# CHAPTER

# Introduction





# §1.3 Strength and Serviceability



Photo 1.1 Wind damage



Figure 1.4 Chemical storage tank supported by tension hangers carrying force T



## Figure 1.5



M

V

♦

au

 $R_A$ 

 $\sigma_{c}$ 

 $\sigma_t$ 

bending

stresses

Beam deflects into a shallow curve

Internal forces (shear V and moment *M*)



arm

internal couple

T



**I**-shaped steel section (d)

**Glue-laminated** wood I-beam

Figure 1.6 (continued)



Figure 1.7 Trusses



Bolted joint detail

Truss bridge showing cross-bracing needed to stabilize the two main trusses

**Steel Design** 

Figure 1.8



Fixed-end arch carries roadway over a canyon where rock walls provide a natural support for arch thrust *T* 



Large abutments provided to carry arch thrust

### Figure 1.9 Truss bridge



Tension tie added at base to carry horizontal thrust, foundations designed only for vertical reaction *R* 

Foundation placed on piles, batter piles used to transfer horizontal component of thrust into ground

Figure 1.9 Truss bridge (continued)





#### Photo 1.4 Golden Gate Bridge and Rhine River Bridge





(*a*) One story rigid frame

Vierendeel truss, loads transmitted both by direct stress and bending

#### Figure 1.13 Rigid-jointed structures



*(C)* 

Details of a welded joint at the corner of a steel rigid frame



*(d)* 

Reinforcing detail for corner of concrete frame

Figure 1.13 Rigid-jointed structures (continued)

# **<u>§1.5 Basic Structural Elements</u>**



#### Beam and slab system

#### Influence of boundaries on curvature



slab

A folded plate roof



Slab and beams act as a unit

## **Figure 1.14**



Figure 1.15 Membrane stresses acting on a small shell element

# §1.6 Assembling Basic Elements to Form a Stable Structural System



*(a)* 

Three-dimensional view of building (arrow indicates direction in which roof deck spans Details of cross braced frame with bolted joints

*(b)* 

Figure 1.17

Details of beam-tocolumn connections

(c)

# §1.6 Assembling Basic Elements to Form a Stable Structural System





 $P \xrightarrow{D} buckles$   $0 \xrightarrow{V} tension -V$   $P \xrightarrow{V} F = 0$  V V (f)

-P

Idealized model of structural system transmitting gravity loads from roof Model of beam CD

(e)

Idealized model of truss system for transmitting lateral load acting to the right. Diagonal member *DB* buckles and is ineffective.

Figure 1.17 (continued)

# CHAPTER



# Design Loads



# **Outline**

- §2.1 Building and Design Code
- §2.2 Loads
- §2.3 Dead Loads
- §2.4 Live Loads
- §2.5 Snow Loads
- §2.6 Wind Loads
- §2.7 Earthquake Loads
- §2.8 Other Loads
- §2.9 Load Combinations

# CHAPTER

# Statics of Structures— Reactions







Right end is free to expand laterally, no stresses created by temperature change Both ends are restrained, compressive and bending stresses develop in beam

Figure 3.10 Influence of supports



Photo 3.1 Pin support for 2.1-mile long steel box-girder San Diego-Coronado Bridge





Photo 3.2 Roller support for the San Diego-Coronado Bridge

Туре	Sketch	Symbol	Movements Allowed or Prevented	Reaction Forces	Unknowns Created
(a) Pin	OR	OR	<i>Prevented:</i> horizontal translation, vertical translation <i>Allowed:</i> rotation	A single linear force of unknown direction or equivalently A horizontal force and a vertical force which are the components of the single force of unknown direction	$R_x$ $R_y$
(b) Hinge			<i>Prevented:</i> relative displacement of member ends <i>Allowed:</i> both rotation and horizontal and vertical displacement	Equal and oppositely directed horizontal and vertical forces	$R_{y} \stackrel{R_{x}}{\longleftarrow} R_{y}$
(c) Roller		0	<i>Prevented:</i> vertical translation <i>Allowed:</i> horizontal translation, rotation	A single linear force (either upward or downward <sup>*</sup> )	R

Table 3.1 Characteristics of Supports

Туре	Sketch	Symbol	Movements Allowed or Prevented	Reaction Forces	Unknowns Created
(d) Rocker		OR			
(e) Elastomeric pad					
(f) Fixed end			<i>Prevented:</i> horizontal translation, vertical translation, rotation <i>Allowed:</i> none	Horizontal and vertical components of a linear resultant; moment	

## Table 3.1 Characteristics of Supports (continued)

Туре	Sketch	Symbol	Movements Allowed or Prevented	Reaction Forces	Unknowns Created
(g) Link	θ	θ	<i>Prevented:</i> translation in the direction of link <i>Allowed:</i> translation perpendicular to link, rotation	A single linear force in the direction of the link	R H H H
(h) Guide			<i>Prevented:</i> vertical translation, rotation <i>Allowed:</i> horizontal translation	A single vertical linear force; moment	

## Table 3.1 Characteristics of Supports (continued)



Figure 3.11 Fixed-end beam produced by embedding its left end in a reinforced concrete wall



Figure 3.12 A steel column supported on a stiffened baseplate, which is bolted to a concrete foundation, producing a fixed-end condition at its base



Figure 3.13 A reinforced concrete beam with a fixed end



Figure 3.13 A reinforced concrete column whose lower end is detailed to act as a pin (continued)

# §3.4 Idealizing Structures





Welded rigid frame with snow load

Idealized frame on which analysis is based

# §3.4 Idealizing Structures



Figure 3.15 Bolted web connection idealized as a pin support

# §3.4 Idealizing Structures



Since the connection supplies only vertical restraint (its capacity for lateral restraint is not mobilized), we are free to model the joint as a pin or roller support



Figure 3.15 Bolted web connection idealized as a pin support (continued)

## §3.5 Free Body Diagrams





Free-body diagram of three-hinged arch

Simplified free body of arch

Figure 3.16 Free-body diagrams

## §3.5 Free Body Diagrams



Figure 3.16 Free-body diagrams (continued)
## §3.6 Equations of Static Equilibrium



#### Figure 3.17 Equivalent planar force systems acting on a rigid body

#### Example 3.5

Compute the reactions for the beam.



## Example 3.5 Solution



• Resolve the force at *C* into components and assume directions for the reactions at *A* and *B*.

$$\rightarrow + \quad \Sigma F_x = 0 \qquad -A_x + 6 = 0$$

$$\uparrow^+ \quad \Sigma F_y = 0 \qquad A_y + B_y - 8 = 0$$

$$\bigcirc^+ \quad \Sigma M_A = 0 \qquad -10B_y + 8(15) = 0$$

• Solving Equations 1, 2, and 3 gives

 $A_x = 6$  kips  $B_y = 12$  kips  $A_y = -4$  kips Ans.

### **Example 3.5 Solution (continued)**

- Recompute reactions, using equilibrium equations that contain only one unknown. One possibility is
  - C<sup>+</sup>  $\Sigma M_A = 0$   $-B_y(10) + 8(15) = 0$ C<sup>+</sup>  $\Sigma M_B = 0$   $A_y(10) + 8(5) = 0$ →+  $\Sigma F_x = 0$   $-A_x + 6 = 0$
- Solving again gives  $A_x = 6$  kips,  $B_y = 12$  kips,  $A_y = -4$  kips

#### Example 3.6

Compute the reactions for the truss.



## **Example 3.6 Solution**



 $C_y = 15.43$  kips **Ans.** 

#### Example 3.7

The frame in Figure 3.20 carries a distributed load that varies from 4 to 10 kN/m. Compute the reactions.



## **Example 3.7 Solution**



• Divide the distributed load into a triangular and a rectangular distributed load. Replace the distributed loads by their resultant

$$R_1 = 10(4) = 40 \text{ kN}$$
  
 $R_2 = \frac{1}{2}(10)(6) = 30 \text{ kN}$ 

• Compute  $A_v$ 

$$C^+ \quad \Sigma M_C = 0$$
$$A_y(4) - R_1(5) - R_2\left(\frac{20}{3}\right) = 0$$
$$A_y = 100 \text{ kN}$$

#### Ans.

## **Example 3.7 Solution (continued)**



• Compute  $C_y$   $\uparrow^+ \Sigma F_y = 0$   $100 - R_1 - R_2 + C_y = 0$  $C_y = -30 \text{ kN} \downarrow$ 

• Compute  $C_x$ 

$$\sum F_x = 0$$

$$C_x = 0$$
**Ans.**

#### Example 3.8

Compute the reactions for the beam in Figure 3.21*a*, treating member *AB* as a link.



### **Example 3.8 Solution**



Free body of link AB

• First compute the forces in the link.

$$\rightarrow + \quad \Sigma F_x = 0 \qquad 0 = F_A - F_B$$

$$\uparrow^{+} \Sigma F_{y} = 0 \qquad 0 = V_{A} - V_{B}$$

 $\bigcirc^+ \quad M_A = 0 \qquad 0 = V_B(5)$ 

#### Solving the equations above gives

 $F_A = F_B (\text{call } F_{AB}) \text{ and } V_A = V_B = 0$ 

## **Example 3.8 Solution (continued)**



• Resolve  $F_{AB}$  into components at B and sum moments about C

$$\begin{array}{ll} \bigcirc^{+} & \Sigma M_{c} = 0 & 0 = 0.8F_{AB}(10) - 36(2) \\ \rightarrow + & \Sigma F_{x} = 0 & 0 = 0.6F_{AB} - C_{x} \\ \uparrow & \Sigma F_{y} = 0 & 0 = 0.8F_{AB} - 36 + C_{y} \end{array}$$

• Solving gives *F*<sub>AB</sub>

 $F_{AB} = 9$  kips,  $C_x = 5.4$  kips, and  $C_y = 28.8$  kips.

#### Example 3.9

Compute the reactions for the beam in Figure 3.22*a*. A load of 12 kips is applied directly to the hinge at *C*.



## **Example 3.9 Solution**



• Compute  $E_y$  by summing moments about C

$$C^+ \quad \Sigma M_c = 0$$
  
 
$$0 = 24(5) - E_v(10) \quad \text{and} \quad E_v = 12 \text{ kips} \quad \text{Ans.}$$

#### **Example 3.9 Solution (continued)**

• Complete the analysis

(down).

$$\rightarrow + \quad \Sigma F_x = 0 \qquad 0 + E_x = 0$$

$$E_x = 0 \qquad \text{Ans.}$$

C<sup>+</sup>  $\Sigma M_A = 0$   $0 = -B_y(10) + 12(15) + 24(20) - 12(25)$   $B_y = 36 \text{ kips}$  Ans.  $\stackrel{+}{\uparrow} \Sigma F_y = 0$   $0 = A_y + B_y - 12 - 24 + E_y$ Substituting  $B_y = 36$  kips and  $E_y = 12$  kips, we compute  $A_y = -12$  kips

#### Example 3.10

Compute the reactions for the beams.



# Example 3.10 Solution



• Applying the equations of equilibrium to member *BD*,

$$\rightarrow + \quad \Sigma F_x = 0 \qquad 0 = 15 - D_x$$

$$\bigcirc^+ \quad \Sigma M_D = 0 \qquad 0 = B_y(10) - 20(5)$$

$$\stackrel{+}{\uparrow} \quad \Sigma F_y = 0 \qquad 0 = B_y - 20 + D_y$$

• Solving Equations, compute  $D_x = 15$ kips,  $B_y = 10$  kips, and  $D_y = 10$  kips

#### **Example 3.10 Solution (continued)**



• With  $B_y$  evaluated, apply the equations of equilibrium to member AC

$$\rightarrow + \Sigma F_x = 0 \qquad 0 = A_x$$

$$\bigcirc^+ \Sigma M_A = 0 \qquad 0 = 10(10) - 15C_y$$

$$\stackrel{+}{\uparrow} \Sigma F_y = 0 \qquad 0 = A_y - 10 + C_y$$

• Static check: To verify the accuracy of the computations, apply  $\Sigma F_y = 0$  to the entire structure

$$A_{y} + C_{y} + D_{y} - 0.8(25) = 0$$
$$\frac{10}{3} + \frac{20}{3} + 10 - 20 = 0$$
$$0 = 0 \qquad \text{OK}$$





Figure 3.24 (continued)



#### Figure 3.24 (continued)



Geometrically unstable, reactions form a parallel force system

Equilibrium position, horizontal reaction develops as link elongates and changes slope

#### Figure 3.25



Geometrically unstable—reactions form a concurrent force system passing through the pin at *A*  Indeterminate beam

Figure 3.25 (continued)



## **Example 3.11**

Investigate the stability of the structure. Hinges at joints *B* and *D*.



# Example 3.11 Solution



member DE

- A necessary condition for stability requires R = 3 + C
- Check the equilibrium of DE

 $\begin{array}{ccc} \rightarrow + & \Sigma F_x = 0 & E_x - D_x = 0 \\ & E_x = D_x \\ \bigcirc^+ & \Sigma M_D = 0 & 8(2) - 4E_y = 0 \\ & E_y = 4 \text{ kips} \\ \uparrow^+ & \Sigma F_y = 0 & D_y + E_y - 8 = 0 \end{array}$ 

 $D_{\rm y} = 4$  kips

#### **Example 3.11 Solution (continued)**



• Check the equilibrium of member BD

C<sup>+</sup>  $\Sigma M_c = 0$   $4D_y - 4B_y = 0$   $B_y = D_y = 4$  kips Ans. →+  $\Sigma F_x = 0$   $D_x - B_x = 0$   $D_x = B_x$ <sup>+</sup>  $\Sigma F_y = 0$   $-B_y + C_y - D_y = 0$  $C_y = 8$  kips Ans.

## **Example 3.11 Solution (continued)**



Free body of member AB

**Unstable structure** 

• Check the equilibrium of member AB

 $\bigcirc^+ \Sigma M_A = 0$   $0 = -B_y(6)$  (inconsistent equation)

• Unstable structure



Figure 3.28 Examples of stable and unstable structures



Figure 3.28 Examples of stable and unstable structures (continued)





Figure 3.29 (continued)



Figure 3.29 (continued)



Stable and indeterminate

Unstable, reactions on *BCDE* equivalent to a parallel force system

Figure 3.29 (continued)

## §3.10 Comparison between Determinate and Indeterminate Structures





Figure 3.30 Alternative modes of transmitting load to supports

## §3.10 Comparison between Determinate and Indeterminate Structures



Photo 3.3 The collapse of a bridge composed of simply supported beams during the 1964 Nigata earthquake
## §3.10 Comparison between Determinate and Indeterminate Structures



Figure 3.31 Comparison of flexibility between a determinate and indeterminate structure

## §3.10 Comparison between Determinate and Indeterminate Structures



Column extends beyond support because girder is too long

Reactions produced by forcing the bottom of the column into the supports

Figure 3.32 Consequences of fabrication error

## §3.10 Comparison between Determinate and Indeterminate Structures



Support *B* settles, creating reactions



Moment curve produced by support settlement

Figure 3.33