CHAPTER 13

Moment Distribution

Outline

§13.1 Introduction

§13.2 Development of the Moment Distribution Method

§13.3 Summary of the Moment Distribution Method with No Joint Translation

§13.4 Analysis of Beams by Moment Distribution

§13.5 Modification of Member Stiffness

Outline (continued)

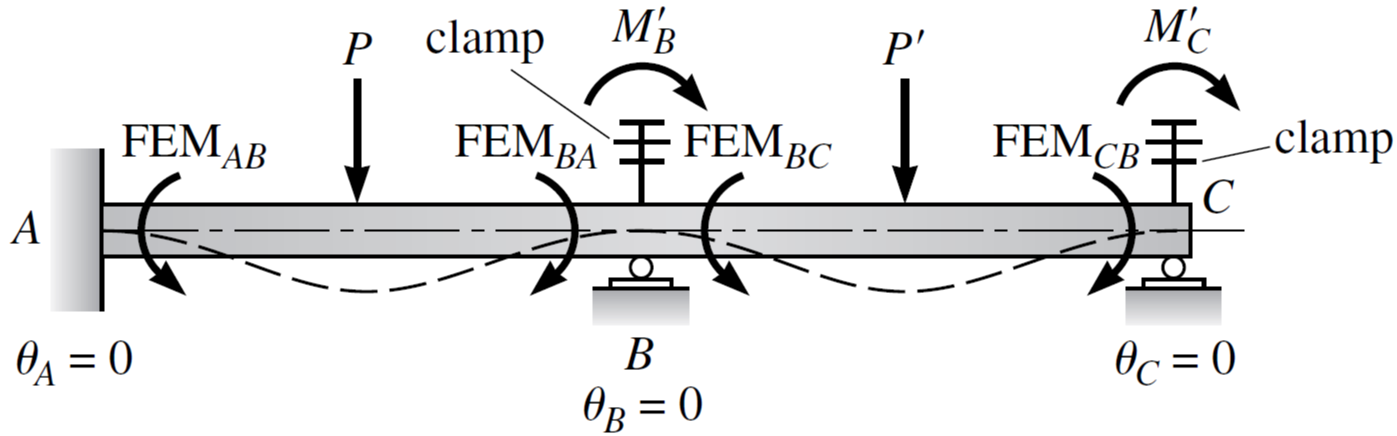
§13.6 Analysis of Frames That Are Free to Sidesway

§13.7 Analysis of an Unbraced Frame for General Loading

§13.8 Analysis of Multistory Frames

§13.9 Nonprismatic Members

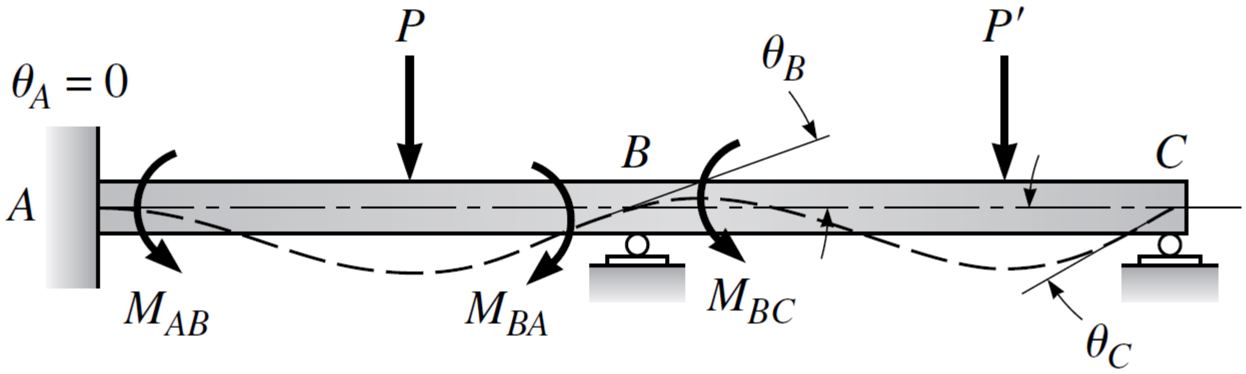
§13.1 Introduction



Temporary clamps added at joints *B* and *C* to produce a restrained structure consisting of two fixed end beams

Figure 13.1 Continuous beam analyzed by moment distribution

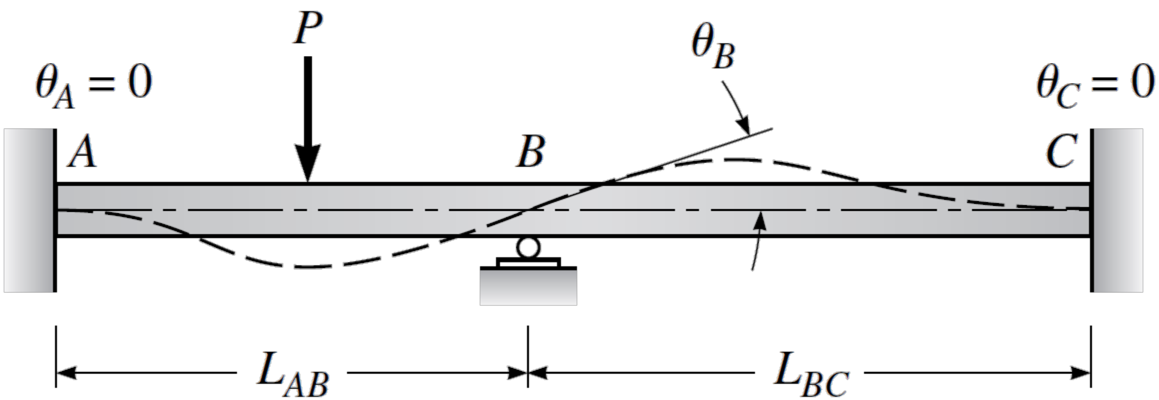
§13.1 Introduction



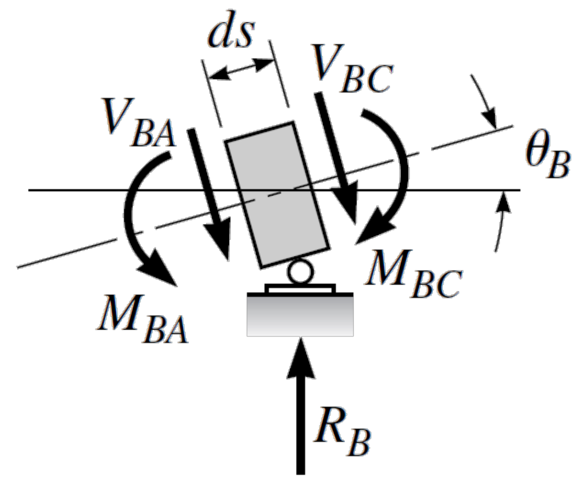
Clamps removed and beam deflected into its equilibrium position

Figure 13.1 Continuous beam analyzed by moment distribution (continued)

§13.2 Development of the Moment Distribution Method



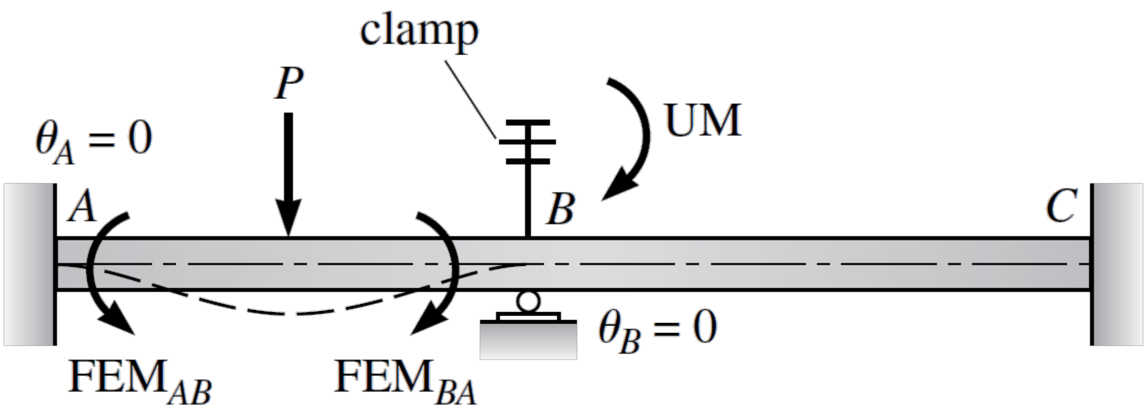
Loaded beam in deflected position



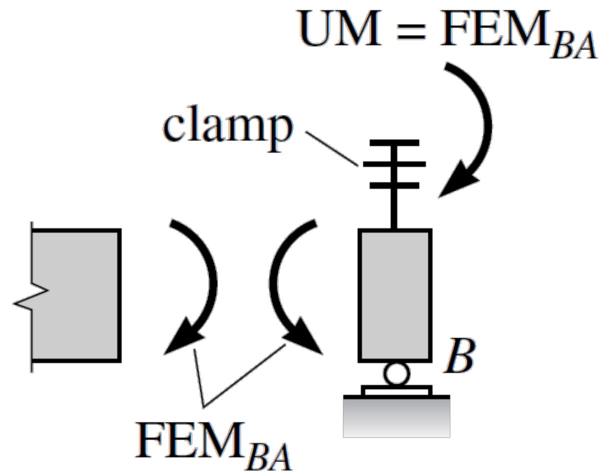
Free-body diagram of joint *B* in deflected position

Figure 13.2 Various stages in the analysis of a beam by moment distribution

§13.2 Development of the Moment Distribution Method



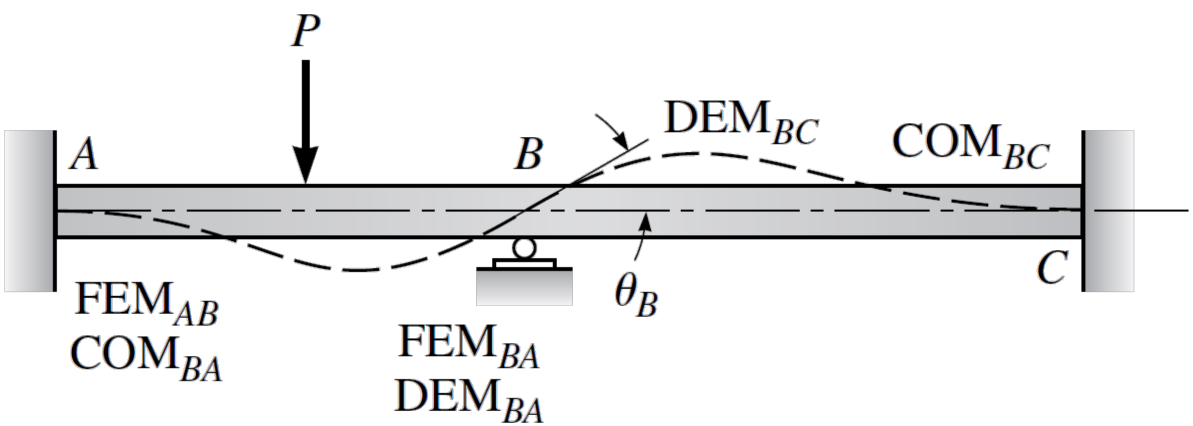
Fixed-end moments in restrained beam (joint *B* clamped)



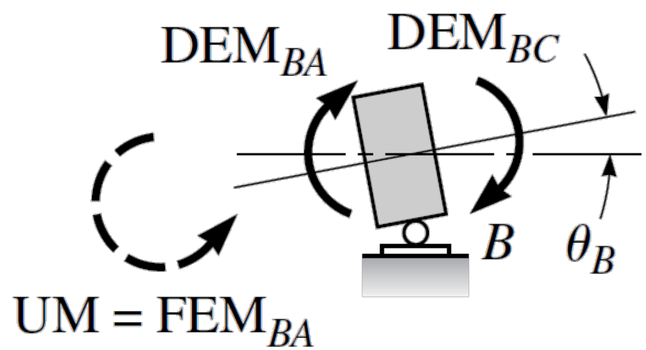
Free-body diagram of joint *B* before clamp removed

Figure 13.2 Various stages in the analysis of a beam by moment distribution (continued)

§13.2 Development of the Moment Distribution Method



Moments in beam after clamp removed

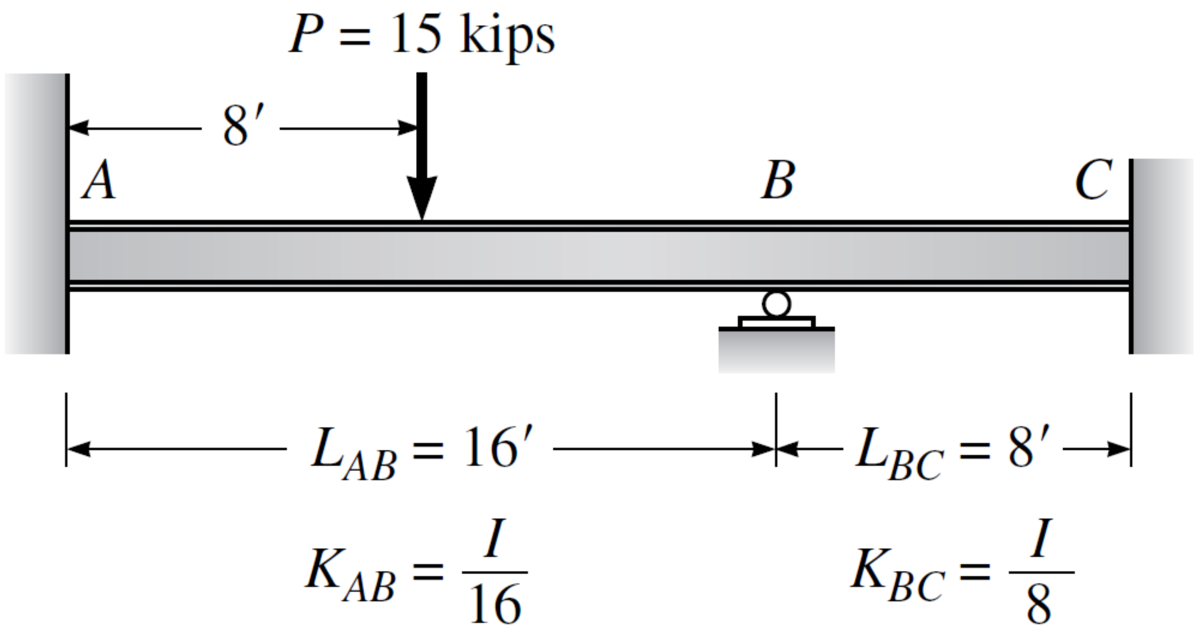


Distributed end moments (dems) produced by joint rotation θ_B to balance the unbalanced moment (UM)

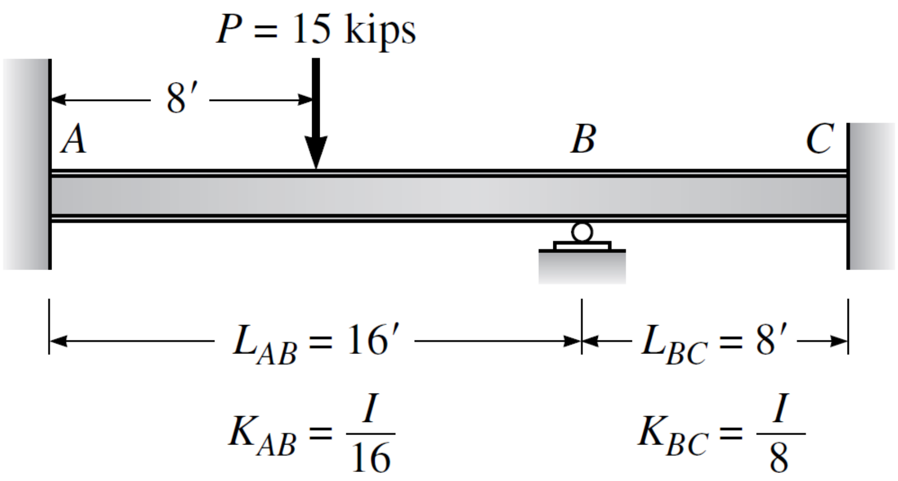
Figure 13.2 Various stages in the analysis of a beam by moment distribution (continued)

Example 13.1

Determine the member end moments in the continuous beam shown in Figure 13.3 by moment distribution. Note that *EI* of all members is constant.



Example 13.1 Solution



- Compute the stiffness *K* of each member connected to joint *B*.

$$K_{AB} = \frac{I}{L_{AB}} = \frac{I}{16} \qquad K_{BC} = \frac{I}{L_{BC}} = \frac{I}{8}$$

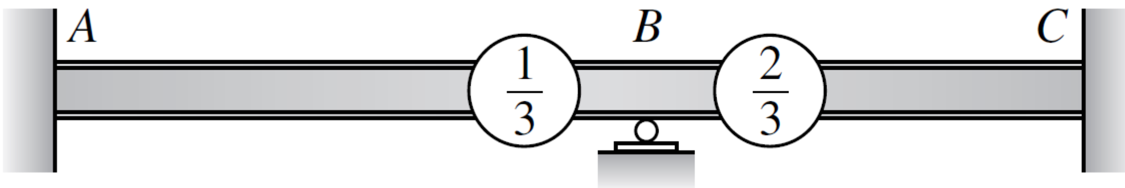
$$\Sigma K = K_{AB} + K_{BC} = \frac{I}{16} + \frac{I}{8} = \frac{3I}{16}$$

- Evaluate the distribution factors at joint *B* and record on Figure 13.4.

$$\mathrm{DF}_{BA} = \frac{K_{AB}}{\Sigma K} = \frac{I/16}{3I/16} = \frac{1}{3}$$

$$\mathrm{DF}_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{I/8}{3I/16} = \frac{2}{3}$$

Example 13.1 Solution (continued)



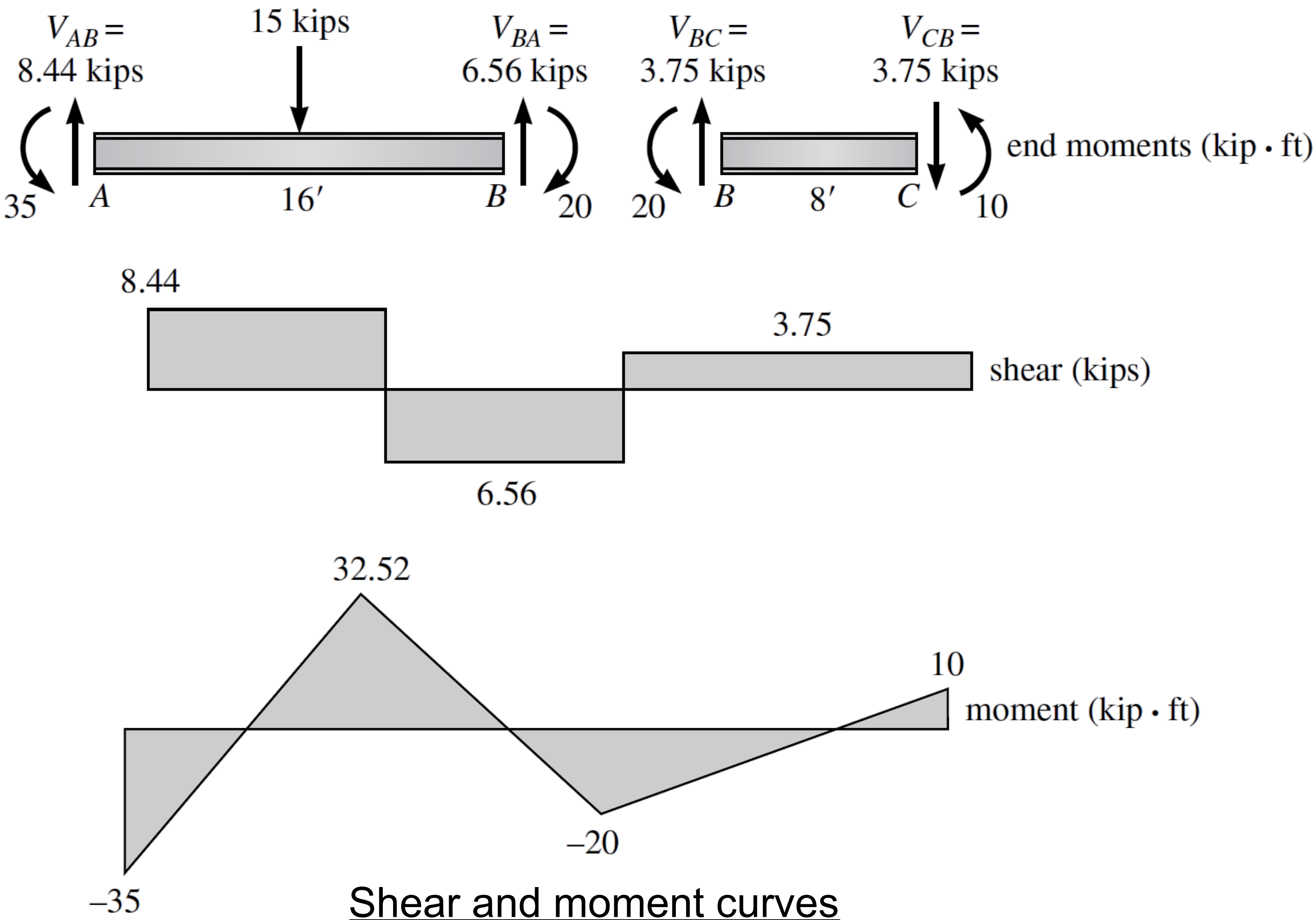
–30	+30			FEM (joint *B* clamped)
1/2	–10	–20	1/2	DEM (clamp removed)
–5			–10	COM
–35	+20	–20	–10	final moments (kip • ft)

- Compute the fixed-end moments at each end of member *AB* (see Figure 12.5) and record on Figure 13.4.

$$\mathrm{FEM}_{AB} = \frac{-PL}{8} = \frac{-15(16)}{8} = -30 \text{ kip} \cdot \text{ft}$$

$$\mathrm{FEM}_{BA} = \frac{+PL}{8} = \frac{15(16)}{8} = +30 \text{ kip} \cdot \text{ft}$$

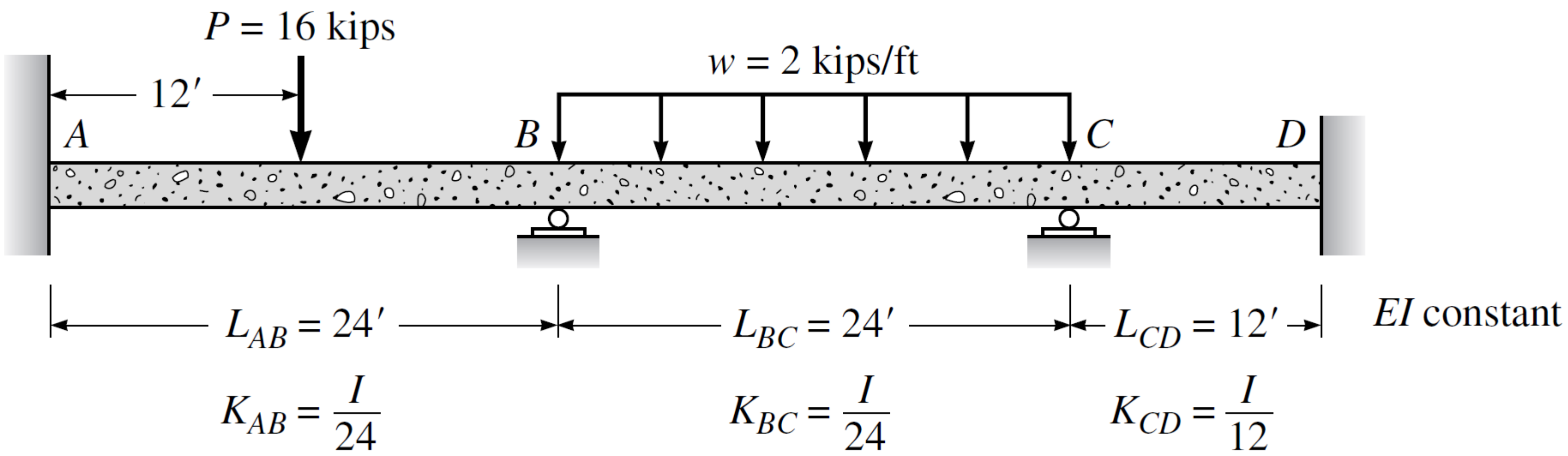
Example 13.1 Solution (continued)



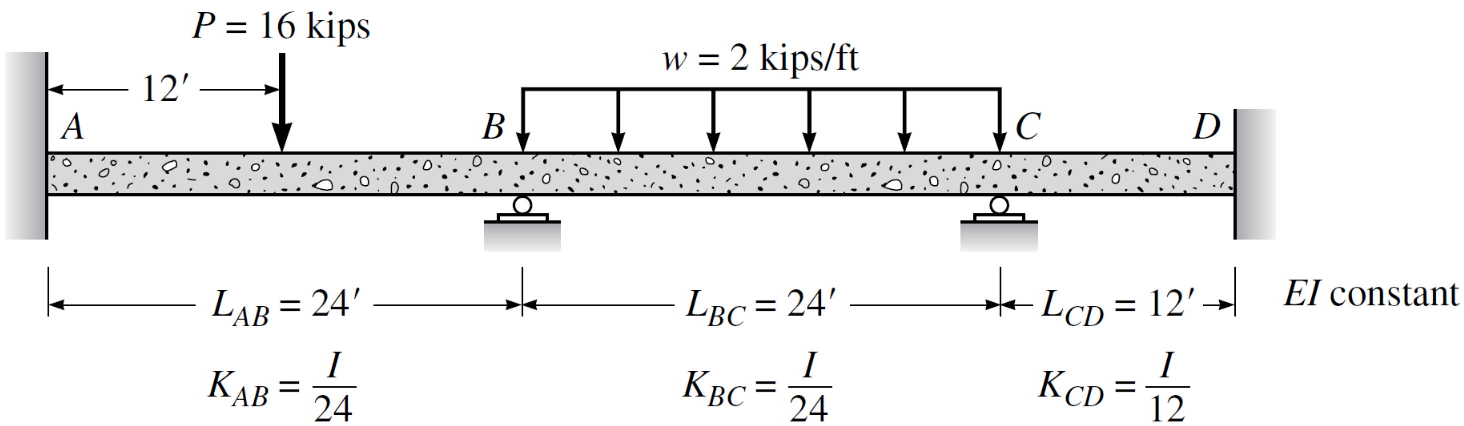
Shear and moment curves

Example 13.2

Analyze the continuous beam in Figure 13.6 by moment distribution. The *EI* of all members is constant.



Example 13.2 Solution



- Compute distribution factors at joints *B* and *C* and record on Figure 13.7. At joint *B*:

$$K_{AB} = \frac{I}{24} \qquad K_{BC} = \frac{I}{24} \qquad \Sigma K = K_{AB} + K_{BC} = \frac{2I}{24}$$

$$\mathrm{DF}_{BA} = \frac{K_{AB}}{\Sigma K} = \frac{I/24}{2I/24} = 0.5$$

$$\mathrm{DF}_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{I/24}{2I/24} = 0.5$$

Example 13.2 Solution (continued)

- At joint *C*:

$$K_{BC} = \frac{I}{24} \qquad K_{CD} = \frac{I}{12} \qquad \Sigma K = K_{BC} + K_{CD} = \frac{3I}{24}$$

$$\mathrm{DF}_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{I/24}{3I/24} = \frac{1}{3} \qquad \mathrm{DF}_{CD} = \frac{K_{CD}}{\Sigma K} = \frac{I/12}{3I/24} = \frac{2}{3}$$

- Fixed-end moments (see Figure 12.5):

$$\mathrm{FEM}_{AB} = \frac{-PL}{8} = \frac{-16(24)}{8} = -48 \text{ kip} \cdot \text{ft}$$

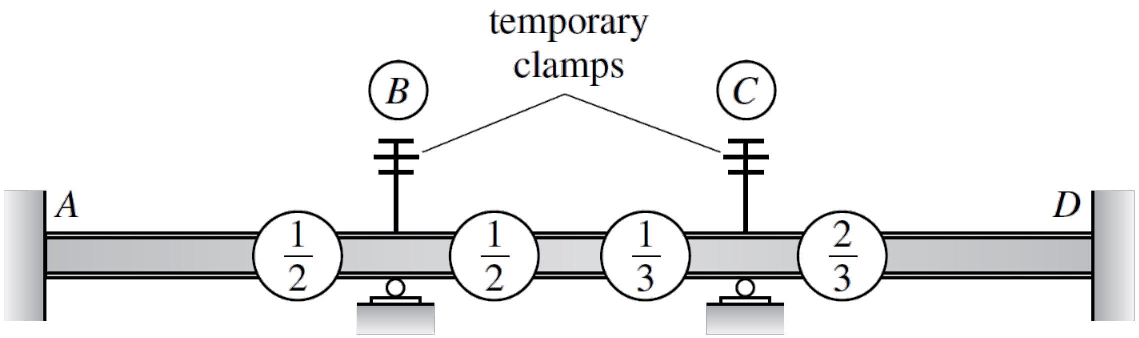
$$\mathrm{FEM}_{BA} = \frac{+PL}{8} = +48 \text{ kip} \cdot \text{ft}$$

$$\mathrm{FEM}_{BC} = \frac{-wL^2}{12} = \frac{-2(24)^2}{12} = -96 \text{ kip} \cdot \text{ft}$$

$$\mathrm{FEM}_{CB} = \frac{+wL^2}{12} = +96 \text{ kip} \cdot \text{ft}$$

Since span *CD* is not loaded,
$FEM_{CD} = FEM_{DC} = 0$

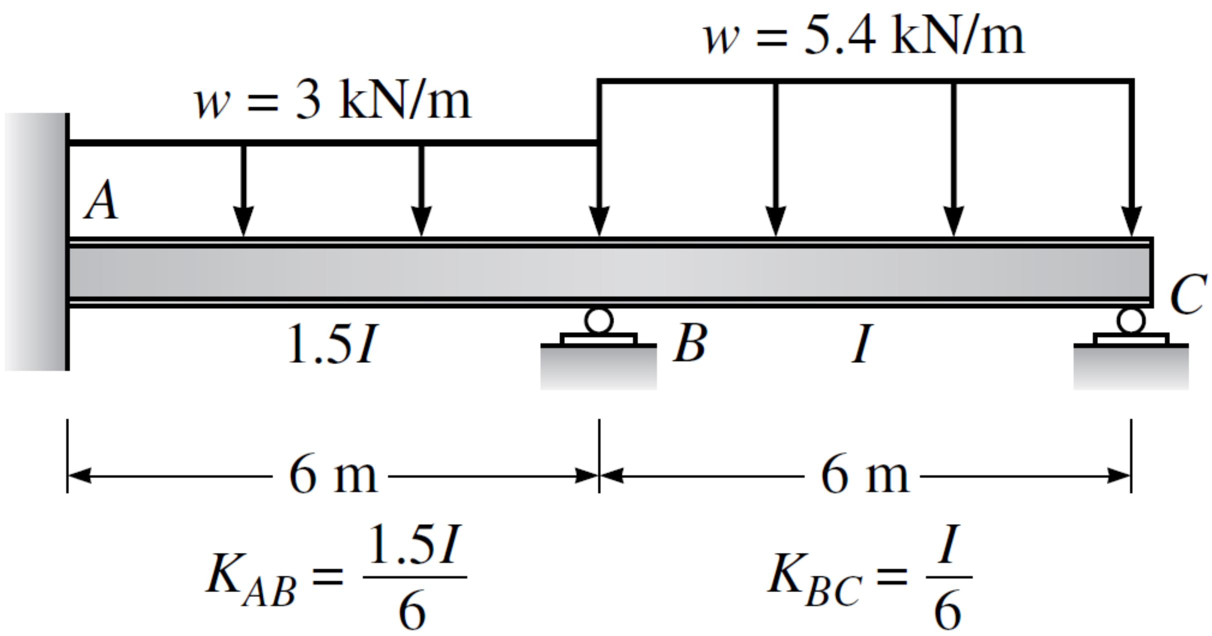
Example 13.2 Solution (continued)



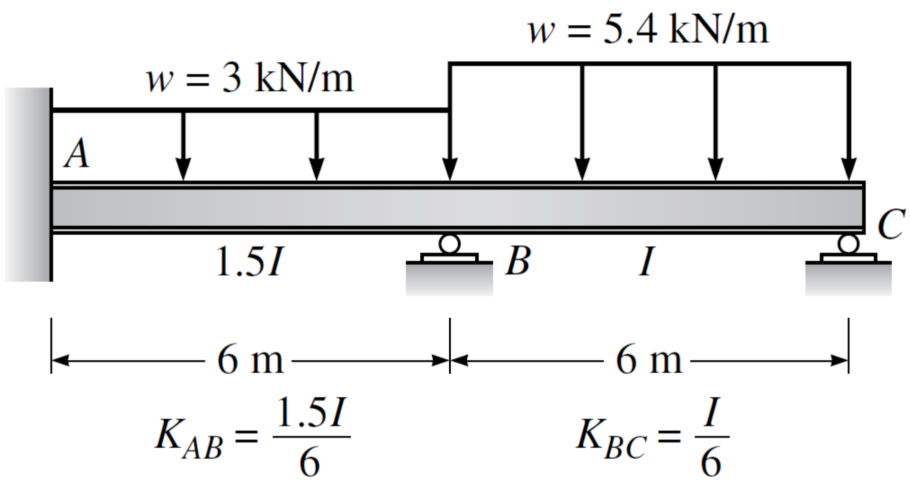
–48	+48	–96	+96			FEM (all joints locked)
	+24	+24				DEM (joint *B* unlocked)
+12			+12			COM
			–36	–72		DEM (joint *C* unlocked)
		–18			–36	COM
	+9	+9				DEM (joint *B* unlocked)
+4.5			+4.5			COM
			–1.5	–3		DEM (joint *C* unlocked)
		–0.76			–1.5	COM
	+0.38	+0.38				DEM (joint *B* unlocked)
+0.2			+0.2			COM
			–0.07	–0.13		DEM (joint *C* unlocked)
–31.3	+81.38	–81.38	+75.13	–75.13	–37.5	final moments (kip • ft)

Example 13.3

Analyze the beam in Figure 13.8 by moment distribution, and draw the shear and moment curves.



Example 13.3 Solution



$$K_{AB} = \frac{1.5I}{6} \qquad K_{BC} = \frac{I}{6}$$

then $$\Sigma K = K_{AB} + K_{BC} = \frac{2.5I}{6}$$

- Compute distribution factors at joint *B*:

$$\text{DF}_{AB} = \frac{K_{AB}}{\Sigma K} = \frac{1.5I/6}{2.5I/6} = 0.6 \qquad \text{DF}_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{I/6}{2.5I/6} = 0.4$$

$$\text{FEM}_{AB} = -\frac{wL^2}{12} = -\frac{3(6)^2}{12} = -9\ \text{kN}\cdot\text{m}$$

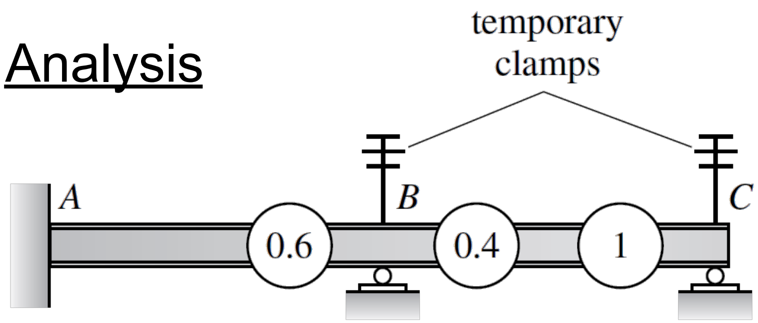
$$\text{FEM}_{BA} = -\text{FEM}_{AB} = +9\ \text{kN}\cdot\text{m}$$

$$\text{FEM}_{BC} = -\frac{wL^2}{12} = -\frac{5.4(6)^2}{12} = -16.2\ \text{kN}\cdot\text{m}$$

$$\text{FEM}_{CB} = -\text{FEM}_{BC} = +16.2\ \text{kN}\cdot\text{m}$$

Example 13.3 Solution (continued)

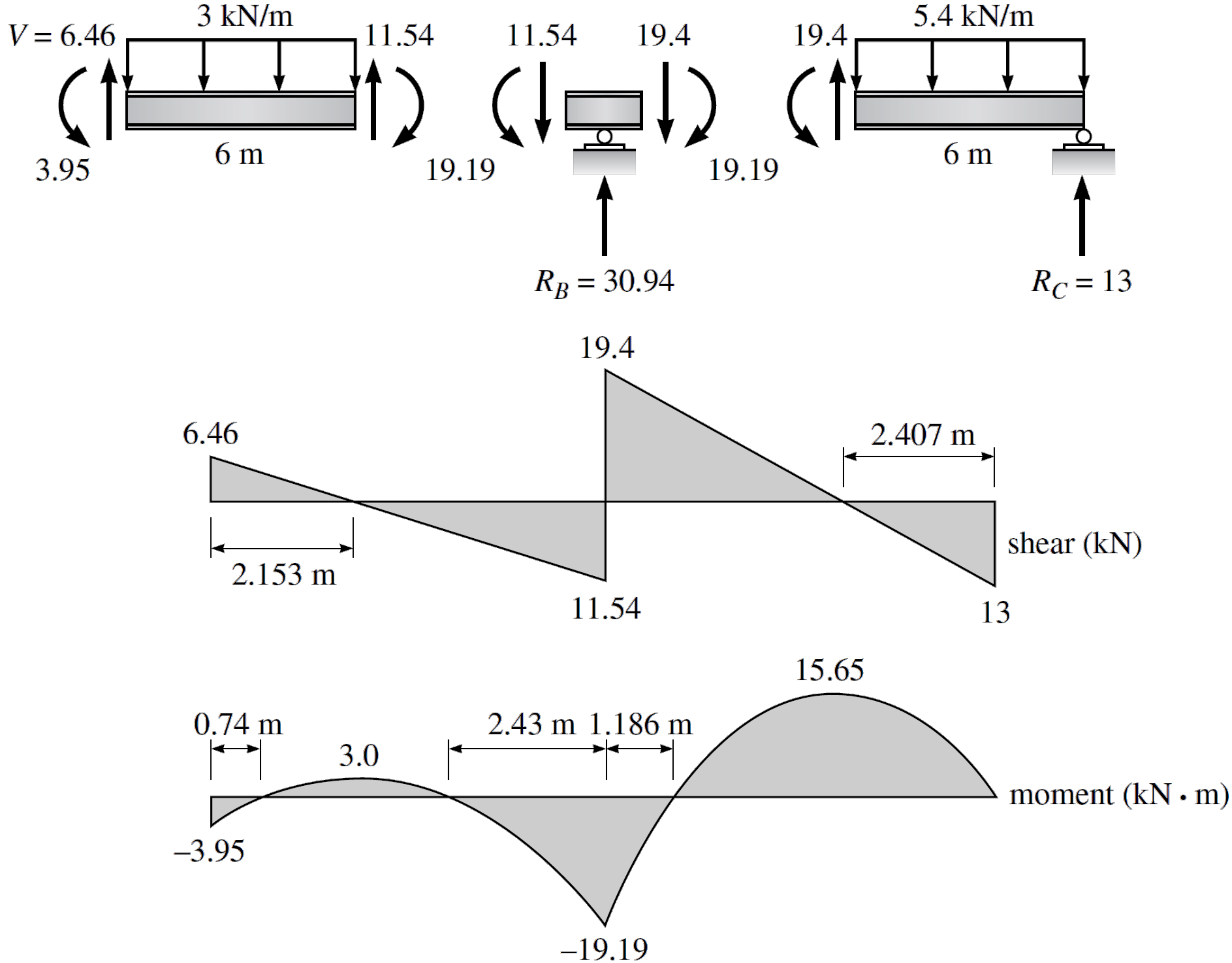
Analysis



–9	+9	–16.2	+16.2	FEM
			–16.2	
		–8.1		
	+9.18	+6.12		
+4.59			+3.06	
			–3.06	
		–1.53		
	+0.92	+0.61		
+0.46			+0.3	
			–0.3	
		–0.15		
	+0.09	+0.06		
–3.95	+19.19	–19.19	0	

final end
moments

Example 13.3 Solution (continued)



Shear and Moment Curves

§13.5 Modification of Member Stiffness

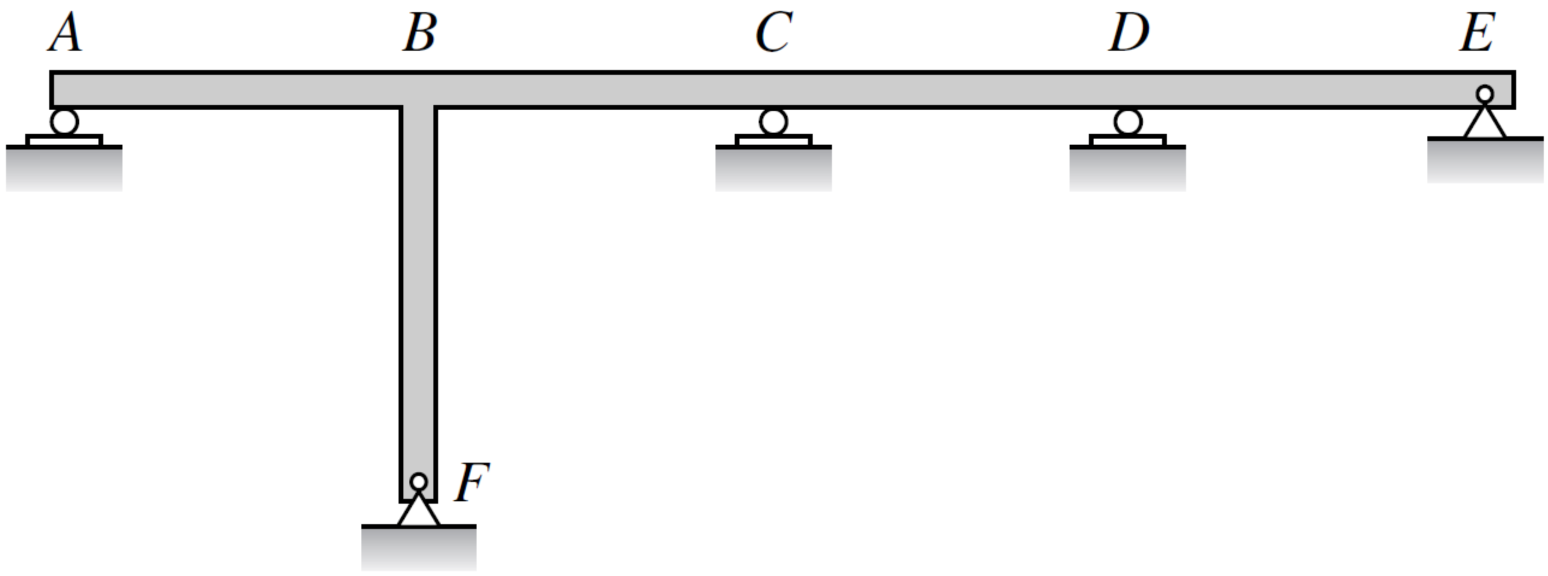
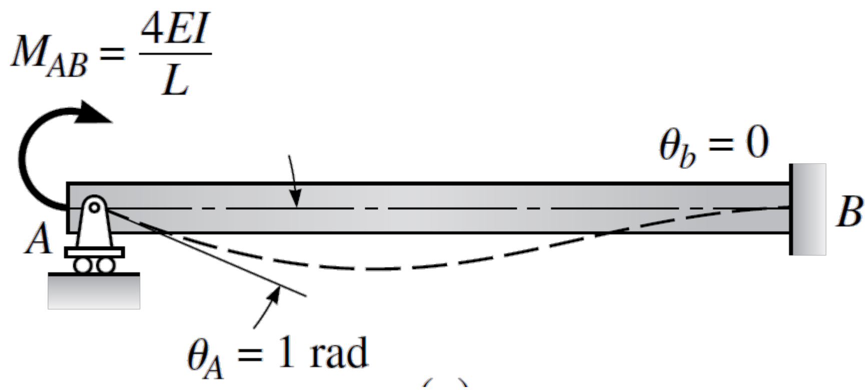
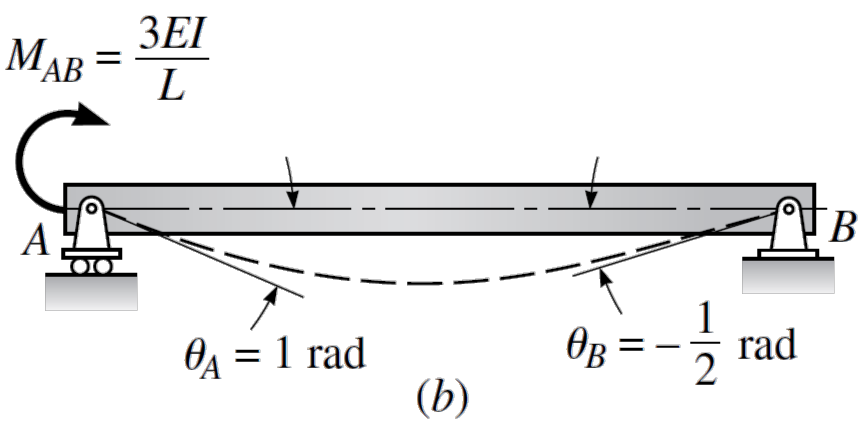


Figure 13.11

§13.5 Modification of Member Stiffness



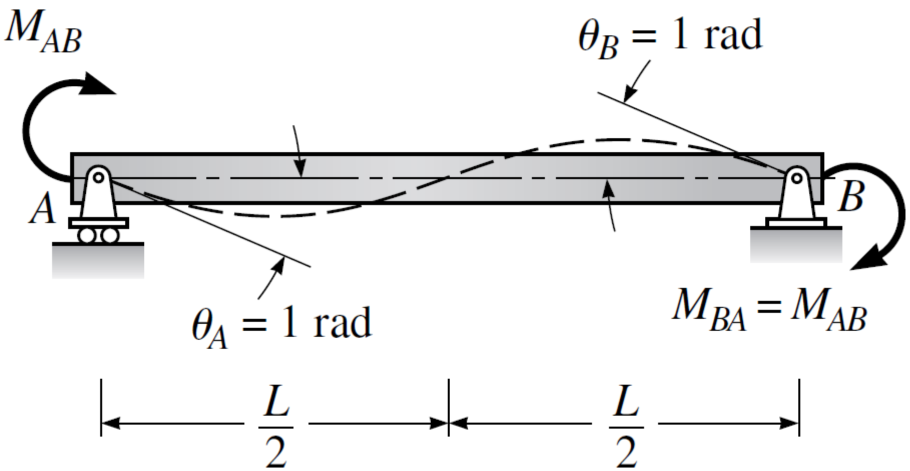
Beam with far end fixed



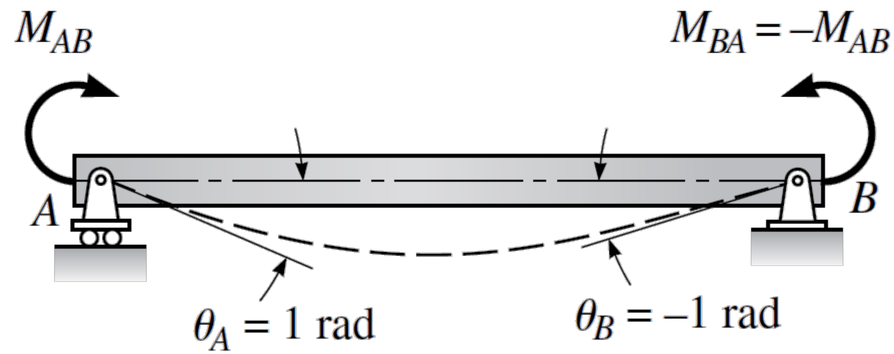
Beam with far end unrestrained against rotation

Figure 13.12

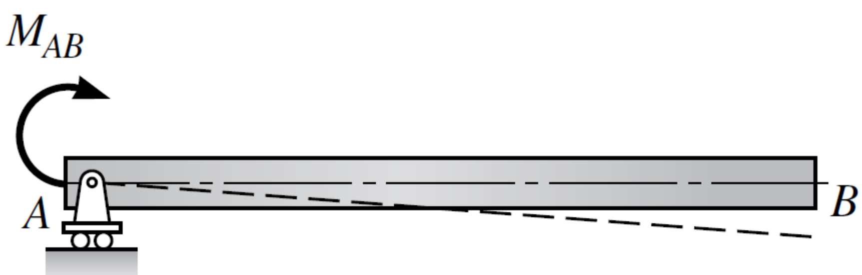
§13.5 Modification of Member Stiffness



Equal values of clockwise moment at each end



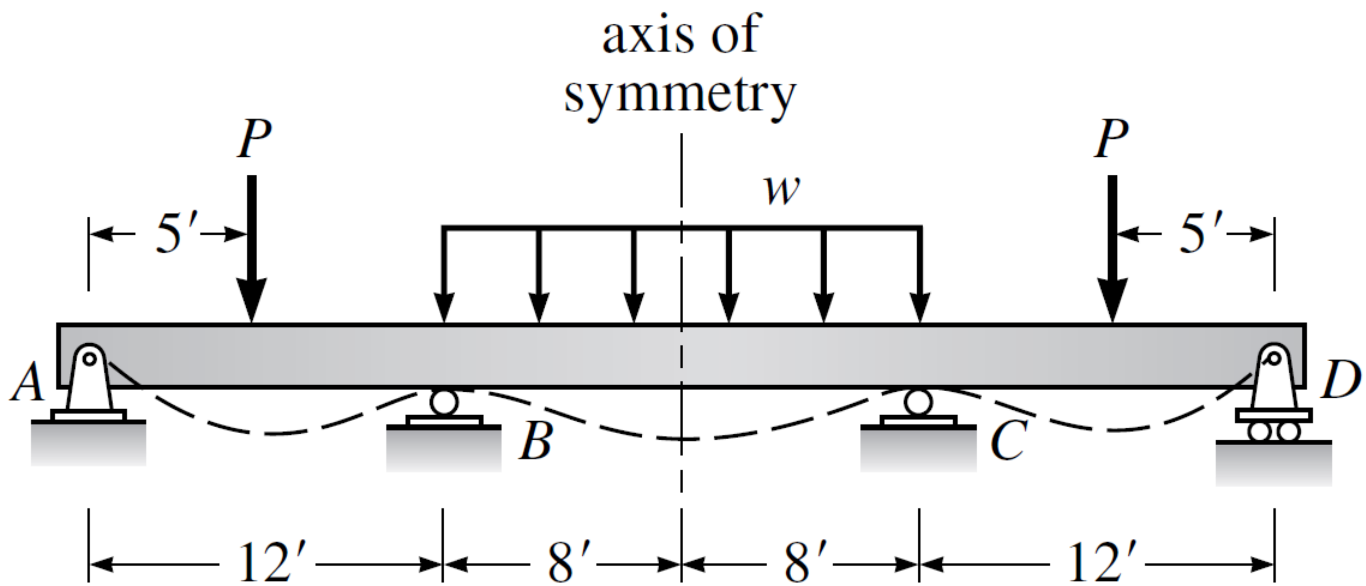
Single curvature bending by equal values of end moments



Cantilever loaded at supported end

Figure 13.12 (continued)

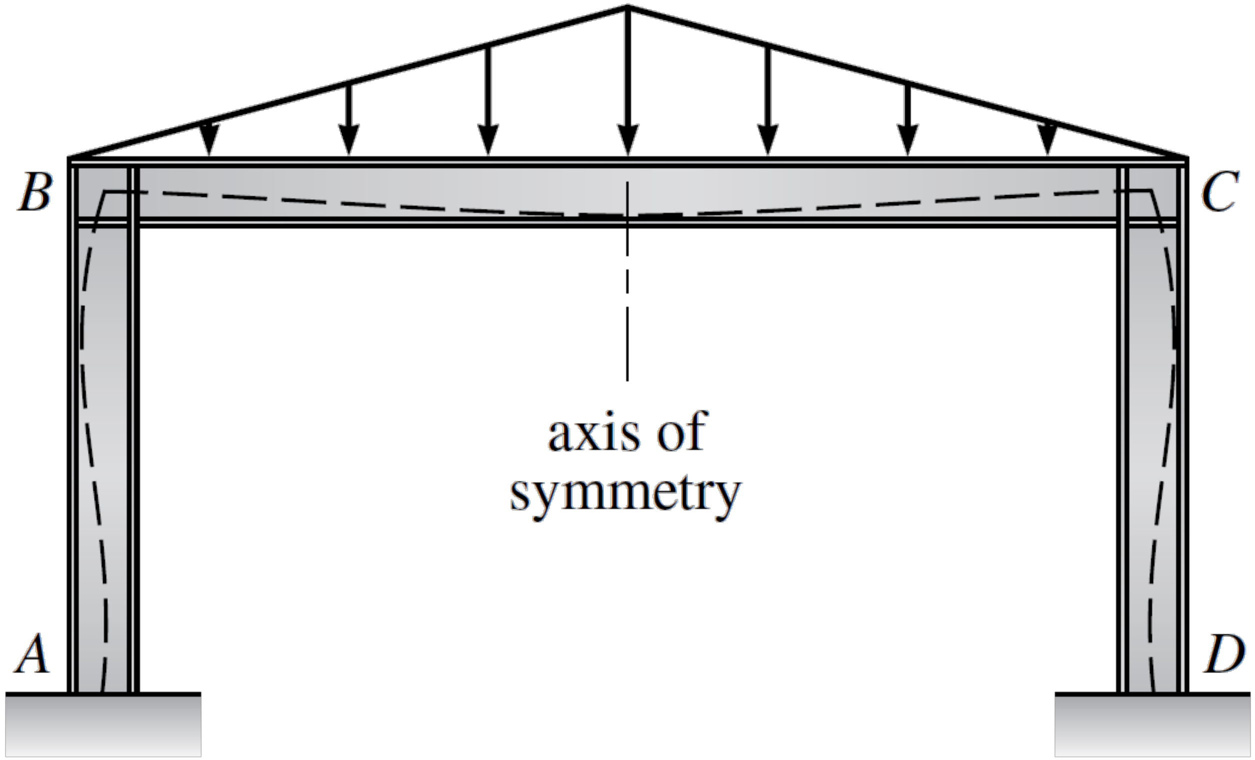
§13.5 Modification of Member Stiffness



Beam *BC* of the continuous beam

Figure 13.13 Examples of symmetric structures, symmetrically loaded, that contain members whose end moments are equal in magnitude and produce single-curvature bending

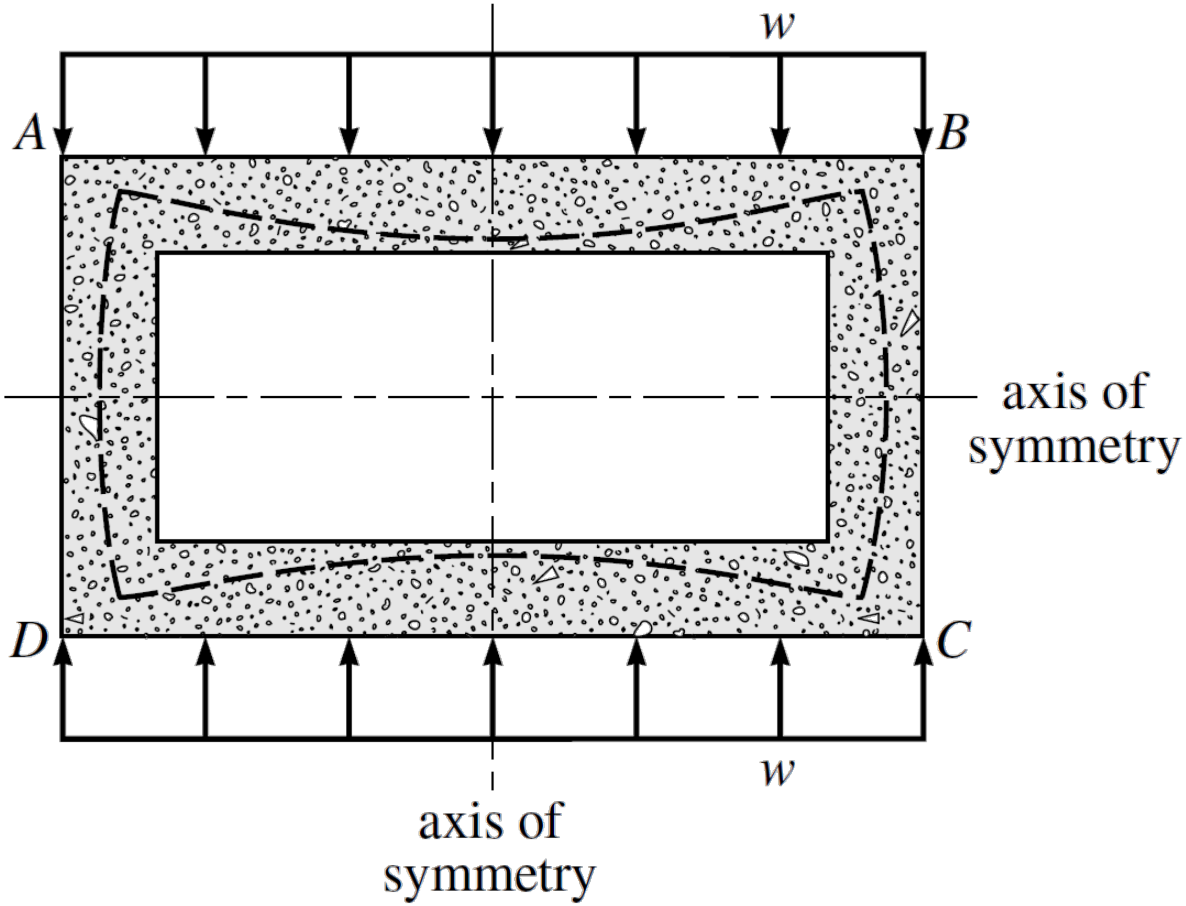
§13.5 Modification of Member Stiffness



Beam *BC* of the rigid frame

Figure 13.13 Examples of symmetric structures, symmetrically loaded, that contain members whose end moments are equal in magnitude and produce single-curvature bending (continued)

§13.5 Modification of Member Stiffness

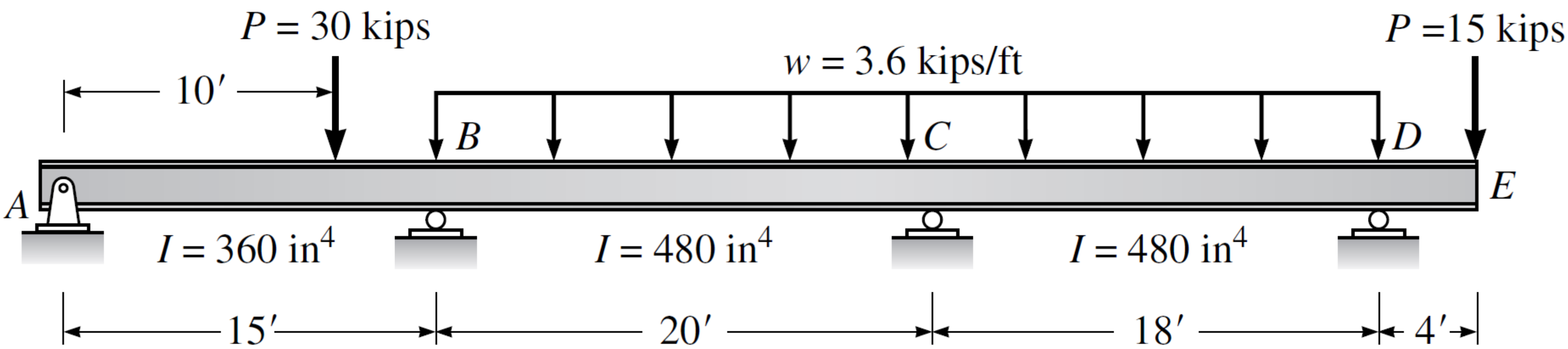


All four members of the box beam

Figure 13.13 Examples of symmetric structures, symmetrically loaded, that contain members whose end moments are equal in magnitude and produce single-curvature bending (continued)

Example 13.4

Analyze the beam in Figure 13.14*a* by moment distribution, using modified flexural stiffnesses for members *AB* and *CD*. Given: *EI* is constant.



Example 13.4 Solution

$$K_{AB} = \frac{3}{4}\left(\frac{360}{15}\right) = 18 \qquad K_{BC} = \frac{480}{20} = 24$$

$$K_{CD} = \frac{3}{4}\left(\frac{480}{18}\right) = 20 \qquad K_{DE} = 0$$

- Compute the distribution factors.

Joint *B*: $$\Sigma K = K_{AB} + K_{BC} = 18 + 24 = 42$$

$$DF_{BA} = \frac{K_{AB}}{\Sigma K} = \frac{18}{42} = 0.43 \qquad DF_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{24}{42} = 0.57$$

Joint *C*: $$\Sigma K = K_{BC} + K_{CD} = 24 + 20 = 44$$

$$\mathrm{DF}_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{24}{44} = 0.55 \qquad \mathrm{DF}_{CD} = \frac{K_{CD}}{\Sigma K} = \frac{20}{44} = 0.45$$

Example 13.4 Solution (continued)

- Compute the fixed-end moments (see Figure 12.5).

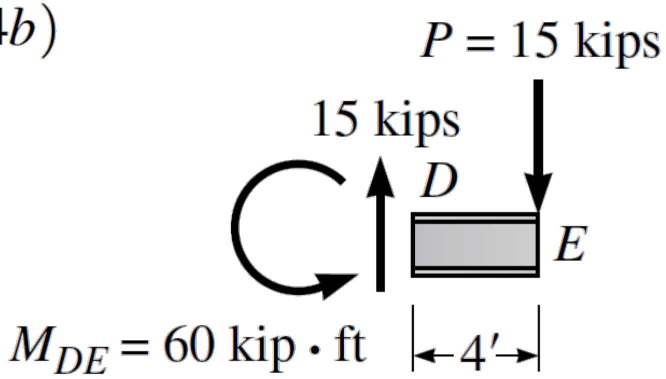
$$\text{FEM}_{AB} = -\frac{Pab^2}{L^2} = -\frac{30(10)(5^2)}{15^2} = -33.3 \text{ kip}\cdot\text{ft} \qquad \text{FEM}_{BA} = \frac{Pba^2}{L^2} = \frac{30(5)(10^2)}{15^2} = +66.7 \text{ kip}\cdot\text{ft}$$

$$\text{FEM}_{BC} = -\frac{wL^2}{12} = -120 \text{ kip}\cdot\text{ft} \qquad \text{FEM}_{CB} = -\text{FEM}_{BC} = 120 \text{ kip}\cdot\text{ft}$$

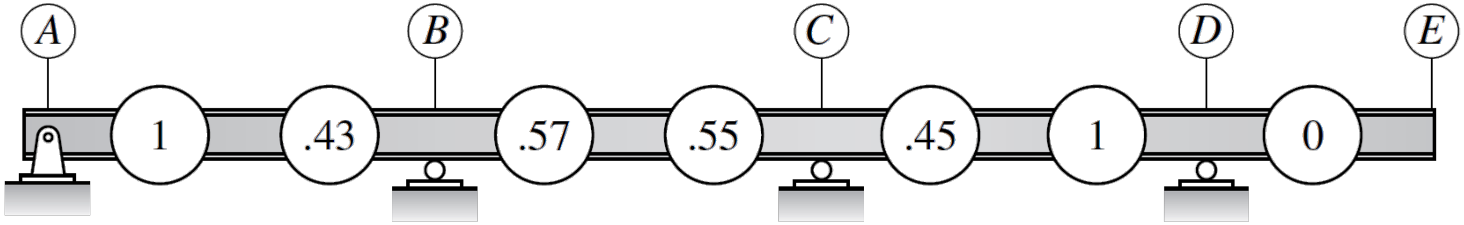
$$\text{FEM}_{CD} = -\frac{wL^2}{12} = -97.2 \text{ kip}\cdot\text{ft} \qquad \text{FEM}_{DC} = -\text{FEM}_{CD} = 97.2 \text{ kip}\cdot\text{ft}$$

$$\text{FEM}_{DE} = -60 \text{ kip}\cdot\text{ft} \quad (\text{see Figure 13.14}b)$$

The minus sign is required because the moment acts counterclockwise on the end of the member.



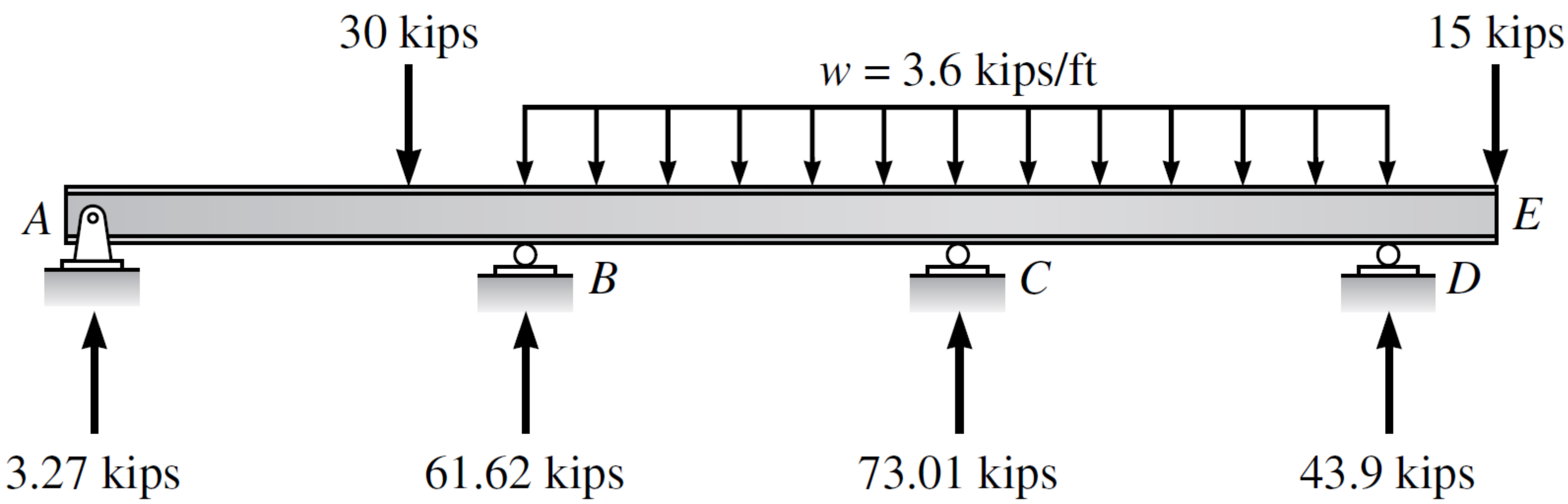
Example 13.4 Solution (continued)



–33.3	+66.7	–120	+120	–97.2	+97.2	–60	FEM
+33.3					–37.2		
	+16.7			–18.6			
			–2.3	–1.9			
		–1.1					
	+16.2	+21.5					
			+10.8				
			–5.9	–4.9			
		–3.0					
	+1.3	+1.7					
			+0.8				
			–0.4	–0.4			
0	+100.9	–100.9	+123	–123	+60	–60	final moments (kip • ft)

Moment distribution details

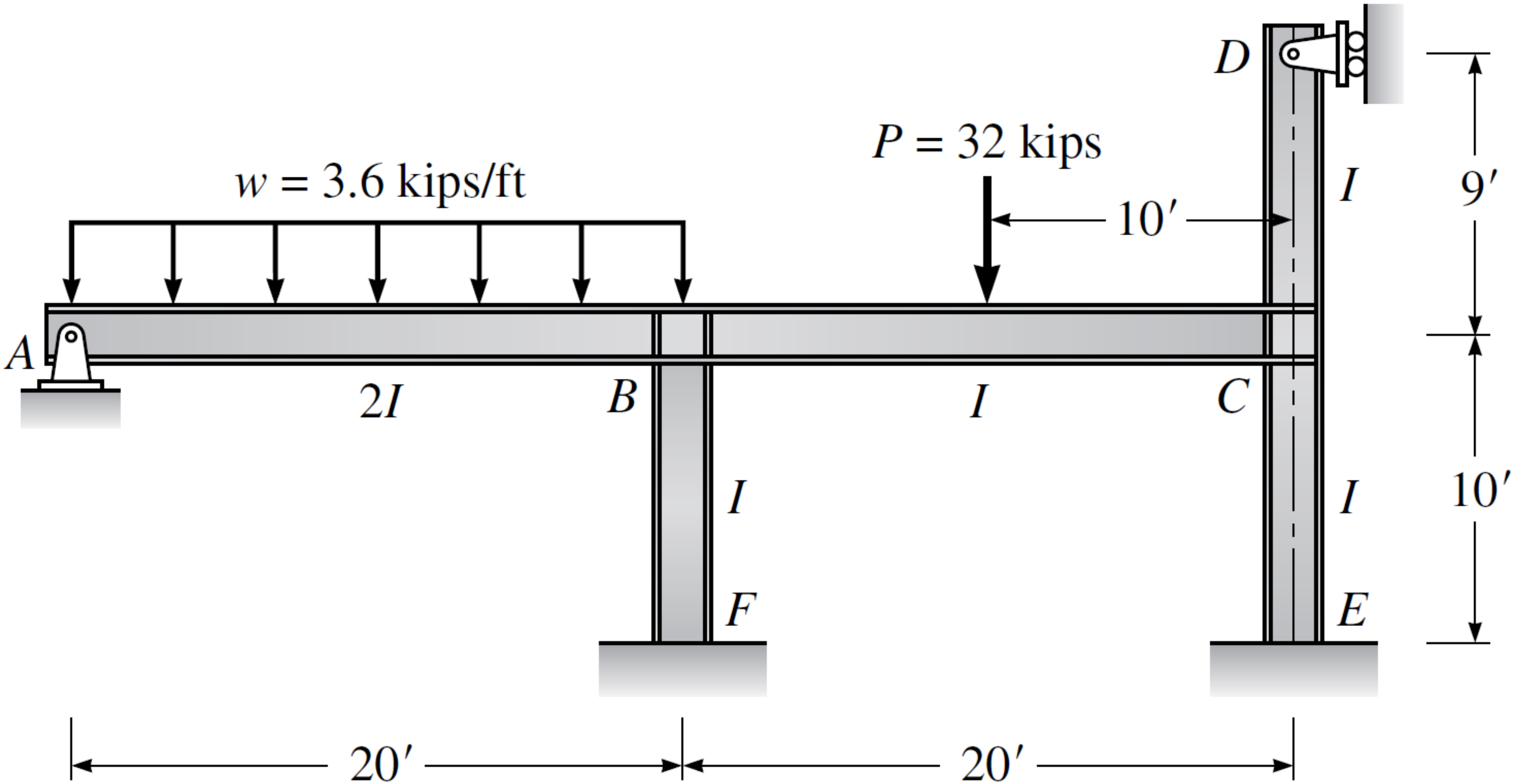
Example 13.4 Solution (continued)



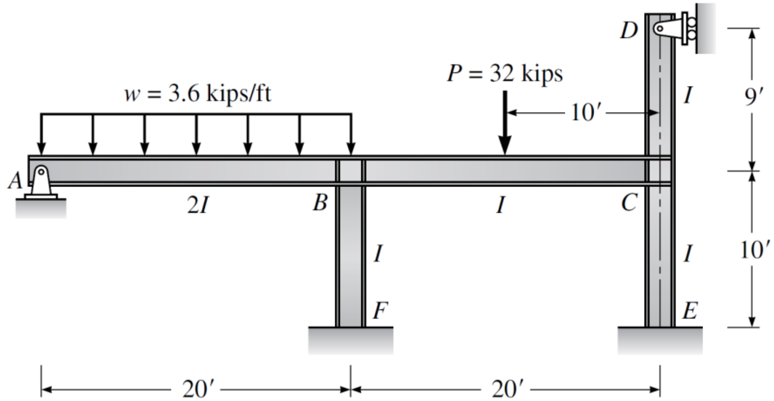
Reactions

Example 13.5

Analyze the frame in Figure 13.16 by moment distribution.



Example 13.5 Solution



- Compute the distribution factors at joint *B*.

$$K_{AB} = \frac{3}{4}\left(\frac{2I}{20}\right) = \frac{3I}{40} \qquad K_{BC} = \frac{I}{20} \qquad K_{BF} = \frac{I}{10} \qquad \Sigma K = \frac{9I}{40}$$

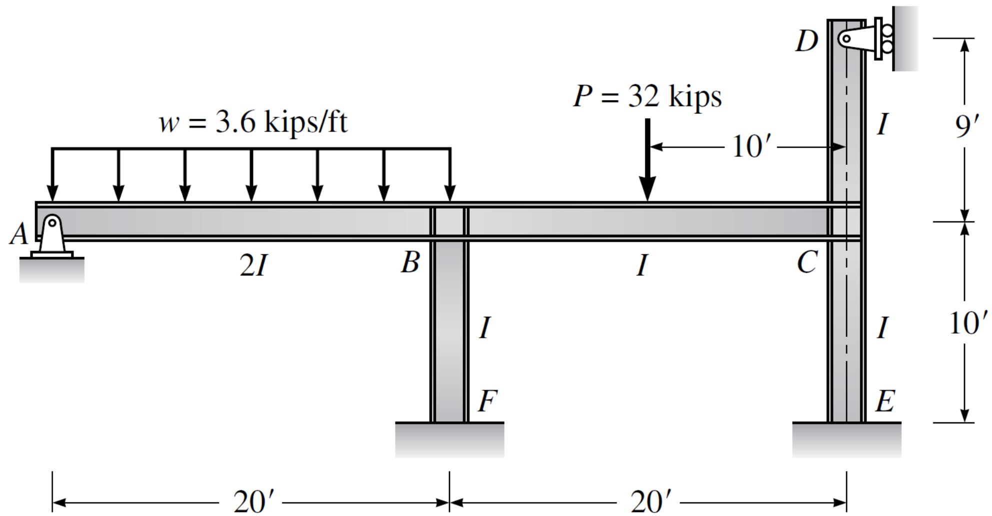
$$\mathrm{DF}_{BA} = \frac{K_{AB}}{\Sigma K} = 0.33 \qquad \mathrm{DF}_{BC} = \frac{K_{BC}}{\Sigma K} = 0.22 \qquad \mathrm{DF}_{BF} = \frac{K_{BF}}{\Sigma K} = 0.45$$

- Compute the distribution factors at joint *C*.

$$K_{CB} = \frac{I}{20} \qquad K_{CD} = \frac{3}{4}\left(\frac{I}{9}\right) \qquad K_{CE} = \frac{I}{10} \qquad \Sigma K = \frac{14I}{60}$$

$$\mathrm{DF}_{CB} = 0.21 \qquad \mathrm{DF}_{CD} = 0.36 \qquad \mathrm{DF}_{CE} = 0.43$$

Example 13.5 Solution (continued)



- Compute the fixed-end moments in spans *AB* and *BC* (see Figure12.5).

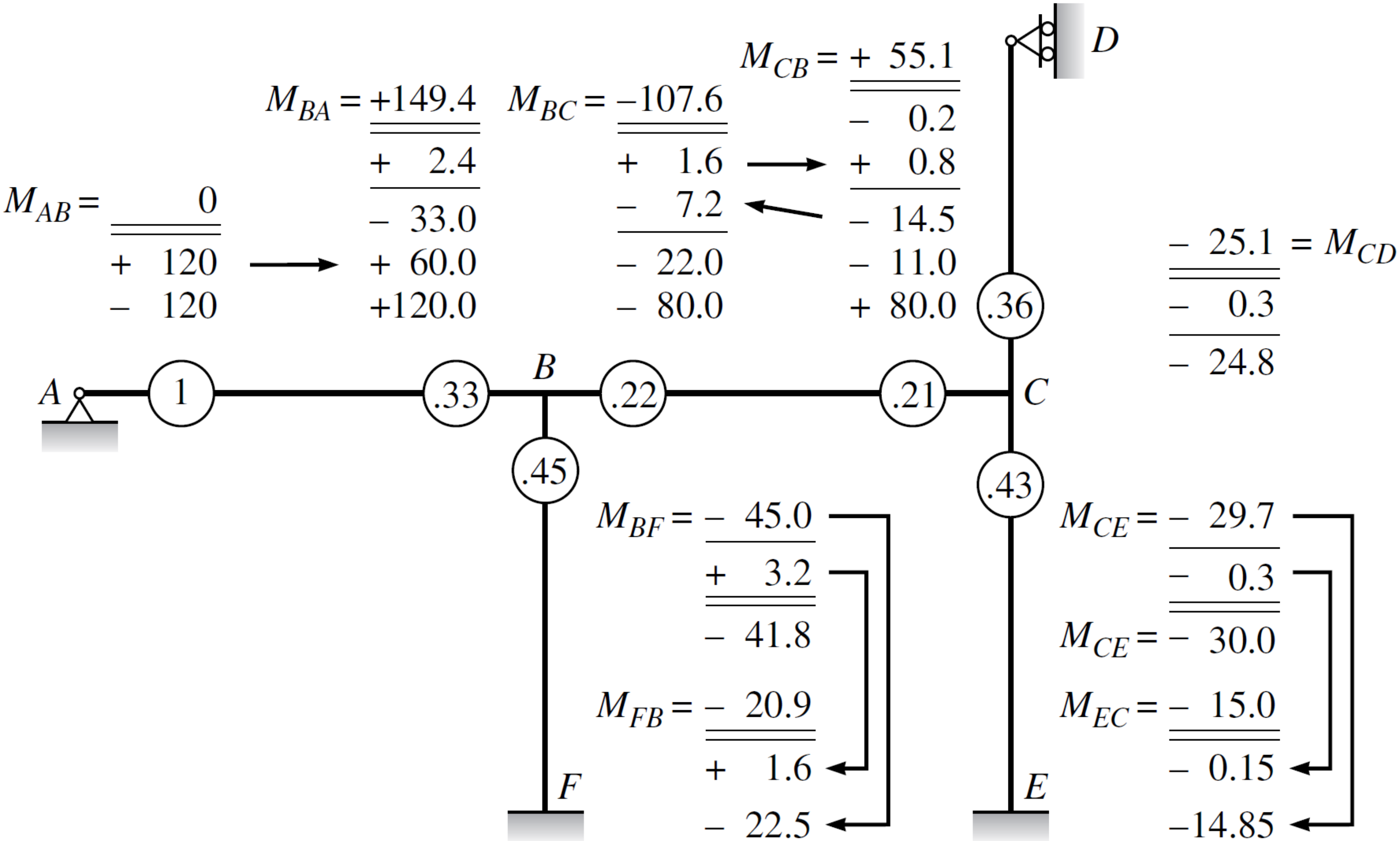
$$\mathrm{FEM}_{AB} = \frac{wL^2}{12} = \frac{-3.6(20)^2}{12} = -120 \text{ kip} \cdot \text{ft}$$

$$\mathrm{FEM}_{BA} = -\mathrm{FEM}_{AB} = +120 \text{ kip} \cdot \text{ft}$$

$$\mathrm{FEM}_{BC} = \frac{-PL}{8} = \frac{-32(20)}{8} = -80 \text{ kip} \cdot \text{ft}$$

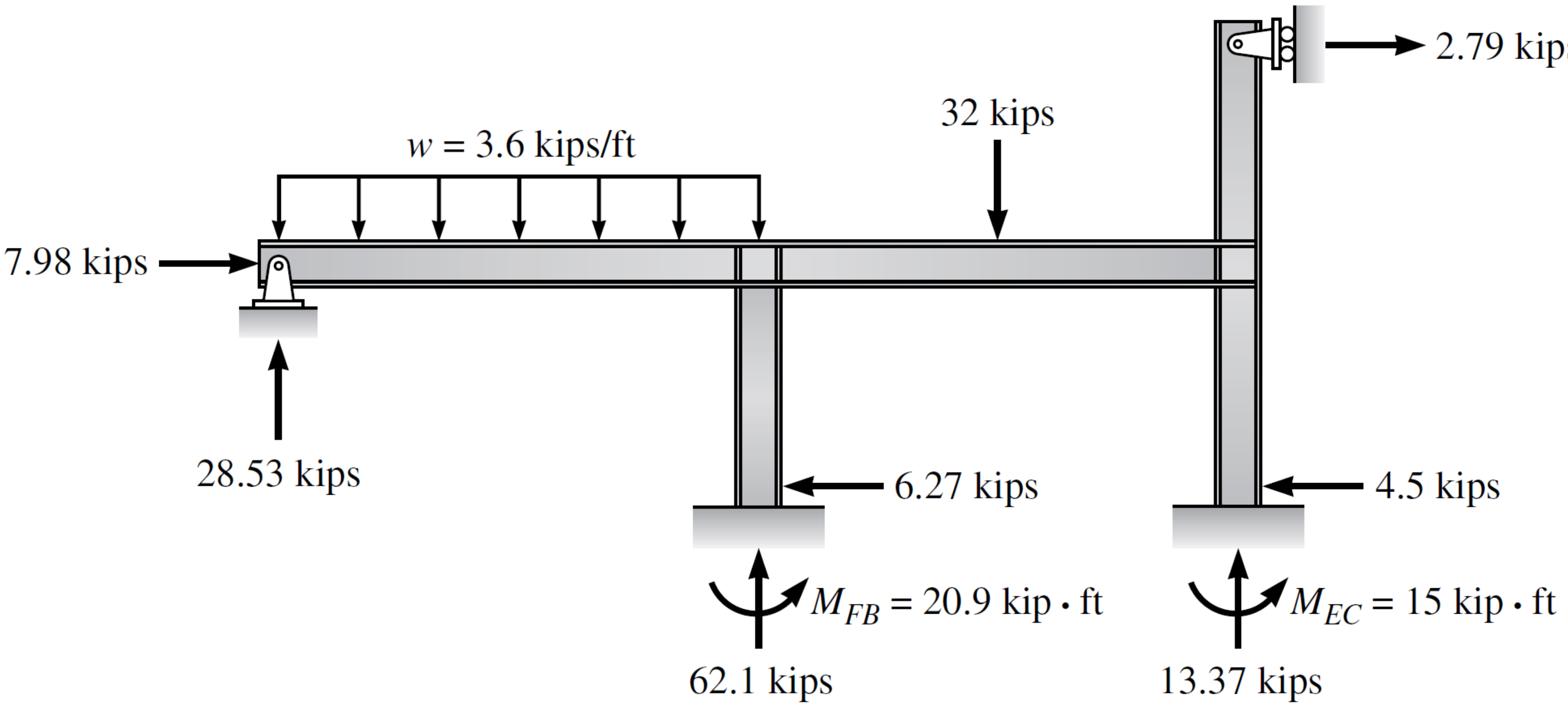
$$\mathrm{FEM}_{CB} = -\mathrm{FEM}_{BC} = +80 \text{ kip} \cdot \text{ft}$$

Example 13.5 Solution (continued)



Analysis by moment distribution

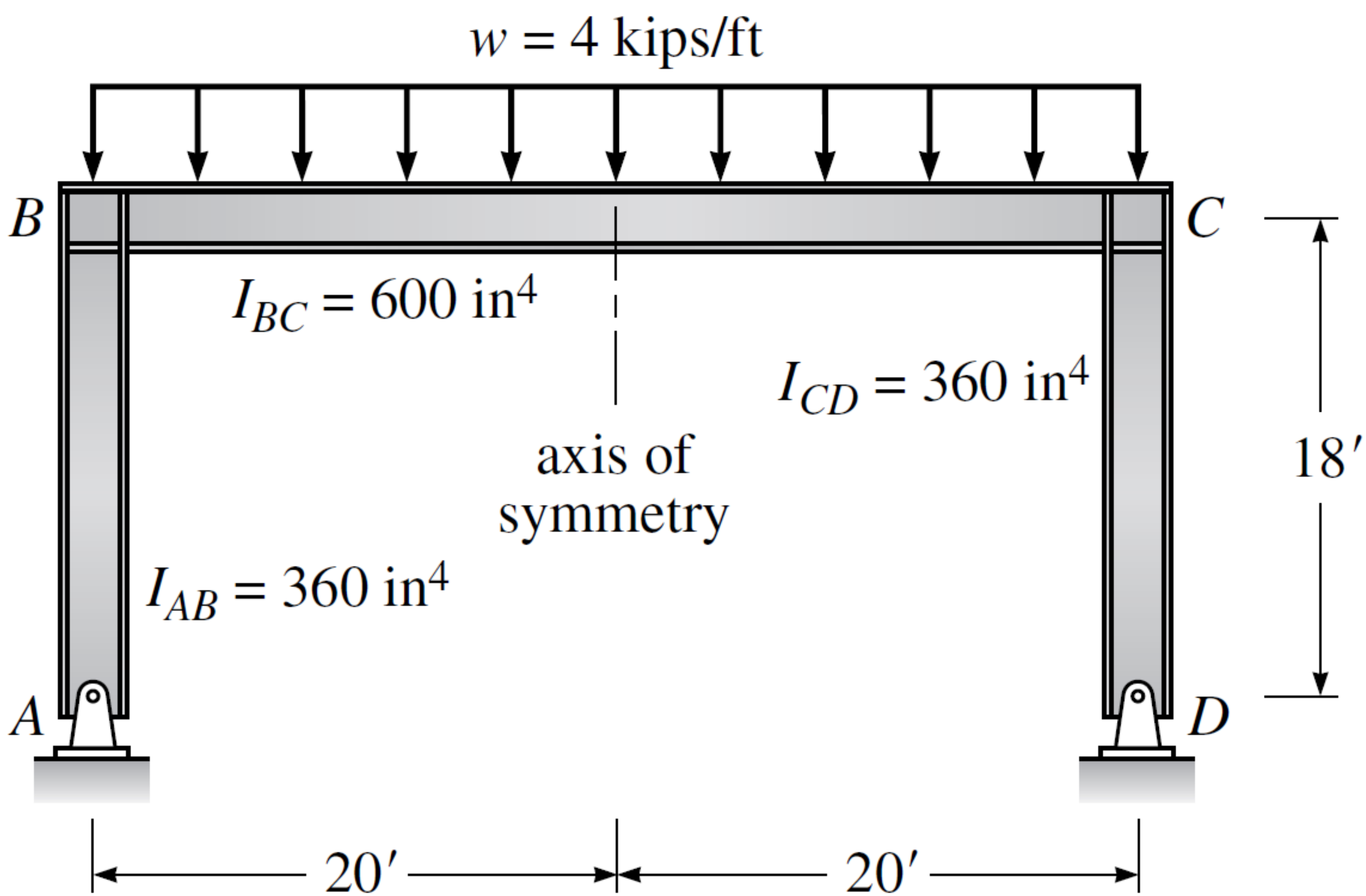
Example 13.5 Solution (continued)



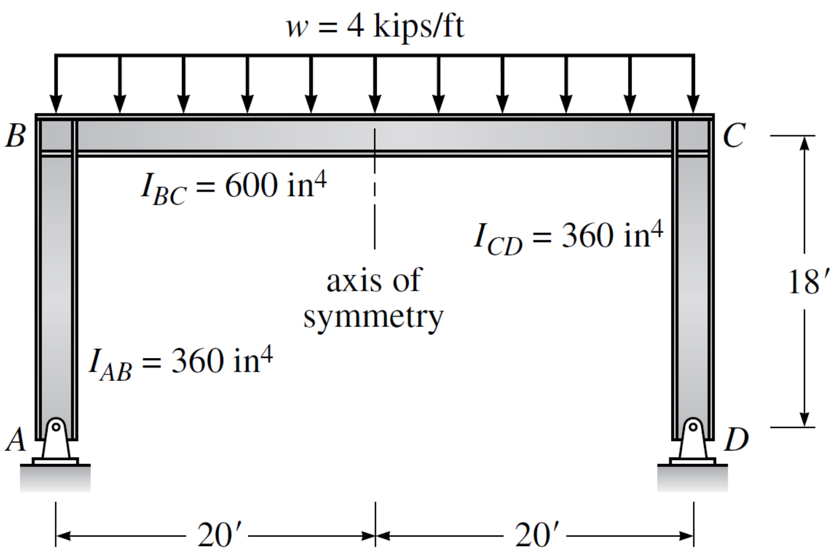
Reactions computed from free bodies of members

Example 13.6

Analyze the frame in Figure 13.18*a* by moment distribution, modifying the stiffness of the columns and girder by the factors discussed in Section 13.5 for a symmetric structure, symmetrically loaded.



Example 13.6 Solution



- **STEP 1** Modify the stiffness of the columns by for a pin support at points *A* and *D*.

$$K_{AB} = K_{CD} = \frac{3}{4}\frac{I}{L} = \frac{3}{4}\quad\frac{360}{18} = 15$$

- Modify the stiffness of girder *BC* by ½ (joints *B* and *C* will be unclamped simultaneously and no carryover moments are distributed).

$$K_{BC} = \frac{1}{2}\frac{I}{L} = \frac{1}{2}\quad\frac{600}{40} = 7.5$$

Example 13.6 Solution (continued)

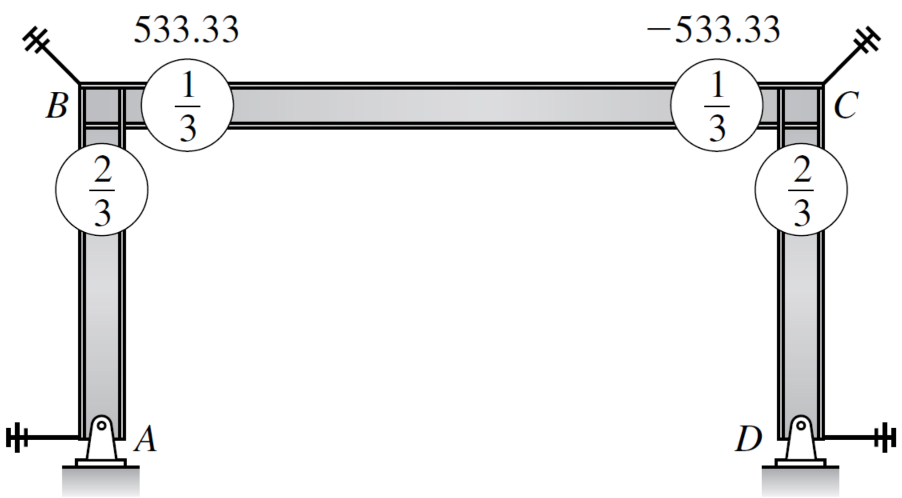
- **STEP 2** Compute the distribution factors at joints *B* and *C*.

$$\mathrm{DF}_{BA} = \mathrm{DF}_{CD} = \frac{K_{AB}}{\Sigma K_s'} = \frac{15}{15 + 7.5} = \frac{2}{3}$$

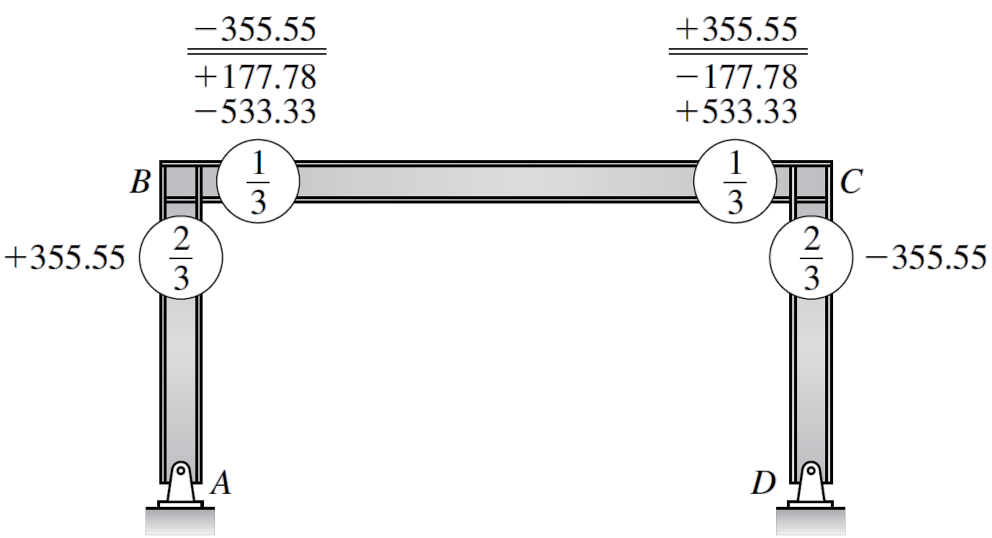
$$\mathrm{DF}_{BC} = \mathrm{DF}_{CB} = \frac{K_{BC}}{\Sigma K_s'} = \frac{7.5}{15 + 7.5} = \frac{1}{3}$$

$$\mathrm{FEM}_{BC} = \mathrm{FEM}_{CB} = \frac{WL^2}{12} = \frac{4(40)^2}{12} = \pm 533.33 \text{ kip} \cdot \text{ft}$$

Example 13.6 Solution (continued)

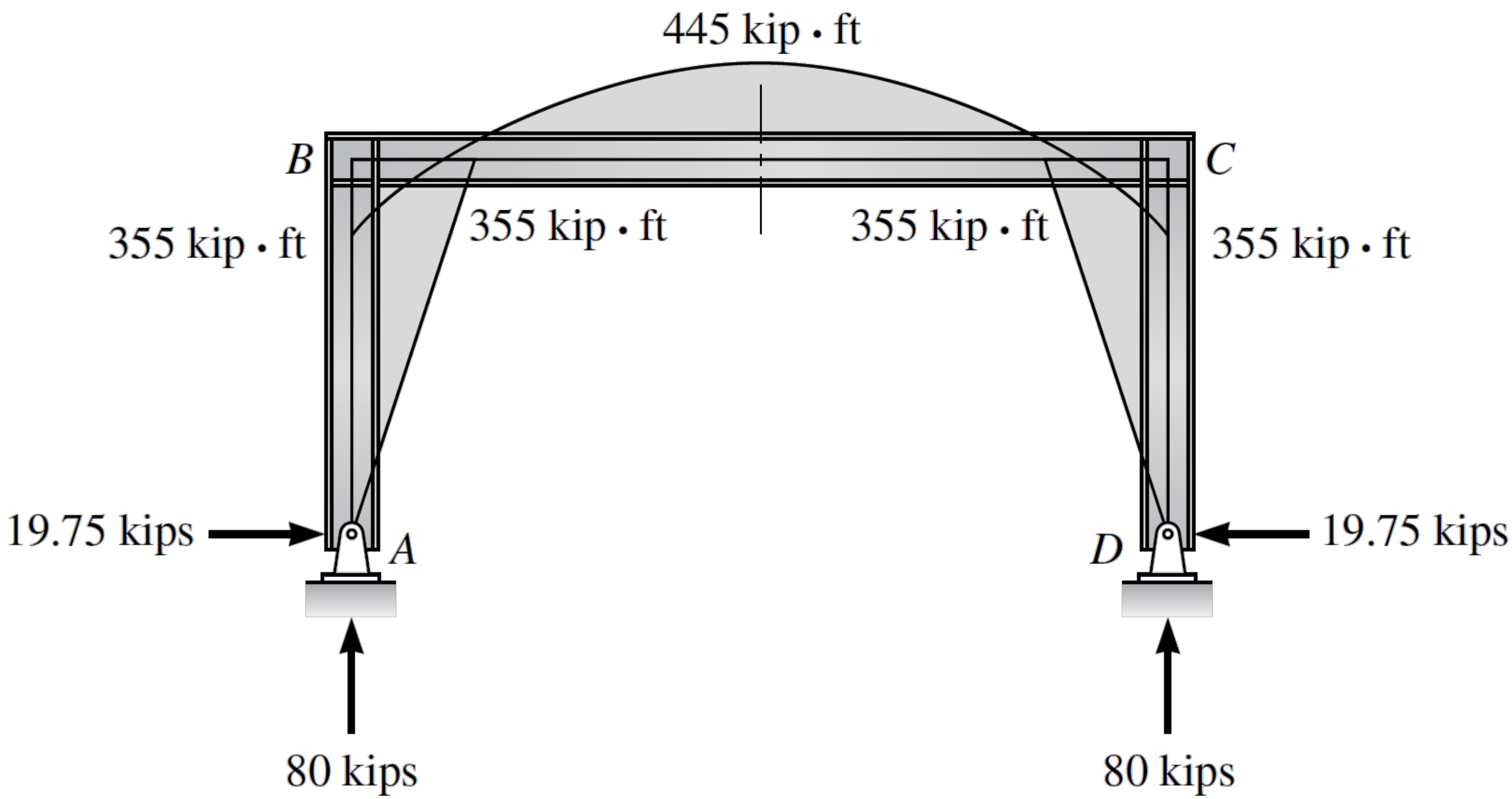


- **STEP 3** *(a)* Clamp all joints and apply the uniform load to girder *BC*. *(b)* Remove clamps at supports *A* and *D*. The stiffness of each column may be reduced by a factor of ¾ .



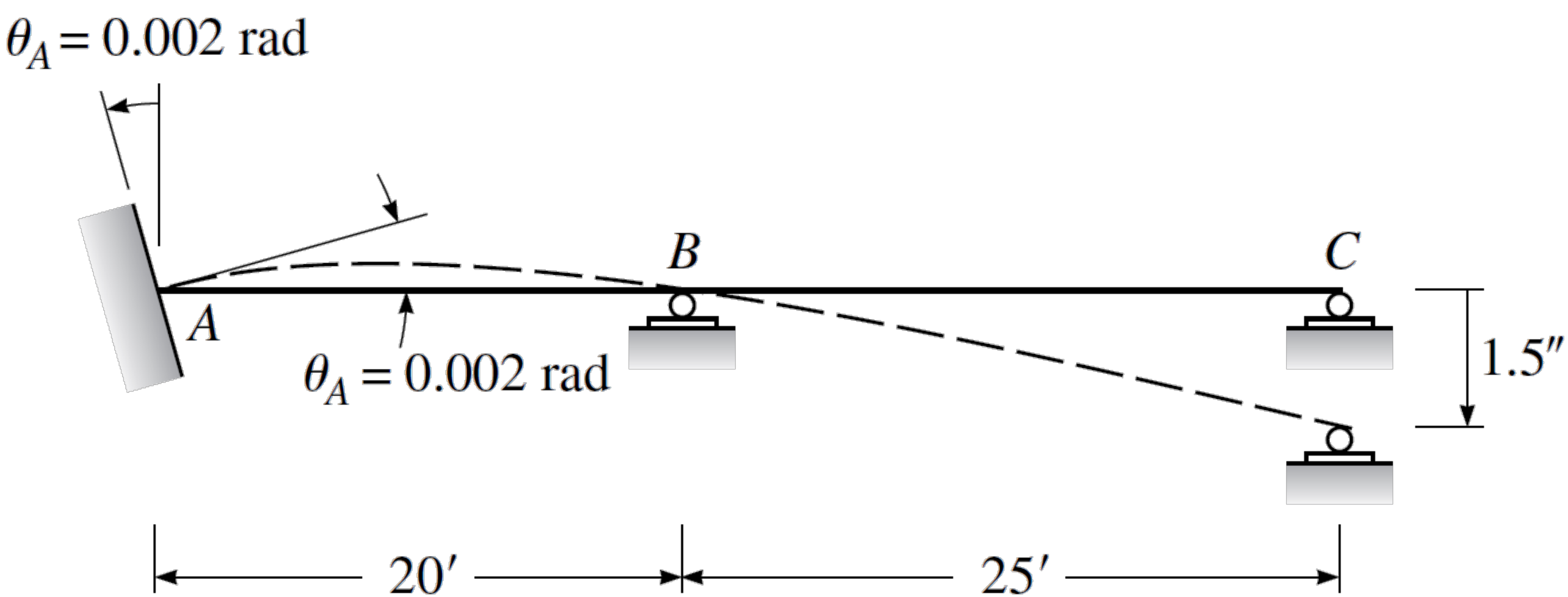
- **STEP 4** Clamps at joints *B* and *C* are next removed simultaneously. Joints *B* and *C* rotate equally and equal values of end moment develop at each end of girder *BC.*

Example 13.6 Solution (continued)

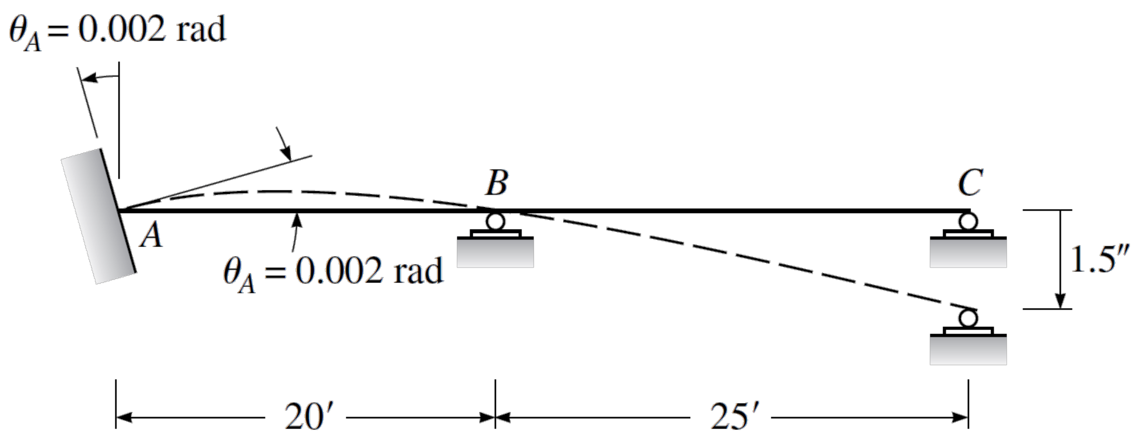


Example 13.7

Determine the reactions and draw the shear and moment curves for the continuous beam in Figure 13.19*a*. The fixed support at *A* is accidentally constructed incorrectly at a slope of 0.002 radian counterclockwise from a vertical axis through *A*, and the support at *C* is accidentally constructed 1.5 in below its intended position. Given: E = 29,000 kips/in^2 and I = 300 in^4.



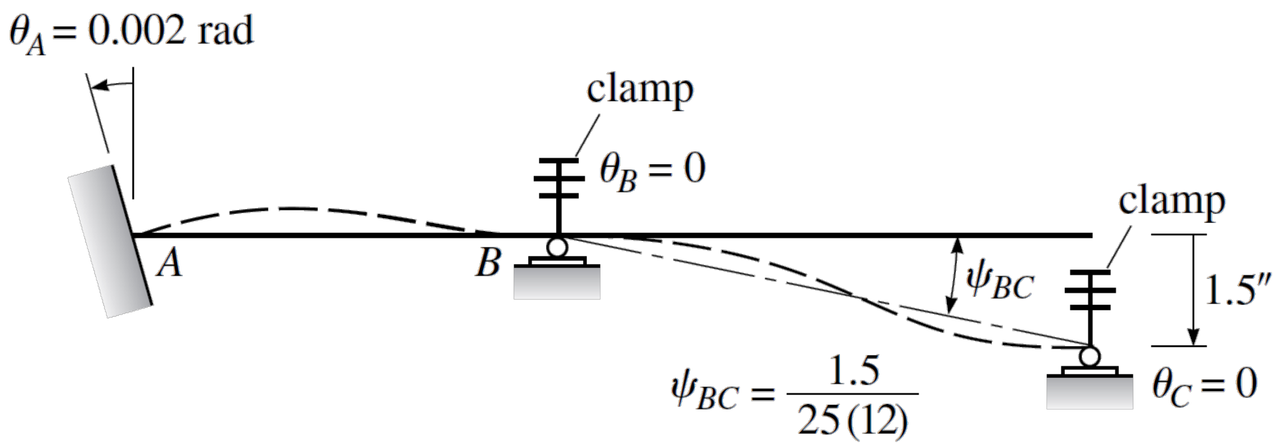
Example 13.7 Solution



- Use the slope-deflection equation to compute the moments at each end of the restrained beams.

$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi) + \text{FEM}_{NF}$$

Example 13.7 Solution (continued)



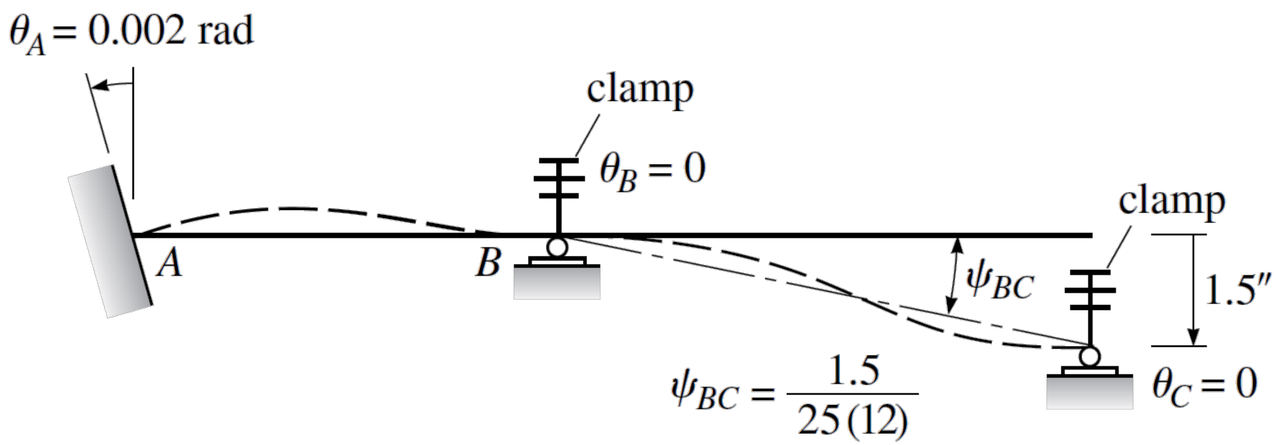
Restrained beam locked in position by temporary clamps at joints *B* and *C*

- Compute moments in span *AB*: θ_A = -0.002 rad, θ_B = 0, and ψ_{AB} = 0. Since no transverse loads are applied to span *AB*, FEM_{AB} = FEM_{BA} = 0.

$$M_{AB} = \frac{2(29{,}000)(300)}{20(12)}[2(-0.002)] = -290 \text{ kip}\cdot\text{in} = -24.2 \text{ kip}\cdot\text{ft}$$

$$M_{BA} = \frac{2(29{,}000)(300)}{20(12)}(-0.002) = -145 \text{ kip}\cdot\text{in} = -12.1 \text{ kip}\cdot\text{ft}$$

Example 13.7 Solution (continued)



Restrained beam locked in position by temporary clamps at joints *B* and *C*

- Compute moments in span *BC*: $\theta_B = 0$, $\theta_C = 0$, $\psi = 1.5$ in/[25(12)] = 0.005.

FEM$_{BC}$ = FEM$_{CB}$ = 0 since no transverse loads applied to span *BC*.

$$M_{BC} = M_{CB} = \frac{2(29{,}000)(300)}{12(25)}[2(0) + 0 - 3(0.005)]$$

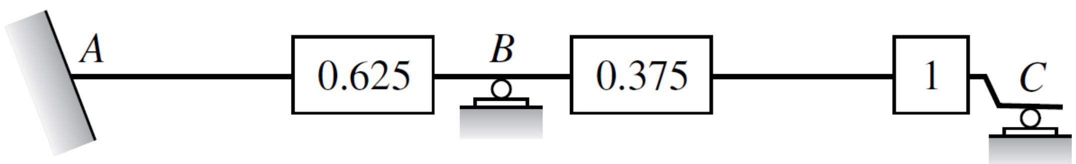
$$= -870 \text{ kip} \cdot \text{in} = -72.5 \text{ kip} \cdot \text{ft}$$

Example 13.7 Solution (continued)

- Compute the distribution factors at joint *B.*

$$K_{AB} = \frac{300}{20} = 15 \qquad K_{BC} = \frac{3}{4}\left(\frac{300}{25}\right) = 9 \qquad \Sigma K = 24$$

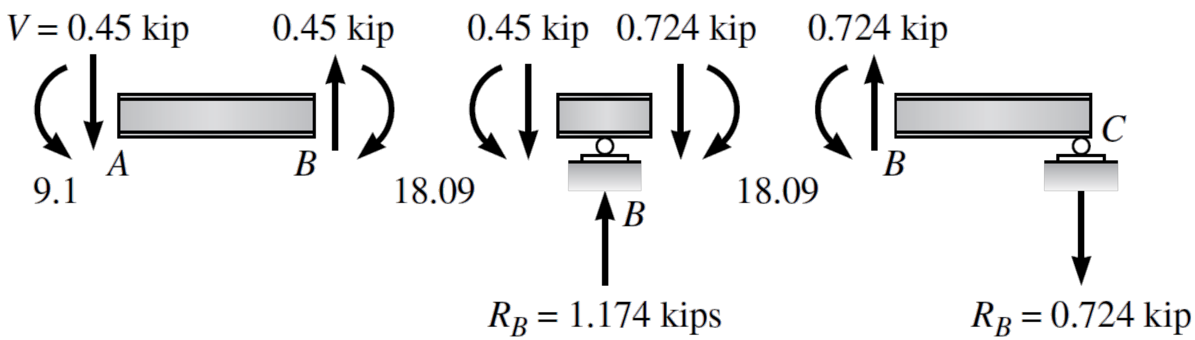
$$\mathrm{DF}_{BA} = \frac{K_{AB}}{\Sigma K} = \frac{15}{24} = 0.625 \qquad \mathrm{DF}_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{9}{24} = 0.375$$



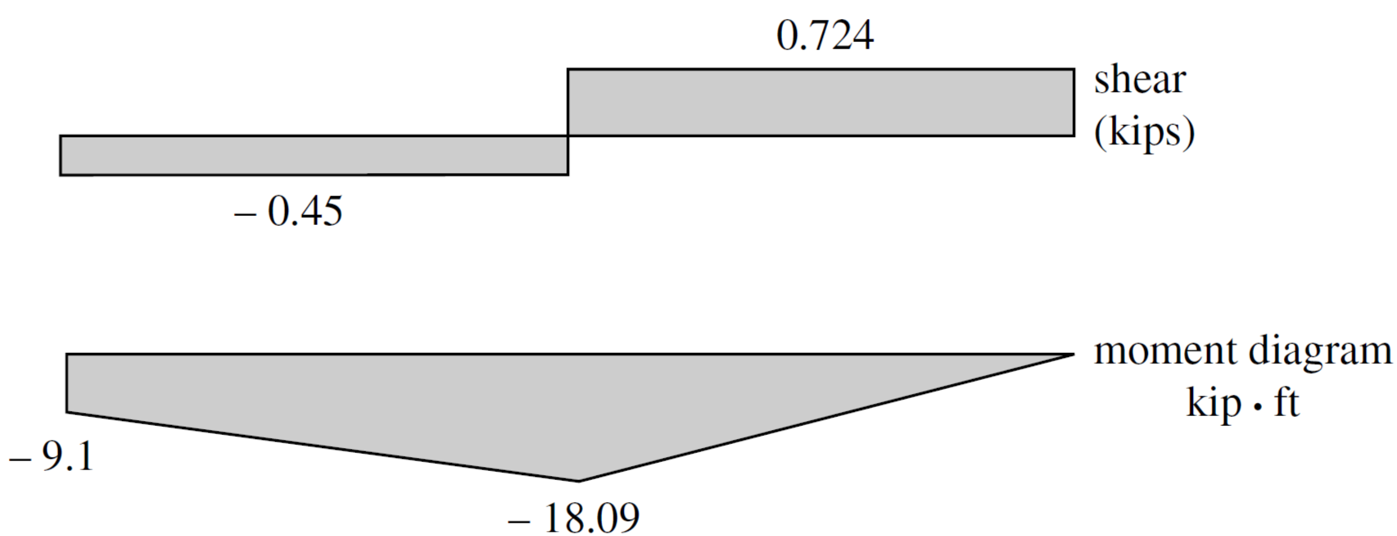
–24.2	–12.1	–72.5	–72.5	FEM
		1/2	+72.5	
		+36.3		
1/2	+30.2	+18.11		
+15.1				
–9.1	+18.09	–18.09	0	final moment (kip • ft)

Moment distribution

Example 13.7 Solution (continued)



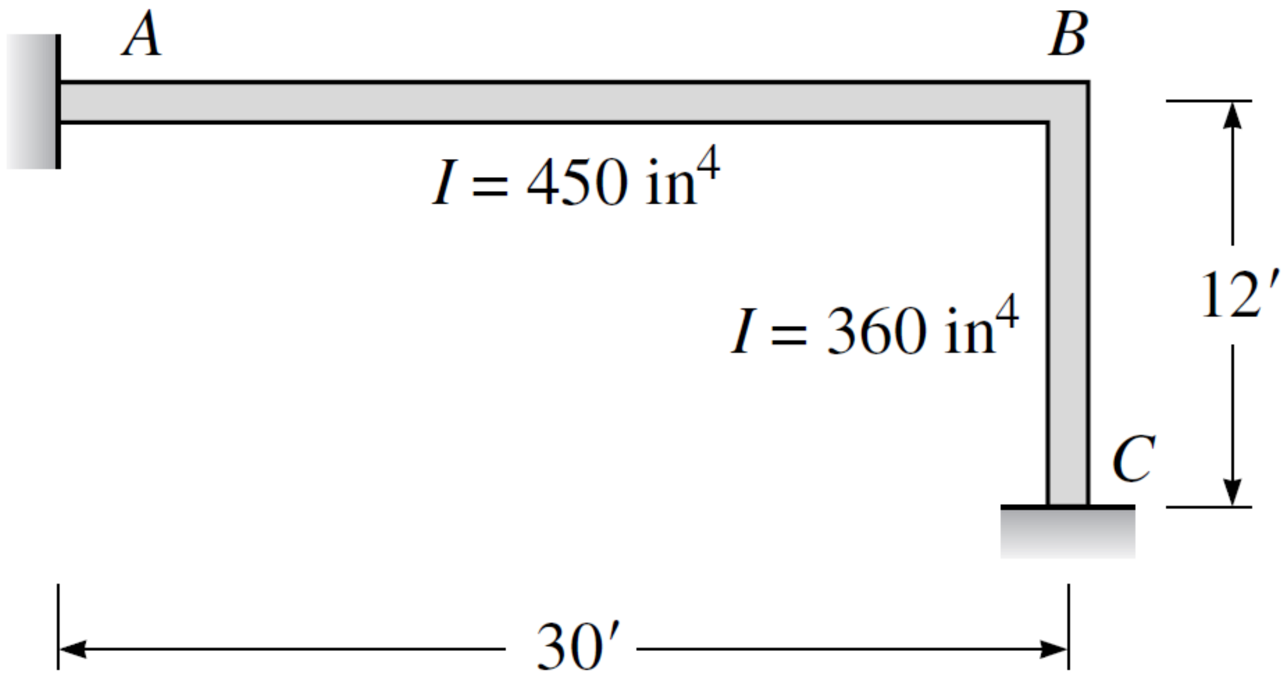
Free bodies used to evaluate shears and reactions



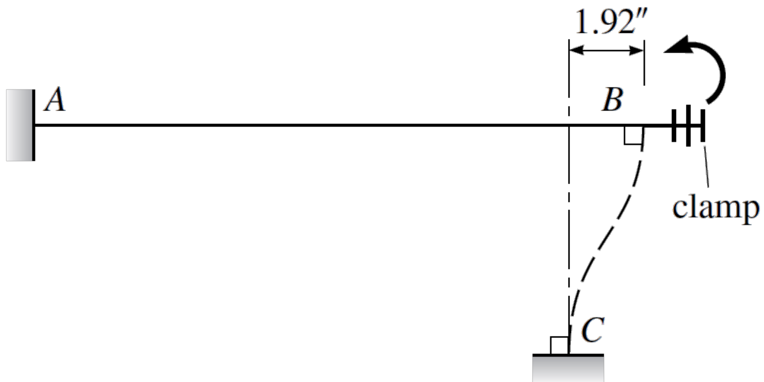
Moment curve produced by support movements

Example 13.8

If girder *AB* of the rigid frame in Figure 13.21*a* is fabricated 1.92 in too long, what moments are created in the frame when it is erected? Given: E = 29,000 kips/in^2.



Example 13.8 Solution



Deformation introduced and joint *B* clamped against rotation (θ_B = 0)

- Add 1.92 in to the end of girder *AB*, and erect the frame with a clamp at joint *B* to prevent rotation. Compute the fixed-end moments in the clamped structure using the slope-deflection equation.

$$\text{Column } BC: \quad \theta_B = 0 \qquad \theta_C = 0$$

$$\psi_{BC} = \frac{1.92}{12(12)} = +0.0133 \text{ rad}$$

- FEM_{BC} = FEM_{CB} = 0 since no loads are applied between joints.

$$M_{BC} = M_{CB} = \frac{2EI}{L}(-3\psi_{BC})$$

$$= \frac{2(29{,}000)(360)}{12(12)}[-3(0.0133)]$$

$$= -5785.5 \text{ kip} \cdot \text{in} = -482.13 \text{ kip} \cdot \text{ft}$$

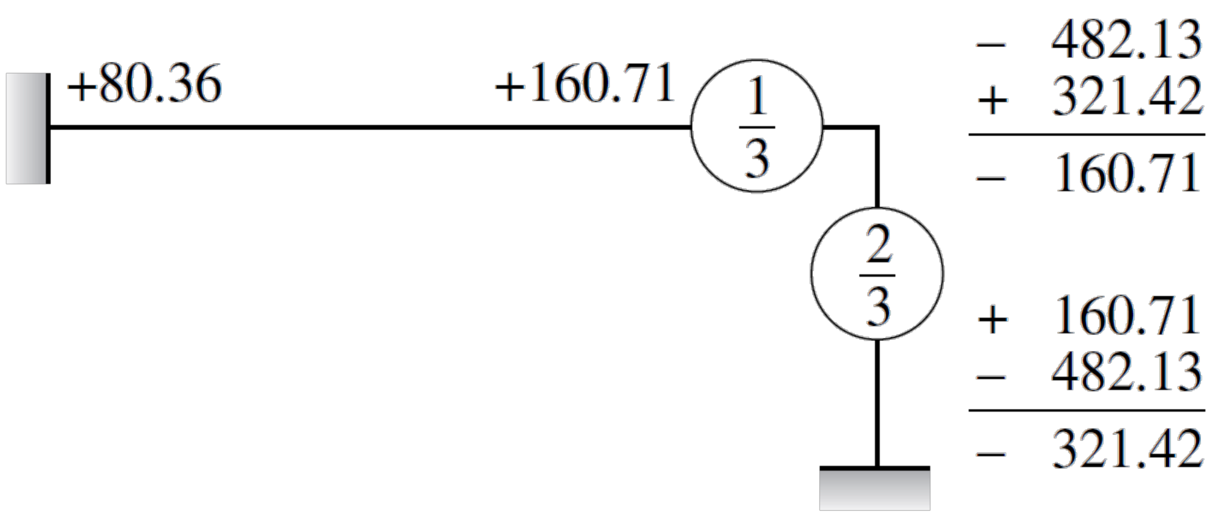
No moments develop in member *AB* because ψ_{AB} = θ_A = θ_B = 0.

Example 13.8 Solution (continued)

- Compute the distribution factors.

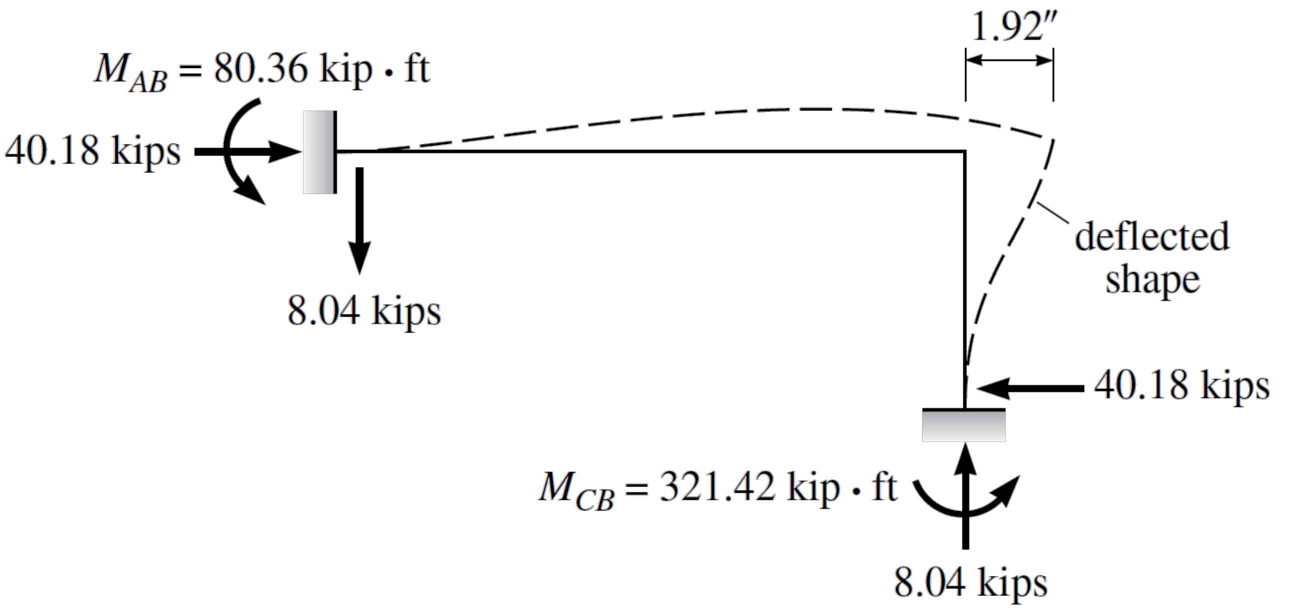
$$K_{AB} = \frac{I}{L} = \frac{450}{30} = 15 \qquad K_{BC} = \frac{360}{12} = 30 \qquad \Sigma K = 15 + 30 = 45$$

$$\mathrm{DF}_{BA} = \frac{K_{AB}}{\Sigma K} = \frac{15}{45} = \frac{1}{3} \qquad \mathrm{DF}_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{30}{45} = \frac{2}{3}$$

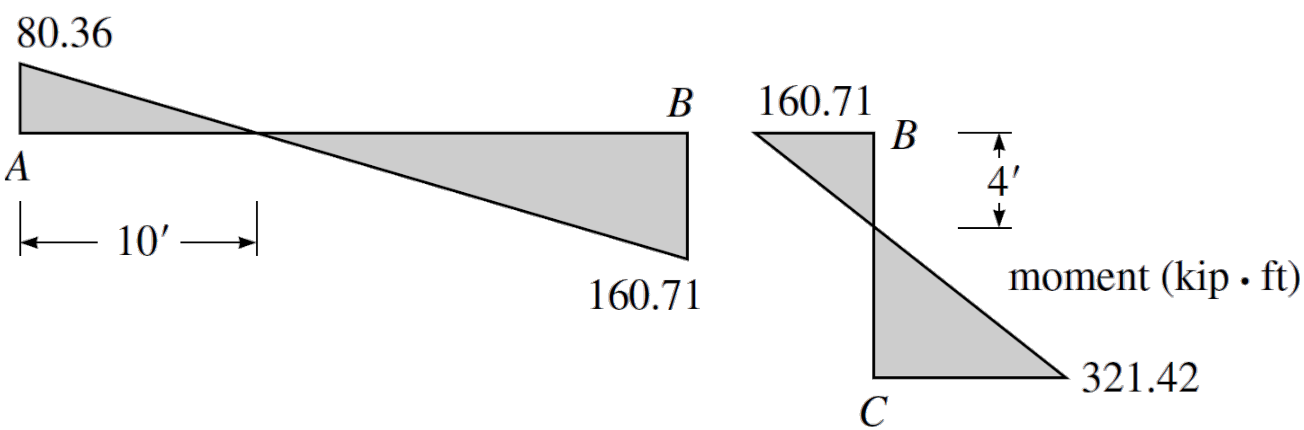


Analysis by moment distribution (moments in kip-ft)

Example 13.8 Solution (continued)

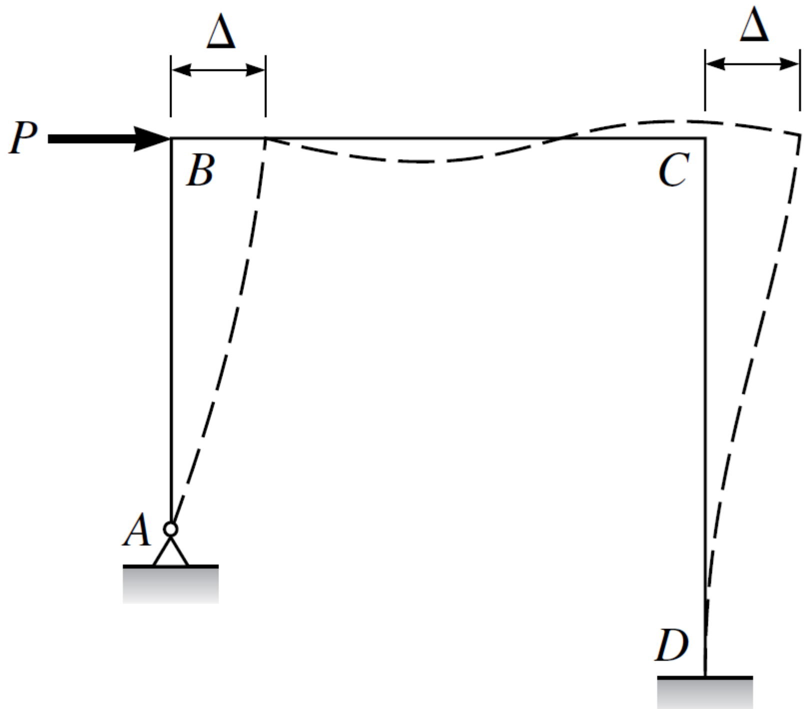


Reactions and deflected shape

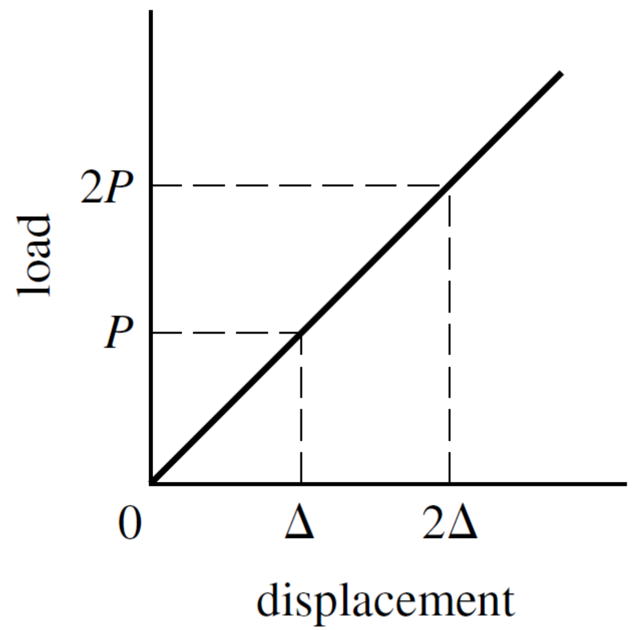


Moment diagrams

§13.6 Analysis of Frames That Are Free to Sidesway



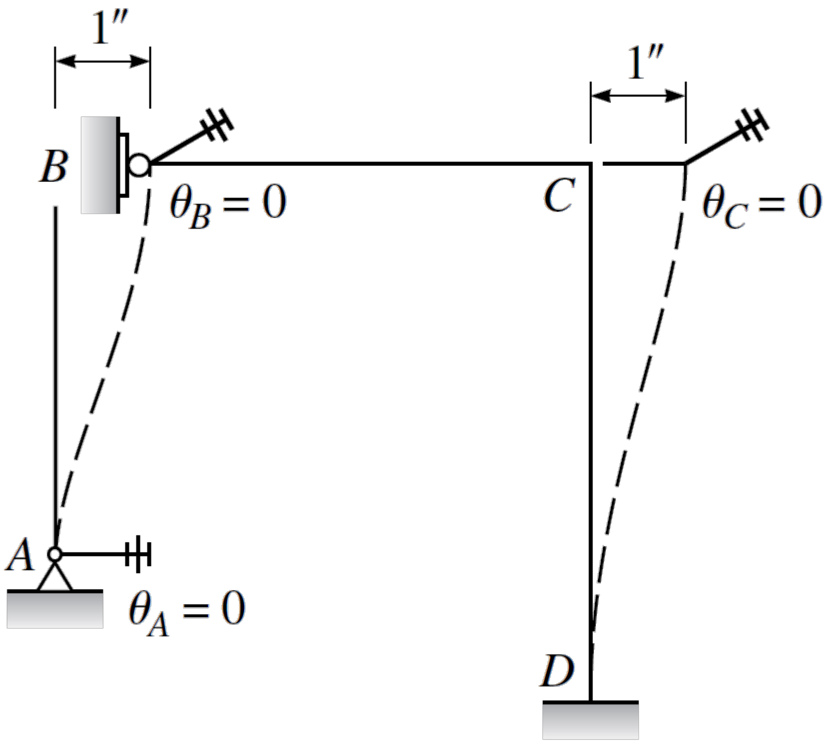
Displacement of loaded frame



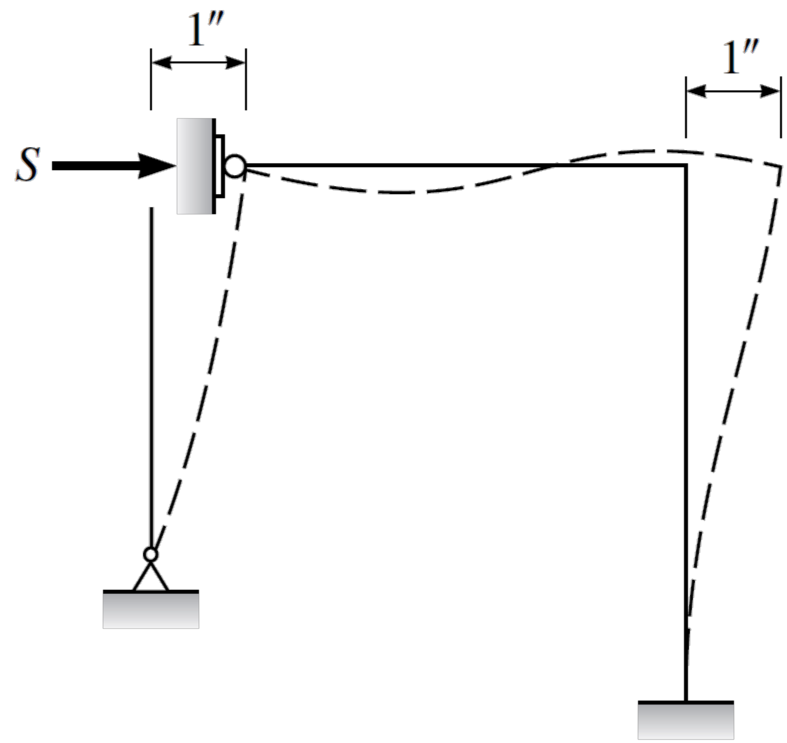
Linear elastic load displacement curve

Figure 13.22

§13.6 Analysis of Frames That Are Free to Sidesway



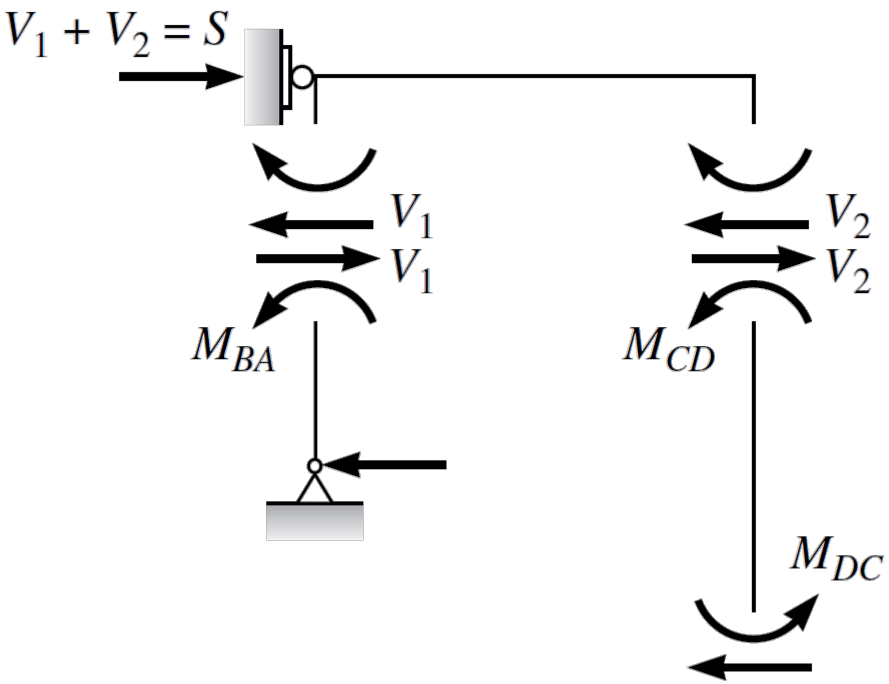
Unit displacement of frame, temporary roller, and clamps introduced to restrain frame



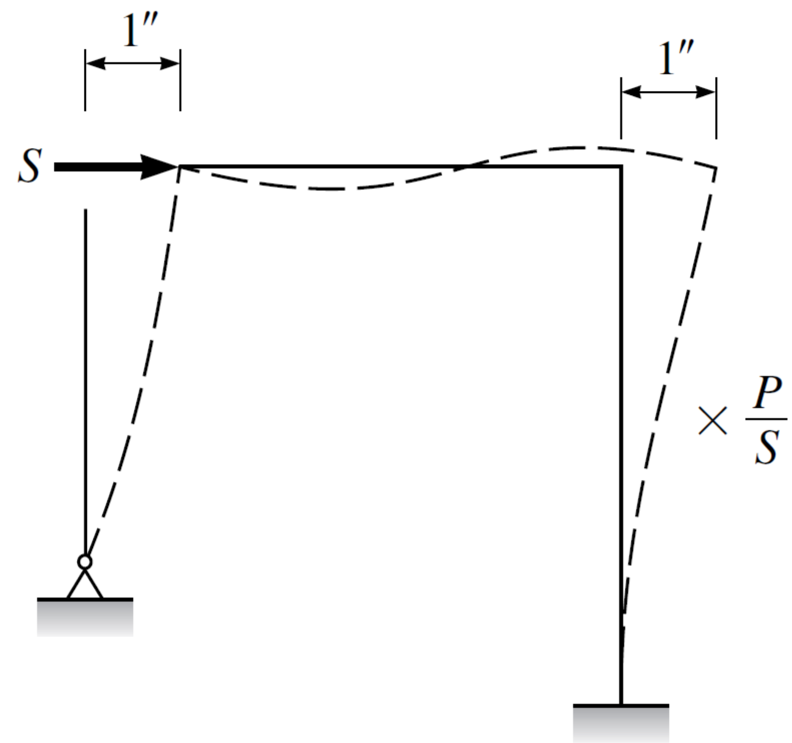
Displaced frame with clamps removed, joints rotated into equilibrium position; All member end moments are known

Figure 13.22 (continued)

§13.6 Analysis of Frames That Are Free to Sidesway



Computation of reaction *(S)* at roller after column shears computed; axial forces in columns omitted for clarity

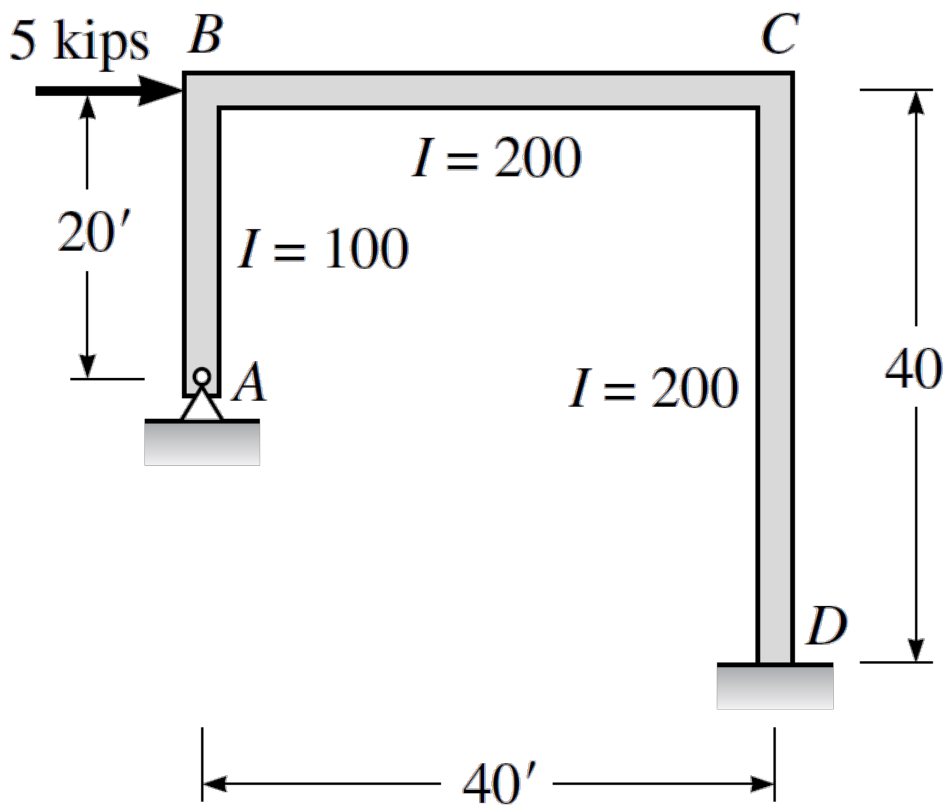


Frame displaced 1 in by a horizontal force *S*, multiply all forces by *P/S* to establish forces and deflections produced in *(a)* by force *P*

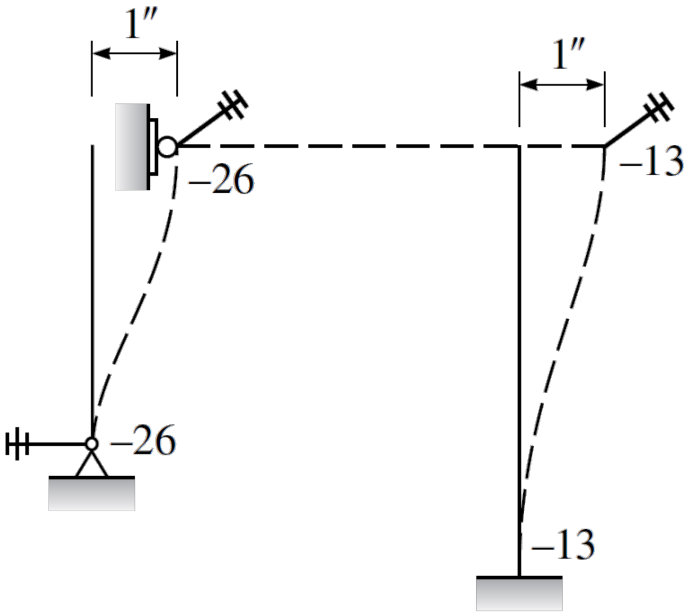
Figure 13.22 (continued)

Example 13.9

Determine the reactions and the member end moments produced in the frame shown in Figure 13.23*a* by a load of 5 kips at joint *B*. Also determine the horizontal displacement of girder *BC*. Given: E = 30,000 kips/in^2. Units of I are in in^4.



Example 13.9 Solution



- Displace the frame 1 in to the right with all joints clamped against rotation and introduce a temporary roller at *B*. The column moments in the restrained structure are computed using Equation 13.21.

$$M_{AB} = M_{BA} = -\frac{6EI}{L^2} = -\frac{6(30{,}000)(100)}{(20 \times 12)^2} = -312 \text{ kip} \cdot \text{in}$$

$$= -26 \text{ kip} \cdot \text{ft}$$

$$M_{CD} = M_{DC} = -\frac{6EI}{L^2} = -\frac{6(30{,}000)(200)}{(40 \times 12)^2} = -166 \text{ kip} \cdot \text{in}$$

$$= -13 \text{ kip} \cdot \text{ft}$$

Example 13.9 Solution (continued)

- The clamps are now removed and the column moments distributed until all joints are in equilibrium. The distribution factors at joints *B* and *C* are computed.

Joint *B*:

$$K_{AB} = \frac{3}{4}\left(\frac{I}{L}\right) = \frac{3}{4}\left(\frac{100}{20}\right) = \frac{15}{4}$$

$$K_{BC} = \frac{I}{L} = \frac{200}{40} = \frac{20}{4}$$

$$\Sigma K = \frac{35}{4}$$

Distribution factors

$$\frac{K_{AB}}{\Sigma K} = \frac{3}{7}$$

$$\frac{K_{BC}}{\Sigma K} = \frac{4}{7}$$

Joint *C*:

$$K_{CB} = \frac{I}{L} = \frac{200}{40} = 5$$

$$K_{CD} = \frac{I}{L} = \frac{200}{40} = 5$$

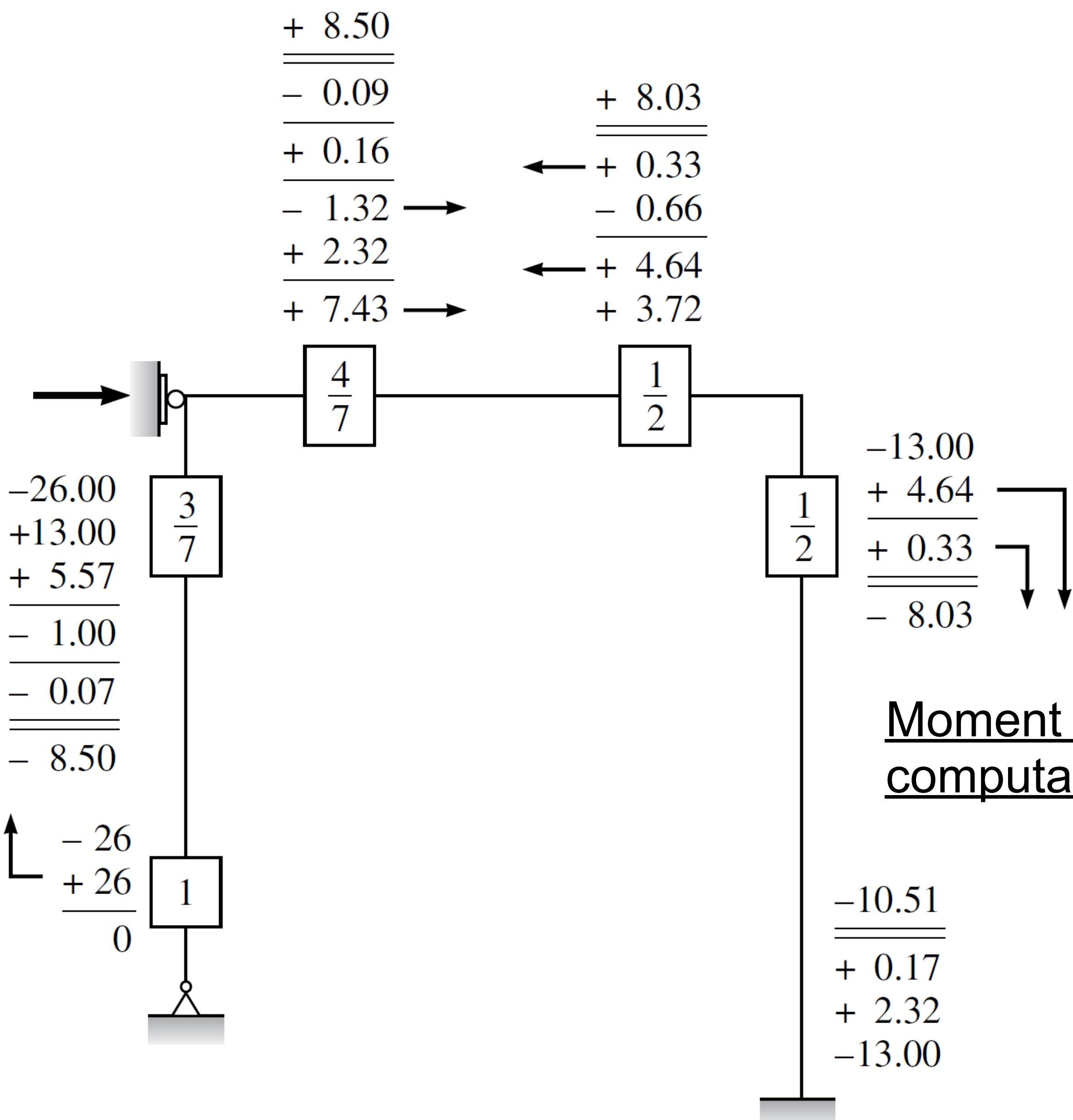
$$\Sigma K = 10$$

Distribution factors

$$\frac{5}{10} = \frac{1}{2}$$

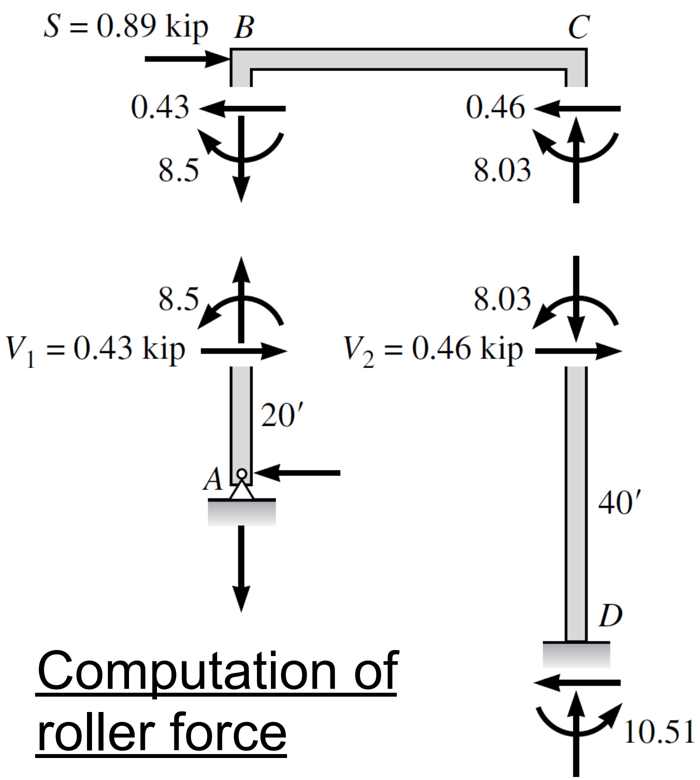
$$\frac{5}{10} = \frac{1}{2}$$

Example 13.9 Solution (continued)



Moment distribution computations

Example 13.9 Solution (continued)



Computation of roller force

- Compute the column shears by summing moments about an axis through the base of each column.

- Compute V_1.

$$\circlearrowright^{+} \quad \Sigma M_A = 0 \qquad 20V_1 - 8.5 = 0$$

$$V_1 = 0.43 \text{ kip}$$

- Compute V_2.

$$\circlearrowright^{+} \quad \Sigma M_D = 0 \qquad 40V_2 - 8.03 - 10.51 = 0$$

$$V_2 = 0.46 \text{ kip}$$

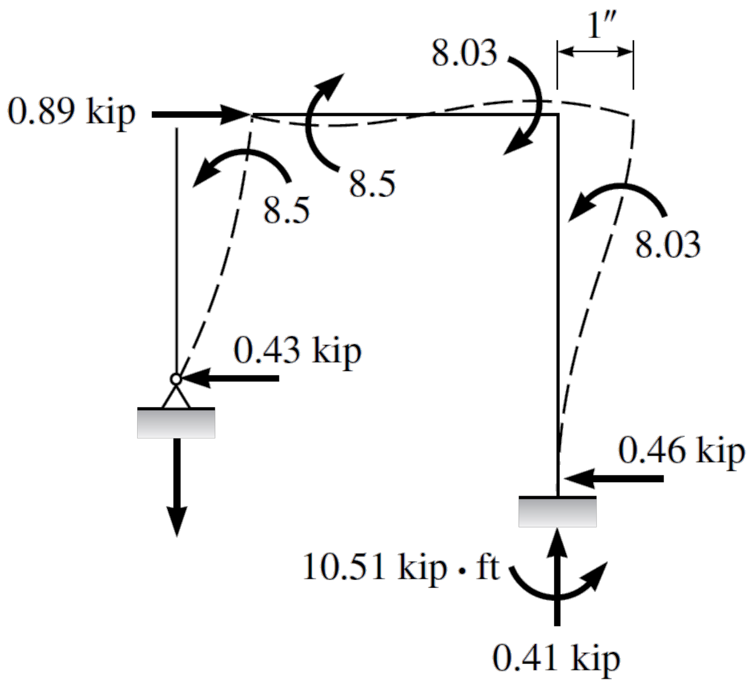
- Considering horizontal equilibrium of the free body of the girder, compute the roller reaction at *B*.

$$\rightarrow + \quad \Sigma F_x = 0 \qquad S - V_1 - V_2 = 0$$

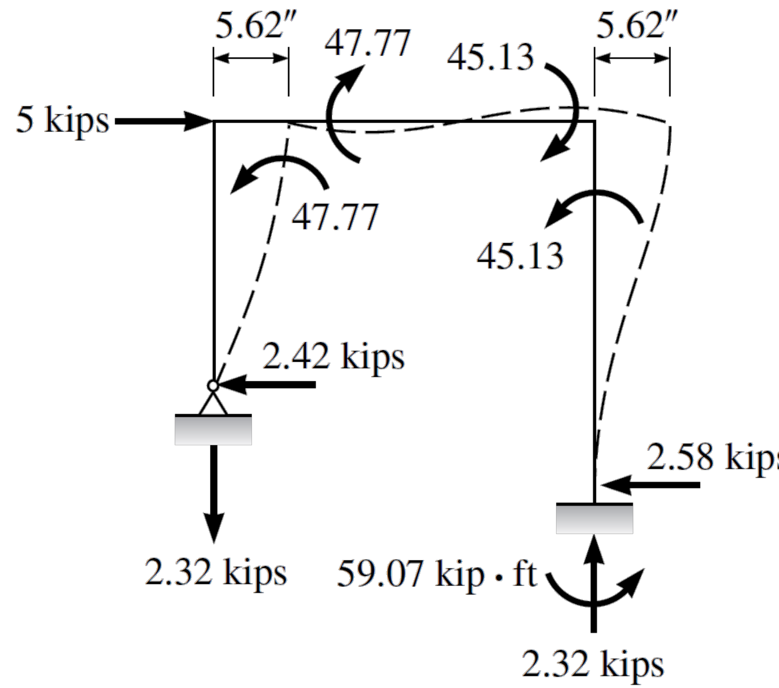
$$S = 0.46 + 0.43 = 0.89 \text{ kip}$$

Example 13.9 Solution (continued)

- To compute the forces and displacements produced by a 5-kip load, scale all forces and displacements by the ratio of $P/S = 5/0.89 = 5.62$. The displacement of the girder (P/S) (1 in) = 5.62 in.

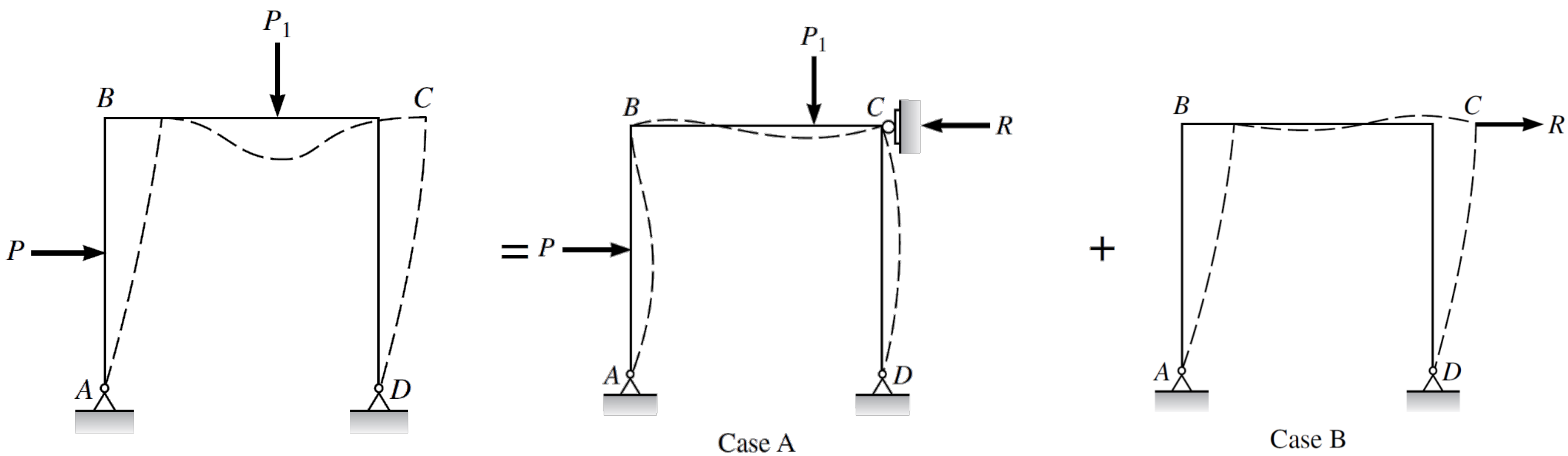


Forces created in the frame by a unit displacement after clamps removed (moments in kip-ft and forces in kips)



Reactions and member end moments produced by 5-kip load

§13.7 Analysis of an Unbraced Frame for General Loading



Deformations of an unbraced frame

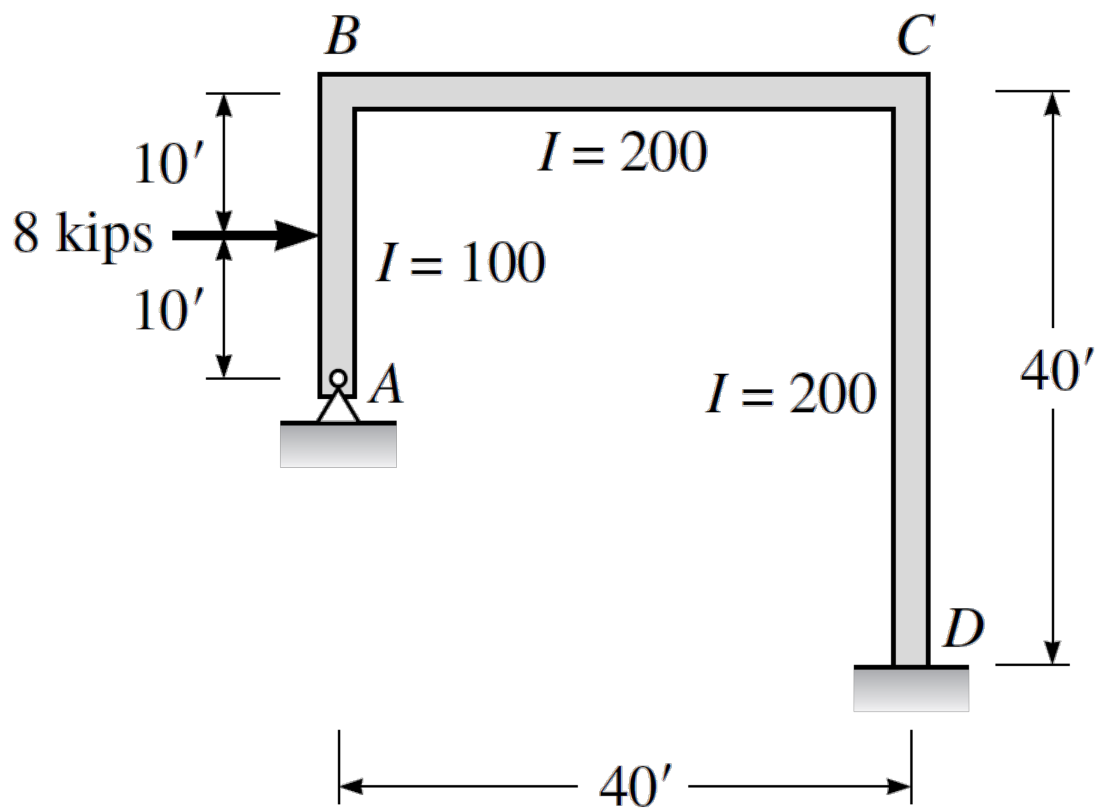
Sidesway prevented by adding a temporary roller that provides a holding force R at C

Sidesway correction, holding force reversed and applied to structure at joint C

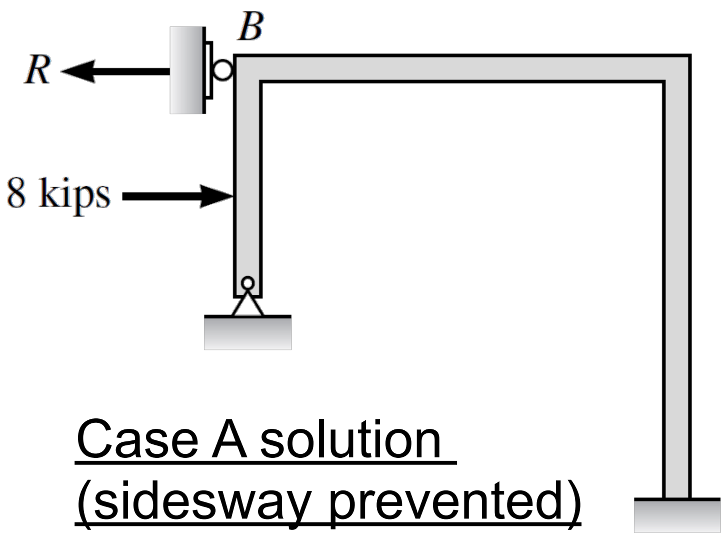
Figure 13.24

Example 13.10

Determine the reactions and member end moments produced in the frame shown in Figure 13.25*a* by the 8-kip load. Also determine the horizontal displacement of joint *B*. Values of moment of inertia of each member in units of in^4 are shown on Figure 13.23*a*. E = 30,000 kips/in^2.



Example 13.10 Solution



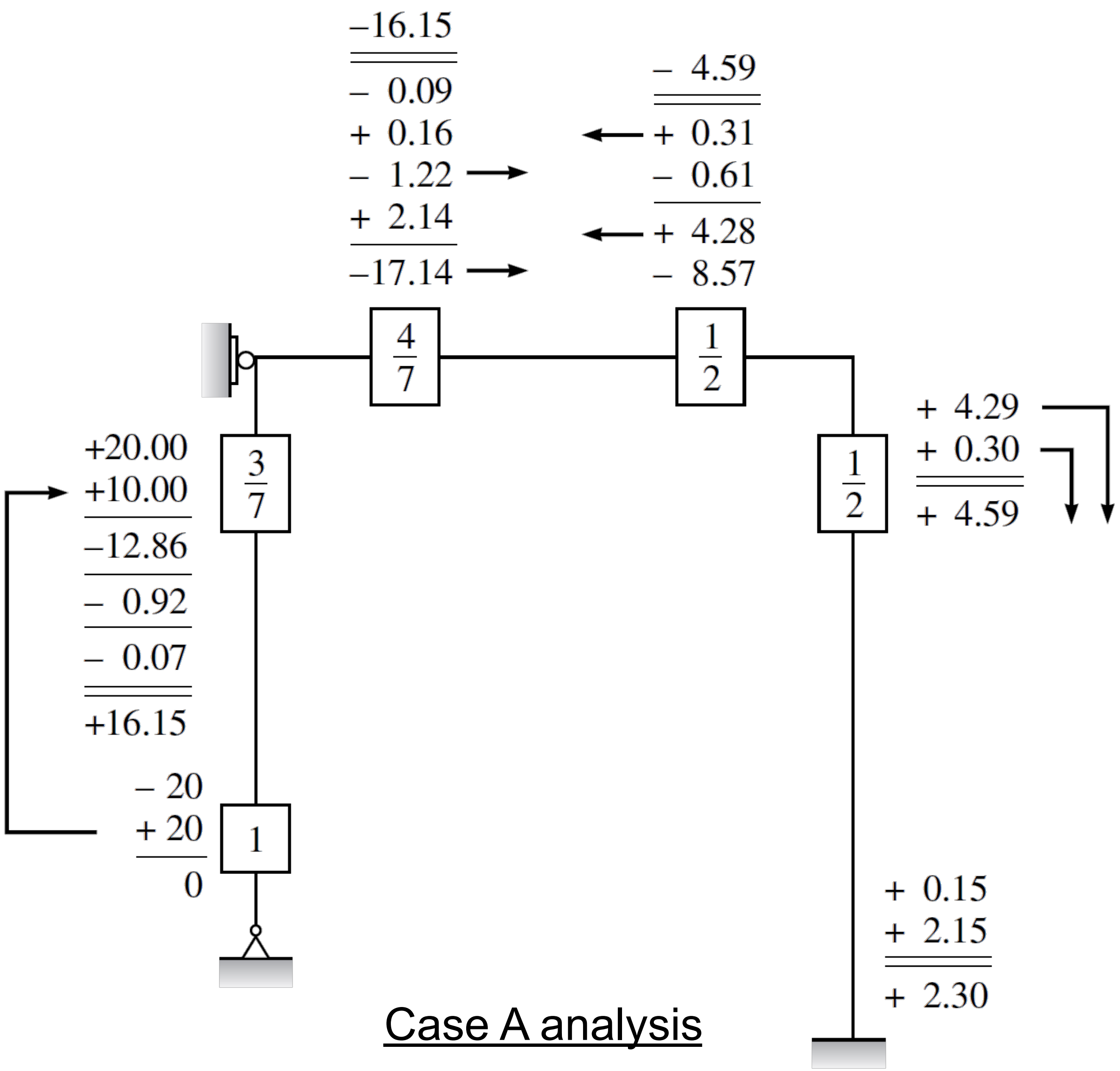
Case A solution
(sidesway prevented)

- An imaginary roller is introduced at support *B* to prevent sidesway. The fixed-end moments produced by the 8-kip load are equal to

$$\text{FEM} = \pm\frac{PL}{8} = \pm\frac{8(20)}{8} = \pm 20 \text{ kip} \cdot \text{ft}$$

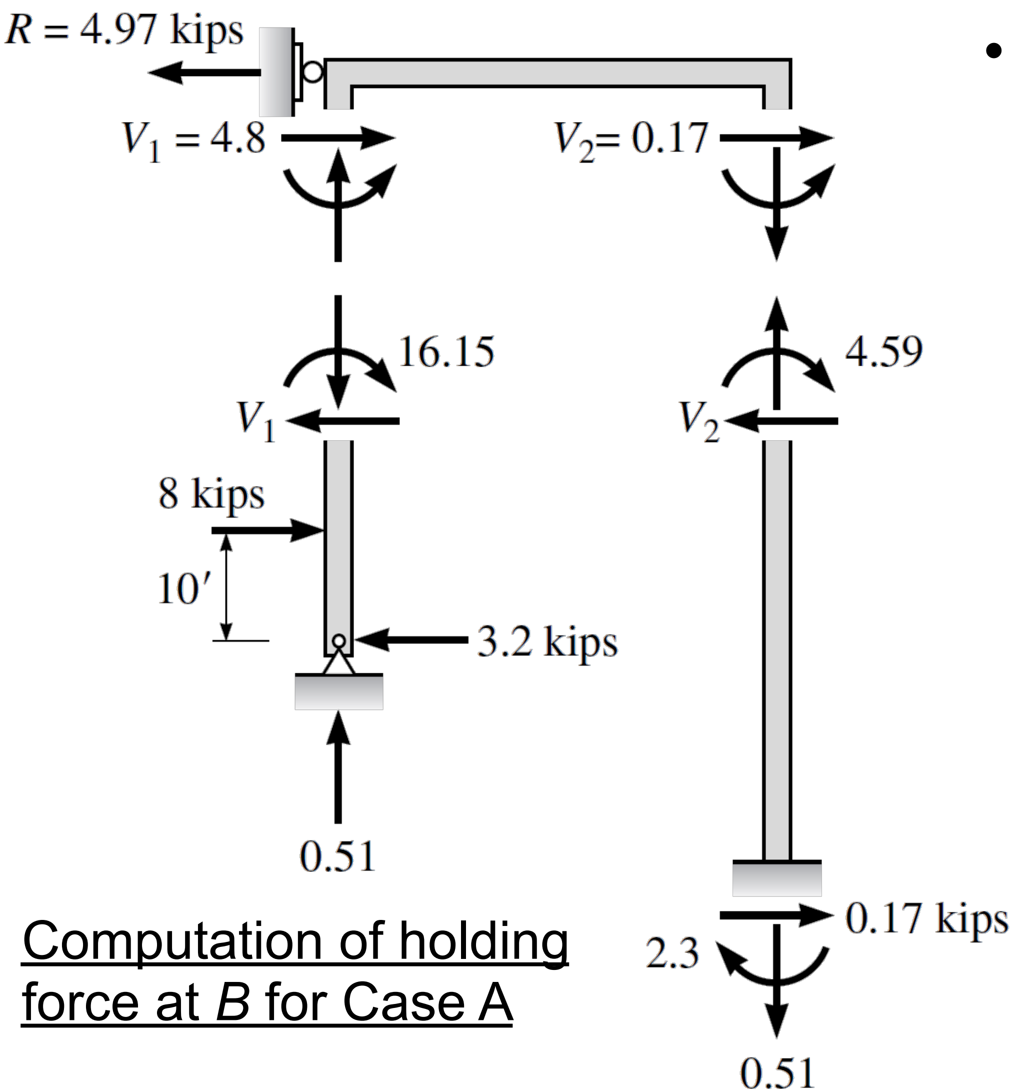
- The analysis of the restrained frame for the 8-kip load is carried out

Example 13.10 Solution (continued)



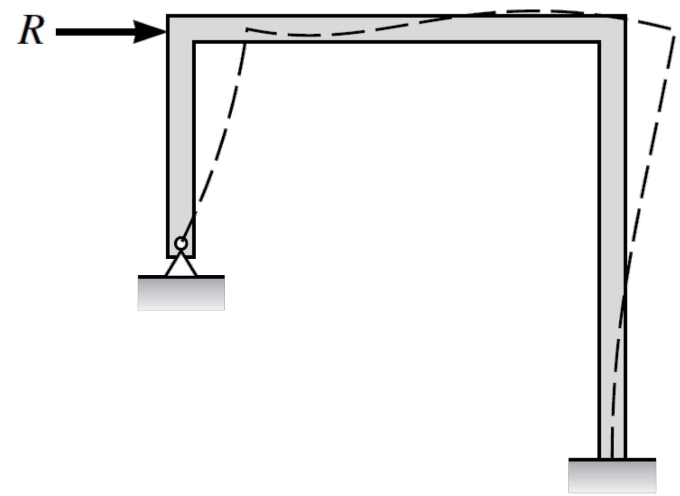
Case A analysis

Example 13.10 Solution (continued)



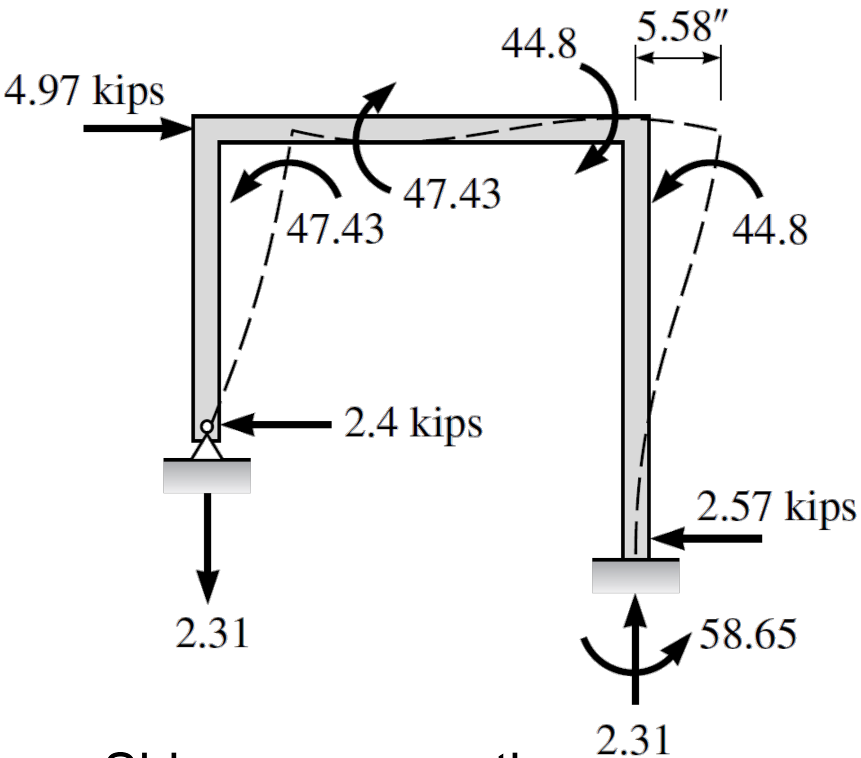
Computation of holding force at *B* for Case A

- Since the roller force at *B* equals 4.97 kips, add the Case B sidesway correction shown in Figure 13.25*c*.



Case B (sidesway correction)

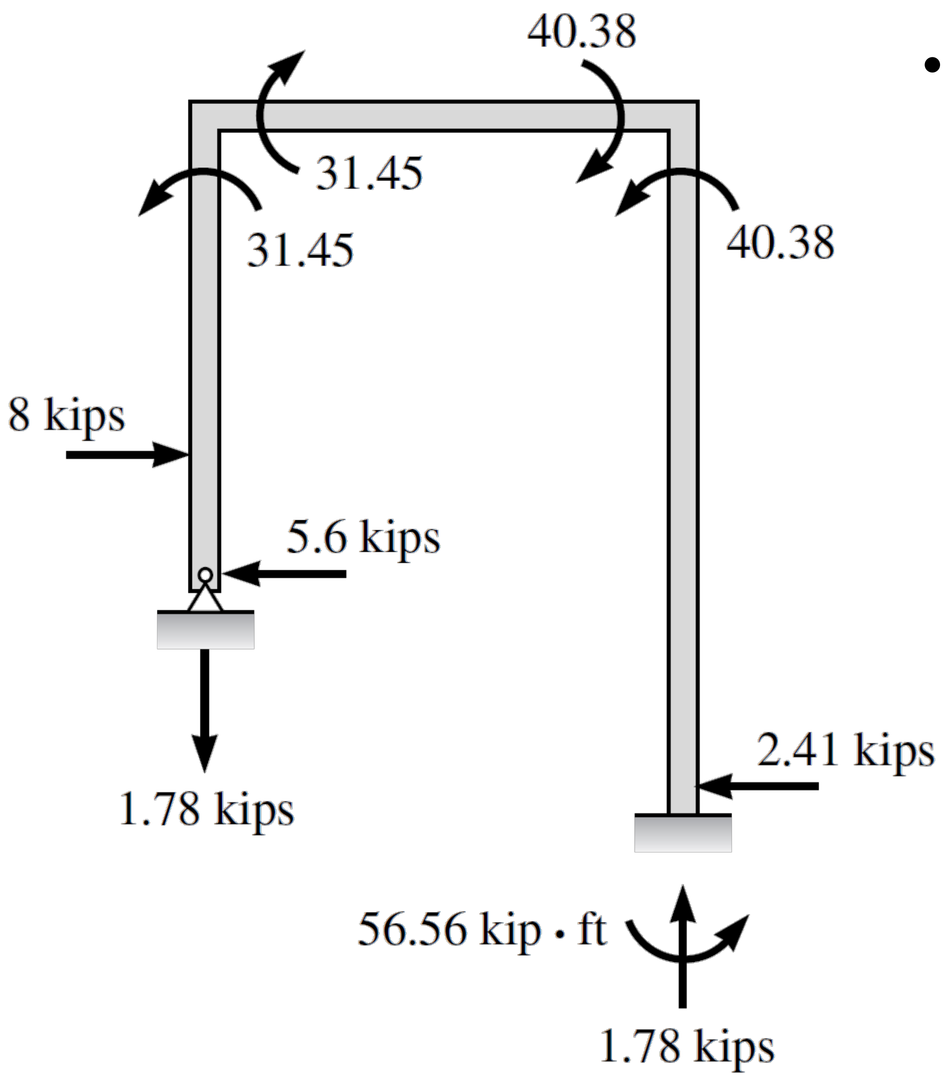
Example 13.10 Solution (continued)



Sidesway correction forces, Case B

- All forces and displacements in Figure 13.23*e* are multiplied by a scale factor 4.97/0.89 (from Example 12.9) = 5.58.

Example 13.10 Solution (continued)

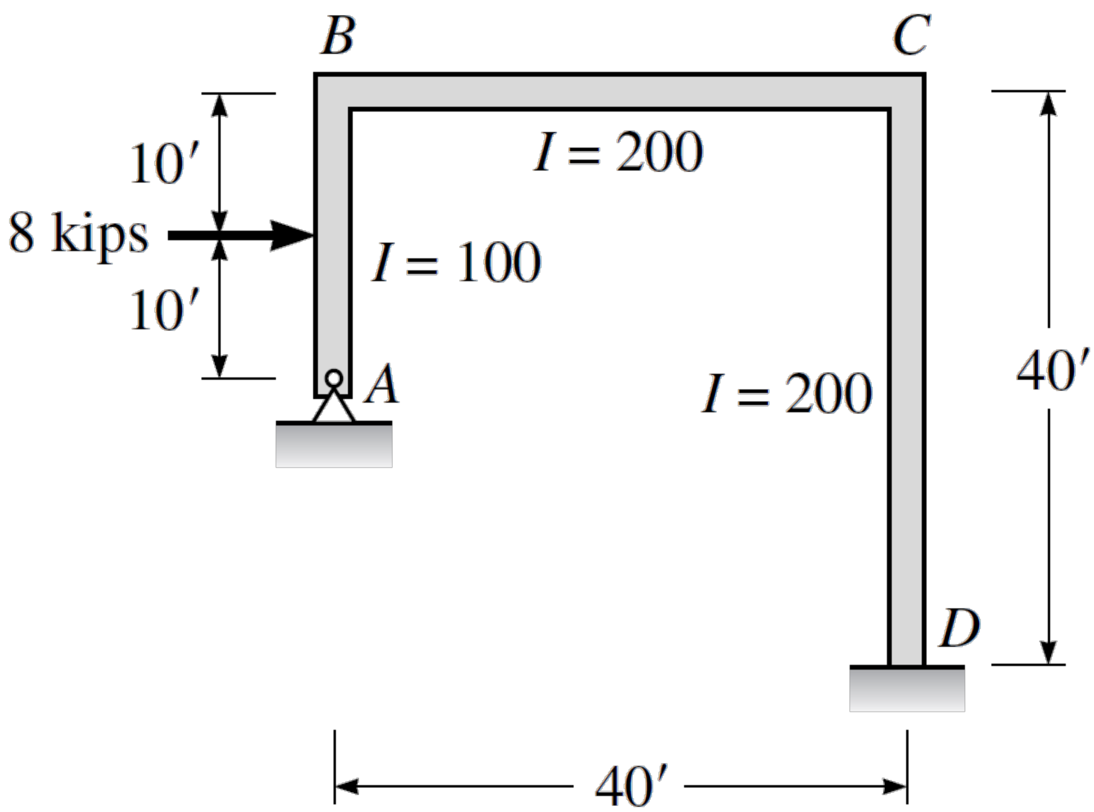


- The final forces in the frame produced by summing the Case A and Case B solutions are shown in Figure 13.25*g*. The displacement of the girder is 5.58 in to the right.

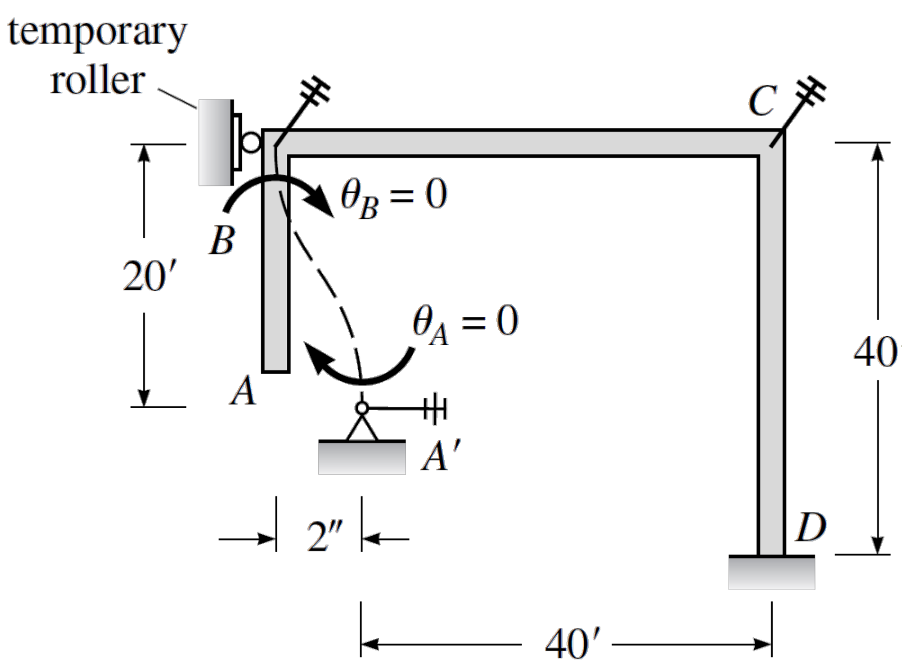
<u>Final results from superposition of Case A and Case B</u>

Example 13.11

If a member *BC* of the frame in Example 13.9 is fabricated 2 in too long, determine the moments and reactions that are created when the frame is connected to its supports. Properties, dimensions of the frame, distribution factors, and so forth are specified or computed in Example 13.9.



Example 13.11 Solution



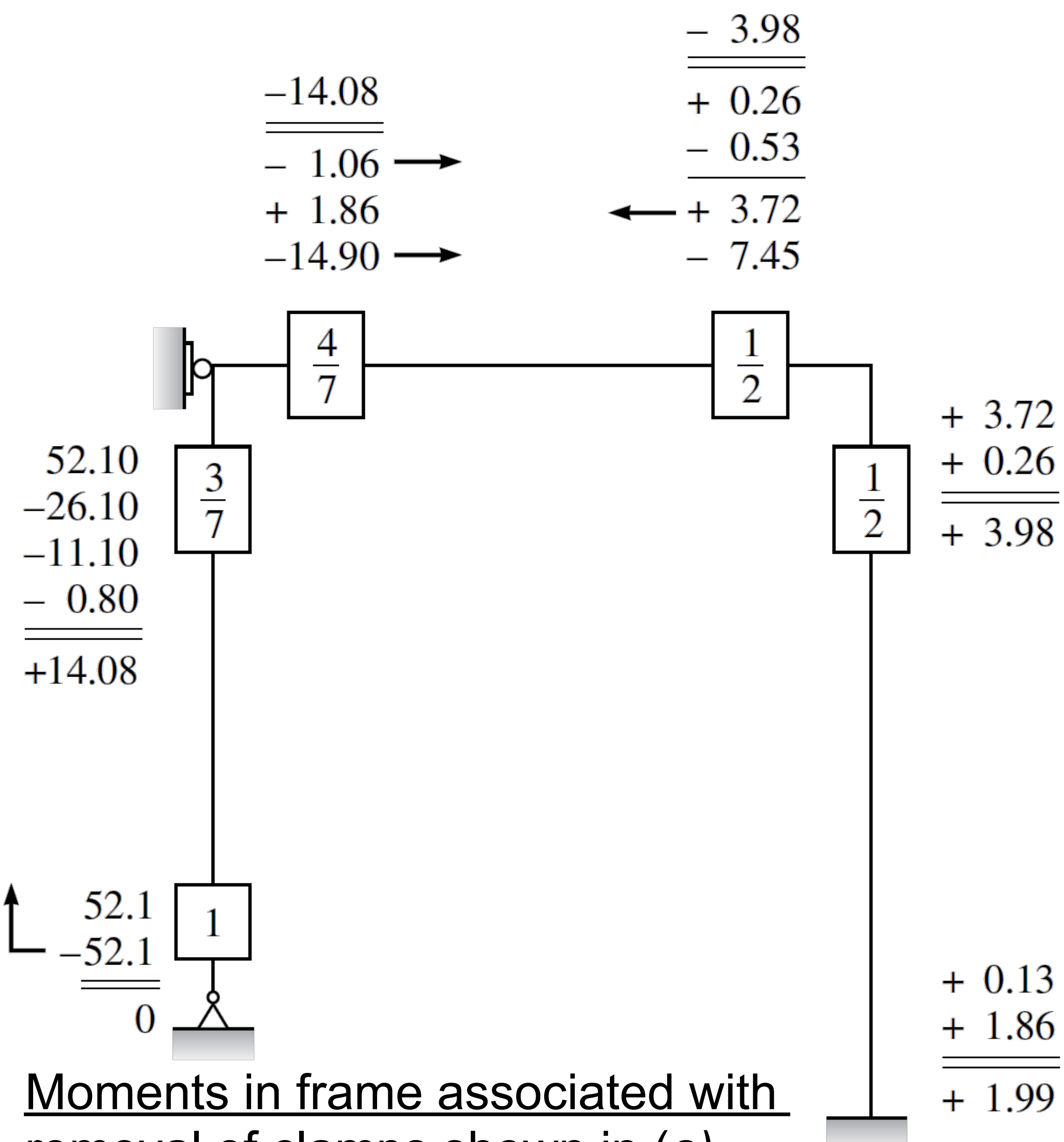
- Compute the end moments in column *AB* due to the chord rotation, using the modified form of the slope-deflection equation given by Equation 13.20

$$\psi_{AB} = -\frac{2}{20(12)} = -\frac{1}{20} \text{ rad}$$

$$M_{AB} = M_{BA} = -\frac{6EI}{L}\psi_{AB} = -\frac{6(30{,}000)(100)}{20 \times 12}\left(-\frac{1}{120}\right)$$

$$= 625 \text{ kip} \cdot \text{in} = 52.1 \text{ kip} \cdot \text{ft}$$

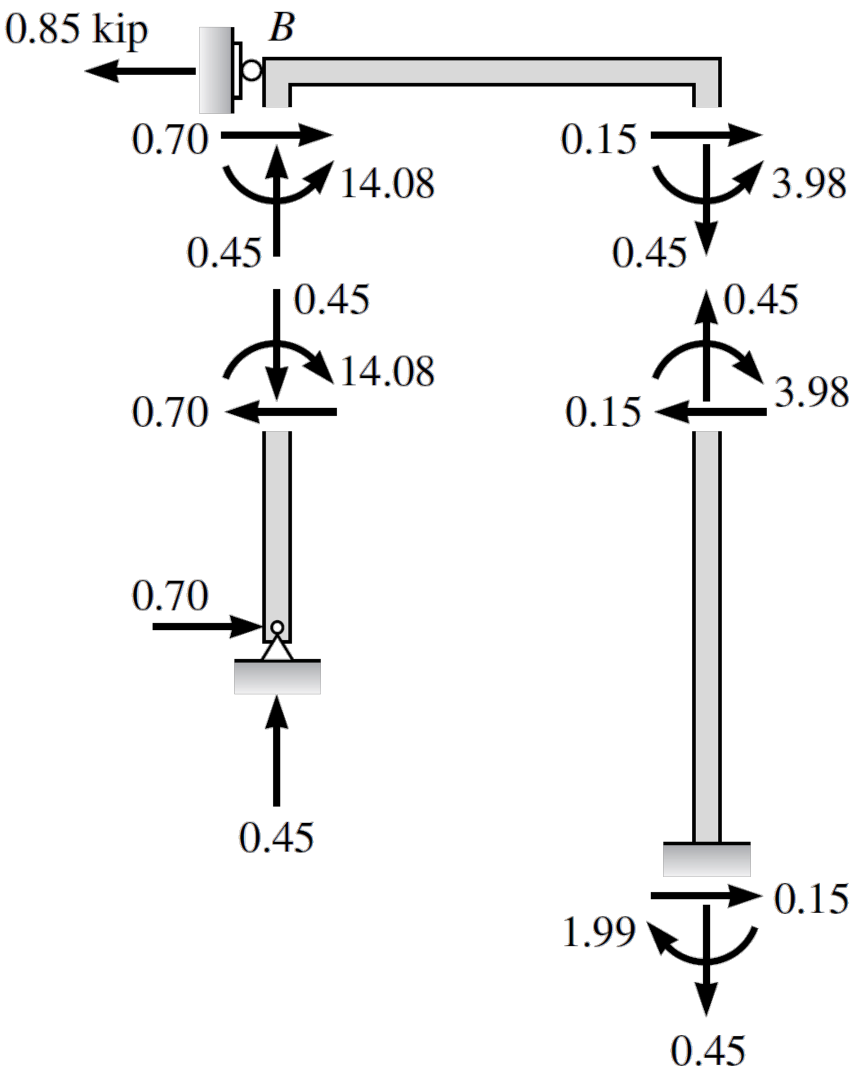
Example 13.11 Solution (continued)



Moments in frame associated with removal of clamps shown in *(a)*

- Carry out a moment distribution until the frame has absorbed the clamp moments

Example 13.11 Solution (continued)

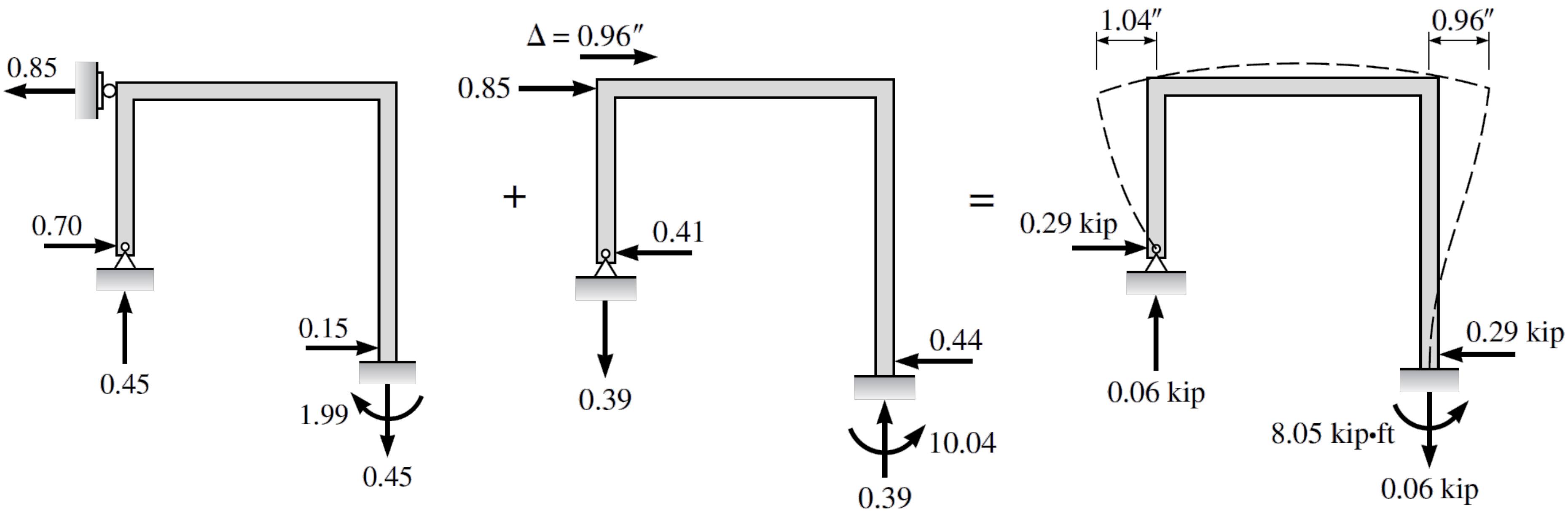


Computation of holding force in temporary roller at *B*

- The reaction at the roller is next computed from the free-body diagrams of the columns and girder

Example 13.11 Solution (continued)

- Add the sidesway correction

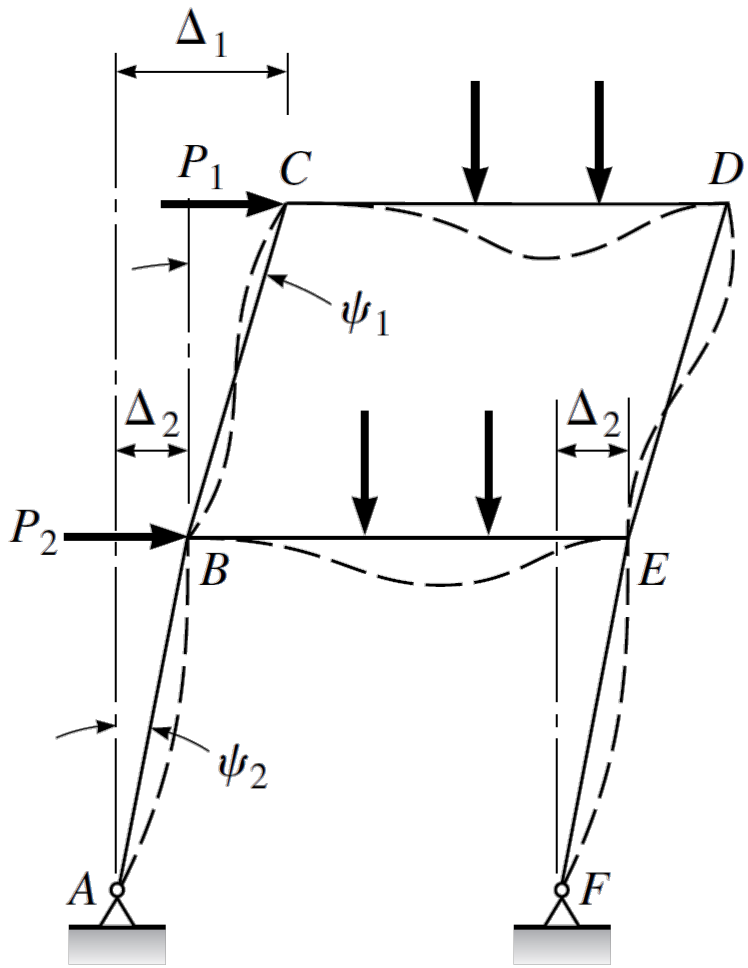


Results of analysis

Sidesway correction made by multiplying results in Figure 13.23*e* by 0.85/0.89

Final results

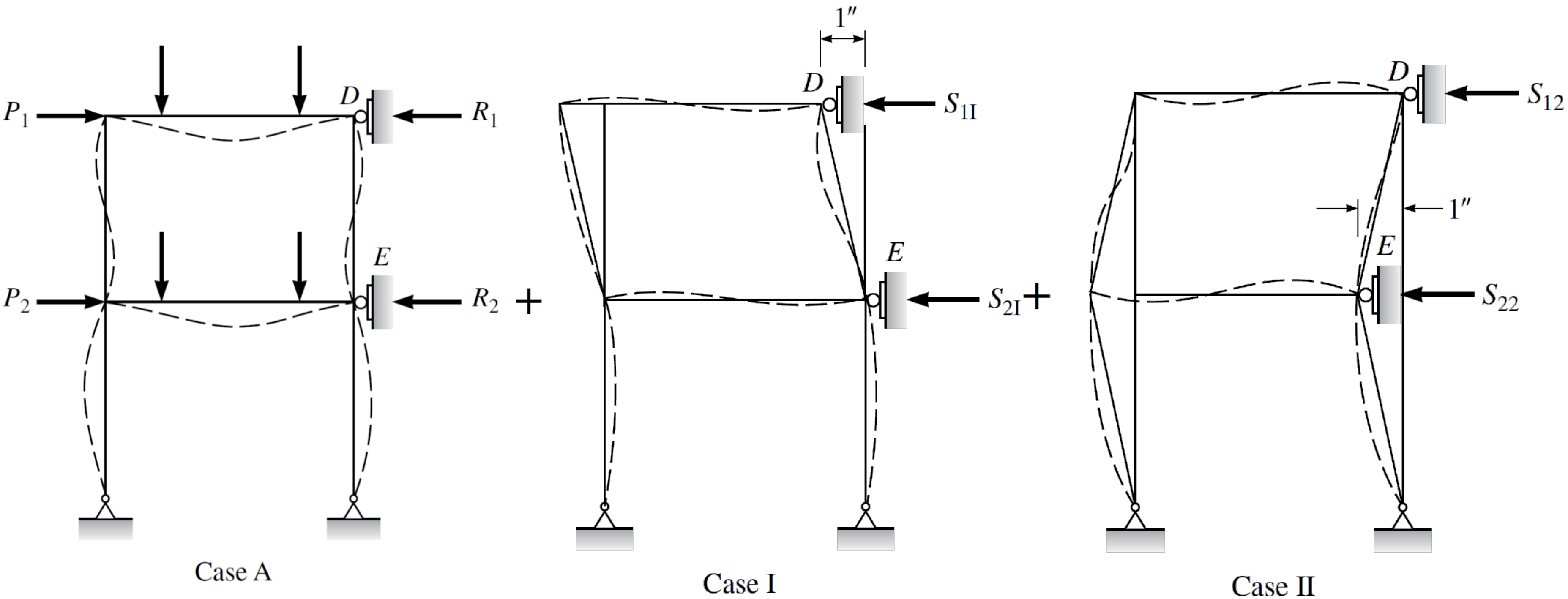
§13.8 Analysis of Multistory Frames



Building frame with two degrees of sidesway

Figure 13.27

§13.8 Analysis of Multistory Frames



Restraining forces introduced at joints *D* and *E*

Case I correction unit displacement introduced at joint D

Case II correction, unit displacement introduced at joint E

Figure 13.27 (continued)