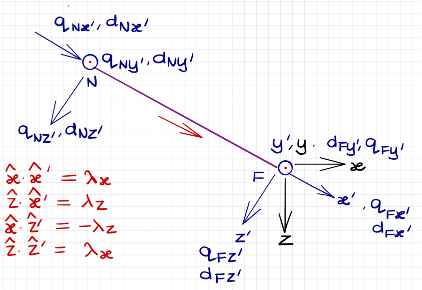
Grid Frames

The degrees of freedom are as follows,
$d_{nx'}$, $d_{ny'}$, $d_{nz'}$, $d_{fx'}$, $d_{fy'}$, $d_{fz'}$ as shown in the figure and the corresponding forces with d replaced with q.



The local stiffness matrix connection connects q and d.

$$\begin{pmatrix} q_{nx'} \\ q_{ny'} \\ q_{nz'} \\ q_{fx'} \\ q_{fy'} \\ q_{fx'} \end{pmatrix} = \begin{pmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ 0 & \frac{6\,EI}{L^2} & \frac{4\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{pmatrix} \begin{pmatrix} d_{nx'} \\ d_{ny'} \\ d_{nz'} \\ d_{fx'} \\ d_{fy'} \\ d_{fx'} \end{pmatrix}$$

In[1]:=
```
k[GJ_, EI_, L_] = {{GJ/L, 0, 0, -GJ/L, 0, 0}, {0, 12 EI/L^3, 6 EI/L^2, 0, -12 EI/L^3, 6 EI/L^2},
   {0, 6 EI/L^2, 4 EI/L, 0, -6 EI/L^2, 2 EI/L}, {-GJ/L, 0, 0, GJ/L, 0, 0},
   {0, -12 EI/L^3, -6 EI/L^2, 0, 12 EI/L^3, -6 EI/L^2}, {0, 6 EI/L^2, 2 EI/L, 0, -6 EI/L^2, 4 EI/L}};
MatrixForm[k[GJ, EI, L]]
```

Out[2]//MatrixForm=

$$\begin{pmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ 0 & \frac{6\,EI}{L^2} & \frac{4\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{pmatrix}$$

In[3]:=
```
T[λx_, λz_] =
  {{λx, 0, λz, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {-λz, 0, λx, 0, 0, 0},
   {0, 0, 0, λx, 0, λz}, {0, 0, 0, 0, 1, 0}, {0, 0, 0, -λz, 0, λx}};
T[λx, λz] // MatrixForm
```

Out[4]//MatrixForm=

$$\begin{pmatrix} \lambda x & 0 & \lambda z & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\lambda z & 0 & \lambda x & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda x & 0 & \lambda z \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\lambda z & 0 & \lambda x \end{pmatrix}$$

The basic transformation rules are as follows:

$\{d\} = [T]\{D\}$

$\{Q\} = [T]^T\{q\}$

$\{q\} = [k]\{d\}$

$\{Q\} = [K]\{D\}$, where

$[K] = [T]^T[k][T]$

[*K*] is obtained as

```
In[32]:= K = Transpose[T[λx, λz]].k[GJ, EI, L].T[λx, λz];
MatrixForm[K]
```

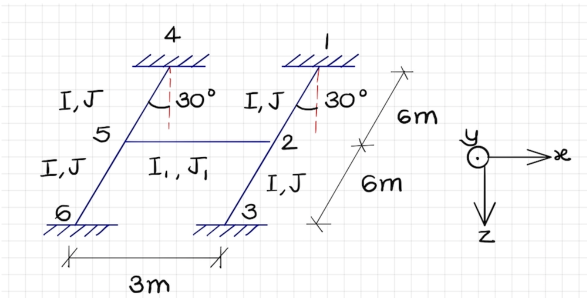
Out[33]//MatrixForm=

$$\begin{pmatrix} \frac{GJ\,\lambda x^2}{L}+\frac{4\,EI\,\lambda z^2}{L} & -\frac{6\,EI\,\lambda z}{L^2} & -\frac{4\,EI\,\lambda x\,\lambda z}{L}+\frac{GJ\,\lambda x\,\lambda z}{L} & -\frac{GJ\,\lambda x^2}{L}+\frac{2\,EI\,\lambda z^2}{L} & \frac{6\,EI\,\lambda z}{L^2} & -\frac{2\,EI\,\lambda x\,\lambda z}{L}-\frac{GJ\,\lambda x\,\lambda z}{L} \\ -\frac{6\,EI\,\lambda z}{L^2} & \frac{12\,EI}{L^3} & \frac{6\,EI\,\lambda x}{L^2} & -\frac{6\,EI\,\lambda z}{L^2} & -\frac{12\,EI}{L^3} & \frac{6\,EI\,\lambda x}{L^2} \\ -\frac{4\,EI\,\lambda x\,\lambda z}{L}+\frac{GJ\,\lambda x\,\lambda z}{L} & \frac{6\,EI\,\lambda x}{L^2} & \frac{4\,EI\,\lambda x^2}{L}+\frac{GJ\,\lambda z^2}{L} & -\frac{2\,EI\,\lambda x\,\lambda z}{L}-\frac{GJ\,\lambda x\,\lambda z}{L} & -\frac{6\,EI\,\lambda x}{L^2} & \frac{2\,EI\,\lambda x^2}{L}-\frac{GJ\,\lambda z^2}{L} \\ -\frac{GJ\,\lambda x^2}{L}+\frac{2\,EI\,\lambda z^2}{L} & -\frac{6\,EI\,\lambda z}{L^2} & -\frac{2\,EI\,\lambda x\,\lambda z}{L}-\frac{GJ\,\lambda x\,\lambda z}{L} & \frac{GJ\,\lambda x^2}{L}+\frac{4\,EI\,\lambda z^2}{L} & \frac{6\,EI\,\lambda z}{L^2} & -\frac{4\,EI\,\lambda x\,\lambda z}{L}+\frac{GJ\,\lambda x\,\lambda z}{L} \\ \frac{6\,EI\,\lambda z}{L^2} & -\frac{12\,EI}{L^3} & -\frac{6\,EI\,\lambda x}{L^2} & \frac{6\,EI\,\lambda z}{L^2} & \frac{12\,EI}{L^3} & -\frac{6\,EI\,\lambda x}{L^2} \\ -\frac{2\,EI\,\lambda x\,\lambda z}{L}-\frac{GJ\,\lambda x\,\lambda z}{L} & \frac{6\,EI\,\lambda x}{L^2} & \frac{2\,EI\,\lambda x^2}{L}-\frac{GJ\,\lambda z^2}{L} & -\frac{4\,EI\,\lambda x\,\lambda z}{L}+\frac{GJ\,\lambda x\,\lambda z}{L} & -\frac{6\,EI\,\lambda x}{L^2} & \frac{4\,EI\,\lambda x^2}{L}+\frac{GJ\,\lambda z^2}{L} \end{pmatrix}$$

The global stiffness matrix
is:

$$\begin{pmatrix} \frac{GJ\,\lambda x^2}{L}+\frac{4\,EI\,\lambda z^2}{L} & -\frac{6\,EI\,\lambda z}{L^2} & -\frac{4\,EI\,\lambda x\,\lambda z}{L}+\frac{GJ\,\lambda x\,\lambda z}{L} & -\frac{GJ\,\lambda x^2}{L}+\frac{2\,EI\,\lambda z^2}{L} & \frac{6\,EI\,\lambda z}{L^2} & -\frac{2\,EI\,\lambda x\,\lambda z}{L}-\frac{GJ\,\lambda x\,\lambda z}{L} \\ -\frac{6\,EI\,\lambda z}{L^2} & \frac{12\,EI}{L^3} & \frac{6\,EI\,\lambda x}{L^2} & -\frac{6\,EI\,\lambda z}{L^2} & -\frac{12\,EI}{L^3} & \frac{6\,EI\,\lambda x}{L^2} \\ -\frac{4\,EI\,\lambda x\,\lambda z}{L}+\frac{GJ\,\lambda x\,\lambda z}{L} & \frac{6\,EI\,\lambda x}{L^2} & \frac{4\,EI\,\lambda x^2}{L}+\frac{GJ\,\lambda z^2}{L} & -\frac{2\,EI\,\lambda x\,\lambda z}{L}-\frac{GJ\,\lambda x\,\lambda z}{L} & -\frac{6\,EI\,\lambda x}{L^2} & \frac{2\,EI\,\lambda x^2}{L}-\frac{GJ\,\lambda z^2}{L} \\ -\frac{GJ\,\lambda x^2}{L}+\frac{2\,EI\,\lambda z^2}{L} & -\frac{6\,EI\,\lambda z}{L^2} & -\frac{2\,EI\,\lambda x\,\lambda z}{L}-\frac{GJ\,\lambda x\,\lambda z}{L} & \frac{GJ\,\lambda x^2}{L}+\frac{4\,EI\,\lambda z^2}{L} & \frac{6\,EI\,\lambda z}{L^2} & -\frac{4\,EI\,\lambda x\,\lambda z}{L}+\frac{GJ\,\lambda x\,\lambda z}{L} \\ \frac{6\,EI\,\lambda z}{L^2} & -\frac{12\,EI}{L^3} & -\frac{6\,EI\,\lambda x}{L^2} & \frac{6\,EI\,\lambda z}{L^2} & \frac{12\,EI}{L^3} & -\frac{6\,EI\,\lambda x}{L^2} \\ -\frac{2\,EI\,\lambda x\,\lambda z}{L}-\frac{GJ\,\lambda x\,\lambda z}{L} & \frac{6\,EI\,\lambda x}{L^2} & \frac{2\,EI\,\lambda x^2}{L}-\frac{GJ\,\lambda z^2}{L} & -\frac{4\,EI\,\lambda x\,\lambda z}{L}+\frac{GJ\,\lambda x\,\lambda z}{L} & -\frac{6\,EI\,\lambda x}{L^2} & \frac{4\,EI\,\lambda x^2}{L}+\frac{GJ\,\lambda z^2}{L} \end{pmatrix}$$

Problems



For members 12, 23, 45, 56

$I = 27.34 \times 10^9\ \text{mm}^4$

$J = 18.77 \times 10^9\ \text{mm}^4$

$L = 6 \times 10^3\ \text{mm}$

$E = 12\ \text{kN}/\text{mm}^2$

$G = 5\ \text{kN}/\text{mm}^2$

for member 25

$I = 18.23 \times 10^9\ \text{mm}^4$

$J = 6.4 \times 10^9\ \text{mm}^4$

$L = 3 \times 10^3\ \text{mm}$

$E = 12\ \text{kN}/\text{mm}^2$

$G = 5\ \text{kN}/\text{mm}^2$

At every node there are three degrees of freedom. So in total there are 18 degrees of freedom.

```
(* Members 1(12), 2(23), 3(45), 4(56) *)
n1 = {1, 2, 3, 4, 5, 6};
n2 = {4, 5, 6, 7, 8, 9};
n3 = {10, 11, 12, 13, 14, 15};
n4 = {13, 14, 15, 16, 17, 18};
vals =
  {EI → 12 × 27.34 × 10^9 10^-6, GJ → 5 × 18.77 × 10^9 10^-6 , L → 6, λx → -1/2, λz → √3/2};
vals = {EI → 12 × 27.34 × 10^9 10^-6, GJ → 5 × 18.77 × 10^9 10^-6 , L → 6, λx → 0, λz → -1};
(* Member 1, 2, 3, 4 *)
K = Transpose[T[λx, λz]].k[GJ, EI, L].T[λx, λz] /. vals;
(* Member 5 (25) *)
n5 = {4, 5, 6, 13, 14, 15};
vals1 =
  {EI → 12 * 18.23 × 10^9 10^-6, GJ → 5 * 6.4 × 10^9 10^-6, L → 3, λx → -1, λz → 0};
K1 = Transpose[T[λx, λz]].k[GJ, EI, L].T[λx, λz] /. vals1;
(* Full Stiffness Matrix *)
Kg = ConstantArray[0, {18, 18}];
(* Assemble stiffness matrix *)
Kg[[n1, n1]] = Kg[[n1, n1]] + K;
Kg[[n2, n2]] = Kg[[n2, n2]] + K;
Kg[[n3, n3]] = Kg[[n3, n3]] + K;
Kg[[n4, n4]]  = Kg[[n4, n4]] + K;
Kg[[n5, n5]] = Kg[[n5, n5]] + K1;
```

```
MatrixForm[Kg]
```

218 720.	54 680.	0.	109 360.	-54 680.	0.	0	0	
54 680.	18 226.7	0.	54 680.	-18 226.7	0.	0	0	
0.	0.	15 641.7	0.	0.	-15 641.7	0	0	
109 360.	54 680.	0.	448 107.	0.	0.	109 360.	-54 680.	
-54 680.	-18 226.7	0.	0.	133 680.	-145 840.	54 680.	-18 226.7	
0.	0.	-15 641.7	0.	-145 840.	322 963.	0.	0.	-15
0	0	0	109 360.	54 680.	0.	218 720.	-54 680.	
0	0	0	-54 680.	-18 226.7	0.	-54 680.	18 226.7	
0	0	0	0.	0.	-15 641.7	0.	0.	15
0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	
0	0	0	-10 666.7	0.	0.	0	0	
0	0	0	0.	-97 226.7	145 840.	0	0	
0	0	0	0.	-145 840.	145 840.	0	0	
0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	

Now extracting the stiffness matrix K_{11}

```
nu = {4, 5, 6, 13, 14, 15};
Kg11 = Kg[[nu, nu]];
MatrixForm[% 10^-5] // FullSimplify
```

4.48107	0.	0.	-0.106667	0.	0.
0.	1.3368	-1.4584	0.	-0.972267	-1.4584
0.	-1.4584	3.22963	0.	1.4584	1.4584
-0.106667	0.	0.	4.48107	0.	0.
0.	-0.972267	1.4584	0.	1.3368	1.4584
0.	-1.4584	1.4584	0.	1.4584	3.22963

```
nmod = {5, 4, 6, 14, 13, 15};
Kg[[nmod, nmod]];
MatrixForm[% 10^-5]
```

1.3368	0.	-1.4584	-0.972267	0.	-1.4584
0.	4.48107	0.	0.	-0.106667	0.
-1.4584	0.	3.22963	1.4584	0.	1.4584
-0.972267	0.	1.4584	1.3368	0.	1.4584
0.	-0.106667	0.	0.	4.48107	0.
-1.4584	0.	1.4584	1.4584	0.	3.22963


```
nmod = {5, 4, 6, 14, 13, 15};
Kg[[nmod, nmod]];
MatrixForm[% 10^-5]
```

1.3368	0.	-1.4584	-0.972267	0.	-1.4584
0.	4.48107	0.	0.	-0.106667	0.
-1.4584	0.	3.22963	1.4584	0.	1.4584
-0.972267	0.	1.4584	1.3368	0.	1.4584
0.	-0.106667	0.	0.	4.48107	0.
-1.4584	0.	1.4584	1.4584	0.	3.22963

Tutorial problem

```
(* connectivity *)
n1 = {1, 2, 3, 4, 5, 6};
n2 = { 7, 8, 9, 4, 5, 6};

(* material properties and geometry *)
val1 = {EI → 200 × 500, GJ → 200/(2 (1 + 0.3)) 200 , L → 7.5, λx → 0.8, λz → 0.6};
val2 = {EI → 200 × 500, GJ → 200/(2 (1 + 0.3)) 200 , L → 4.5, λx → 0, λz → 1};

(* stiffness matrices *)
k1 = Transpose[T[λx, λz]].k[GJ, EI, L].T[λx, λz] /. val1;
k2 = Transpose[T[λx, λz]].k[GJ, EI, L].T[λx, λz] /. val2;
K = ConstantArray[0, {9, 9}];

(* Assembling the matrix *)
K[[n1, n1]] = K[[n1, n1]] + k1;
K[[n2, n2]] = K[[n2, n2]] + k2;

(* Extracting the k11 matrix *)
nu = {4, 5, 6};
k11 = K[[nu, nu]];
MatrixForm[k11]
```

Out[94]//MatrixForm=

109 402.	36 029.6	-24 615.4
36 029.6	16 013.2	-8533.33
-24 615.4	-8533.33	38 290.6

Out[96]= {0.00236418, -0.00724314, -0.0000943573}

In[97]:=
```
(* known load *)
Qk = {0, -30, 0};
Du = LinearSolve[k11, Qk]
```

Out[98]= {0.00236418, -0.00724314, -0.0000943573}

In[105]:=
```
(* support reactions *)
nk = {1, 2, 3, 7, 8, 9};
k21 = K[[nk, nu]];
Qu = k21.Du
```

Out[107]= {-25.463, 4.66679, 27.6782, -109.537, 25.3332, 0.322589}