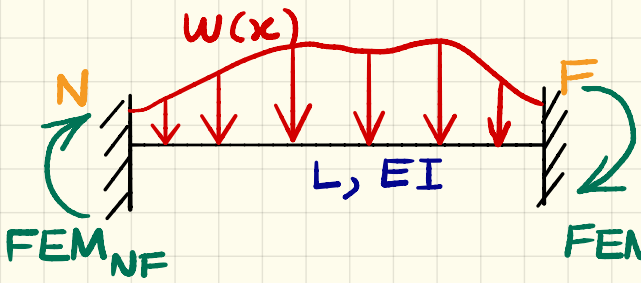
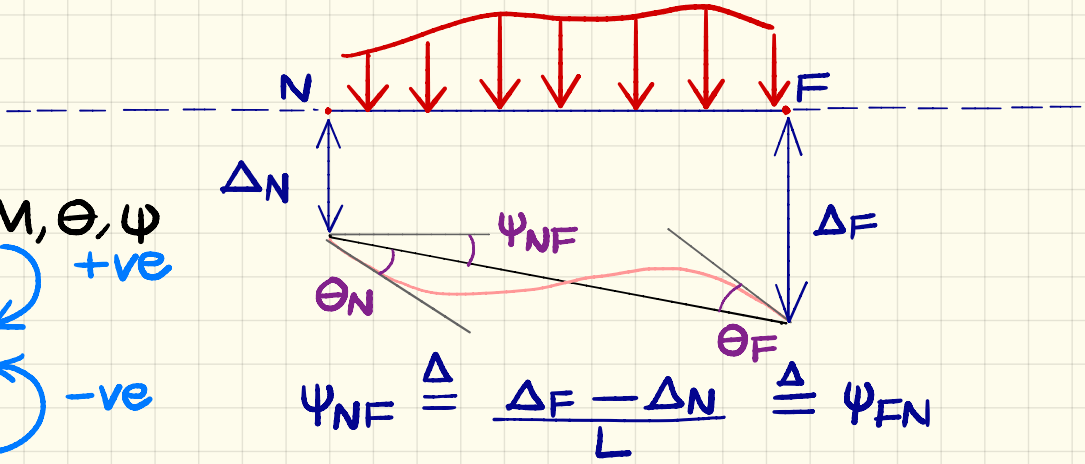
12/08/19





M, θ, ψ

+ve

-ve

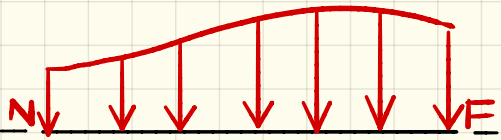
$$\psi_{NF} \overset{\Delta}{=} \frac{\Delta_F - \Delta_N}{L} \overset{\Delta}{=} \psi_{FN}$$

Slope deflection

$$M_{NF} = \frac{2EI}{L}\left(2\theta_N + \theta_F - 3\psi_{NF}\right) + (FEM)_{NF}$$

M_{FN} : replace $F \leftrightarrow N$

If moment @ N or F, M_{NF} or $M_{FN} = 0$



Modified slope-deflection

1>

$$M_{NF} = \frac{2EI}{L}\left[2\theta_N + \theta_F - 3\psi\right] + (FEM)_{NF}$$

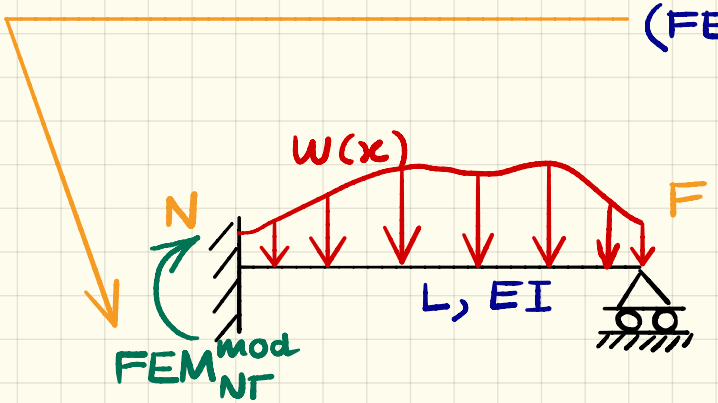
2>

$$M_{FN} = 0 = \frac{2EI}{L}\left[2\theta_F + \theta_N - 3\psi_{NF}\right] + (FEM)_{FN}$$

$$1> - \tfrac{1}{2}\, 2>$$

$$M_{NF} = \frac{3EI}{L}\left[\theta_F - \psi_{NF}\right] + (FEM)_{NF} - \frac{1}{2}(FEM)_{FN}$$

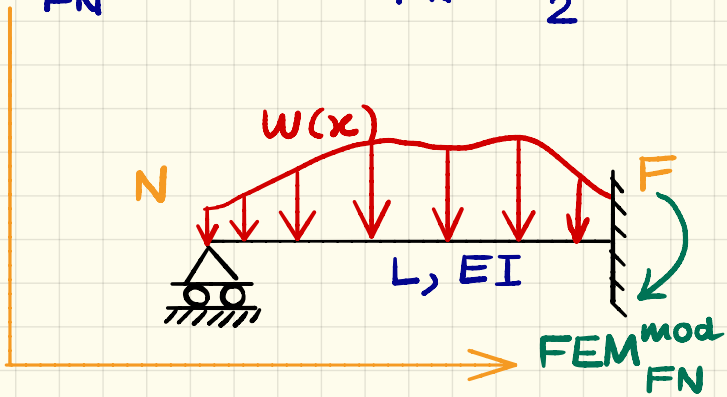
$$(FEM)_{NF}^{mod}$$



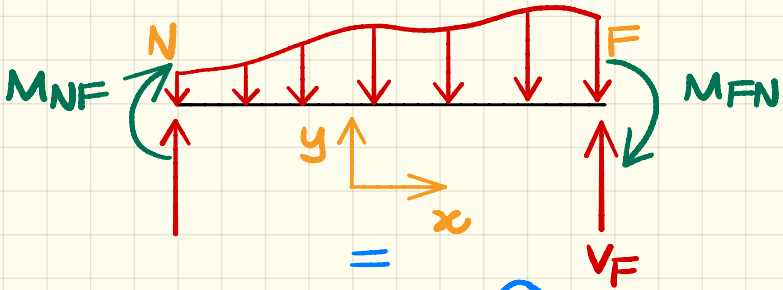
If $M_{FN} = 0$, everything remains the same

$N \leftrightarrow F$

$$(FEM)_{FN}^{mod} = (FEM)_{FN} - \frac{1}{2}(FEM)_{NF}$$



Shear force (only for reactions)



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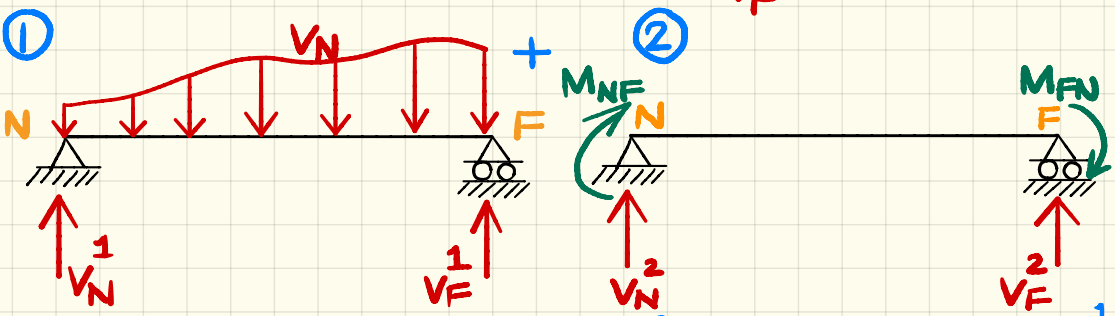
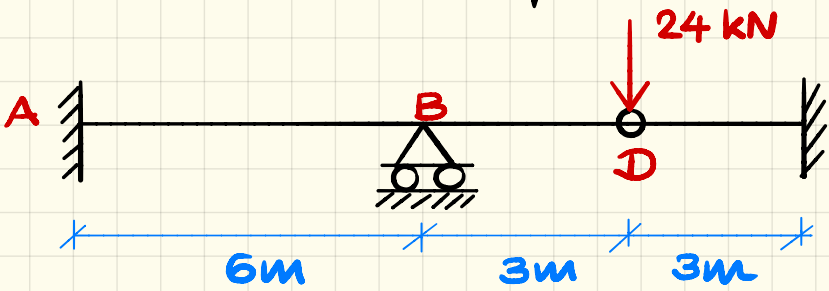


$$V_F^2 = \frac{M_{NF} + M_{NF}}{L} = -V_F^1$$

$$V_N = V_N^1 + V_N^2$$

$$V_F = V_F^1 + V_F^2$$

Tutorial problem



$L = 6m$

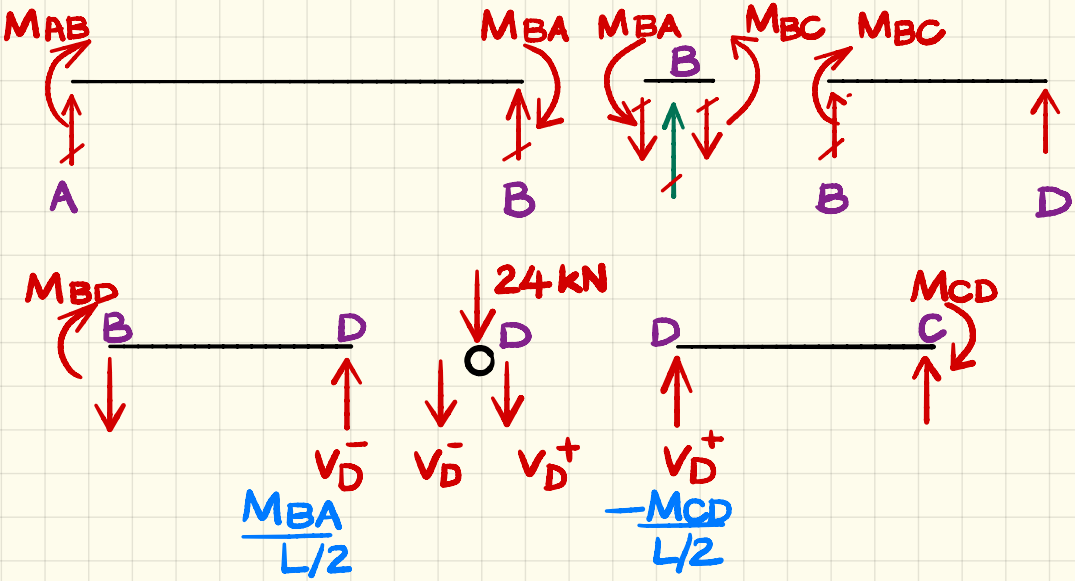
$\frac{EI}{L} = K$

$\psi_{BD} = -\psi_{DC} = \psi$

$$M_{BA} = 2K(2\theta_B) \quad -1\rangle$$

$$M_{BD} = 6K(\theta_B - \psi) \quad -2\rangle$$

$$M_{DC} = 6K(-\psi) \quad -3\rangle$$



$$\circlearrowleft \Sigma M_B = M_{BA} + M_{BD} = 0$$

$$\downarrow \Sigma F_D = V_D^- + V_D^+ + 24 = 0$$

solve, $\theta_b = -\frac{6L}{7K} = -\frac{216}{7EI}$

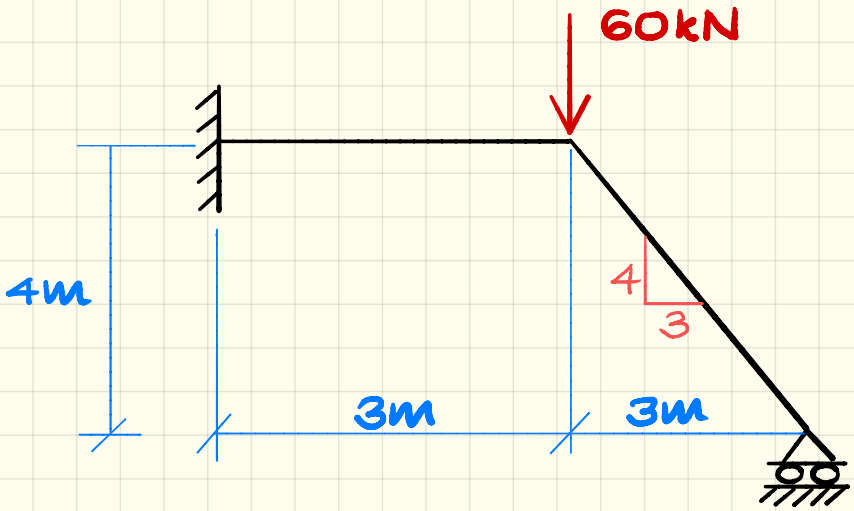
$$\psi = -\frac{360}{7EI}$$

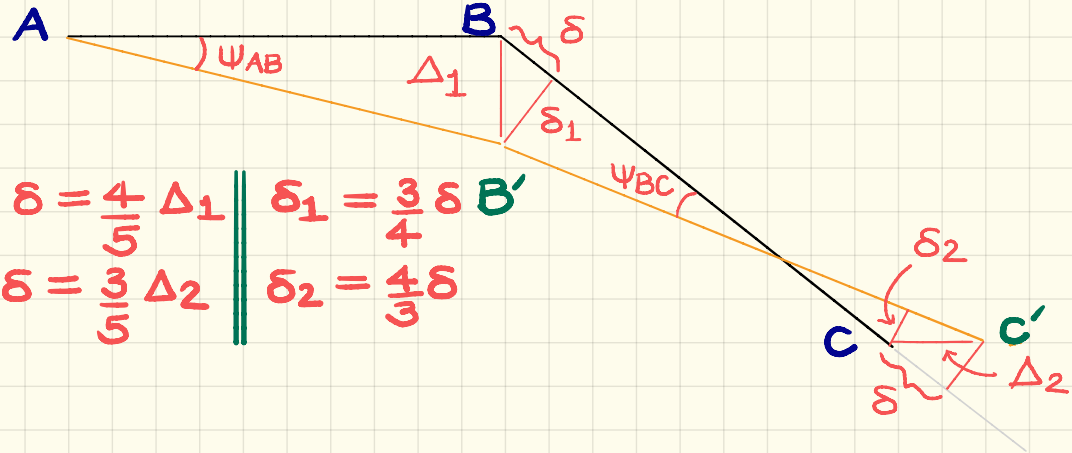
$$M_{BA} = -M_{BC} = -20.57\,kN\cdot m$$

$$M_{AB} = -10.28\,kN\cdot m$$

$$M_{CD} = 51.42\,kN\cdot m$$

Problem:





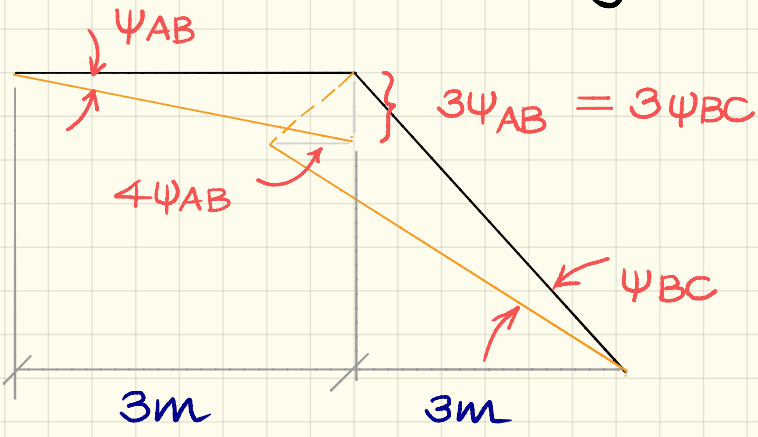
$$\delta = \frac{4}{5}\Delta_1 \quad \delta_1 = \frac{3}{4}\delta$$

$$\delta = \frac{3}{5}\Delta_2 \quad \delta_2 = \frac{4}{3}\delta$$

$$\psi_{AB} = \frac{\Delta_1}{3} = \frac{5}{12}\delta \quad , \quad \psi_{BC} = \frac{-\delta_1 + \delta_2}{5} = \frac{-5}{12}\delta$$

$\psi_{AB} = -\psi_{BC} = \psi$ } Including sign for CW

Alternately



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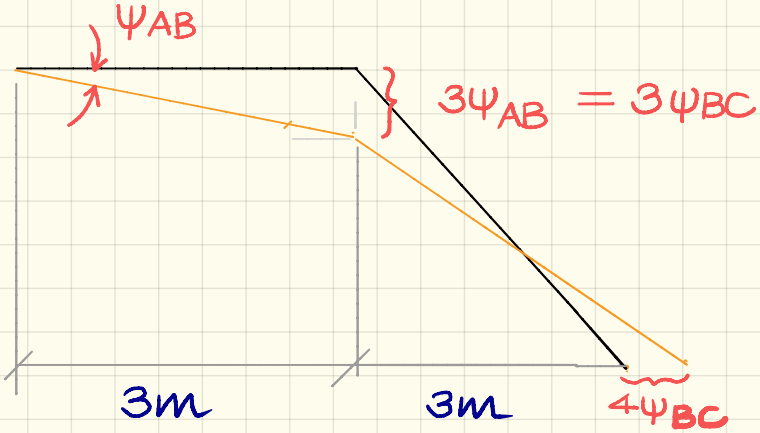


Taking sign convention

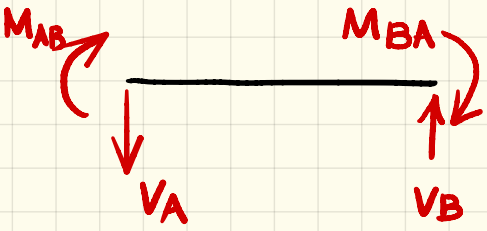
$\psi_{AB} = -\psi_{BC} = \psi$

$$M_{BA} = \frac{2EI}{3}(2\theta_B - \psi)$$

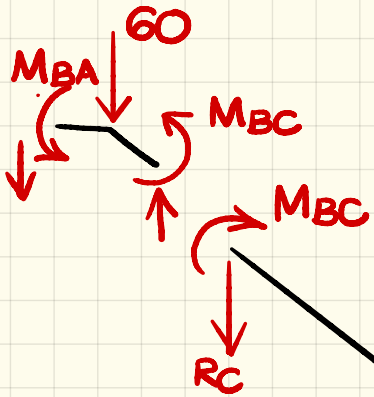
$$M_{BC} = \frac{3EI}{5}(\theta_B - (-\psi))$$

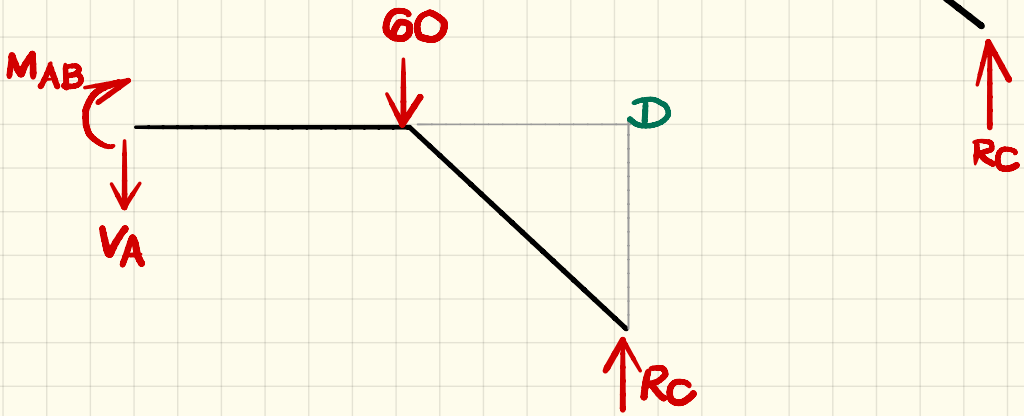
(modified method)

$$M_{AB} = \frac{2EI}{3}(\theta_B - \psi)$$



$$= \frac{M_{AB}+M_{BA}}{3}$$





Joint B

$$\circlearrowleft \Sigma M_B = M_{BA} + M_{BC} = 0$$

full FBD

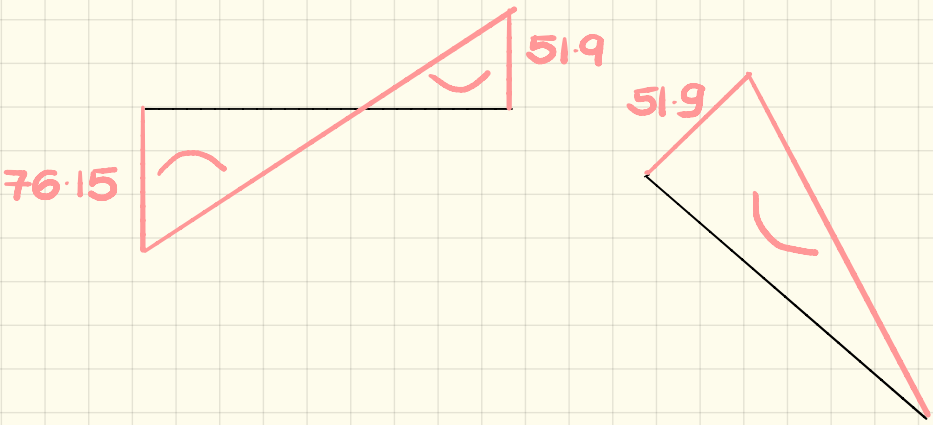
$$\circlearrowleft \Sigma M_D = M_{AB} - V_A \times 6 - 60 \times 3 = 0$$

Solve

$$\theta_b = \frac{945}{26EI}$$

$$\psi = \frac{1305}{26EI}$$

$M_{AB} = -76.15$ kN.m
$M_{BA} = -51.92$ "
$M_{BC} = 51.92$ "



QDS