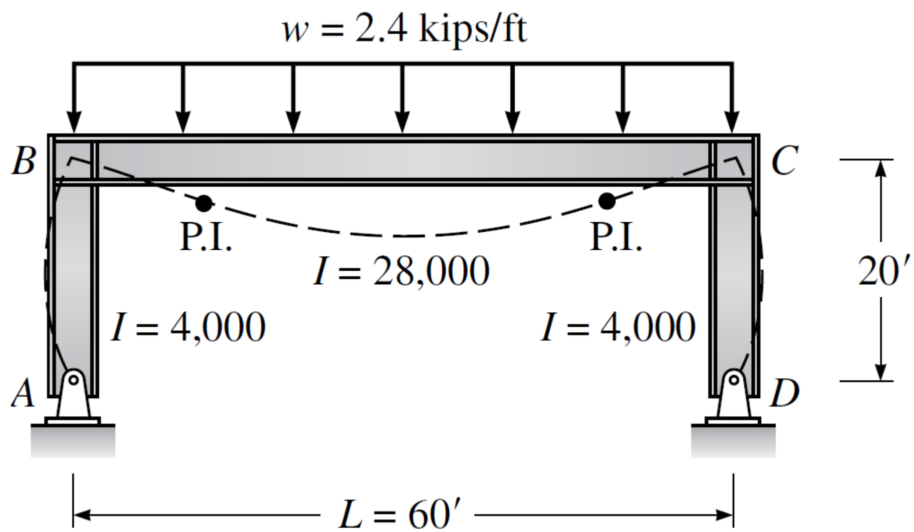
- An exact analysis of the structure indicates that the end moment in the girder is 1113.6 kip-ft and the moment at midspan is 806 kip-ft.

Example 15.4

Estimate the moments in the frame shown in Figure 15.8*a* by guessing the location of the points of inflection in the girder.



Example 15.4 Solution



- Assume the points of inflection in the girder are located 0.12*L* from the ends.
- Compute the distance *L* between points of inflection in the girder.

$$L' = L - (0.12L)(2) = 0.76L = 45.6 \text{ ft}$$

Example 15.4 Solution (continued)

- Since the segment of girder between points of inflection acts as a simply supported beam (i.e., the moments are zero each end), the moment at midspan equals

$$M_c = \frac{wL'^2}{8} = \frac{2.4(45.6)^2}{8} = 623.8 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

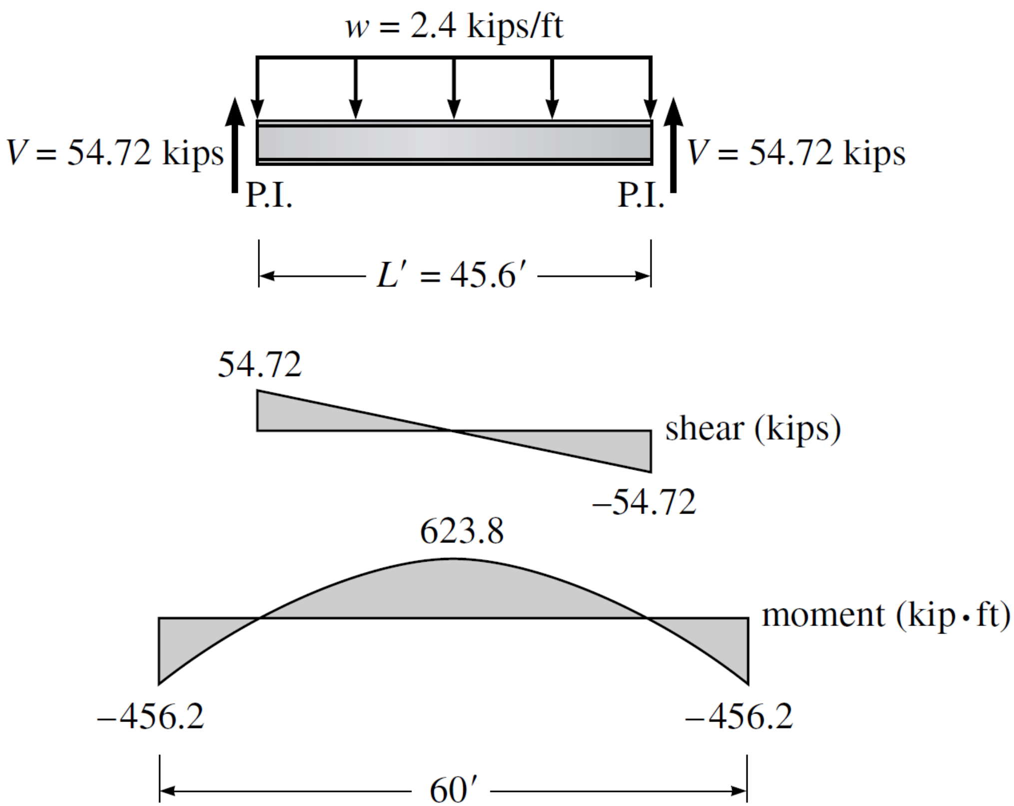
- Using Equation 15.1, compute the girder end moments M_s:

$$M_s + M_c = \frac{wL^2}{8} = \frac{2.4(60)^2}{8} = 1080 \text{ kip} \cdot \text{ft}$$

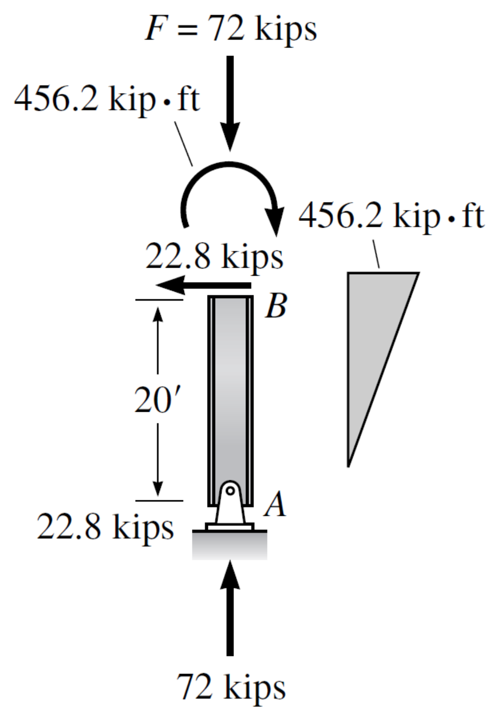
$$M_s = 1080 - 623.8 = 456.2 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

- The exact value of moment at the ends of the girder is 404.64 kip-ft.

Example 15.4 Solution (continued)

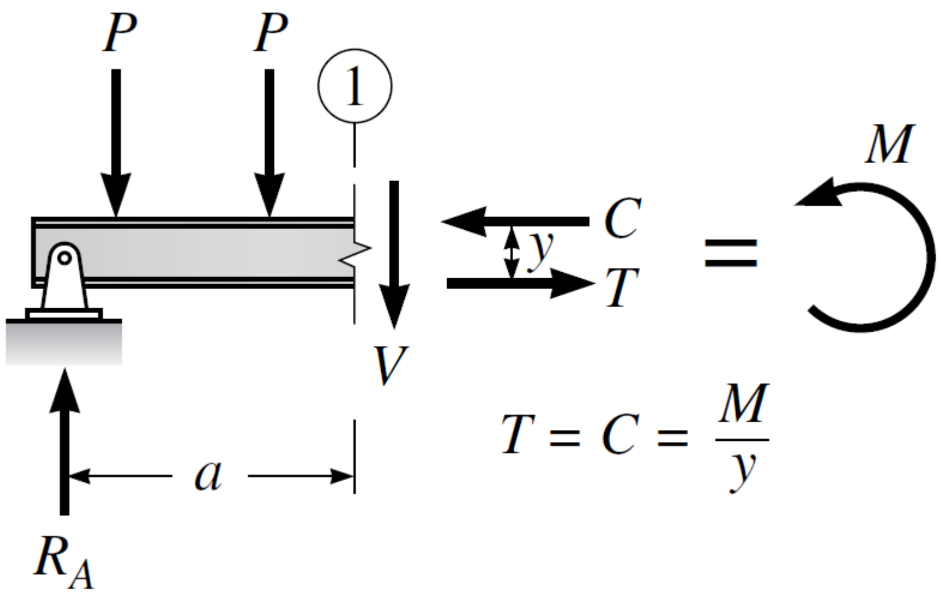


Free body of girder between points of inflection



Free body of column *AB*

§15.4 Approximate Analysis of a Continuous Truss



A beam

Figure 15.9 Internal forces

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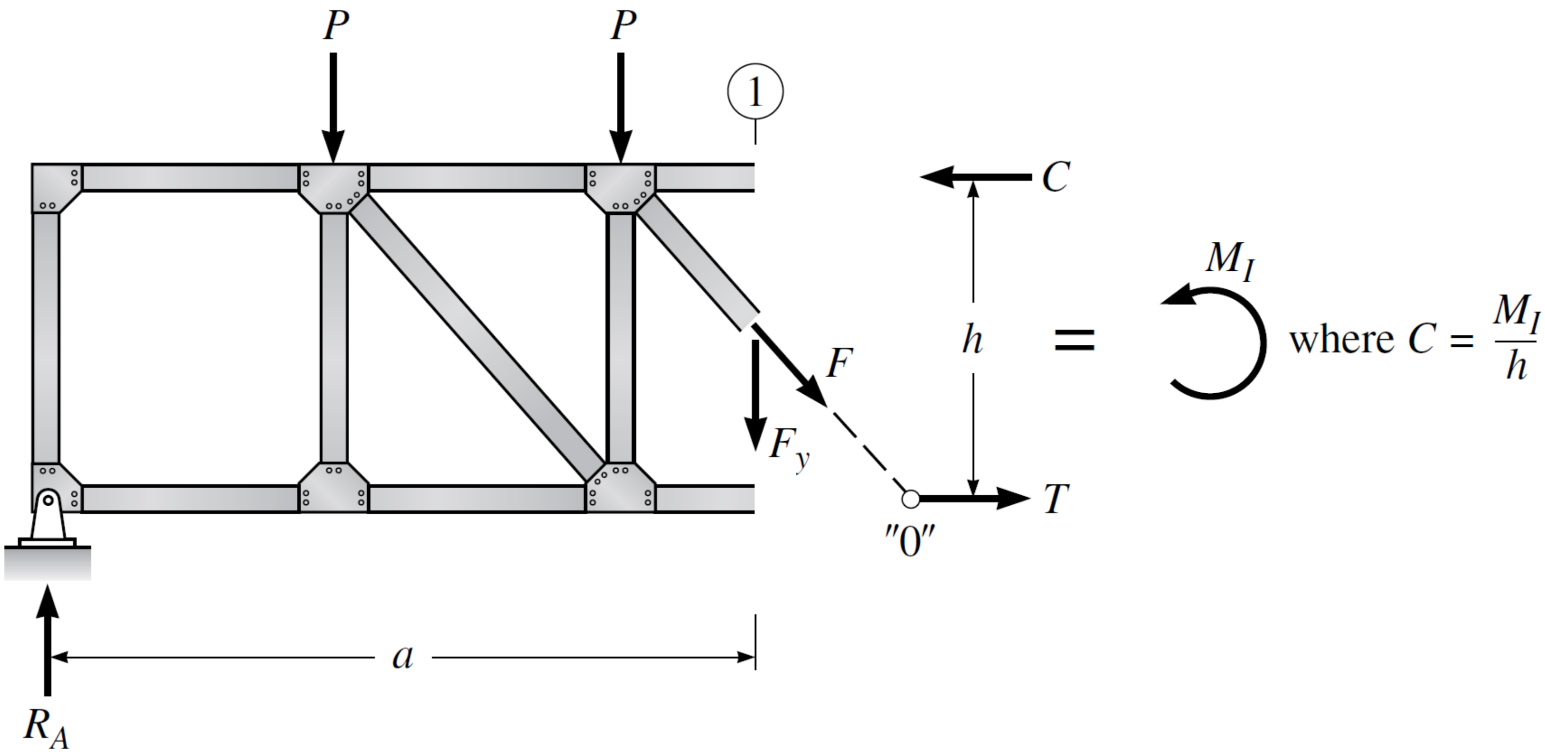
§15.4 Approximate Analysis of a Continuous Truss

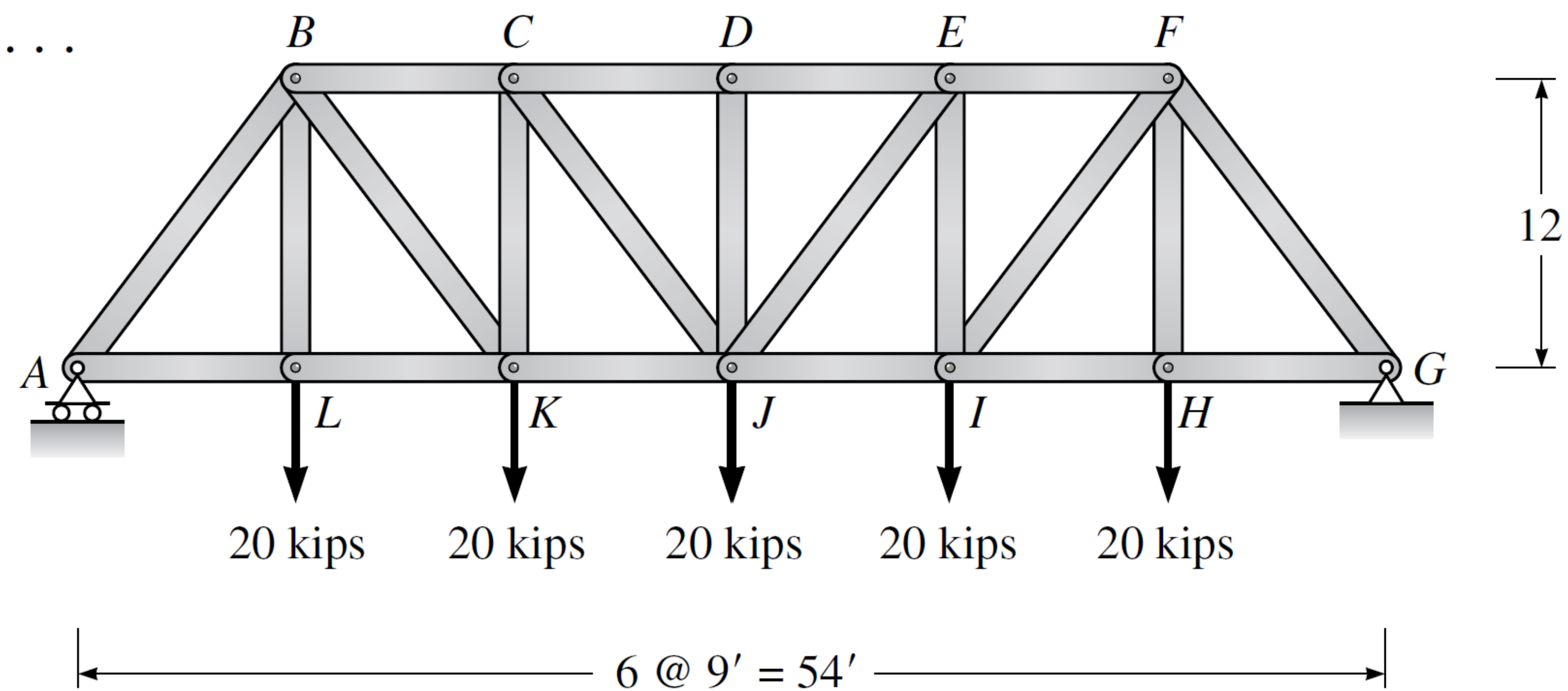


A truss

Figure 15.9 Internal forces (continued)

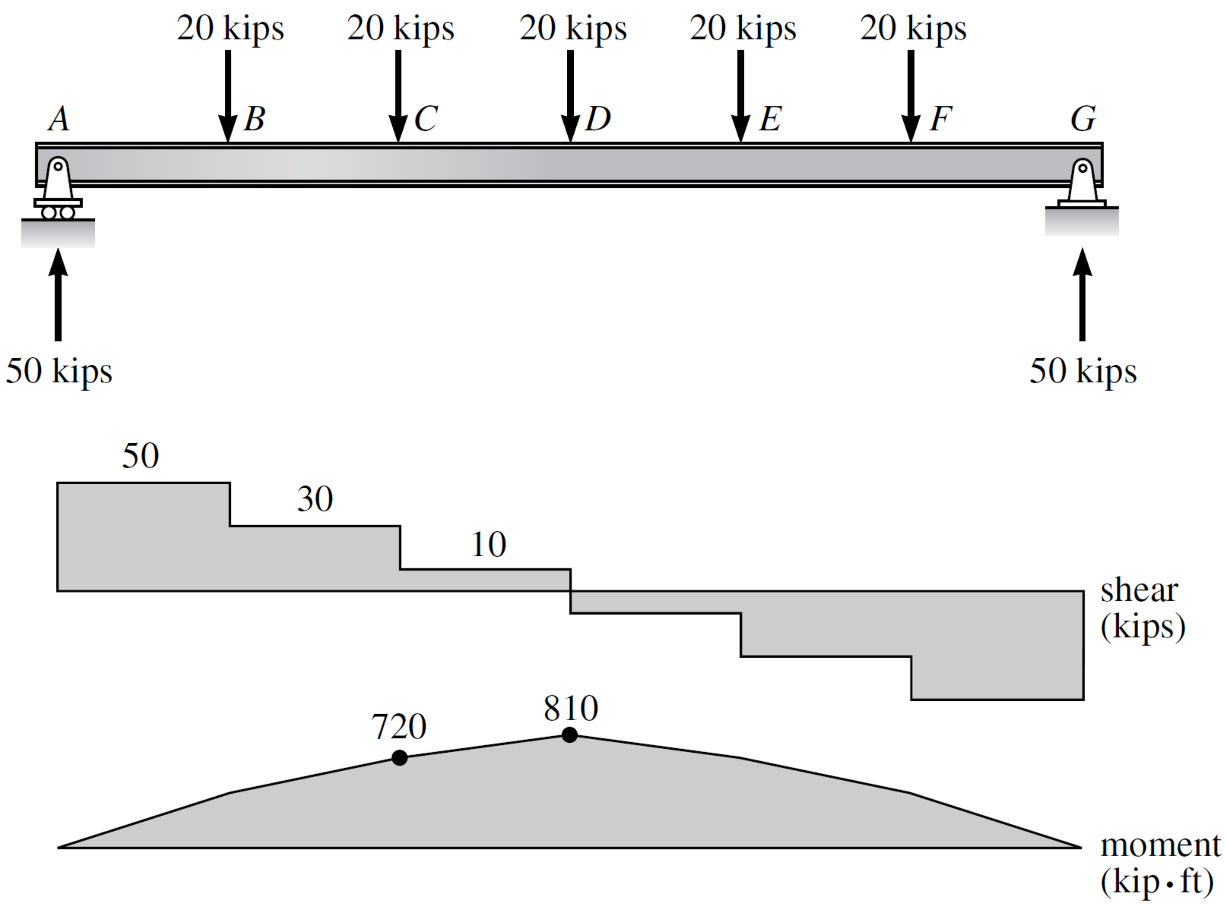
Example 15.5

By analyzing the truss in Figure 15.10*a* as a beam, compute the axial forces in the top chord (member *CD*) and bottom chord (member *JK*) at midspan and in diagonal *BK*. Compare the values of force to those computed by the method of joints or sections.



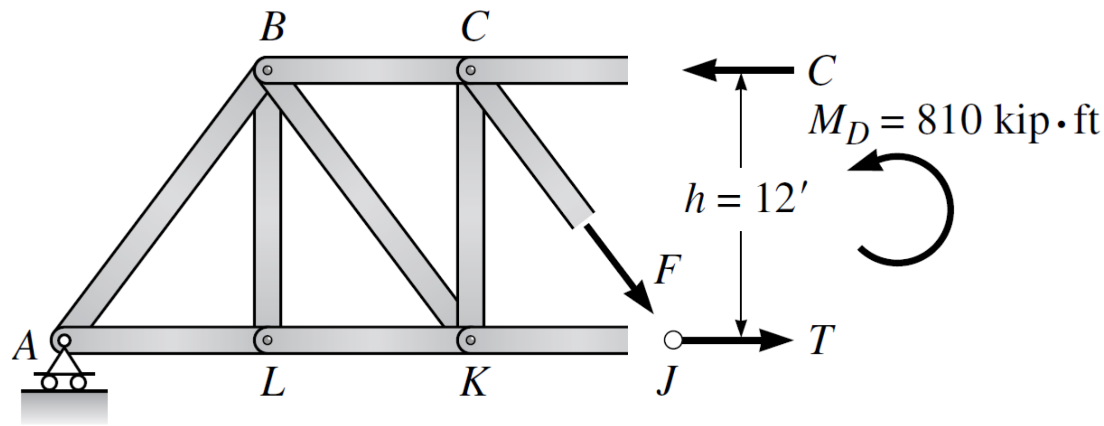
Example 15.5 Solution

- Apply the loads acting at the bottom panel points of the truss to a beam of the same span, and construct the shear and moment curves.



Loads from truss applied to beam of same span

Example 15.5 Solution (continued)



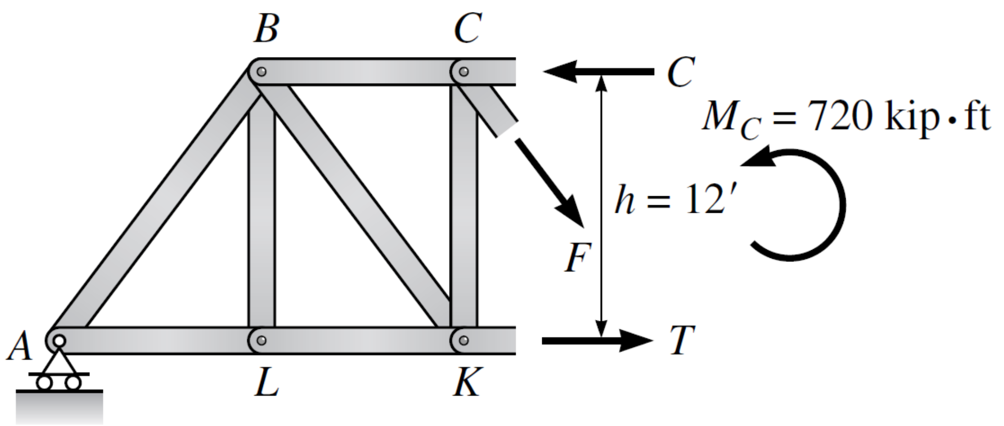
Free body of truss cut by a vertical section an infinitesimal distance to the left of midspan

- Compute the axial force in member *CD* of the truss, using Equation 15.2 and the beam moment at *D*.

$$\Sigma M_J = 0$$

$$F_{CD} = C = \frac{M_D}{h} = \frac{810}{12} = 67.5 \text{ kips} \qquad \textbf{Ans.}$$

Example 15.5 Solution (continued)

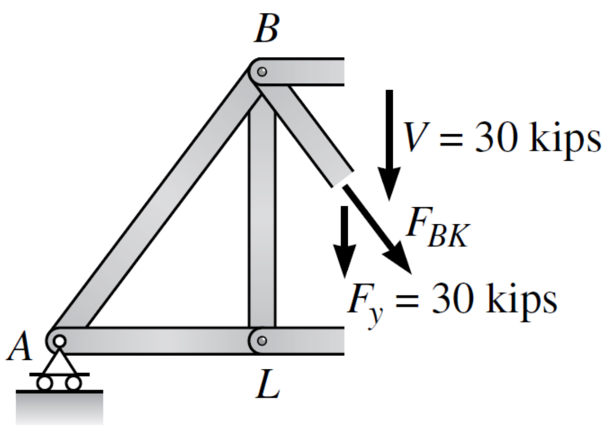


Free body of truss cut by a vertical section at infinitesimal distance to the right of joint *K*

- Compute the axial force in member *JK* of the truss, using Equation 15.2 and the beam moment at *C*.

$$F_{JK} = C = \frac{M_C}{h} = \frac{720}{12} = 60 \text{ kips} \qquad \textbf{Ans.}$$

Example 15.5 Solution (continued)



Free body of truss cut by a vertical section through panel *BC*

- Compute the force in diagonal *BK*. Equate the shear of 30 kips between *BC* of the beam to the vertical component F_y of the axial force in bar *BK*.

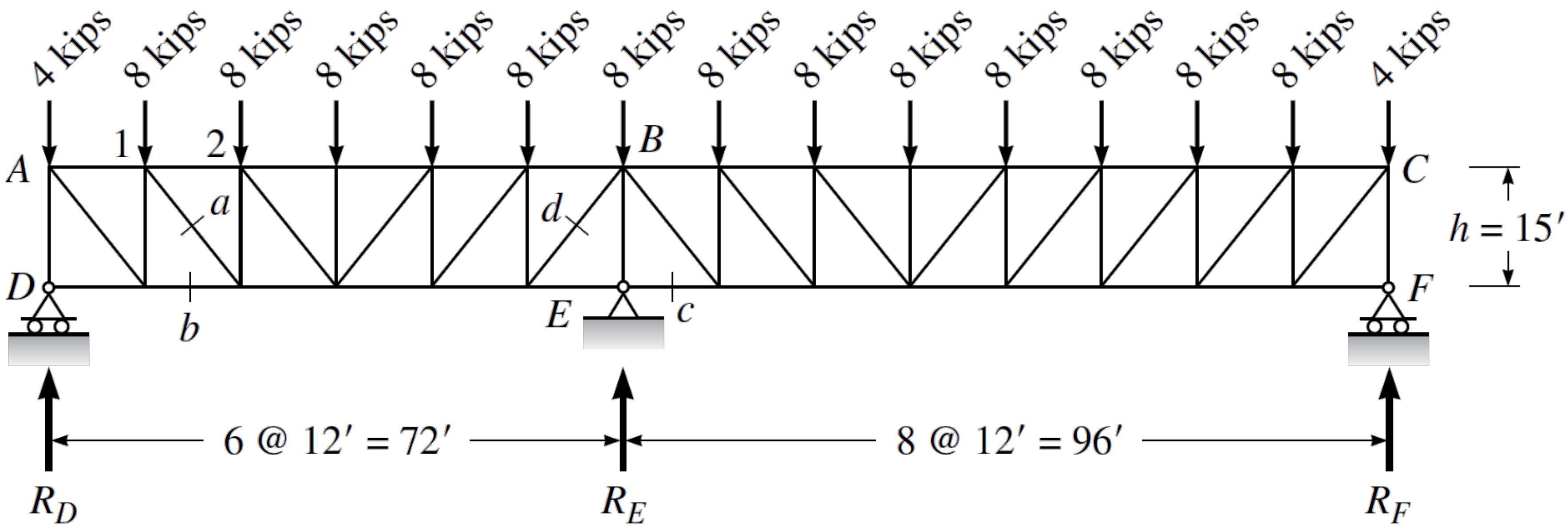
$$F_y = V$$

$$= 30 \text{ kips}$$

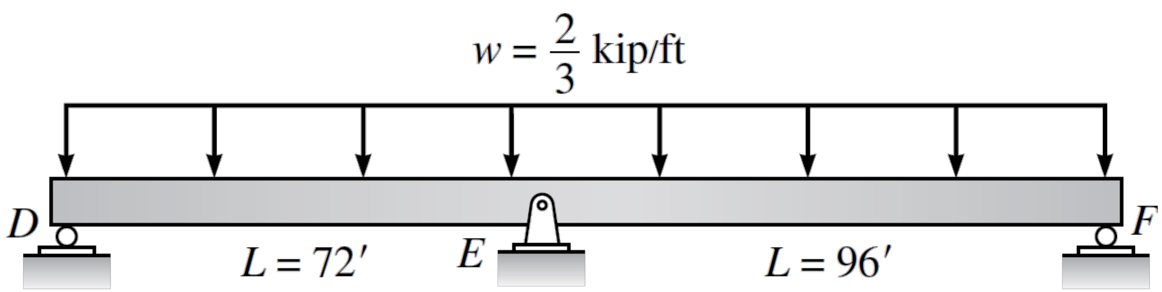
$$F_{BK} = \frac{5}{4} F_y = 37.5 \text{ kips} \qquad \textbf{Ans.}$$

Example 15.6

Estimate the forces in bars *a*, *b*, *c*, and *d* of the continuous truss in Figure 15.11.



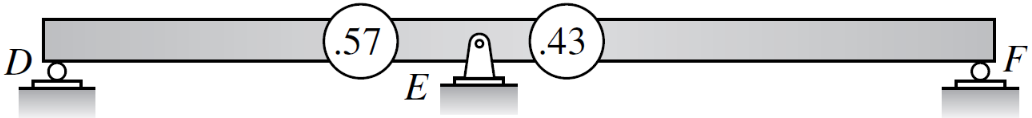
Example 15.6 Solution



- The truss will be analyzed as a continuous beam of constant cross section. Using Equation 15.3, convert the panel loads to a statically equivalent uniform load.

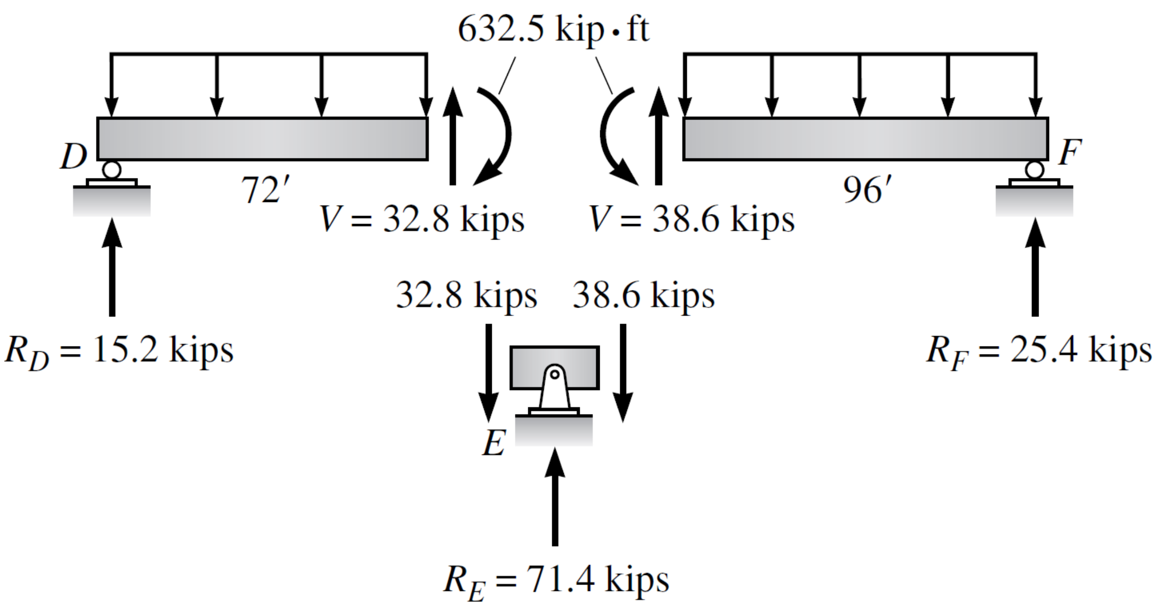
$$w = \frac{\Sigma P}{L} = \frac{(8\text{ kips})(13) + (4\text{ kips})(2)}{72 + 96} = \frac{2}{3}\text{ kip/ft}$$

Example 15.6 Solution (continued)



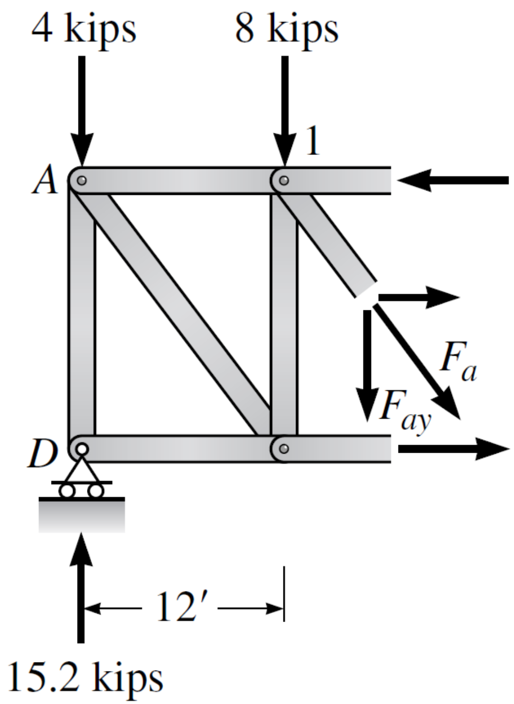
–288.0	+288.0	–512.0	+512.0	FEM
+288.0	+144.0	–256.0	–512.0	
	+191.5	+144.5		
0.0	+632.5	–632.5	0.0	final moments (kip • ft)

- Analyze the beam by moment distribution



- Compute reactions using the free bodies

Example 15.6 Solution (continued)



Computation of force in diagonal bar

- Pass vertical sections through the beam; alternatively, after the reactions are established, analyze the truss directly.

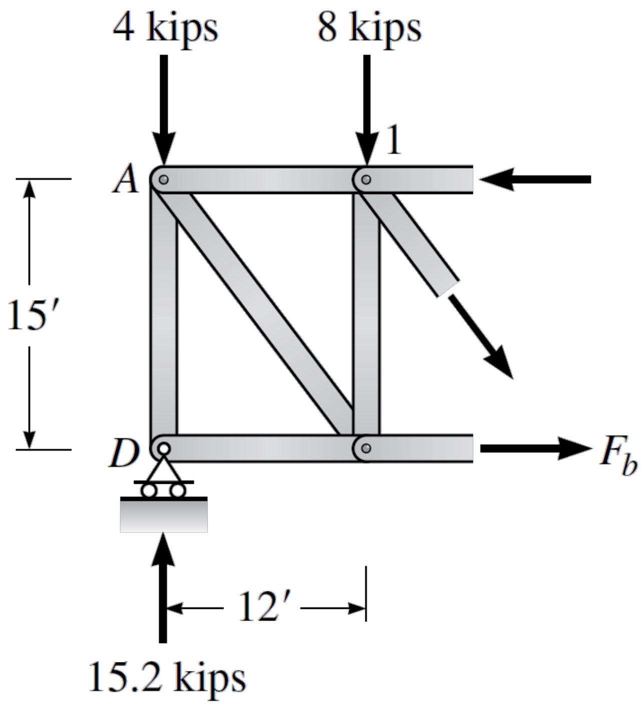
$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$15.2 - 4 - 8 - F_{ay} = 0$$

$$F_{ay} = 3.2 \text{ kips}$$

$$F_a = \frac{5}{4} F_{ay} = \frac{5}{4}(3.2) = 4 \text{ kips} \qquad \textbf{Ans.}$$

Example 15.6 Solution (continued)



Computation of force F_b

- For bar *b*, sum moments about point 1, 12 ft to the right of support *D*:

$$\circlearrowright^{+} \quad \Sigma M_1 = 0$$

$$(15.2)12 - 4(12) - 15F_b = 0$$

$$F_b = \frac{134.4}{15} = 8.96 \text{ kips tension,}$$

round to 9 kips **Ans.**

- For bar *c*,

  Moment at center support = 632.5 kip-ft

$$F_c = \frac{M}{h} = \frac{623.5}{15} = 42.2 \text{ kips}$$ **Ans.**

Example 15.6 Solution (continued)

- Arbitrarily increase by 10 percent to account for the increased stiffness of heavier chords adjacent to the center support in the real truss.

$$F_c = 1.1(42.2) = 46.4 \text{ kips compression}$$

- For bar *d*, consider a free-body diagram just to the left of support *E* cut by a vertical section.

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$15.2 \text{ kips} - 4 \text{ kips} - 5(8 \text{ kips}) + F_{dy} = 0$$

$$F_{dy} = 28.8 \text{ kips (tension)}$$

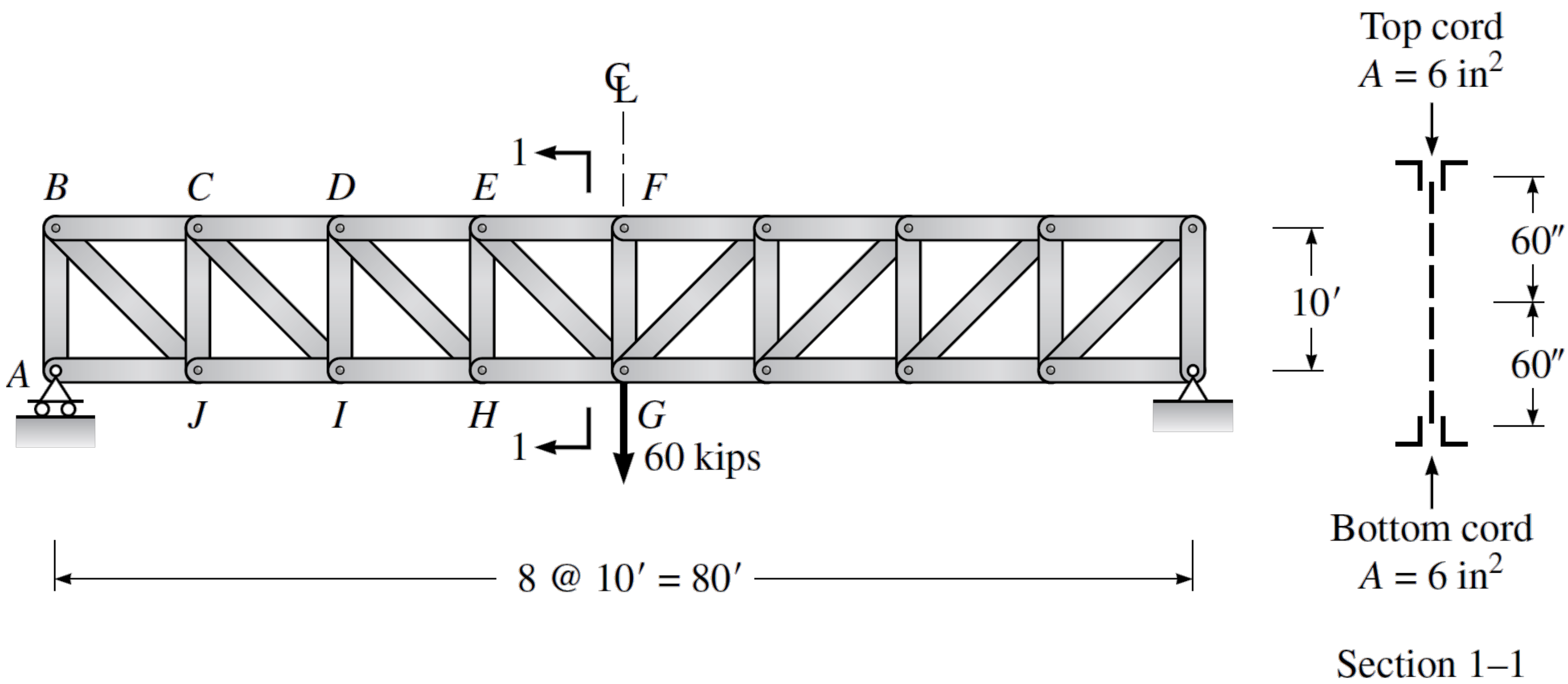
$$F_d = \frac{5}{4} F_{dy} = \frac{5}{4}(28.8) = 36 \text{ kips}$$

- Increase by 10 percent:

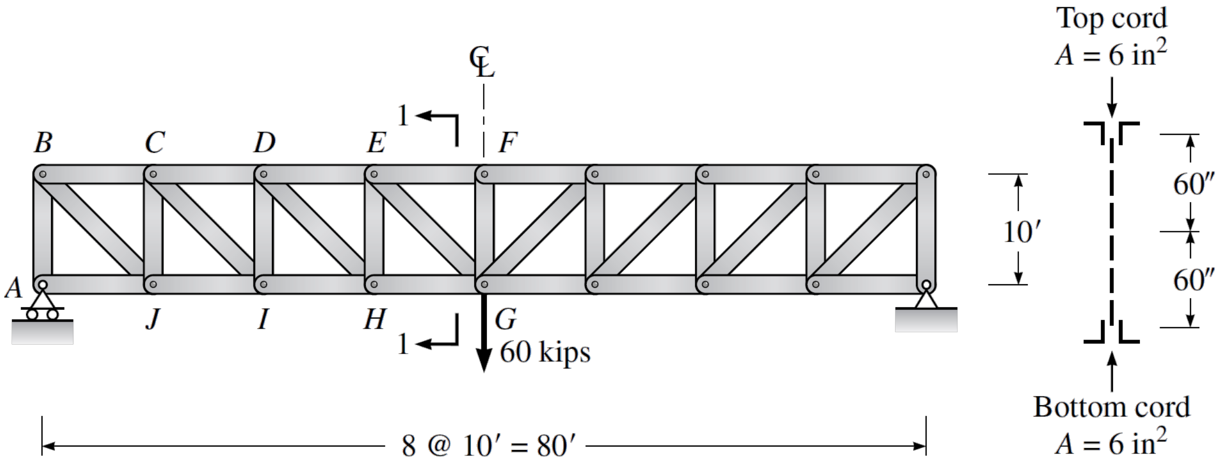
$$F_d = 39.6 \text{ kips} \qquad \textbf{Ans.}$$

Example 15.7

Estimate the midspan deflection of the truss in Figure 15.12 by treating it as a beam of constant cross section. The truss is symmetric about a vertical axis at midspan. The area of the top and bottom chords in the four center panels is 6 in^2. The area of all other chords equals 3 in^2. The area of all diagonals equals 2 in^2; the area of all verticals equals 1.5 in^2. Also E = 30,000 kips/in^2.



Example 15.7 Solution



Section 1–1

- Compute the moment of inertia I of the cross section at midspan based on the area of the top and bottom chords and neglecting the moment of inertia of the chord area about its own centroid (I_{na})

$$I = \Sigma(I_{\text{na}} + Ad^2)$$

$$= 2[6(60)^2] = 43{,}200 \text{ in}^4$$

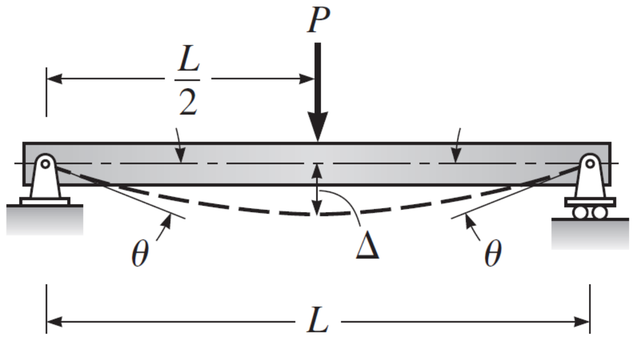
Example 15.7 Solution (continued)

- Compute the deflection at midspan (see Figure 11.3*d* for the equation).

$$\Delta = \frac{PL^3}{48EI}$$

$$= \frac{60(80 \times 12)^3}{48(30{,}000)(43{,}200)}$$

$$= 0.85 \text{ in}$$



$$\theta = \frac{PL^2}{16EI} \qquad \Delta = \frac{PL^3}{48EI}$$

- Double Δ to account for contribution of web members:

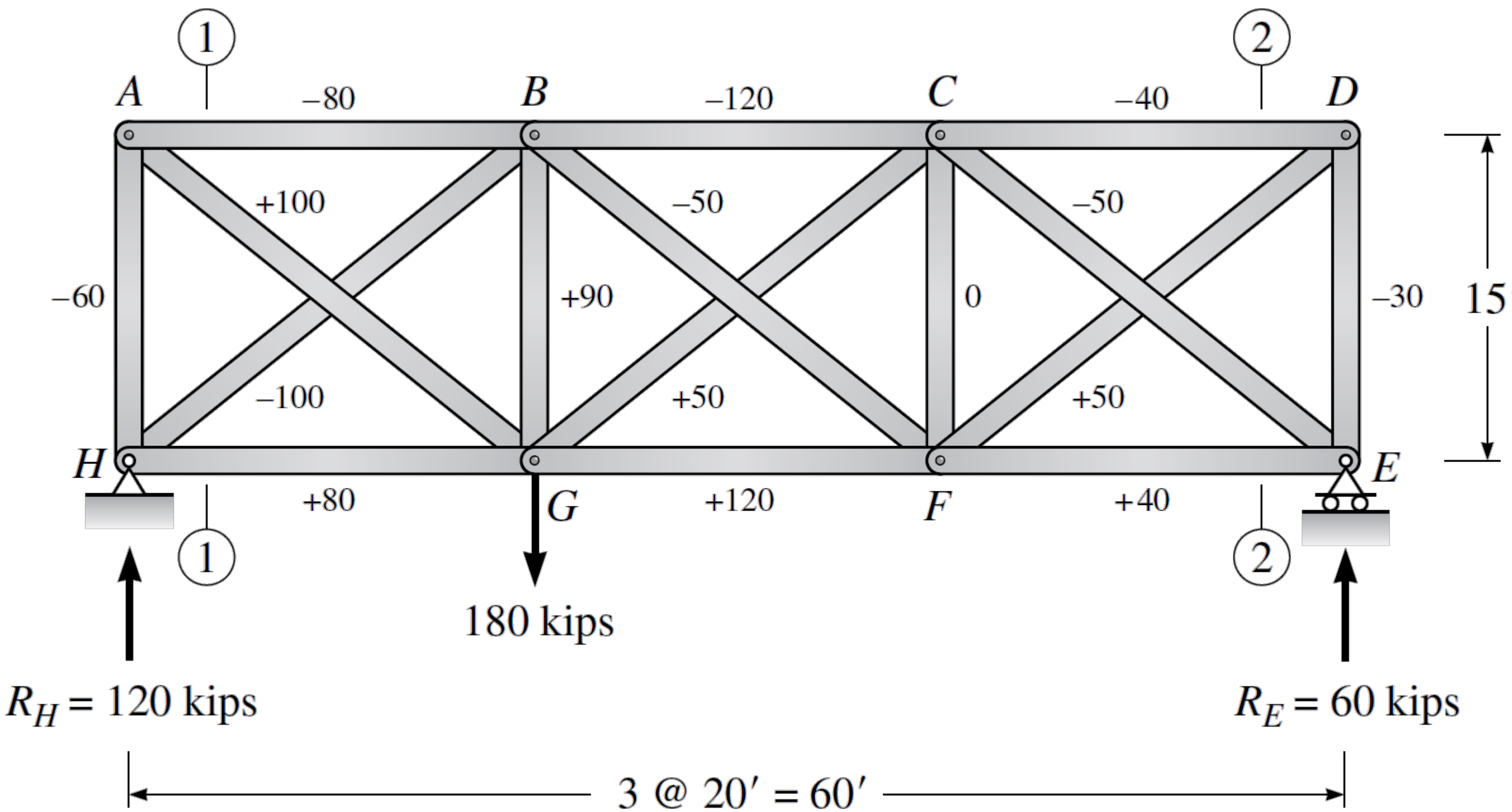
$$\text{Estimated } \Delta_{\text{truss}} = 2\Delta = 2(0.85) = 1.7 \text{ in} \qquad \textbf{Ans.}$$

- Solution by virtual work, which accounts for the reduced area of chords at each end and the actual contribution of the diagonals and verticals to the deflection, gives

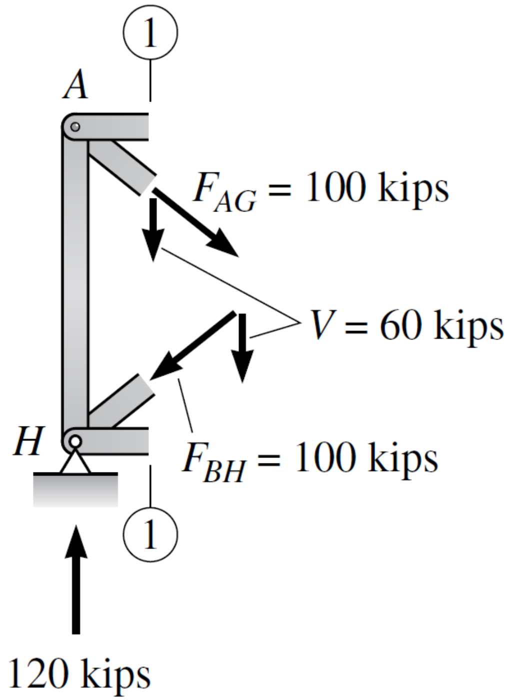
$$\Delta_{\text{truss}} = 2.07 \text{ in.}$$

Example 15.8

Analyze the indeterminate truss in Figure 15.13. Diagonals in each panel are identical and have sufficient strength and stiffness to carry loads in either tension or compression.



Example 15.8 Solution

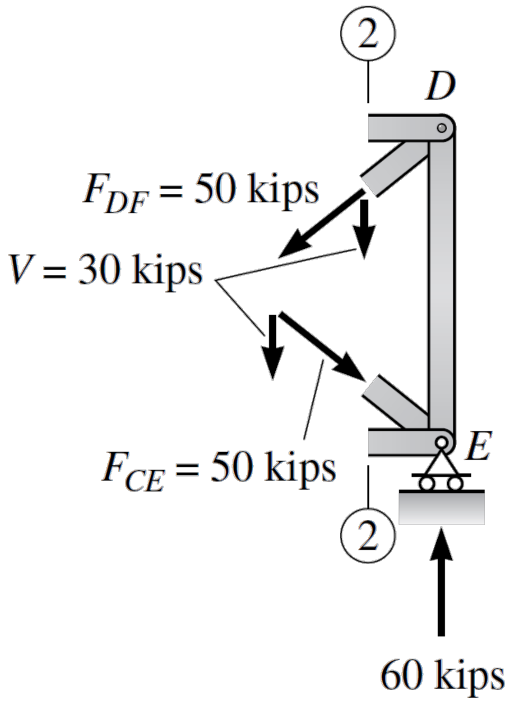


Free body of truss cut by section 1-1

- Pass a vertical Section 1-1 through the first panel of the truss cutting the free body. Assume each diagonal carries one-half the shear in the panel

- *AG* must be in tension and *BH* in compression and since the resultant bar force is 5/3 of the vertical component, the force in each bar equals 100 kips.

Example 15.8 Solution (continued)



Free body of truss cut by section 2-2

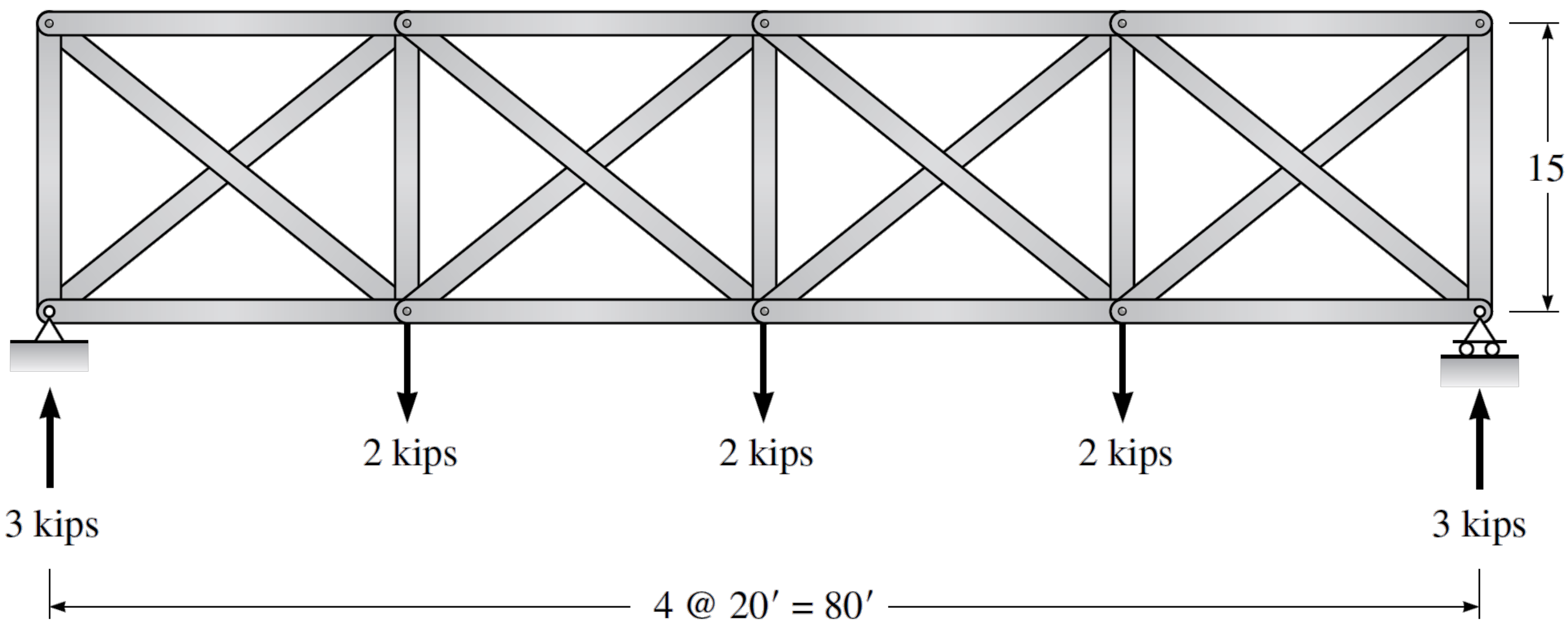
- Pass Section 2-2 through the end panel on the right. From a summation of forces in the vertical direction, there is a tension force of 50 kips in member *DF* and a compression force of 50 kips in member *CE*.

- Consider a free body of the truss to the right of a vertical section through the center panel and observe that the unbalance shear is 60 kips and the forces in the diagonals act in the same direction as those shown.

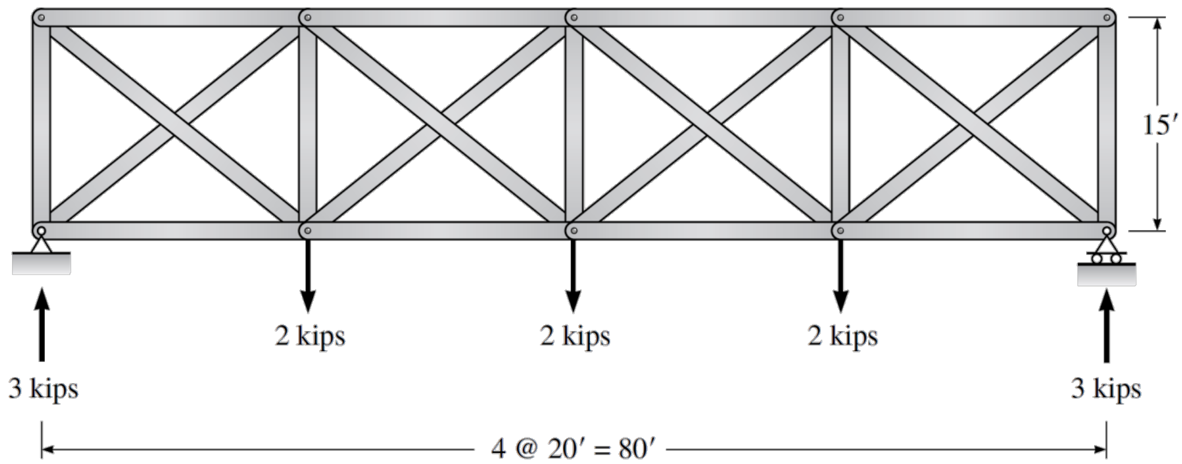
- After the forces in all diagonals are evaluated, the forces in the chords and verticals are computed by the method of joints.

Example 15.9

Small-diameter rods form the diagonal members of the truss in Figure 15.14*a*.The diagonals can transmit tension but buckle if compressed. Analyze the truss for the loading shown.



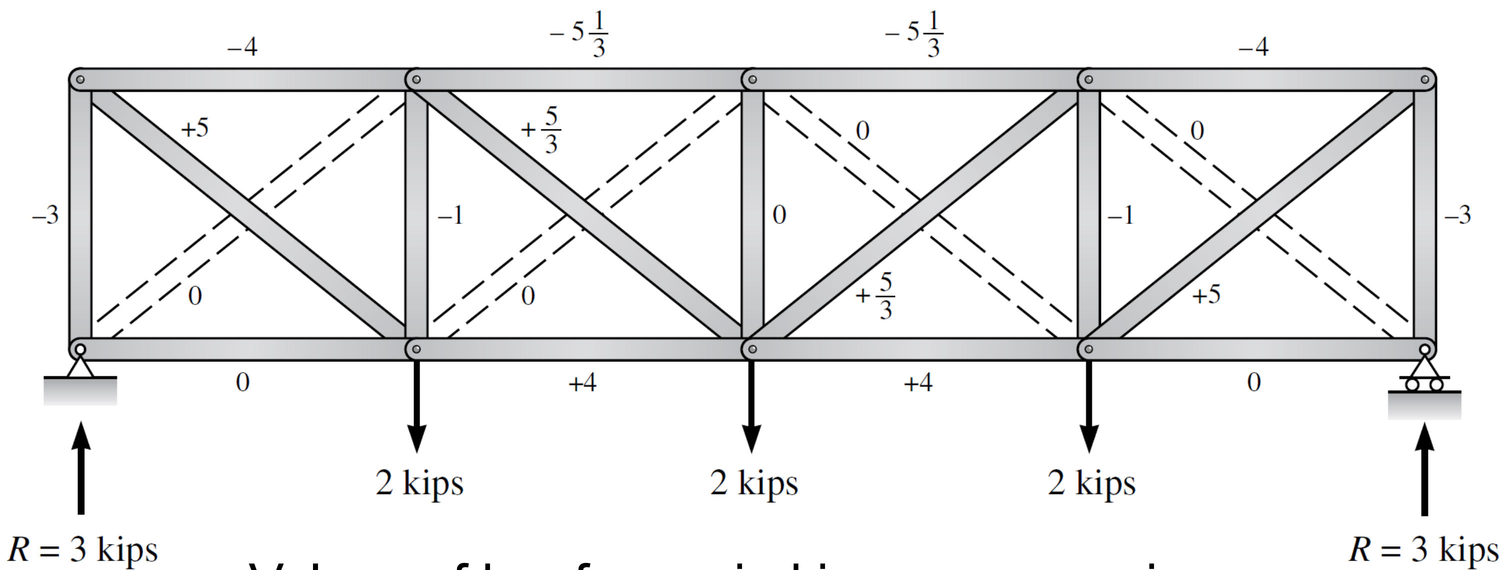
Example 15.9 Solution



- Compute the reactions.

- Pass vertical sections through each panel and establish the direction of the internal force in the diagonal bars required for vertical equilibrium of the shear in each panel.

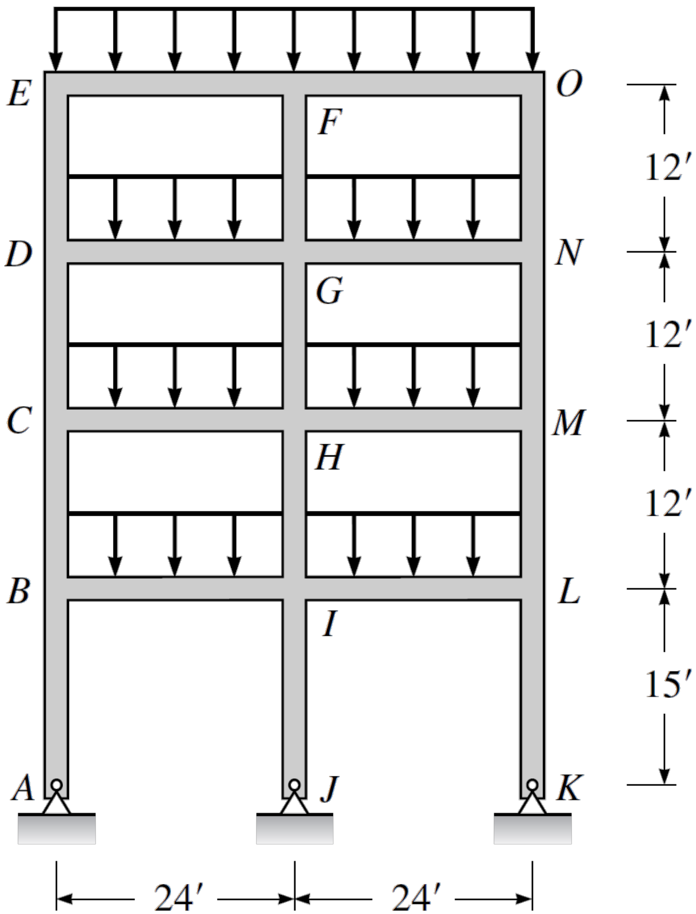
Example 15.9 Solution (continued)



Values of bar forces in kips, compression diagonals indicated by dashed lines

- The tension and compression diagonals are identified. Since the compression diagonals buckle, the entire shear in a panel is assigned to the tension diagonal, and the force in the compression diagonals is set equal to zero.
- Analyzed by the methods of joints or sections.

§15.7 Approximate Analysis of a Multistory Rigid Frame for Gravity Load



Properties of Members

Member	***A* in^2**	***I* in^4**
Exterior columns	100	1000
Interior columns	144	1728
Girders	300	6000

Figure 15.15 Dimensions and member properties of a vertically loaded multistory building frame

§15.7 Approximate Analysis of a Multistory Rigid Frame for Gravity Load

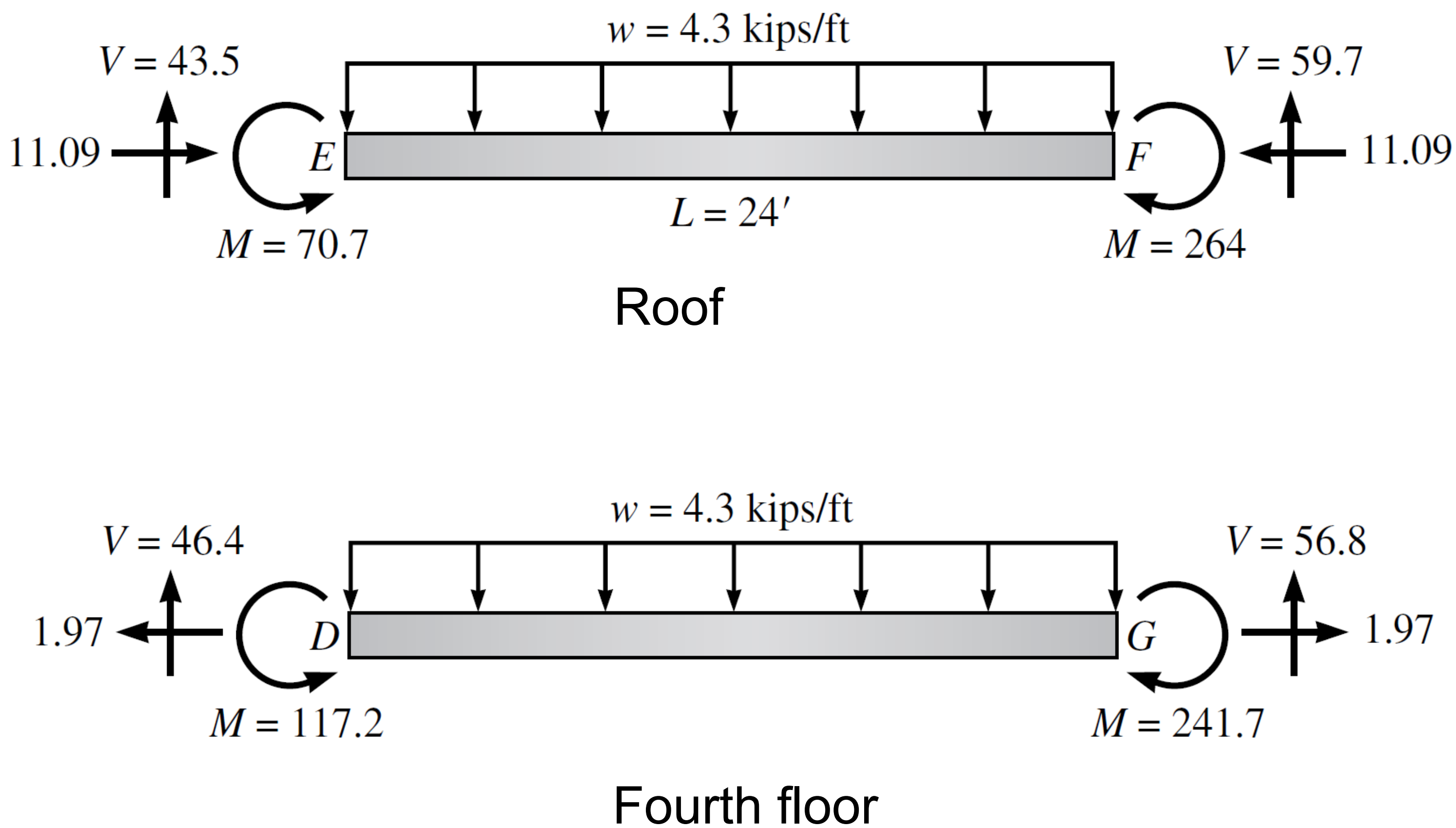
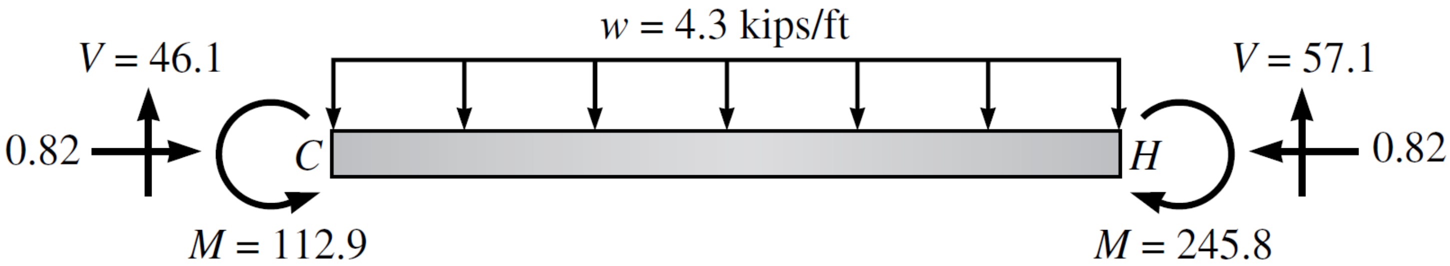
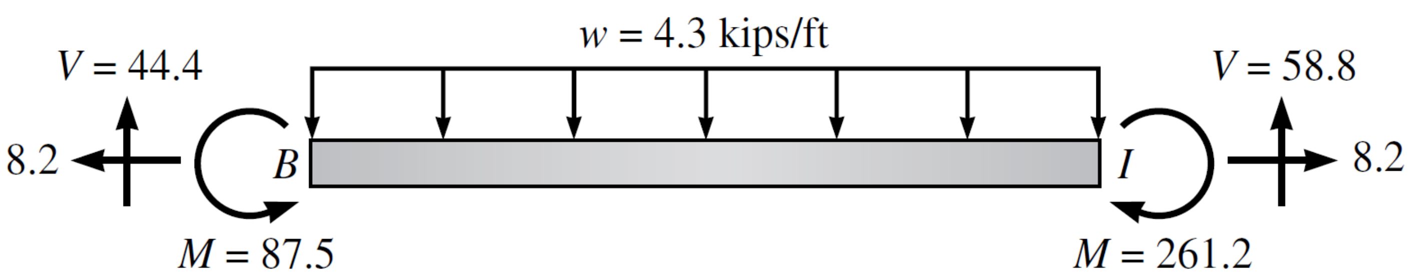


Figure 15.16 Free bodies of floor beams showing forces from an exact analysis

§15.7 Approximate Analysis of a Multistory Rigid Frame for Gravity Load



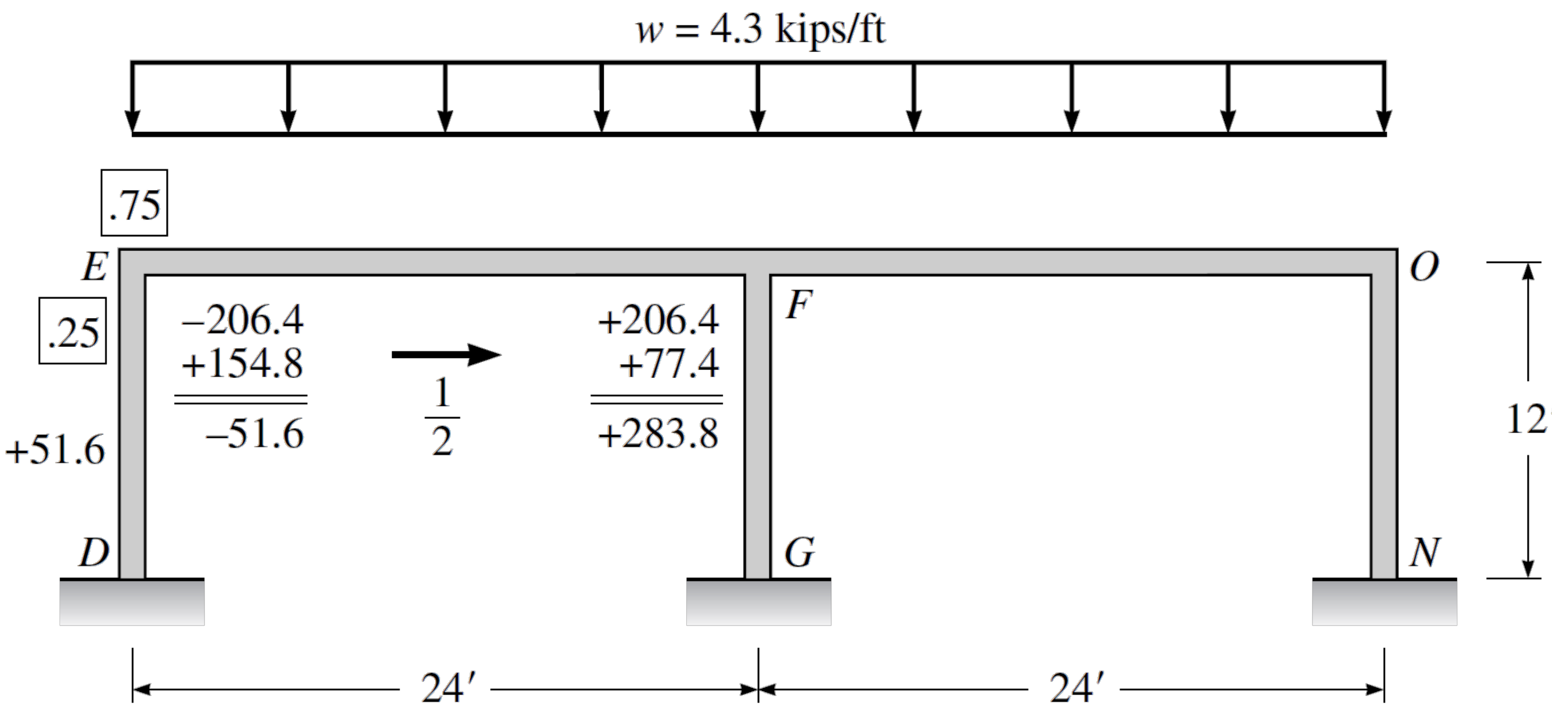
Third floor



Second floor

Figure 15.16 Free bodies of floor beams showing forces from an exact analysis (continued)

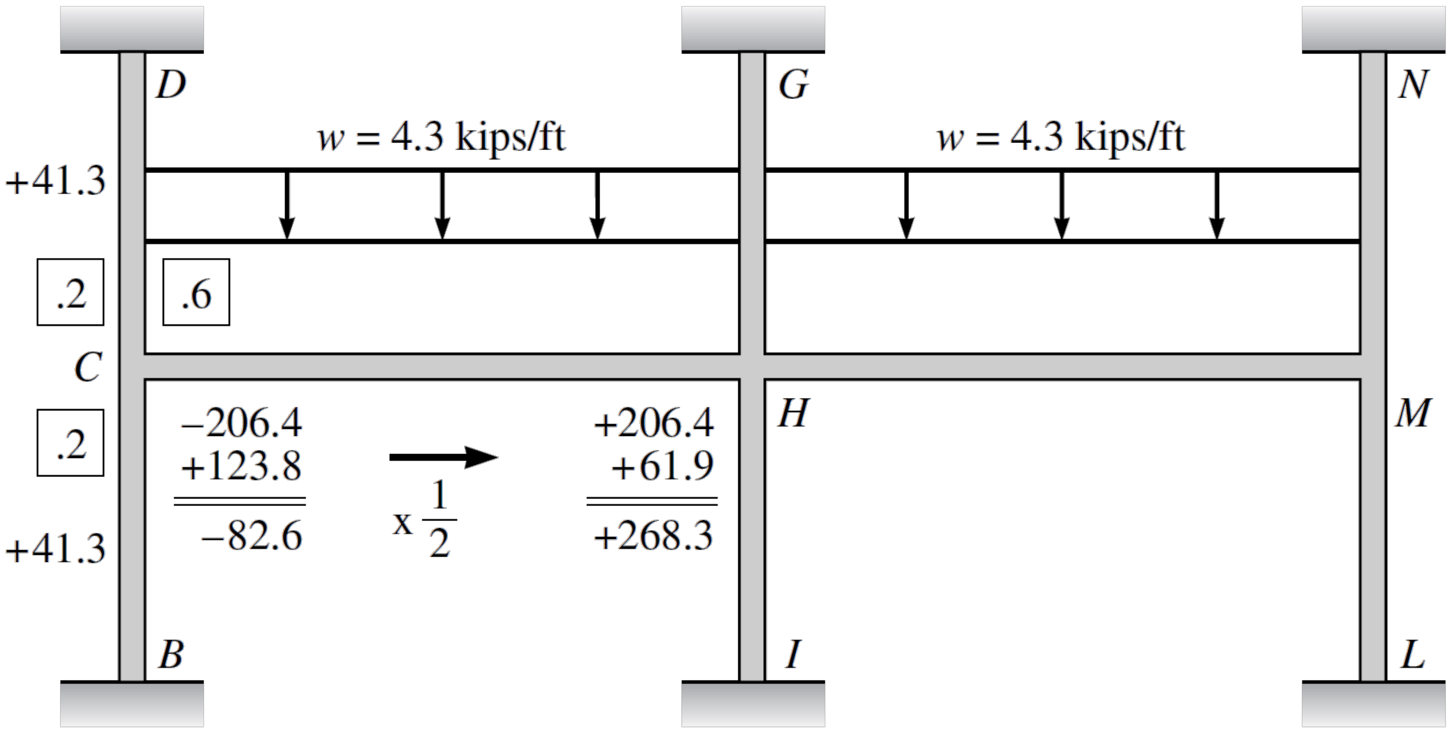
§15.7 Approximate Analysis of a Multistory Rigid Frame for Gravity Load



Rigid frame composed of roof beams and attached columns

Figure 15.17 Approximate analysis of beams in frame for vertical load

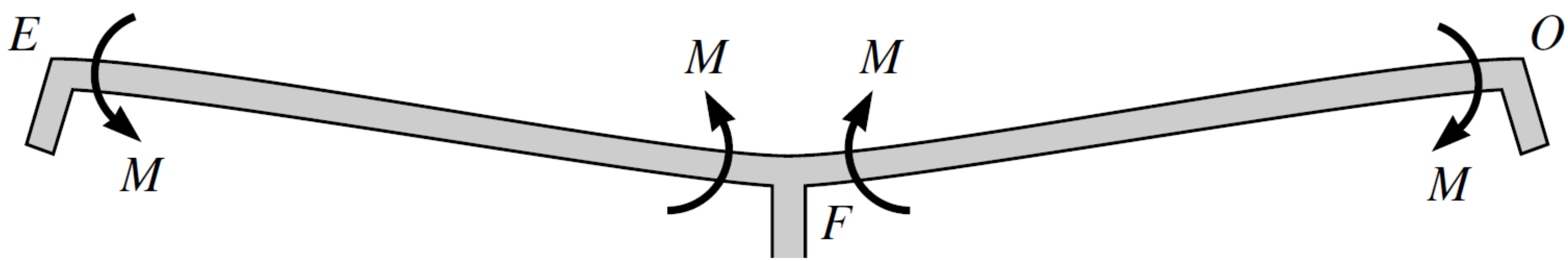
§15.7 Approximate Analysis of a Multistory Rigid Frame for Gravity Load



Rigid frame composed of floor beams and attached columns

Figure 15.17 Approximate analysis of beams in frame for vertical load (continued)

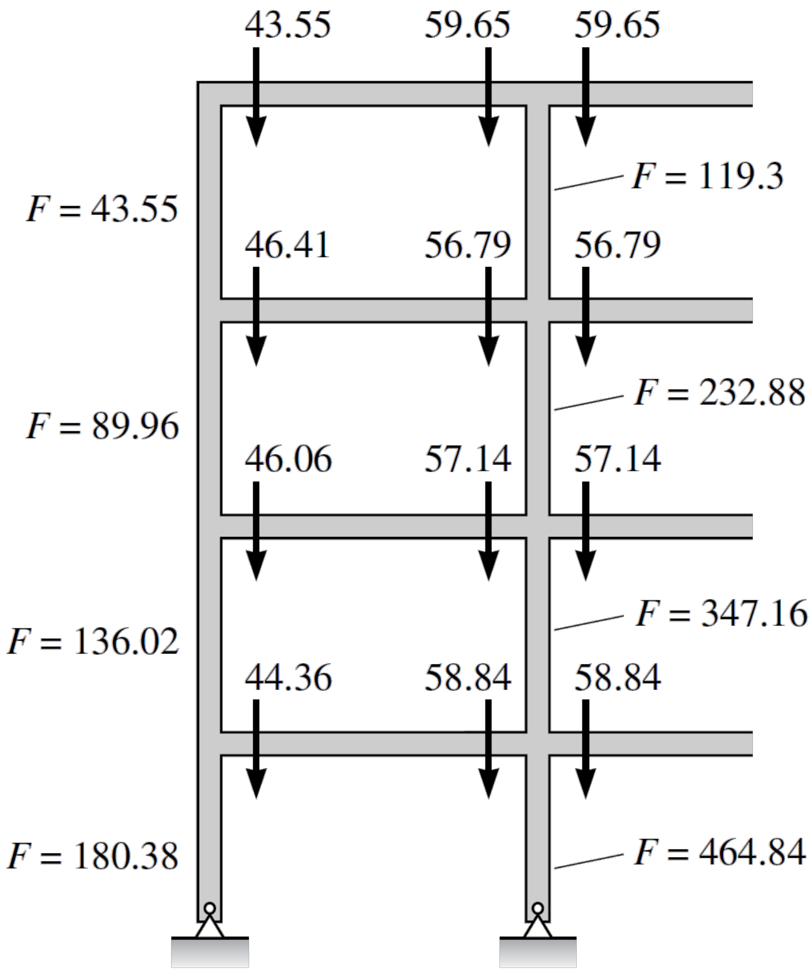
§15.7 Approximate Analysis of a Multistory Rigid Frame for Gravity Load



Moments created by differential displacement of interior and exterior joints

Figure 15.17 Approximate analysis of beams in frame for vertical load (continued)

§15.7 Approximate Analysis of a Multistory Rigid Frame for Gravity Load

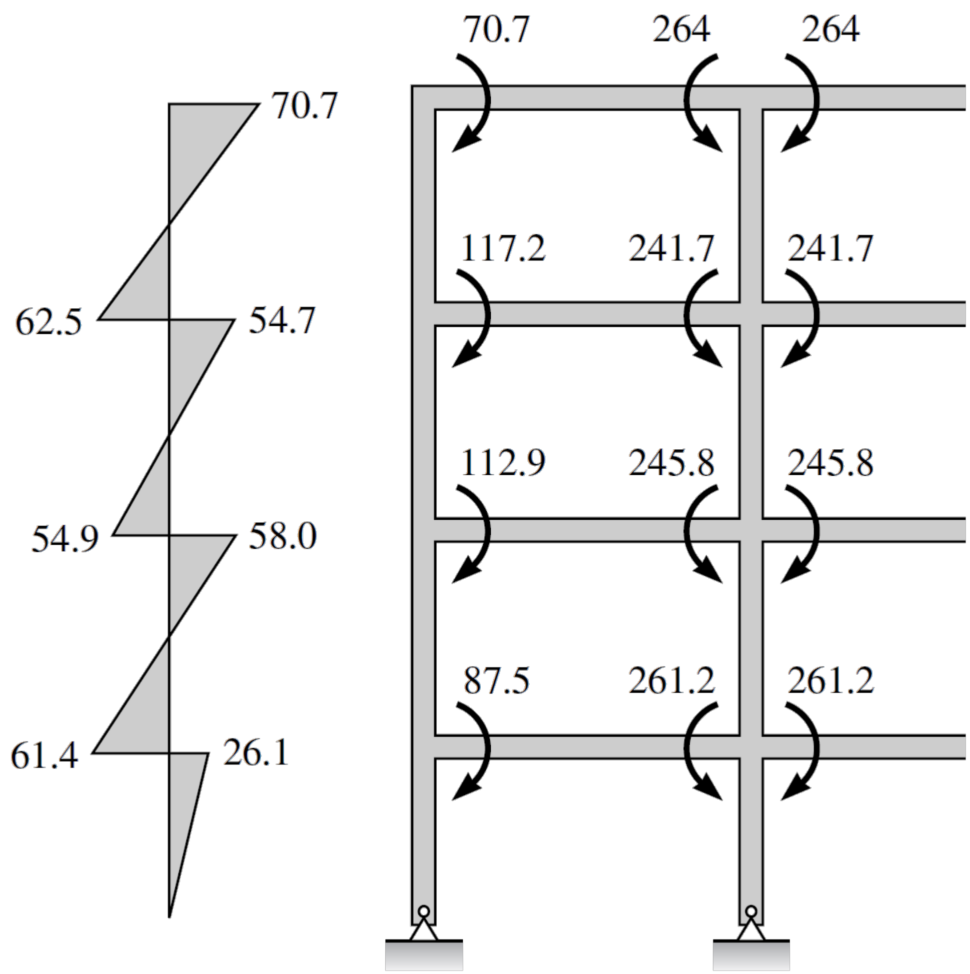


Axial force (kips) in columns created by reactions of beams supporting a uniformly distributed load of 4.3 kips/ft.

Figure 15.18 Results of computer analysis of frame in Figure 15.15

§15.7 Approximate Analysis of a Multistory Rigid Frame for Gravity Load

Moment curve for exterior column (kip-ft)

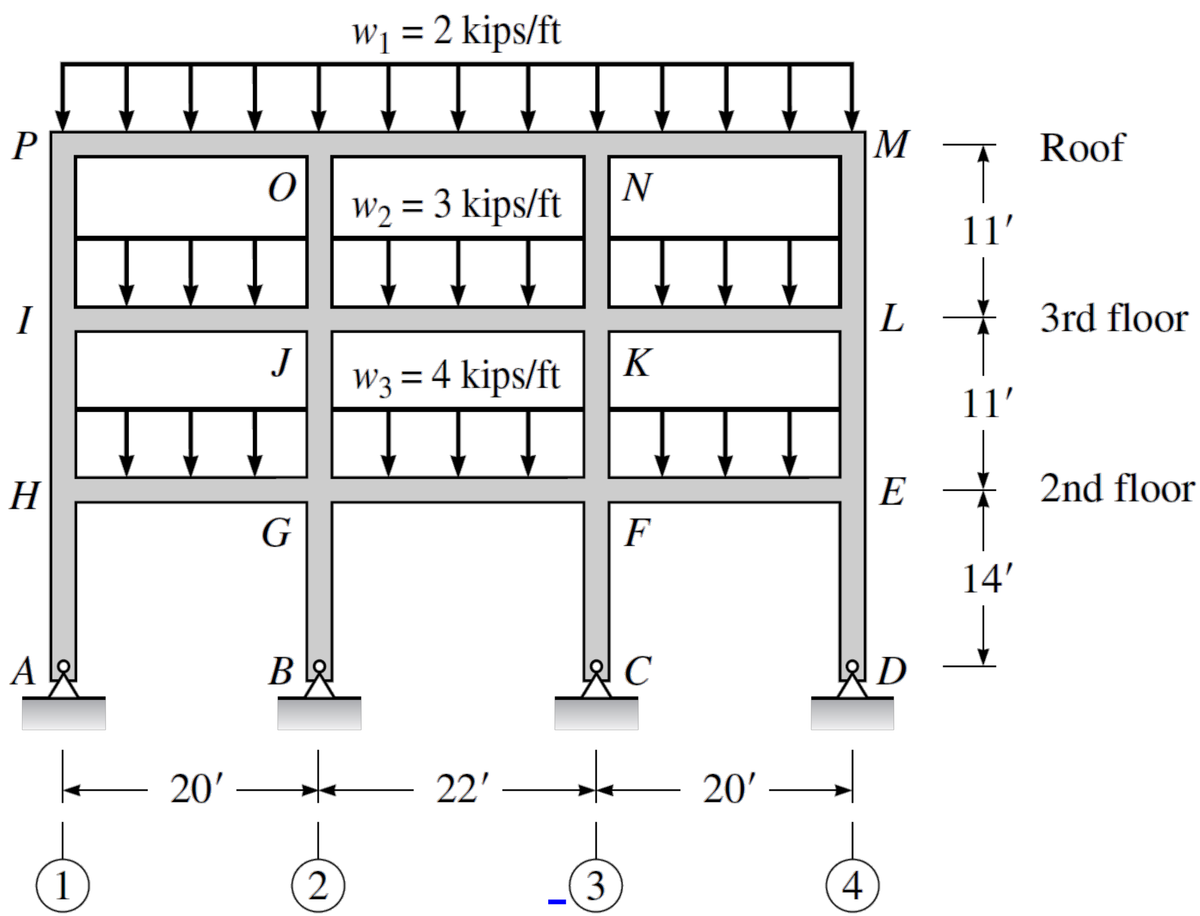


Moments (kip-ft) applied to columns by beams; this moment divides between top and bottom columns

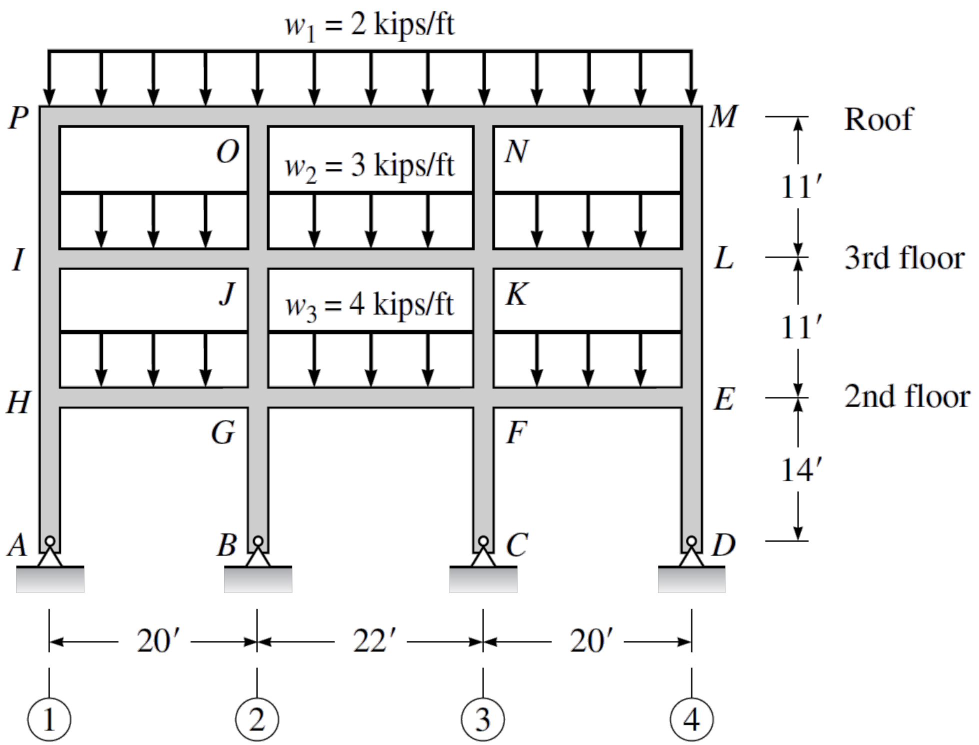
Figure 15.18 Results of computer analysis of frame in Figure 15.15 (continued)

Example 15.10

Using an approximate analysis, estimate the axial forces and moments in columns *BG* and *HI* of the frame in Figure 15.19*a*. Also draw the shear and moment curves for beam *HG*. Assume that *I* of all exterior columns equals 833 in^4, *I* of interior columns equals 1728 in^4, and I of all girders equals 5000 in^4. Circled numbers represent column lines.



Example 15.10 Solution



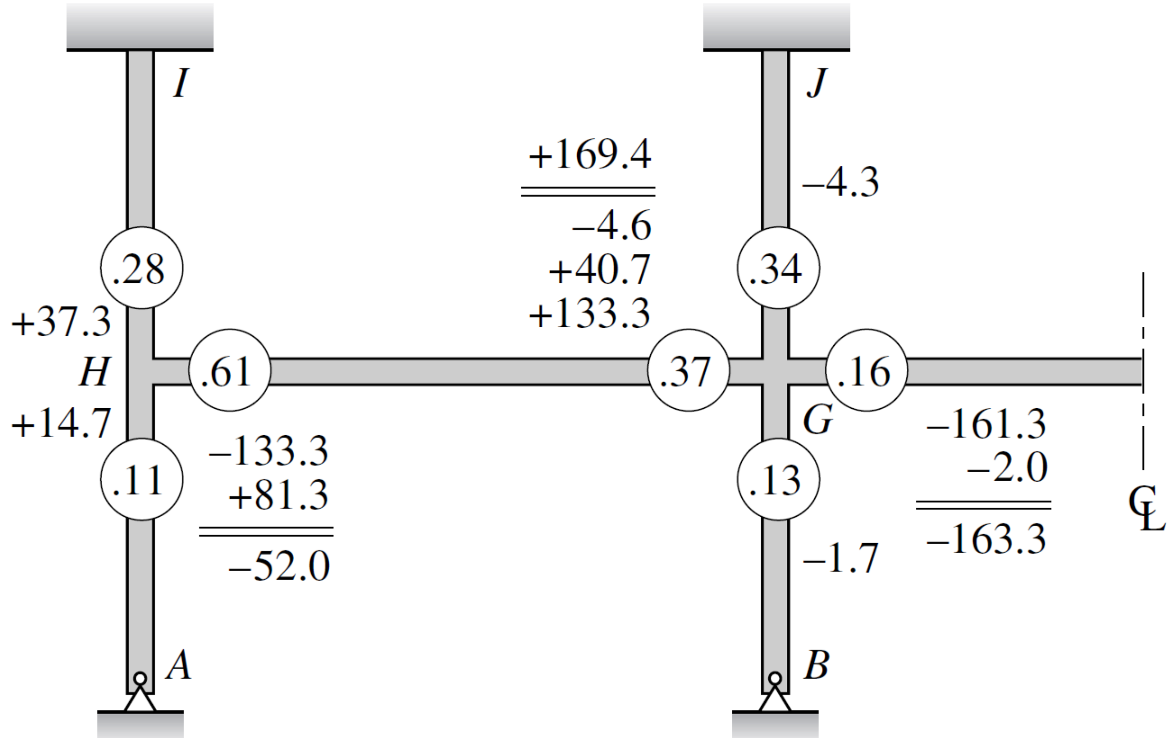
- ***Axial Load in Column HI*** Assume that 45 percent of the uniform load on beams *PO* and *IJ* is carried to the exterior column.

$$F_{HI} = 0.45(w_1L + w_2L)$$
$$= 0.45[2(20) + 3(20)]$$
$$= 45 \text{ kips} \qquad \textbf{Ans.}$$

- ***Axial Load in Column BG*** Assume that 55 percent of the load from exterior beams on the left side of the column and 50 percent of the load from the interior beams on the right side of the column are carried into the column.

$$F_{BG} = 0.55[2(20) + 3(20) + 4(20)] + 0.5[2(22) + 3(22) + 4(22)]$$
$$= 198 \text{ kips} \qquad \textbf{Ans.}$$

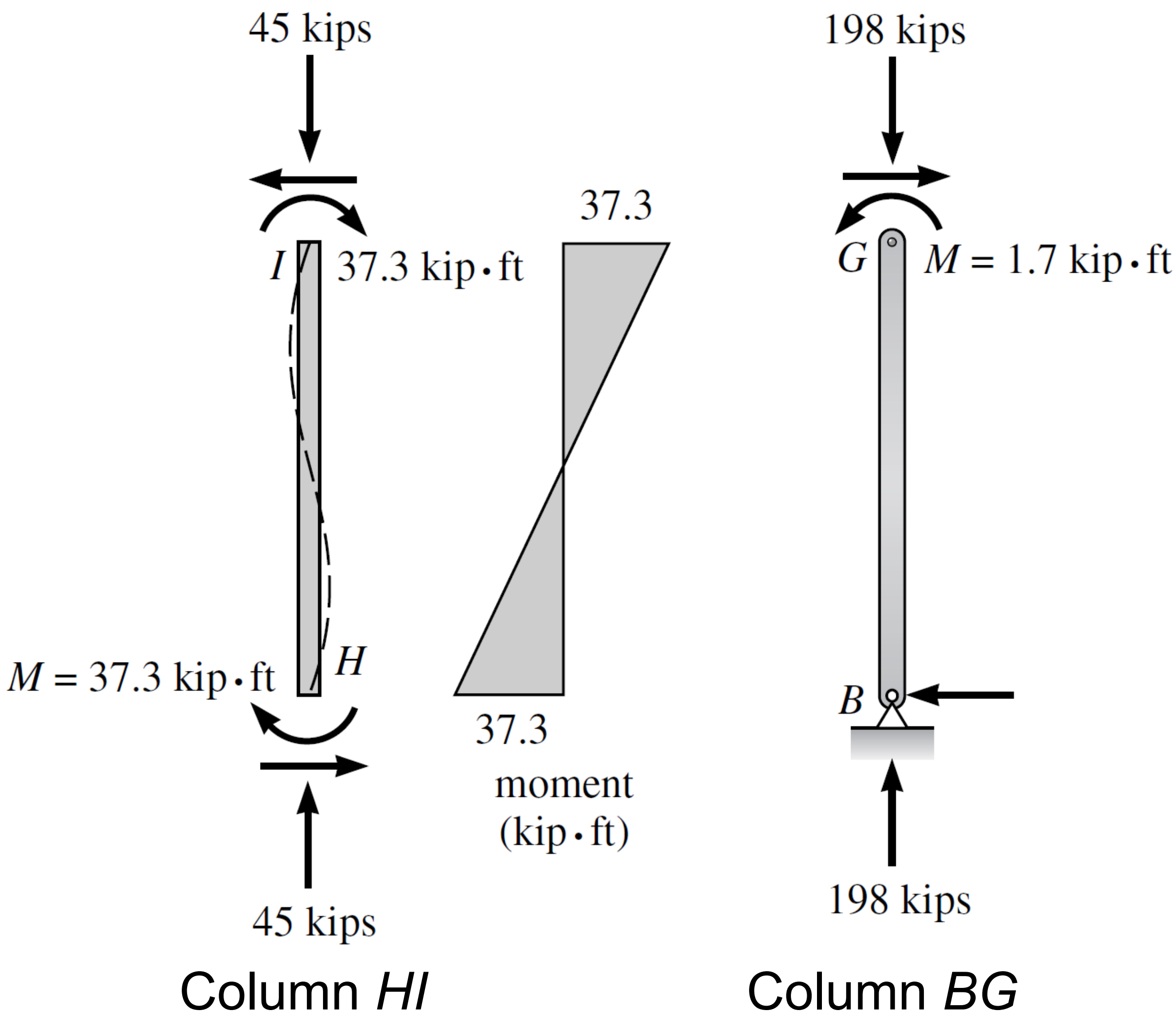
Example 15.10 Solution (continued)



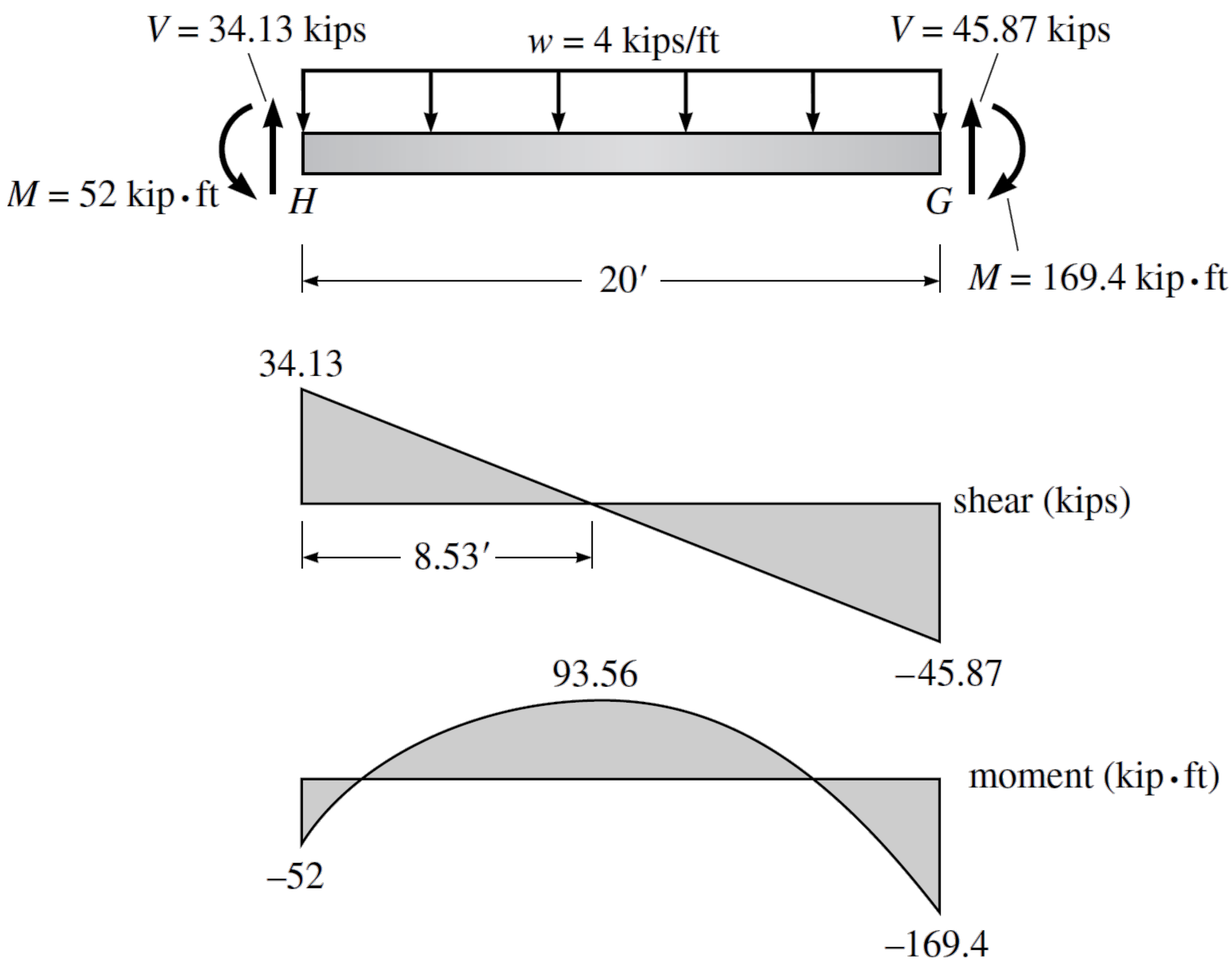
Approximate analysis of second floor by moment distribution to establish moments in beams and columns

- Compute the moments in columns and beam *HG* by analyzing the frame in Figure 15.19*b* by moment distribution.

Example 15.10 Solution (continued)

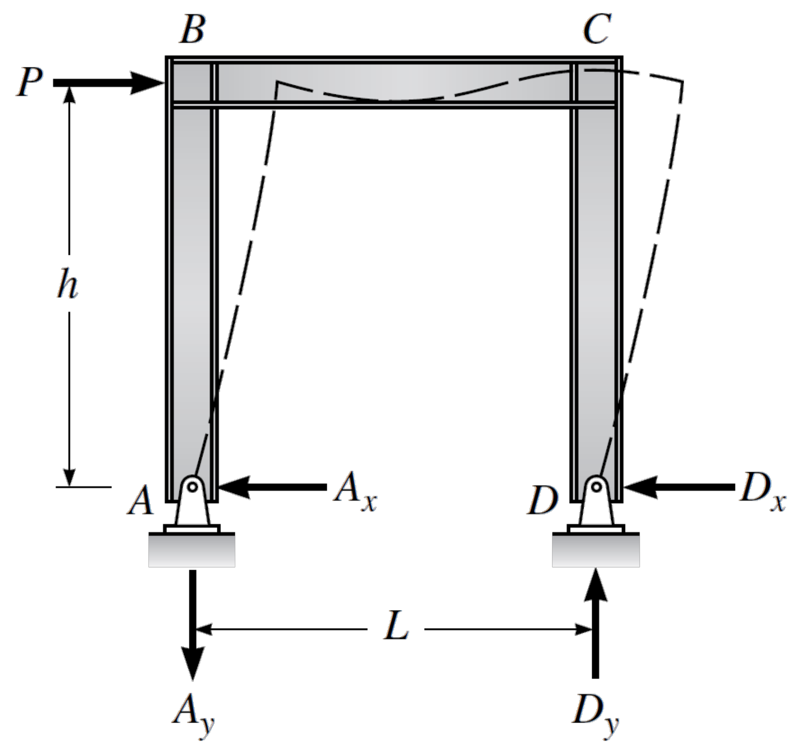


Example 15.10 Solution (continued)

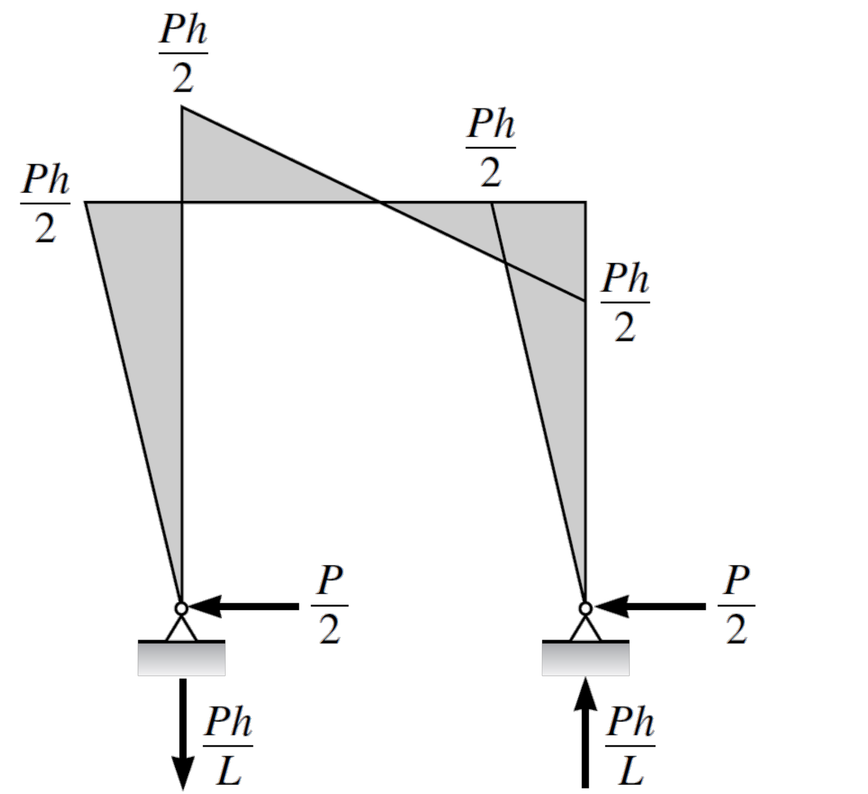


Shear and moment curves for beam *HG*

§15.8 Analysis of Unbraced Frames for Lateral Load



Laterally loaded frame



Reactions and moment curves; point of inflection occurs at midspan of girder

Figure 15.21

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§15.8 Analysis of Unbraced Frames for Lateral Load

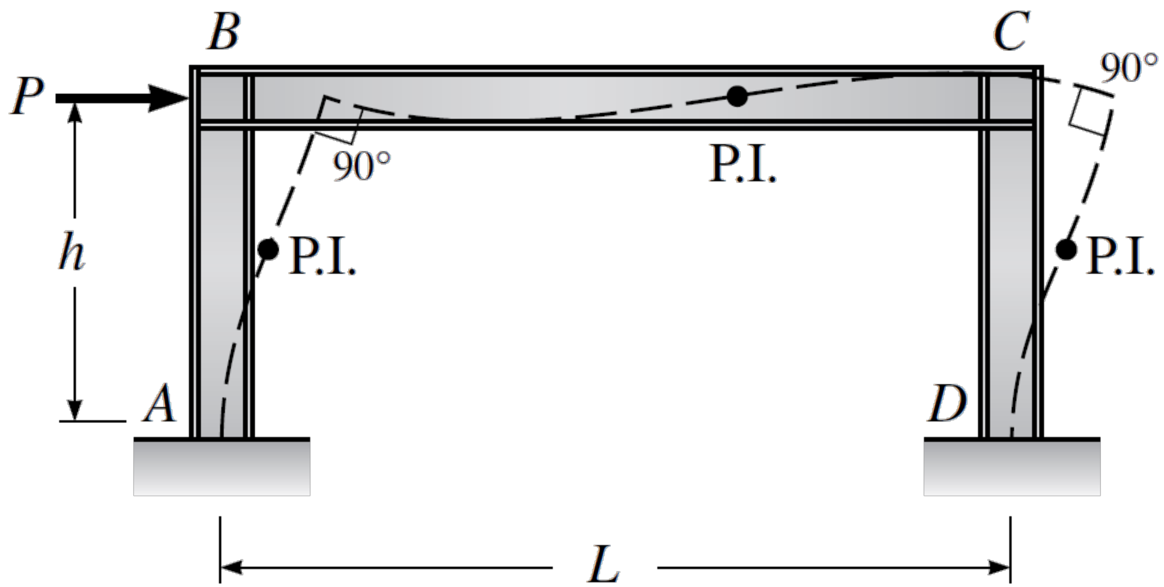
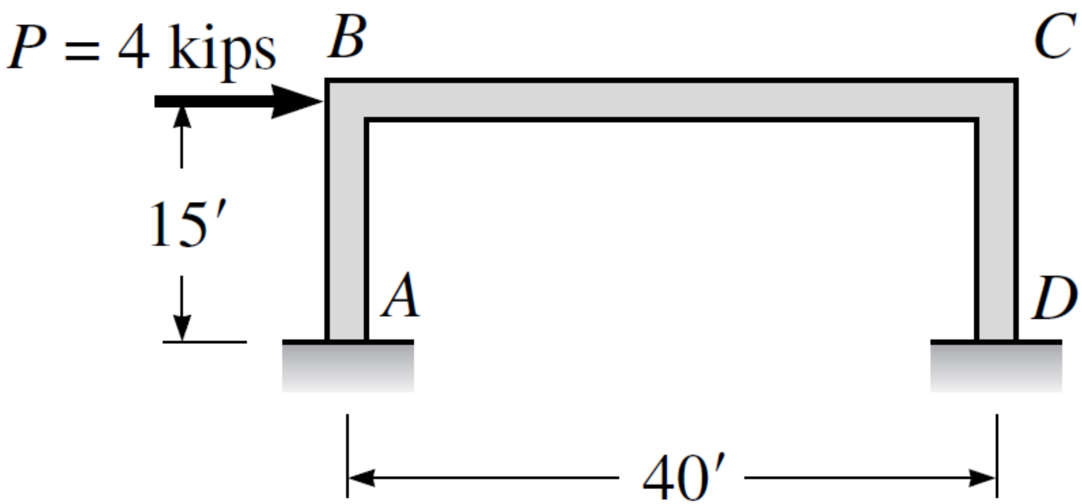


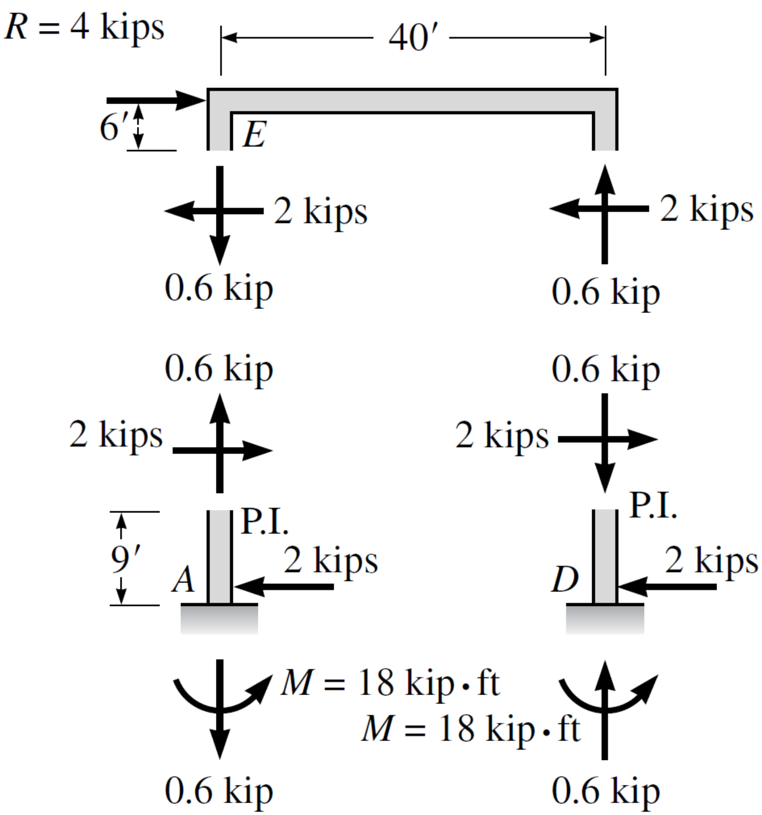
Figure 15.22 A laterally loaded rigid frame with fixed-end columns

Example 15.11

Estimate the reactions at the base of the frame in Figure 15.23*a* produced by the horizontal load of 4 kips at joint *B*. The column legs are identical.



Example 15.11 Solution



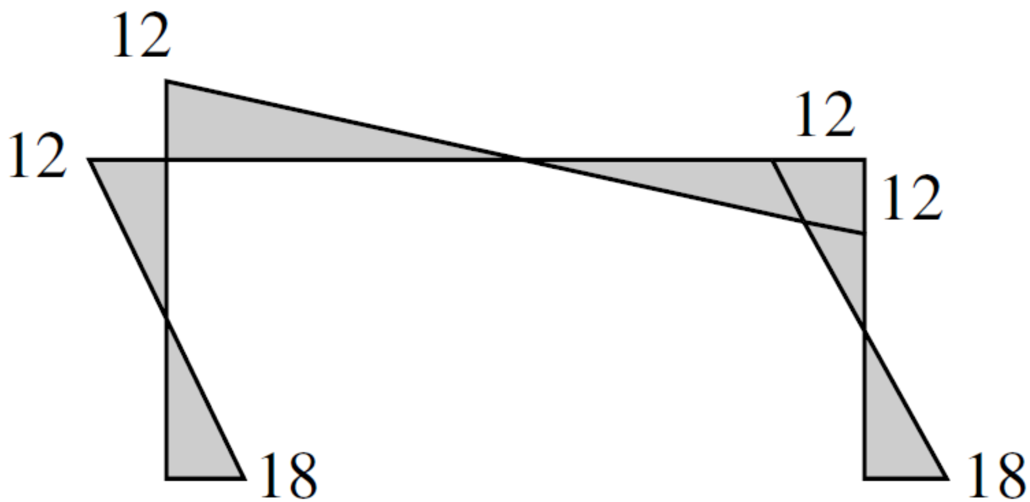
Free bodies above and below the points of inflections in the columns

- Assume that the 4-kip load divides equally between the two columns, producing shears of 2 kips in each column and horizontal reactions of 2 kips at *A* and *D*.
- Assume that the points of inflection (P.I.) in each column are located 0.6 of the column height, or 9 ft, above the base.
- Sum moments about the point of inflection in the left column (point *E*) to compute an axial force *F* = 0.6 kip in the column on the right.

Example 15.11 Solution (continued)

- Reverse the forces at the points of inflection on the upper free body and apply them to the lower column segments. Use the equations of statics to compute the moments at the base.

$$M_A = M_D = (2\ \text{kips})(9\ \text{ft}) = 18\ \text{kip} \cdot \text{ft}$$



Moment diagram

§15.9 Portal Method

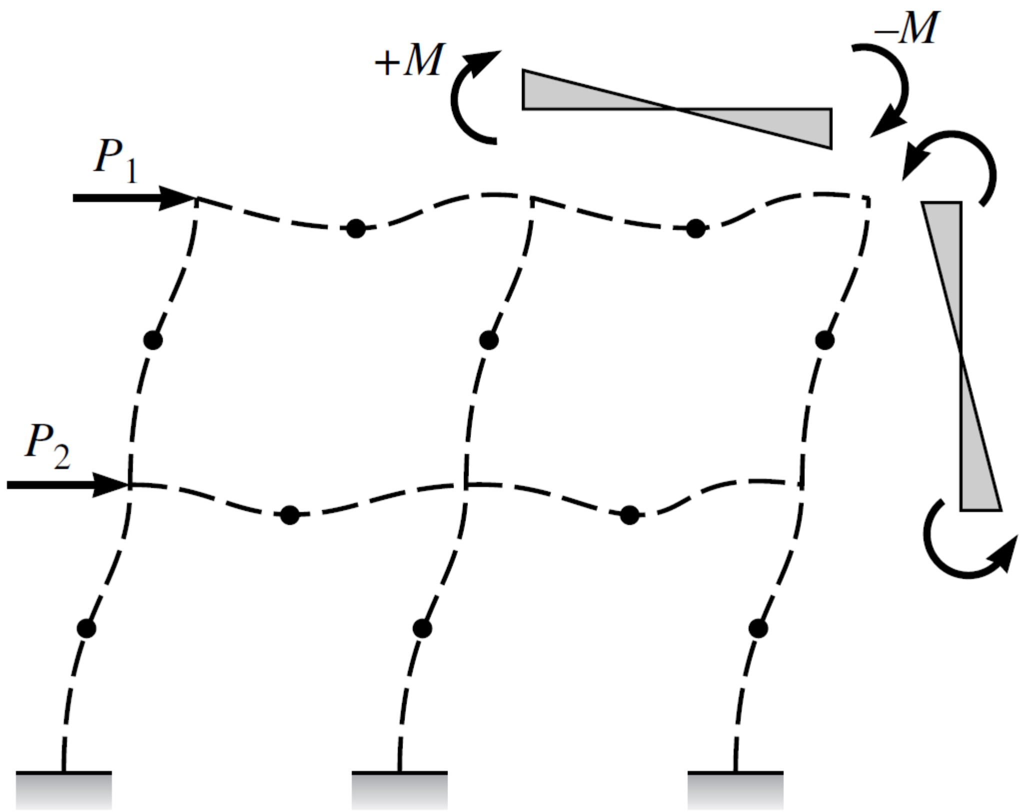
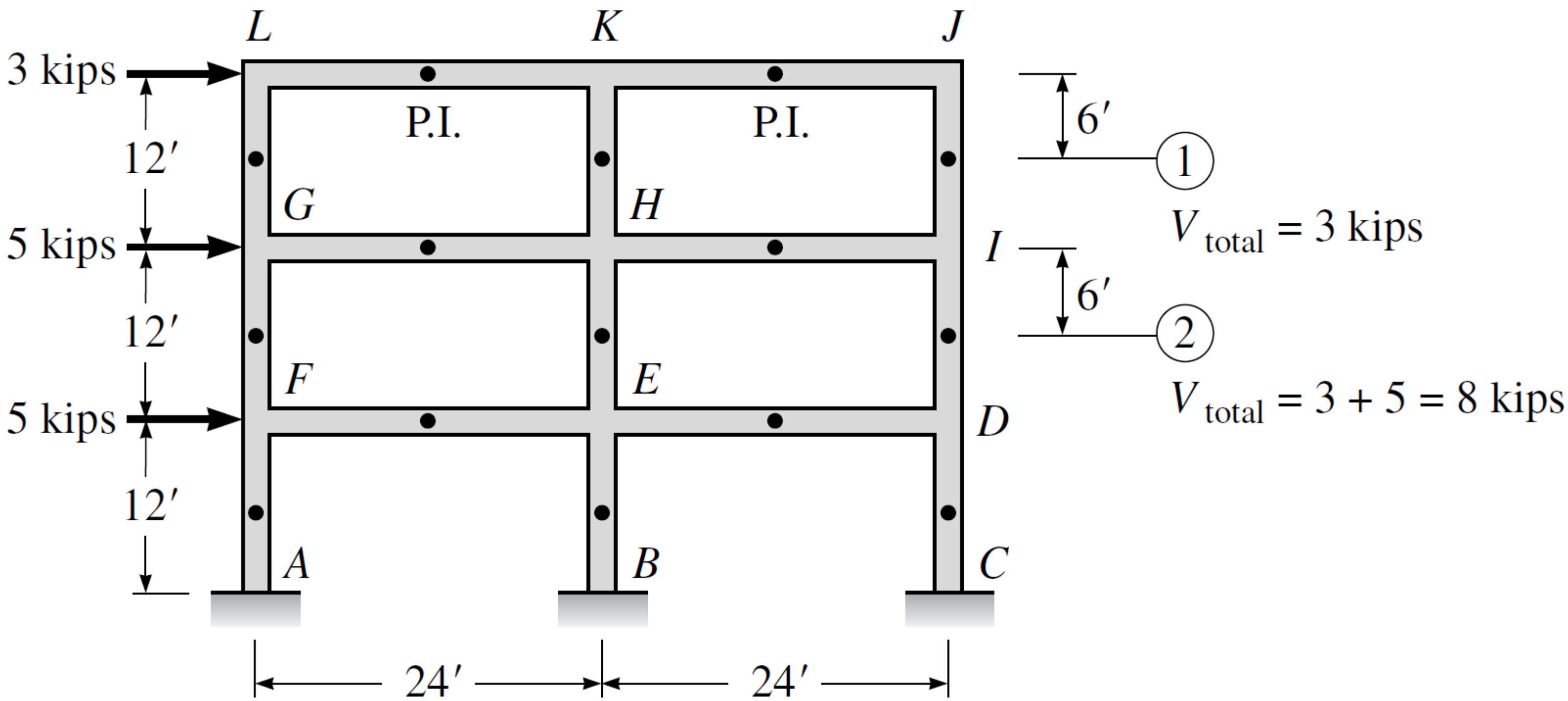


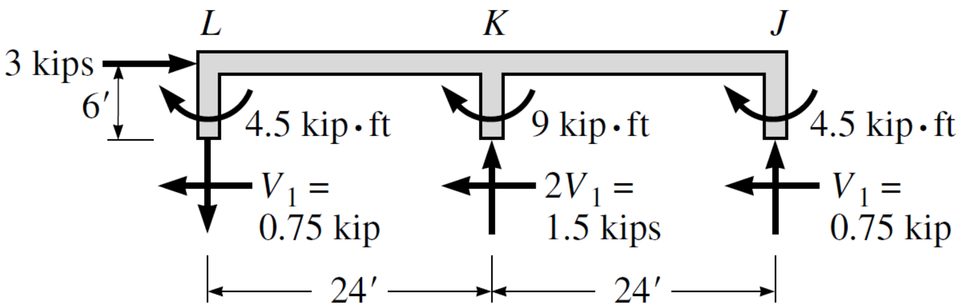
Figure 15.24 Deflected shape of rigid frame; points of inflection shown at center of all members by black dots

Example 15.12

Analyze the frame in Figure 15.25*a*, using the portal method. Assume the reinforced baseplates at supports *A*, *B*, and *C* produce fixed ends.



Example 15.12 Solution



Free body of roof and columns cut by section 1, which passes through points of inflection of columns

- Pass horizontal Section 1 and consider the upper free body.
- Equate the lateral load above the cut (3 kips at joint *L*) to the sum of the column shears.

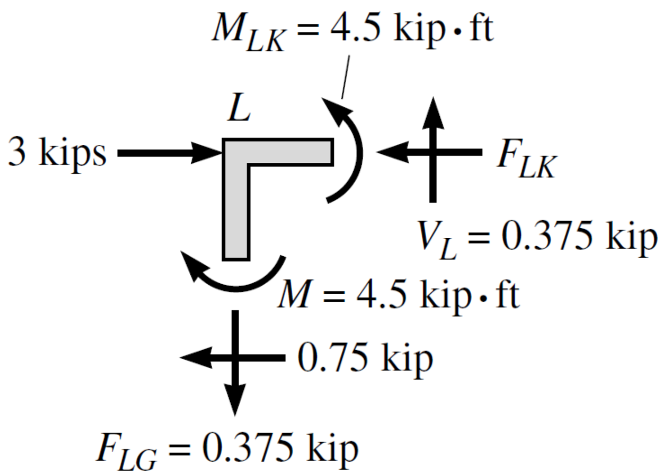
$$\xrightarrow{+} \quad \Sigma F_x = 0$$

$$3 - (V_1 + 2V_1 + V_1) = 0$$

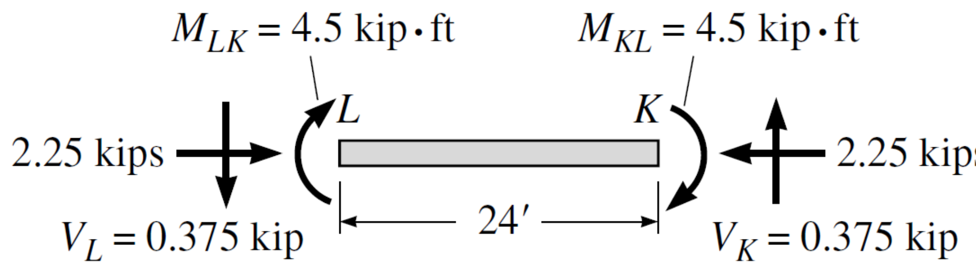
$$\text{and} \qquad V_1 = 0.75 \text{ kip}$$

- Compute moments at the tops of the columns by multiplying the shear forces at the points of inflection by 6 ft, the half-story height.

Example 15.12 Solution (continued)



Free body of joint *L*

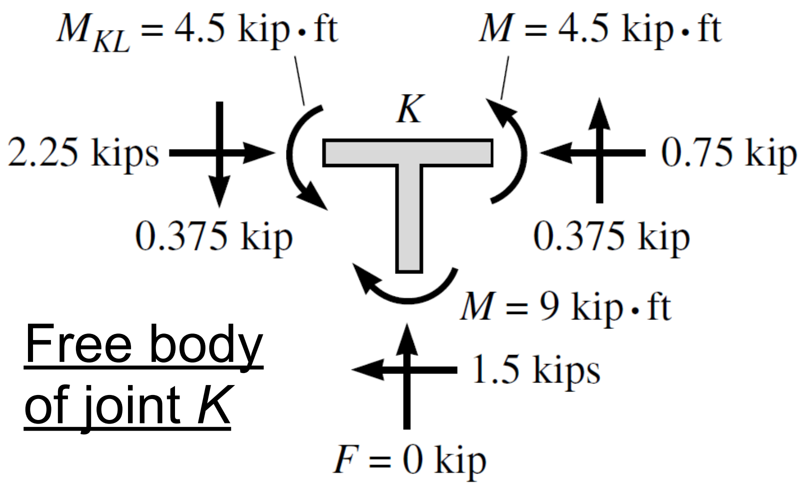


Free body of girder *LK* used to compute shears in girders

- Compute F_{LK} = 2.25 kips by summing forces in the *x* direction.
- Compute V_L and F_{LG} after finding the shear in girder *LK*.
- Compute the shear in the girder by summing moments about *K*.

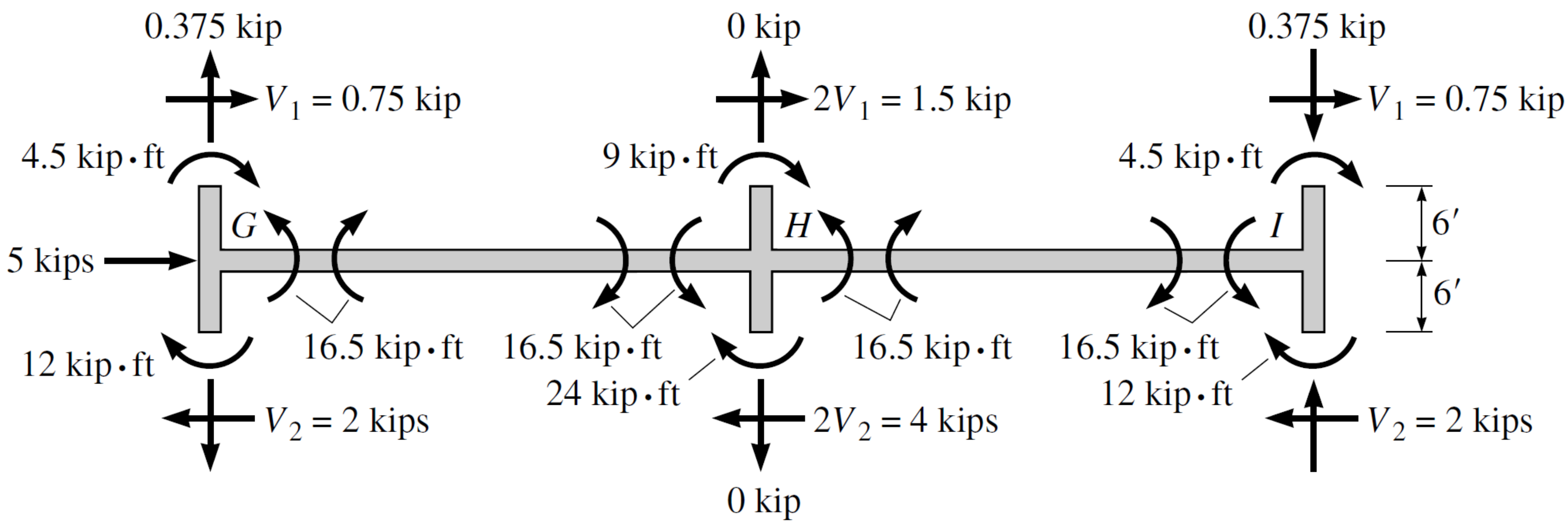
$$V_L = \frac{\Sigma M}{L} = \frac{4.5 + 4.5}{24} = 0.375 \text{ kip}$$

Example 15.12 Solution (continued)



Free body of joint K

- Use the equilibrium equations to evaluate all unknown forces acting on the joint.
- Isolate the next row of girders and columns between Sections 1 and 2



Free body of floor and columns located between Sections 1 and 2

Example 15.12 Solution (continued)

- Evaluate shears at points of inflection in the columns along section 2.

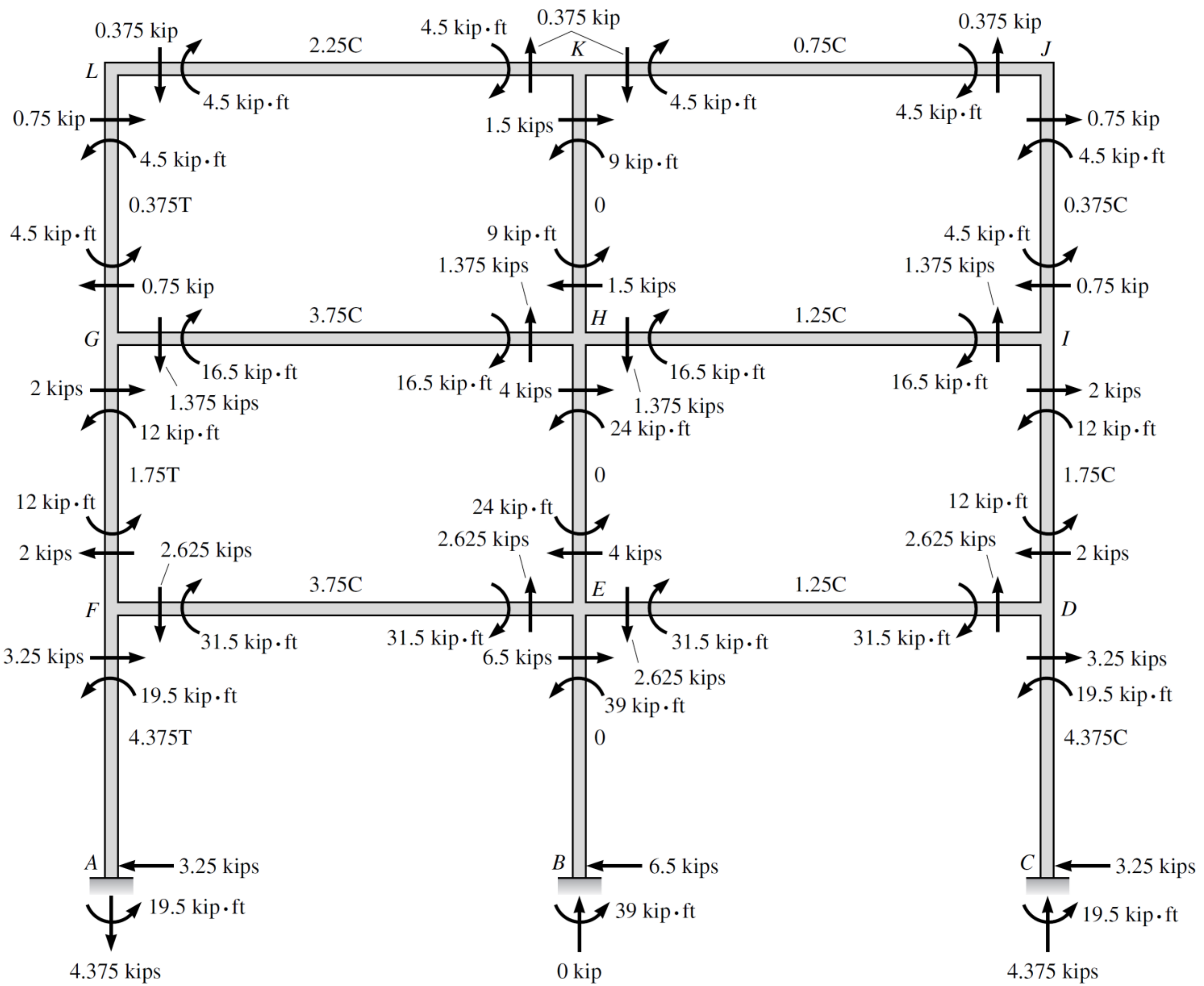
$$\xrightarrow{+} \quad \Sigma F_x = 0$$

$$3 + 5 - 4V_2 = 0$$

$$V_2 = 2 \text{ kips}$$

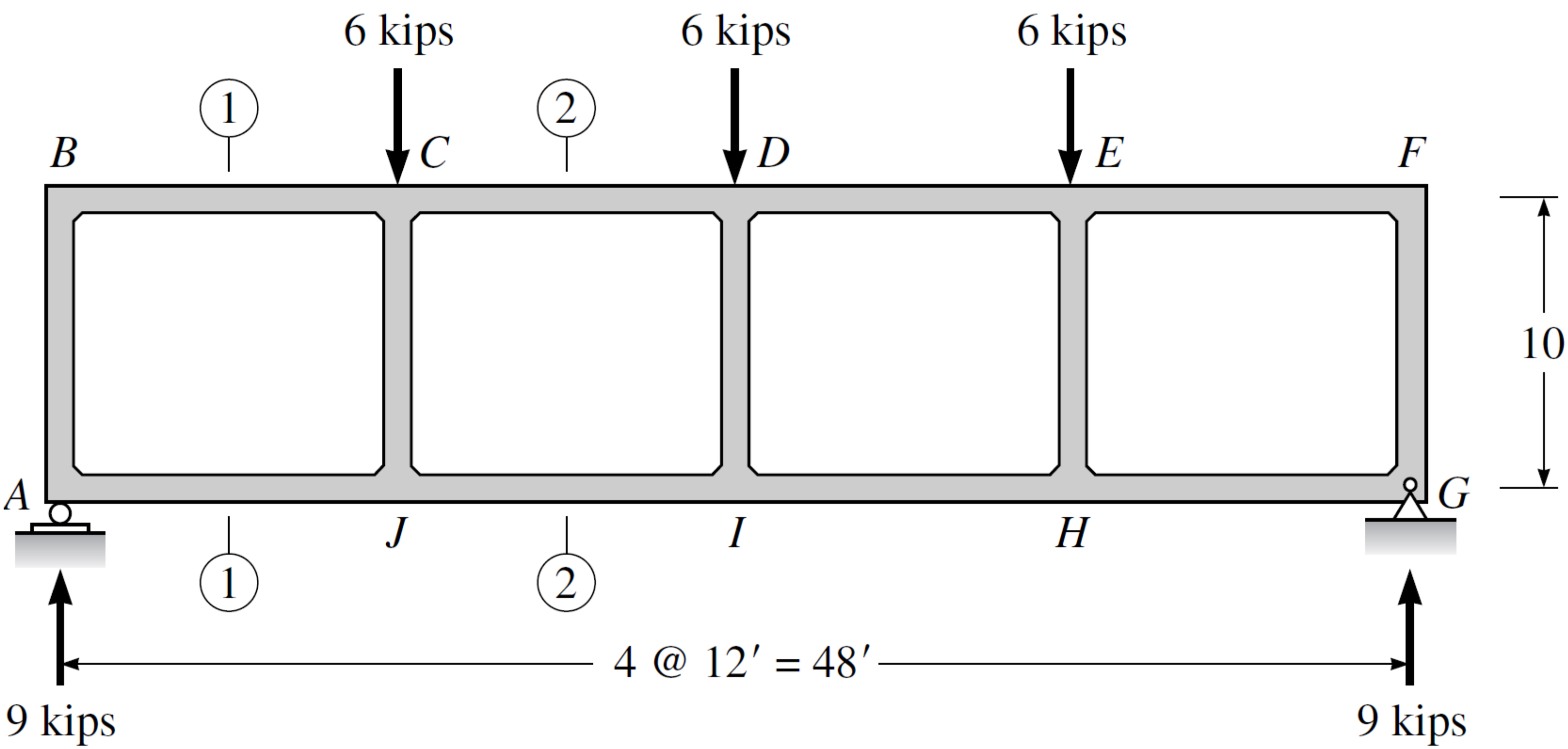
- Evaluate moments applied to joints *G, H,* and *I* by multiplying the shear by the half-column length. Compute the forces in girders and axial loads in columns following the procedure previously used to analyze the top floor.

Example 15.12 Solution (continued)

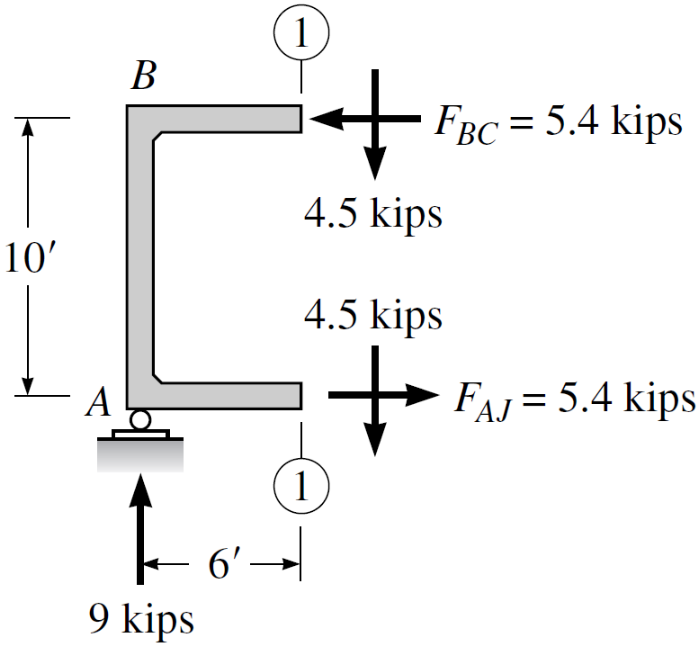


Example 15.13

Carry out an approximate analysis of the Vierendeel truss in Figure 15.27, using the assumptions of the portal method.



Example 15.13 Solution



Free body used to establish the forces at the points of inflection in the first panel

- Compute reactions.
- Pass section 1-1 through the points of inflection in the chords. No moments act on the ends of the members at the cut.
- Sum moments about an axis through the bottom point of inflection to compute an axial force of 5.4 kips compression in the top chord:

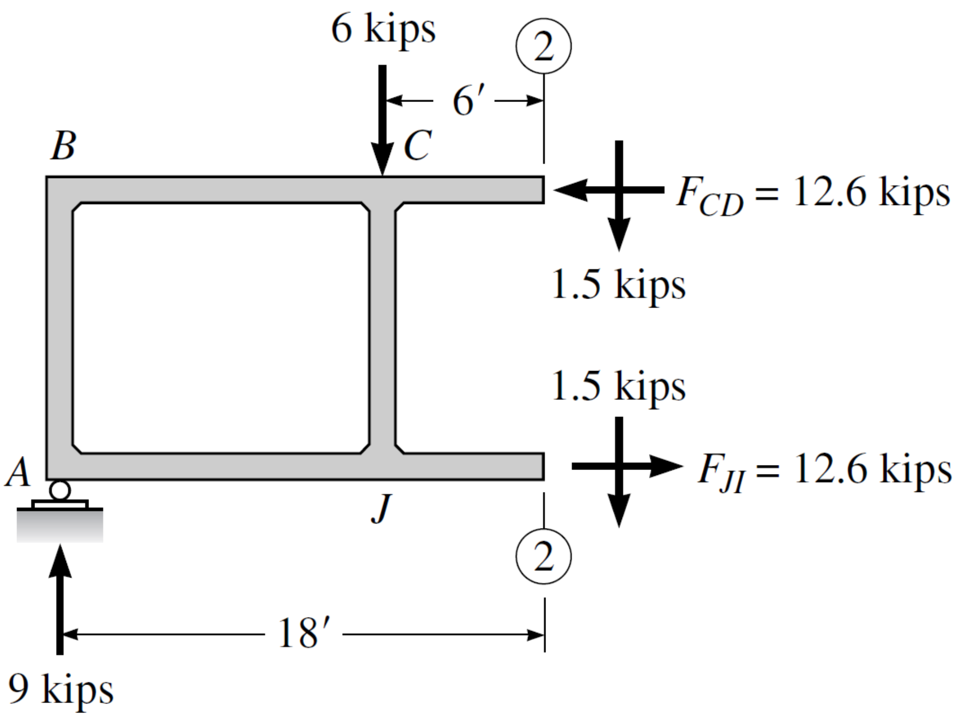
$$\circlearrowright^{+} \Sigma M = 0$$

$$9(6) - F_{BC}(10) = 0$$

$$F_{BC} = 5.4 \text{ kips}$$

- Equilibrium in the *x* direction establishes that a tension force of 5.4 kips acts in the bottom chord.

Example 15.13 Solution (continued)



Free body to compute forces at points of inflection in second panel

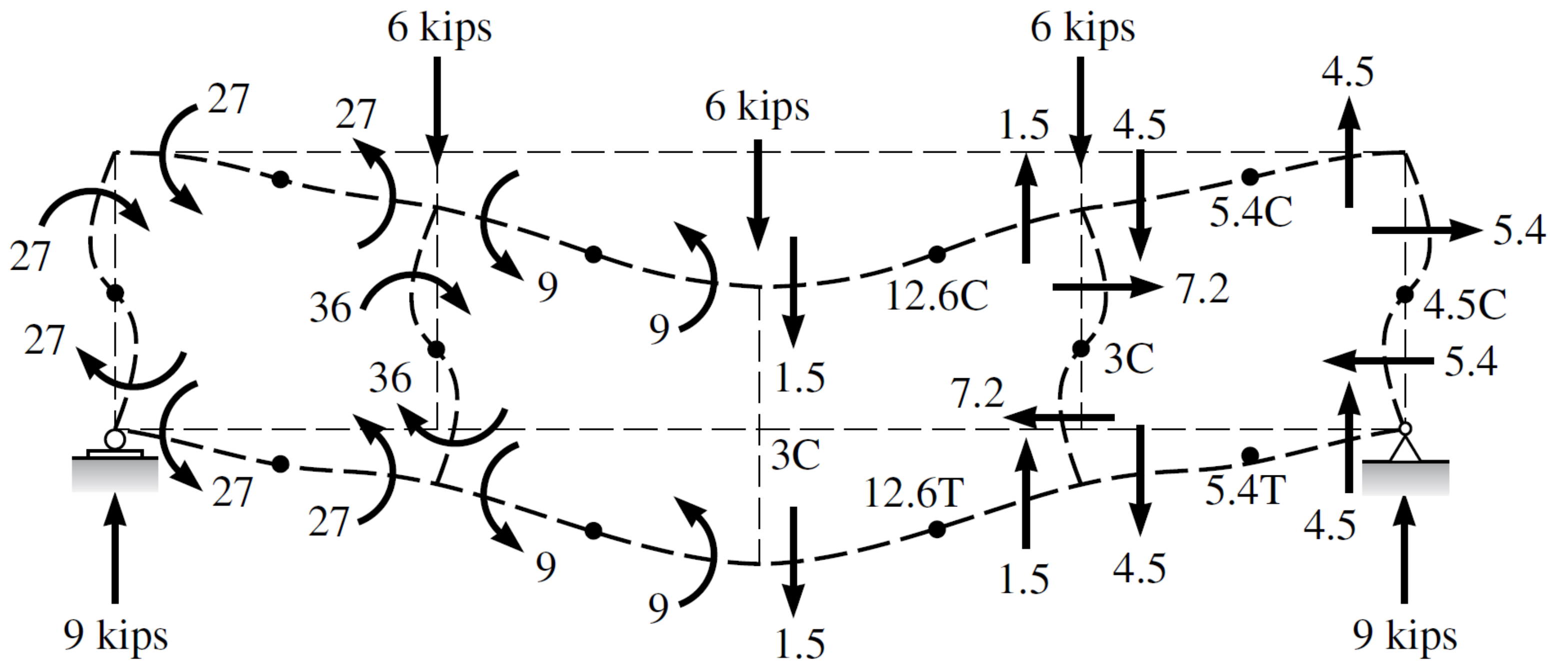
- Pass Section 2-2 through the midpoint of the second panel. As before, divide the unbalanced shear of 3 kips between the two chords and compute the axial forces in the chords by summing moments about the bottom point of inflection:

$$\circlearrowright^{+} \Sigma M = 0$$

$$9(18) - 6(6) - F_{CD}(10) = 0$$

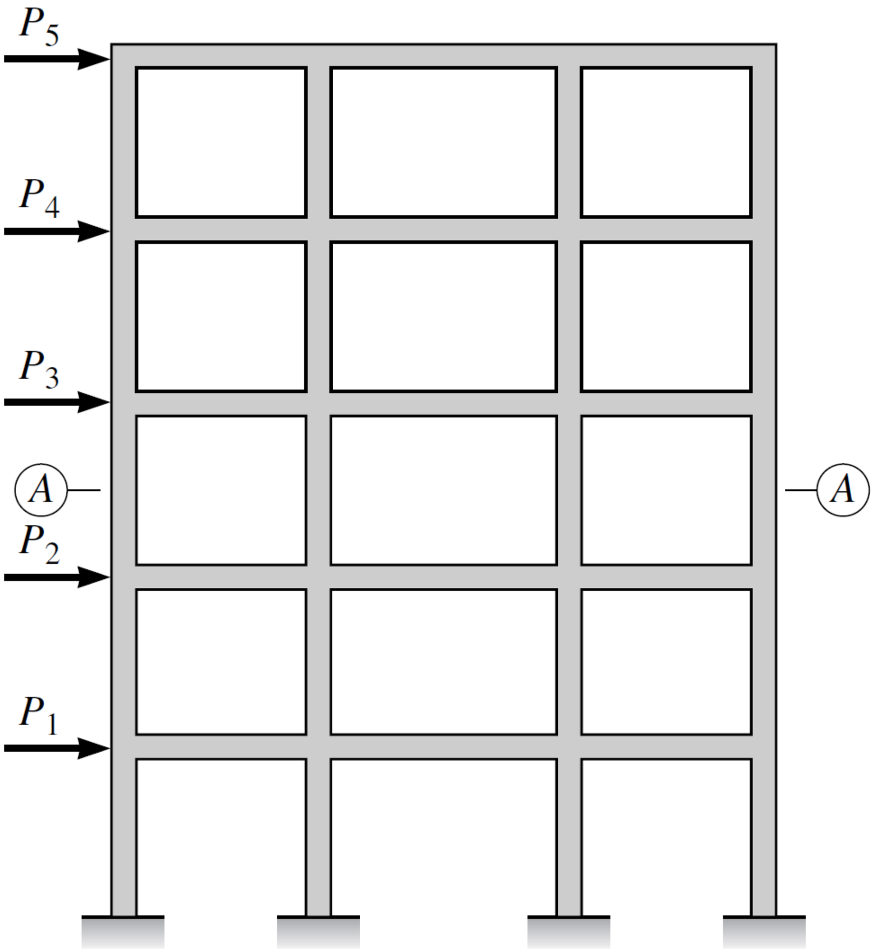
$$F_{CD} = 12.6 \text{ kips}$$

Example 15.13 Solution (continued)



Deflected shape; shears and axial forces in kips, moments in kip-ft.

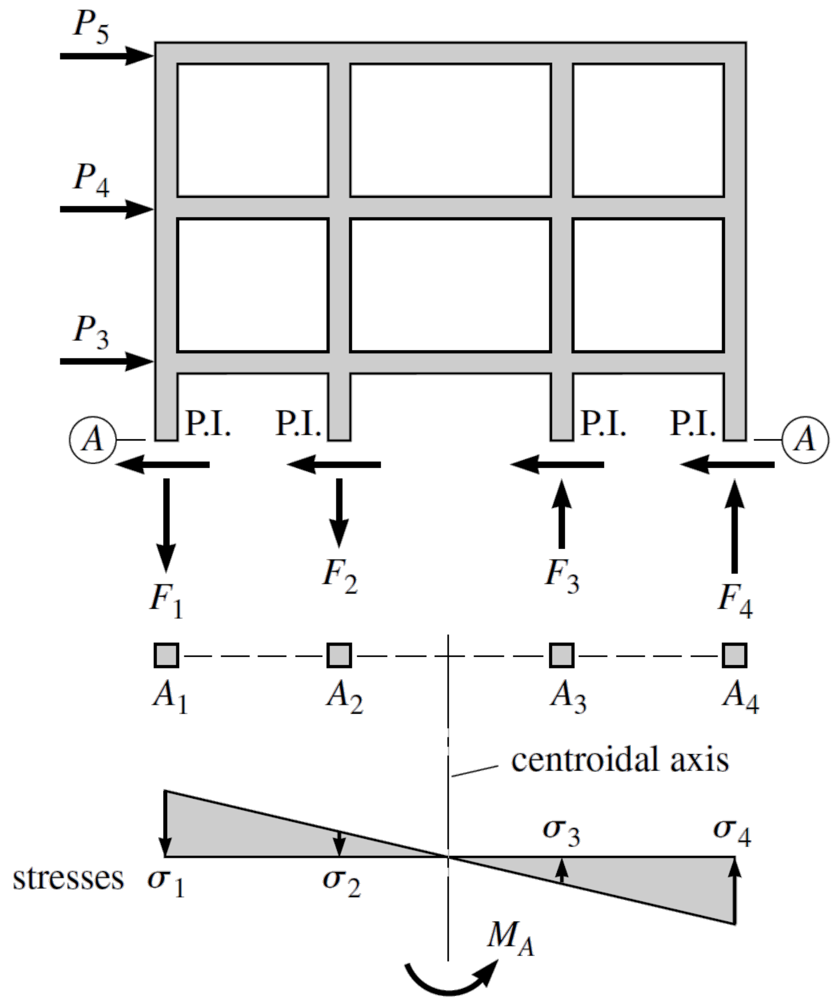
§15.10 Cantilever Method



Laterally loaded frame

Figure 15.28

§15.10 Cantilever Method

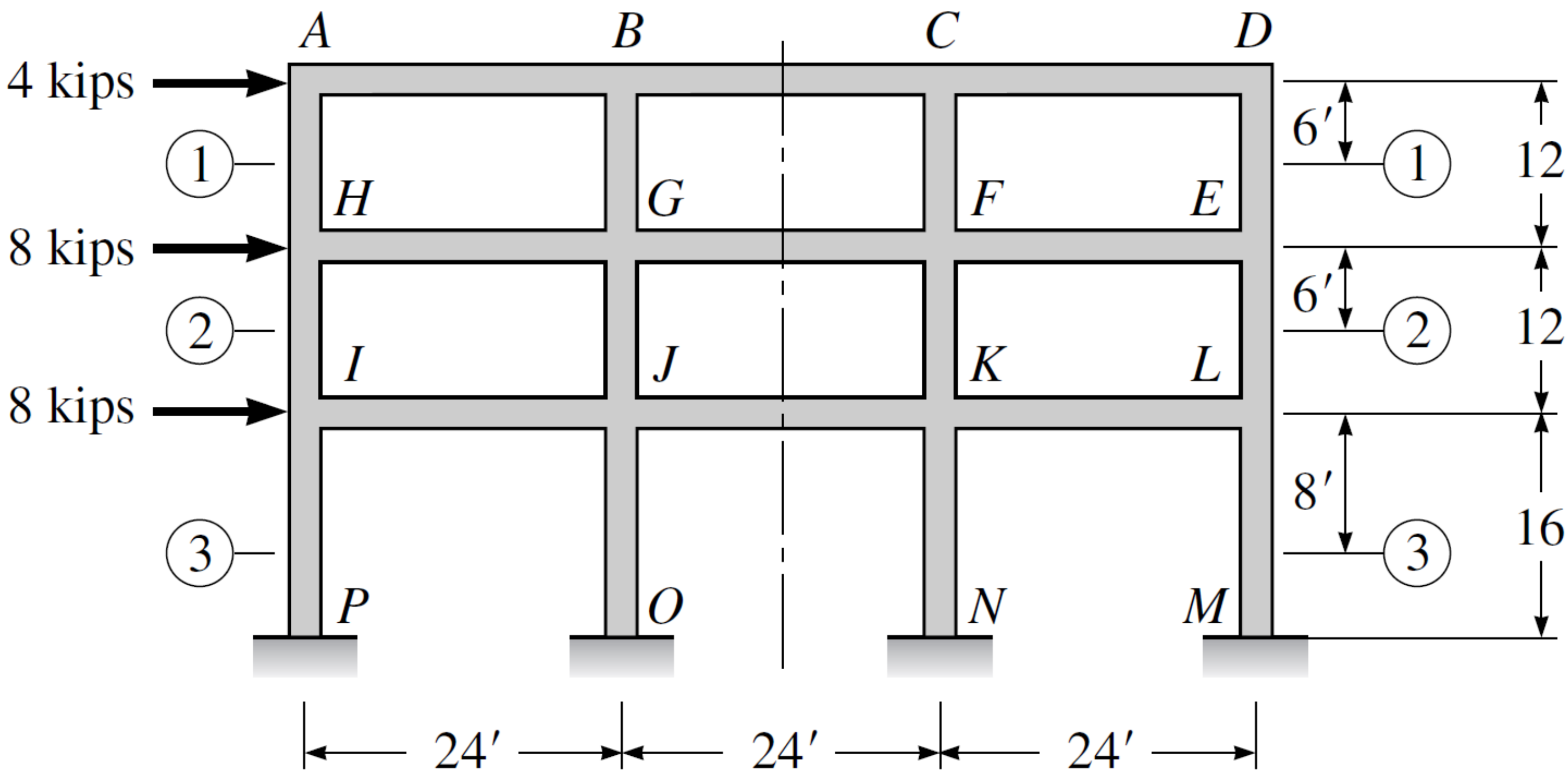


Free body of frame cut by Section *A*-*A*

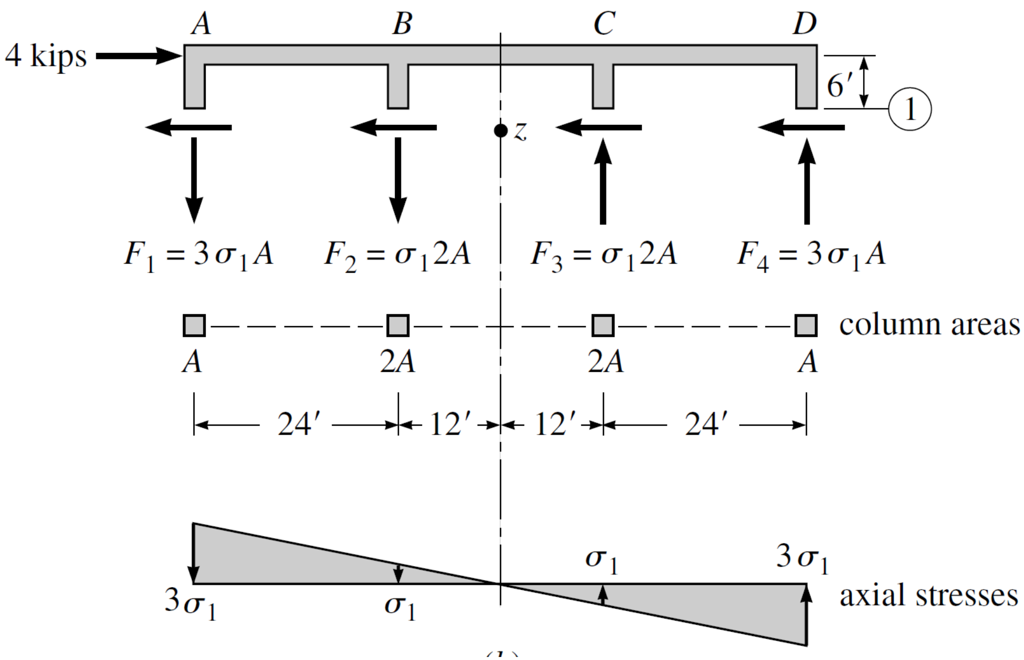
Figure 15.28 (continued)

Example 15.14

Use the cantilever method to estimate the forces in the laterally loaded frame shown in Figure 15.29*a*. Assume that the area of the interior columns is twice as large as the area of the exterior columns.



Example 15.14 Solution



<u>Free body of roof and attached columns cut by Section 1-1, axial stress in columns assumed to vary linearly with distance from centroid of column areas</u>

- Pass Section 1-1 through the points of inflection, so only shear and axial force act on the ends of each column.
- Sum moments about point *z*

External moment M_{ext}

$$= (4 \text{ kips})(6 \text{ ft}) = 24 \text{ kip} \cdot \text{ft}$$

- Compute the internal moment on Section 1-1 produced by axial forces in columns.
- Arbitrarily denote the axial stress in the interior columns as σ_1. The stress in the exterior columns equals $3\sigma_1$.

Example 15.14 Solution (continued)

- Multiply the area of each column by the indicated axial stress. Compute the internal moment by summing moments of the axial forces in the columns about an axis passing through point *z*.

$$M_{\text{int}} = 36F_1 + 12F_2 + 12F_3 + 36F_4$$

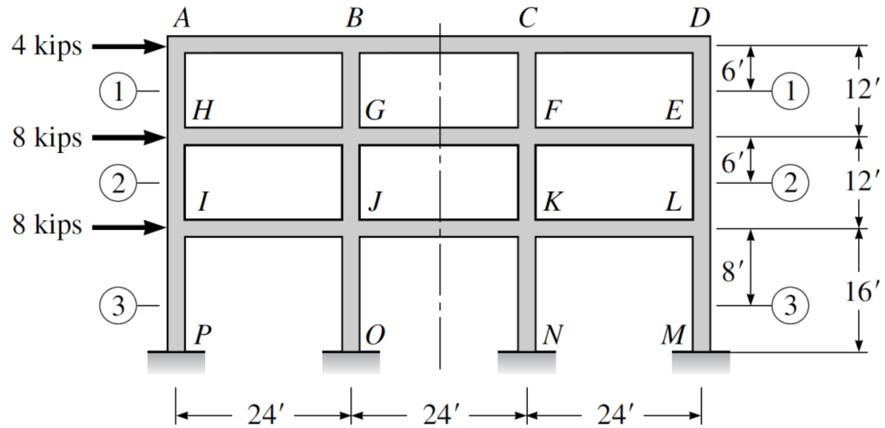
- Expressing the forces in Equation 2 in terms of the stress σ_1 and the column areas

$$M_{\text{int}} = 3\sigma_1 A(36) + 2\sigma_1 A(12) + 2\sigma_1 A(12) + 3\sigma_1 A(36)$$
$$= 264\sigma_1 A$$

- Equating the external moment given by Equation 1 to the internal moment given by Equation 3,

$$24 = 264\sigma_1 A; \quad \sigma_1 A = \frac{1}{11}$$

Example 15.14 Solution (continued)



- Substituting the value of $\sigma_1 A$ into the expressions for column force gives

$$F_1 = F_4 = 3\sigma_1 A = \frac{3}{11} = 0.273 \text{ kip}$$

$$F_2 = F_3 = 2\sigma_1 A = \frac{2}{11} = 0.182 \text{ kip}$$

- Pass Section 2-2 through the points of inflection of the second-floor columns. Compute the moment on Section 2-2 produced by the external loads.

$$M_{ext} = (4 \text{ kips})(12 + 6) + (8 \text{ kips})(6) = 120 \text{ kip} \cdot \text{ft}$$

Example 15.14 Solution (continued)

- Equating the internal and external moments

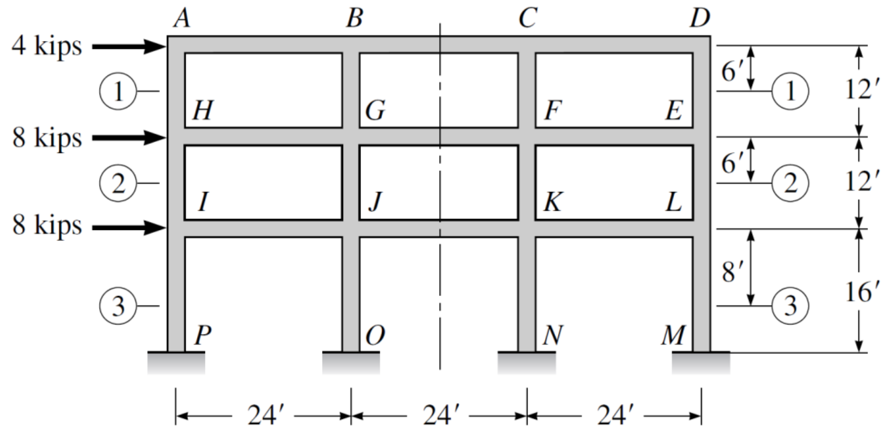
$$120 \text{ kip} \cdot \text{ft} = 264\sigma_2 A; \quad \sigma_2 A = \frac{5}{11}$$

- Axial forces in columns are

$$F_1 = F_4 = 3\sigma_2 A = \frac{15}{11} = 1.364 \text{ kips}$$

$$F_2 = F_3 = 2\sigma_2 A = \frac{10}{11} = 0.91 \text{ kip}$$

Example 15.14 Solution (continued)



- Pass Section 3-3 through the points of inflection. Compute the moment on Section 3 produced by all external loads acting above the section.

$$M_{ext} = (4\text{ kips})(32) + (8\text{ kips})(20) + (8\text{ kips})(8) = 352\text{ kip}\cdot\text{ft}$$

- Equate the external moment of 352 kip-ft to the internal moment given by Equation 3.

$$264\sigma_3 A = 352; \quad \sigma_3 A = \frac{3}{4}$$

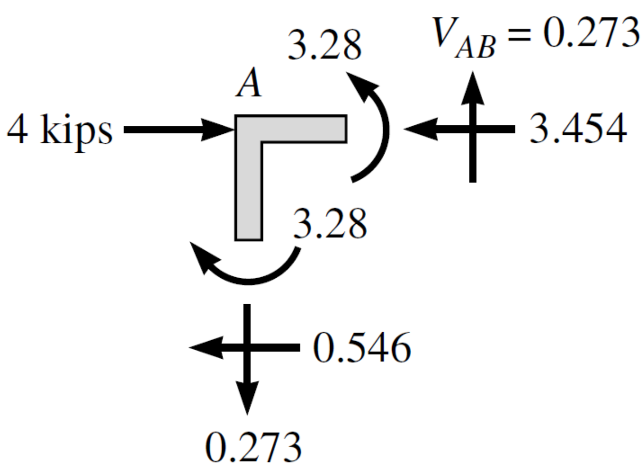
Example 15.14 Solution (continued)

- Compute the forces in the columns.

$$F_1 = F_4 = 3\sigma_3 A = 3\left(\frac{4}{3}\right) = 4 \text{ kips}$$

$$F_2 = F_3 = 2\sigma_3 A = 2\left(\frac{4}{3}\right) = 2.67 \text{ kips}$$

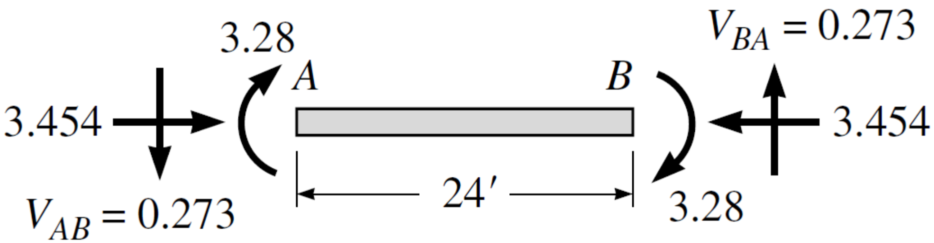
- Compute the shear in girder *AB* by considering equilibrium of vertical forces applied to joint *A.*



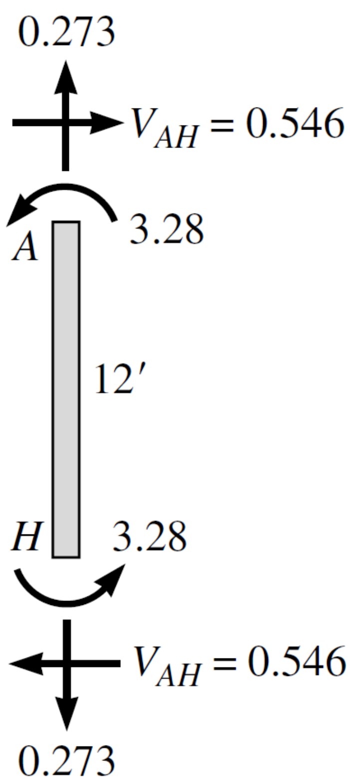
Free body of joint *A* initially used to establish V_{AB} = 0.273 kip

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0 \qquad 0 = -0.273 + V_{AB} \qquad V_{AB} = 0.273 \text{ kip}$$

Example 15.14 Solution (continued)



Free body of beam *AB*, used to establish end moments in beam



Free body of column used to compute shear

- Compute the end moments in girder *AB*.

$$M = V_{AB}\frac{L}{12} = 0.273(12) = 3.28 \text{ kip} \cdot \text{ft}$$

- Apply the girder end moment to joint *A*, and sum moments to establish that the moment at the top of the column equals 3.28 kip·ft.

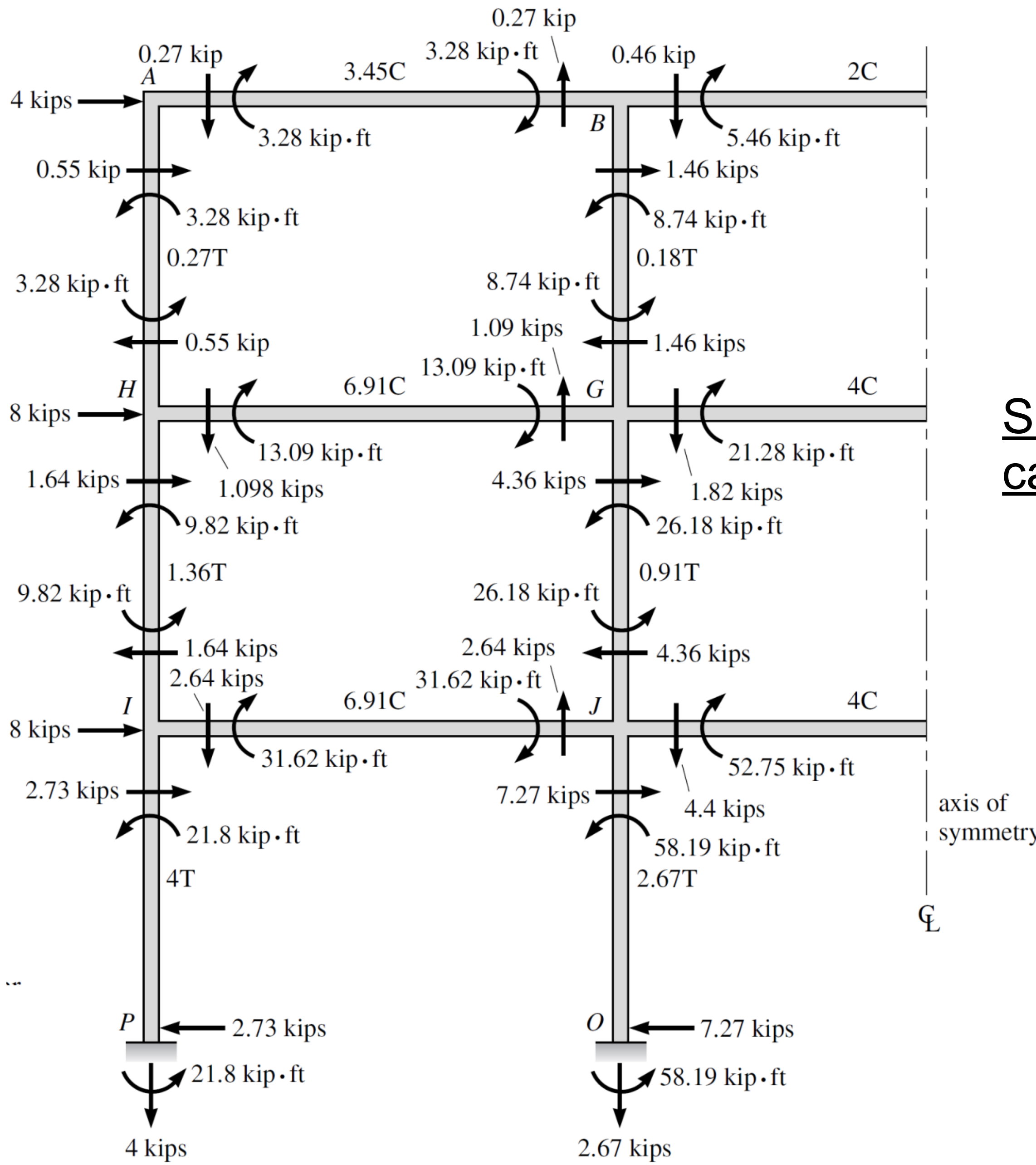
- Compute the shear in column *AH*.

$$V_{AH} = \frac{M}{L/2} = \frac{3.28}{6} = 0.547 \text{ kip}$$

Example 15.14 Solution (continued)

- Compute the axial force in the girder *AB.* Apply the value of column shear from above to joint *A*. The equilibrium of forces in the *x* direction establishes that the axial force in the girder equals the difference between 4 kips and the shear in column *AH*.

Example 15.14 Solution (continued)



Summary of cantilever analysis