

```
In[*]:= k[AE_, EI_, L_] =
  {{ $\frac{AE}{L}$ , 0, 0,  $-\frac{AE}{L}$ , 0, 0}, {0,  $12 \left(\frac{EI}{L^3}\right)$ ,  $6 \left(\frac{EI}{L^2}\right)$ , 0,  $-12 \left(\frac{EI}{L^3}\right)$ ,  $6 \left(\frac{EI}{L^2}\right)$ },
  {0,  $6 \left(\frac{EI}{L^2}\right)$ ,  $4 \left(\frac{EI}{L}\right)$ , 0,  $-6 \left(\frac{EI}{L^2}\right)$ ,  $2 \left(\frac{EI}{L}\right)$ }, { $-\frac{AE}{L}$ , 0, 0,  $\frac{AE}{L}$ , 0, 0},
  {0,  $-12 \left(\frac{EI}{L^3}\right)$ ,  $-6 \left(\frac{EI}{L^2}\right)$ , 0,  $12 \left(\frac{EI}{L^3}\right)$ ,  $-6 \left(\frac{EI}{L^2}\right)$ },
  {0,  $6 \left(\frac{EI}{L^2}\right)$ ,  $2 \frac{EI}{L}$ , 0,  $-6 \frac{EI}{L^2}$ ,  $4 \frac{EI}{L}$ }};
```

```
MatrixForm[k[AE, EI, L]]
```

```
T[λx_, λy_] =
```

```
{{λx, λy, 0, 0, 0, 0}, {-λy, λx, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
  {0, 0, 0, λx, λy, 0}, {0, 0, 0, -λy, λx, 0}, {0, 0, 0, 0, 0, 1}};
```

```
MatrixForm[T[λx, λy]]
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12 EI}{L^3} & \frac{6 EI}{L^2} & 0 & -\frac{12 EI}{L^3} & \frac{6 EI}{L^2} \\ 0 & \frac{6 EI}{L^2} & \frac{4 EI}{L} & 0 & -\frac{6 EI}{L^2} & \frac{2 EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12 EI}{L^3} & -\frac{6 EI}{L^2} & 0 & \frac{12 EI}{L^3} & -\frac{6 EI}{L^2} \\ 0 & \frac{6 EI}{L^2} & \frac{2 EI}{L} & 0 & -\frac{6 EI}{L^2} & \frac{4 EI}{L} \end{pmatrix}$$

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} \lambda x & \lambda y & 0 & 0 & 0 & 0 \\ -\lambda y & \lambda x & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda x & \lambda y & 0 \\ 0 & 0 & 0 & -\lambda y & \lambda x & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[*]:= Vals = {AE -> 0.015 × 200 × 109, EI -> 200 × 109 × 350 × 10-6, L -> 4};
```

```
k1 = Transpose[T[1, 0]].k[AE, EI, L].T[1, 0] /. Vals;
```

```
k1 // MatrixForm
```

```
k2 = Transpose[T[0, -1]].k[AE, EI, L].T[0, -1] /. Vals;
```

```
k2 // MatrixForm
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} 7.5 \times 10^8 & 0 & 0 & -7.5 \times 10^8 & 0 & 0 \\ 0 & 13\,125\,000 & 26\,250\,000 & 0 & -13\,125\,000 & 26\,250\,000 \\ 0 & 26\,250\,000 & 70\,000\,000 & 0 & -26\,250\,000 & 35\,000\,000 \\ -7.5 \times 10^8 & 0 & 0 & 7.5 \times 10^8 & 0 & 0 \\ 0 & -13\,125\,000 & -26\,250\,000 & 0 & 13\,125\,000 & -26\,250\,000 \\ 0 & 26\,250\,000 & 35\,000\,000 & 0 & -26\,250\,000 & 70\,000\,000 \end{pmatrix}$$

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} 13\,125\,000 & 0 & 26\,250\,000 & -13\,125\,000 & 0 & 26\,250\,000 \\ 0 & 7.5 \times 10^8 & 0 & 0 & -7.5 \times 10^8 & 0 \\ 26\,250\,000 & 0 & 70\,000\,000 & -26\,250\,000 & 0 & 35\,000\,000 \\ -13\,125\,000 & 0 & -26\,250\,000 & 13\,125\,000 & 0 & -26\,250\,000 \\ 0 & -7.5 \times 10^8 & 0 & 0 & 7.5 \times 10^8 & 0 \\ 26\,250\,000 & 0 & 35\,000\,000 & -26\,250\,000 & 0 & 70\,000\,000 \end{pmatrix}$$

```

In[ ]:= k11 = IdentityMatrix[5] * 0
k11[[{1, 2, 3, 5}, {1, 2, 3, 5}]] =
  k11[[{1, 2, 3, 5}, {1, 2, 3, 5}]] + k1[[{4, 5, 6, 3}, {4, 5, 6, 3}]] ;
k11[[{1, 2, 3, 4}, {1, 2, 3, 4}]] =
  k11[[{1, 2, 3, 4}, {1, 2, 3, 4}]] + k2[[{1, 2, 3, 6}, {1, 2, 3, 6}]] ;
k11 // MatrixForm

Out[ ]:= {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

```

```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 7.63125 \times 10^8 & 0 & 26\,250\,000 & 26\,250\,000 & 0 \\ 0 & 7.63125 \times 10^8 & -26\,250\,000 & 0 & -26\,250\,000 \\ 26\,250\,000 & -26\,250\,000 & 140\,000\,000 & 35\,000\,000 & 35\,000\,000 \\ 26\,250\,000 & 0 & 35\,000\,000 & 70\,000\,000 & 0 \\ 0 & -26\,250\,000 & 35\,000\,000 & 0 & 70\,000\,000 \end{pmatrix}$$


```

```

In[ ]:= k21 = ConstantArray[0, {4, 5}]
k21[[{3, 4}, {1, 2, 3, 5}]] =
  k21[[{3, 4}, {1, 2, 3, 5}]] + k1[[{1, 2}, {4, 5, 6, 3}]]
k21[[{1, 2}, {1, 2, 3, 4}]] = k21[[{1, 2}, {1, 2, 3, 4}]] + k2[[{4, 5}, {1, 2, 3, 6}]]

```

```

Out[ ]:= {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

```

```

Out[ ]:= {{-7.5 \times 10^8, 0, 0, 0}, {0, -13\,125\,000, 26\,250\,000, 26\,250\,000}}

```

```

Out[ ]:= {{-13\,125\,000, 0, -26\,250\,000, -26\,250\,000}, {0, -7.5 \times 10^8, 0, 0}}

```

```

In[ ]:= Qk = {0, -41.25 \times 10^3, 45 \times 10^3, 0, 0};
Qk = {0, -30, 30, 0, -30} * 10^3;
Du = LinearSolve[k11, Qk] // FullSimplify;
% // MatrixForm
Qu = (k21.Du + {0, 0, 0, 30} * 10^3) 10^-3

```

```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -7.38021 \times 10^{-6} \\ -0.0000473802 \\ 0.000423571 \\ -0.000209018 \\ -0.000658125 \end{pmatrix}$$


```

```

Out[ ]:= {-5.53516, 35.5352, 5.53516, 24.4648}

```

```

{-5.535158680771621`, 35.53515868077162`,
 5.535158680771623`, 5.714841319228374`}

```

```

In[ ]:= k11 // MatrixForm

```

```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 4.79712 \times 10^8 & 6.80192 \times 10^8 & 0. & -2.18413 \times 10^7 & 2.18413 \times 10^7 \\ 6.80192 \times 10^8 & 1.04654 \times 10^9 & 0. & 1.45609 \times 10^7 & -1.45609 \times 10^7 \\ 0. & 0. & 140\,000\,000 & 35\,000\,000 & 35\,000\,000 \\ -2.18413 \times 10^7 & 1.45609 \times 10^7 & 35\,000\,000 & 70\,000\,000 & 0 \\ 2.18413 \times 10^7 & -1.45609 \times 10^7 & 35\,000\,000 & 0 & 70\,000\,000 \end{pmatrix}$$


```

```

Qk[[3]] / (6  $\frac{EI}{L}$  /. Vals) // N

```

```

0.000428571

```

$$45/2./4.$$

$$5.625$$

## Tutorial 6 problems

```
In[*]:= k[AE_, EI_, L_] =
  {{ { AE/L, 0, 0, -AE/L, 0, 0}, {0, 12 (EI/L^3), 6 (EI/L^2), 0, -12 (EI/L^3), 6 (EI/L^2)},
    {0, 6 (EI/L^2), 4 (EI/L), 0, -6 (EI/L^2), 2 (EI/L)}, { -AE/L, 0, 0, AE/L, 0, 0},
    {0, -12 (EI/L^3), -6 (EI/L^2), 0, 12 (EI/L^3), -6 (EI/L^2)},
    {0, 6 (EI/L^2), 2 EI/L, 0, -6 EI/L^2, 4 EI/L}};
```

```
MatrixForm[k[AE, EI, L]]
```

```
T[λx_, λy_] =
  {{λx, λy, 0, 0, 0, 0}, {-λy, λx, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
   {0, 0, 0, λx, λy, 0}, {0, 0, 0, -λy, λx, 0}, {0, 0, 0, 0, 0, 1}};
```

```
ktruss[AE_, L_] := AE/L {{1, -1}, {-1, 1}};
```

```
Ttruss[λx_, λy_] := {{λx, λy, 0, 0}, {0, 0, λx, λy}};
```

```
kbeam[EI_, L_] := {{ { 12 EI/L^3, 6 EI/L^2, -12 EI/L^3, 6 EI/L^2}, { 6 EI/L^2, 4 EI/L, -6 EI/L^2, 2 EI/L},
  { -12 EI/L^3, -6 EI/L^2, 12 EI/L^3, -6 EI/L^2}, { 6 EI/L^2, 2 EI/L, -6 EI/L^2, 4 EI/L}}};
```

Out[\*]//MatrixForm=

$$\begin{pmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12 EI}{L^3} & \frac{6 EI}{L^2} & 0 & -\frac{12 EI}{L^3} & \frac{6 EI}{L^2} \\ 0 & \frac{6 EI}{L^2} & \frac{4 EI}{L} & 0 & -\frac{6 EI}{L^2} & \frac{2 EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12 EI}{L^3} & -\frac{6 EI}{L^2} & 0 & \frac{12 EI}{L^3} & -\frac{6 EI}{L^2} \\ 0 & \frac{6 EI}{L^2} & \frac{2 EI}{L} & 0 & -\frac{6 EI}{L^2} & \frac{4 EI}{L} \end{pmatrix}$$

Geometrical and Material properties of the truss and the beam

```

In[ ]:= val = {AE → 100 × 105, L → √(42 + 6.2)};
valb = {EI → 105 × 109 × 10-6, L → 10};

λ1 = {  $\frac{4}{\sqrt{4^2 + 6.^2}}$ ,  $\frac{6}{\sqrt{4^2 + 6.^2}}$  };

λ2 = { - $\frac{4}{\sqrt{4^2 + 6.^2}}$ ,  $\frac{6}{\sqrt{4^2 + 6.^2}}$  };

(* connectivity for near and far nodes *)
n1 = {6, 7, 1, 2};
n2 = {8, 9, 1, 2};
nb = {4, 5, 2, 3};

(* Initialise the stiffness matrix *)
K = ConstantArray[0, {10, 10}];

In[ ]:= (* for beam *)
K[[nb, nb]] = K[[nb, nb]] + kbeam[EI, L] /. valb // N;

(* for truss member 1 *)
Transpose[Ttruss[λ1[[1]], λ1[[2]]]].
ktruss[AE, L].Ttruss[λ1[[1]], λ1[[2]]] /. val
K[[n1, n1]] = K[[n1, n1]] + %;

(* for truss member 2 *)
Transpose[Ttruss[λ2[[1]], λ2[[2]]]].
ktruss[AE, L].Ttruss[λ2[[1]], λ2[[2]]] /. val
K[[n2, n2]] = K[[n2, n2]] + %;

Out[ ]:= {{426 692., 640 039., -426 692., -640 039.},
{640 039., 960 058., -640 039., -960 058.},
{-426 692., -640 039., 426 692., 640 039.},
{-640 039., -960 058., 640 039., 960 058.}}

Out[ ]:= {{426 692., -640 039., -426 692., 640 039.},
{-640 039., 960 058., 640 039., -960 058.},
{-426 692., 640 039., 426 692., -640 039.},
{640 039., -960 058., -640 039., 960 058.}}

```

Following is the complete stiffness matrix

```
In[ ]:= MatrixForm[K]
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 853385. & 0. & 0 & 0 & 0 & -426692. & -640039. & -426692. \\ 0. & 3.12012 \times 10^6 & -6. \times 10^6 & -1.2 \times 10^6 & -6. \times 10^6 & -640039. & -960058. & 640039. \\ 0 & -6. \times 10^6 & 4. \times 10^7 & 6. \times 10^6 & 2. \times 10^7 & 0 & 0 & 0 \\ 0 & -1.2 \times 10^6 & 6. \times 10^6 & 1.2 \times 10^6 & 6. \times 10^6 & 0 & 0 & 0 \\ 0 & -6. \times 10^6 & 2. \times 10^7 & 6. \times 10^6 & 4. \times 10^7 & 0 & 0 & 0 \\ -426692. & -640039. & 0 & 0 & 0 & 426692. & 640039. & 0 \\ -640039. & -960058. & 0 & 0 & 0 & 640039. & 960058. & 0 \\ -426692. & 640039. & 0 & 0 & 0 & 0 & 0 & 426692. \\ 640039. & -960058. & 0 & 0 & 0 & 0 & 0 & -640039. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The unknown displacements are: 2, 3. The known forces are:

```
In[ ]:= f = {-30 * 10^3, 0};
```

```
K[[{2, 3}, {2, 3}]];
```

```
du = LinearSolve[%, f]
```

```
Out[ ]:= {-0.0135128, -0.00202692}
```

Support reactions for the beam

```
In[ ]:= (kbeam[EI, L] /. valb).{0, 0, du[[1]], du[[2]]}
```

```
Out[ ]:= {4053.84, 40538.4, -4053.84, 0.}
```

```
In[ ]:= λx = λ1[[1]];
```

```
λy = λ1[[2]];
```

```
 $\frac{AE}{L}$  {-λx, -λy, λx, λy}.{0, 0, 0, du[[1]]} /. val
```

```
Out[ ]:= -15591.7
```

Problem 2

```
In[ ]:= kbeam[EI, L];
```

```
val1 = {EI → 10^5 * 10^9 * 10^-6, L → 2};
```

```
val2 = {EI → 10^5 * 10^9 * 10^-6, L → 3};
```

```
n1 = {1, 2, 3, 4};
```

```
n2 = {3, 4, 5, 6};
```

```
q10 = {15, 15  $\frac{2^2}{12}$ , 15, -15  $\frac{2^2}{12}$ } * 1000;
```

```
q20 = {  $\frac{200}{9}$ , 30 *  $\frac{2^2}{3^2}$ ,  $\frac{70}{9}$ , -30 *  $\frac{2}{3^2}$ } * 1000;
```

```
K = ConstantArray[0, {6, 6}];
```

```
q0 = ConstantArray[0, {6}];
```

```
K[[n1, n1]] = K[[n1, n1]] + kbeam[EI, L] /. val1;
```

```
K[[n2, n2]] = K[[n2, n2]] + kbeam[EI, L] /. val2;
```

```
q0[[n1]] = q0[[n1]] + q10;
```

```
q0[[n2]] = q0[[n2]] + q20;
```

```
In[ ]:= MatrixForm[K]
```

```
MatrixForm[10-3 q0]
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 150\,000\,000 & 150\,000\,000 & -150\,000\,000 & 150\,000\,000 & 0 & 0 \\ 150\,000\,000 & 200\,000\,000 & -150\,000\,000 & 100\,000\,000 & 0 & 0 \\ -150\,000\,000 & -150\,000\,000 & \frac{1\,750\,000\,000}{9} & -\frac{250\,000\,000}{3} & -\frac{400\,000\,000}{9} & \frac{200\,000\,000}{3} \\ 150\,000\,000 & 100\,000\,000 & -\frac{250\,000\,000}{3} & \frac{1\,000\,000\,000}{3} & -\frac{200\,000\,000}{3} & \frac{200\,000\,000}{3} \\ 0 & 0 & -\frac{400\,000\,000}{9} & -\frac{200\,000\,000}{3} & \frac{400\,000\,000}{9} & -\frac{200\,000\,000}{3} \\ 0 & 0 & \frac{200\,000\,000}{3} & \frac{200\,000\,000}{3} & -\frac{200\,000\,000}{3} & \frac{400\,000\,000}{3} \end{pmatrix}$$

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 15 \\ 5 \\ \frac{335}{9} \\ \frac{25}{3} \\ \frac{70}{9} \\ -\frac{20}{3} \end{pmatrix}$$

```
In[ ]:= Solve[K[[4, 4]] d4 + q0[[4]] == 0, d4] // N
```

```
10-3 (K[[{1, 2, 3, 5, 6}, 4]] * d4 + q0[[{1, 2, 3, 5, 6}]] /. %[[1]])
```

```
Out[ ]:= {{d4 -> -0.000025}}
```

```
Out[ ]:= {11.25, 2.5, 39.3056, 9.44444, -8.33333}
```

### Problem 3

```
In[ ]:= λ1 = {1, 0};
```

```
λ2 = {1, 0};
```

```
λ3 = {0, 1};
```

```
n1 = {1, 2, 3, 4, 5, 6};
```

```
n2 = {4, 5, 6, 7, 8, 9};
```

```
n3 = {10, 11, 12, 4, 5, 6};
```

```
kp = k[AE, EI, L];
```

```
k1 = Transpose[T[1, 0]].kp.T[1, 0];
```

```
k2 = k1;
```

```
k3 = Transpose[T[0, 1]].kp.T[0, 1];
```

### Material properties

```
In[ ]:= val = {EI -> 105 × 109 × 10-6, AE -> 100 × 105, L -> 2, W -> 1000};
```

### Assembling the matrix

```
In[ ]:= K = ConstantArray[0, {12, 12}];
```

```
K[[n1, n1]] = K[[n1, n1]] + k1;
```

```
K[[n2, n2]] = K[[n2, n2]] + k2;
```

```
K[[n3, n3]] = K[[n3, n3]] + k3;
```

Find the force vector

```
In[*]:= q10 = {0,  $\frac{W}{2}$ ,  $\frac{W(L)}{8}$ , 0,  $\frac{W}{2}$ ,  $-\frac{W(L)}{8}$ };
```

```
q20 = q10 * 0;
```

```
q30 = q30 * 0;
```

```
(* total force vector *)
```

```
q0 = ConstantArray[0, 12];
```

```
q0[[n1]] = q0[[n1]] + q10;
```

```
q0[[n2]] = q0[[n2]] + q20;
```

```
q0[[n3]] = q0[[n3]] + q30;
```

Finding the displacement

```
In[*]:= qk0 = q0[[{4, 5, 6}]];
```

```
k11 = K[[{4, 5, 6}, {4, 5, 6}]];
```

```
du = LinearSolve[k11, -qk0] /. val // N
```

```
Out[*]:= {-5.10204 × 10-7, -1.63934 × 10-6, 5.44218 × 10-7}
```

Unknown reactions

```
In[*]:= K[[{1, 2, 3, 7, 8, 9, 10, 11, 12}, {4, 5, 6}]].du +  
q0[[{1, 2, 3, 7, 8, 9, 10, 11, 12}]] /. val // N
```

```
Out[*]:= {2.55102, 827.534, 550.323, 2.55102,  
164.269, -191.48, -5.10204, 8.19672, -22.1088}
```

Problem 4

Material properties of the beam

```
In[*]:= val1 = {EI → 105 × 109 × 10-6, L → 6};
```

```
val2 = {EI → 105 × 109 × 10-6, L → 1.5};
```

```
val3 = {EI → 105 × 109 × 10-6, L → 3.5};
```

Connectivity for the beam

```
In[*]:= n1 = {1, 2, 3, 4};
```

```
n2 = {3, 4, 5, 6};
```

```
n3 = {5, 7, 8, 9};
```

Stiffness matrix for each beam and the global stiffness matrix 9 × 9.

```
k1 = kbeam[EI, L] /. val1;
```

```
k2 = kbeam[EI, L] /. val2;
```

```
k3 = kbeam[EI, L] /. val3;
```

```
K = ConstantArray[0, {9, 9}];
```

Assembling the stiffness matrix.

```
In[*]:= K[[n1, n1]] = K[[n1, n1]] + k1;
```

```
K[[n2, n2]] = K[[n2, n2]] + k2;
```

```
K[[n3, n3]] = K[[n3, n3]] + k3;
```

Unknown forces, fixed end forces and unknown displacements

```

In[ ]:= P = 100 × 103;
q10 = {  $\frac{P}{2}$ ,  $\frac{P}{8}$ ,  $\frac{P}{2}$ ,  $-\frac{P}{8}$  } /. val1;
q20 = {0, 0, 0, 0};
Q = 80 × 103;
(* Intermediate computations *)
Mcd =  $\frac{Q \cdot 2^2 \times 1.5}{(3.5)^2}$ ;
Mdc = -  $\frac{Q \cdot 2 \times 1.5^2}{(3.5)^2}$ ;
Mtot = Mcd + Mdc;
Vc =  $\frac{Mtot}{(3.5)} + Q \cdot \frac{2}{3.5}$ ;
Vd = -  $\frac{Mtot}{3.5} + \frac{Q \cdot 1.5}{3.5}$ ;

q30 = {Vc, Mcd, Vd, Mdc};

(* global q0 matrix *)

q0 = ConstantArray[0, 9];
Global q0 array

In[ ]:= q0[[n1]] = q0[[n1]] + q10;
q0[[n2]] = q0[[n2]] + q20;
q0[[n3]] = q0[[n3]] + q30;

Unknown displacements

In[ ]:= nu = {4, 5, 6, 7};
K11 = K[[nu, nu]];
qu0 = q0[[nu]];
du = LinearSolve[K11, -qu0]

Out[ ]:= {0.000380155, 0.00019781, 7.73282 × 10-6, -0.000427633}

Unknown reactions

In[ ]:= nqu = {1, 2, 3, 8, 9};
qu = (K[[nqu, nu]].du + q0[[nqu]]) 10-3

Out[ ]:= {56.3359, 87.6718, 76.7683, 46.8958, -44.1353}

```

## Problem 2: Tutorial 8



```
In[1]:= k[AE_, EI_, L_] =
  {{ { $\frac{AE}{L}$ , 0, 0,  $-\frac{AE}{L}$ , 0, 0}, {0, 12  $\left(\frac{EI}{L^3}\right)$ , 6  $\left(\frac{EI}{L^2}\right)$ , 0, -12  $\left(\frac{EI}{L^3}\right)$ , 6  $\left(\frac{EI}{L^2}\right)$ },
    {0, 6  $\left(\frac{EI}{L^2}\right)$ , 4  $\left(\frac{EI}{L}\right)$ , 0, -6  $\left(\frac{EI}{L^2}\right)$ , 2  $\left(\frac{EI}{L}\right)$ }, { $-\frac{AE}{L}$ , 0, 0,  $\frac{AE}{L}$ , 0, 0},
    {0, -12  $\left(\frac{EI}{L^3}\right)$ , -6  $\left(\frac{EI}{L^2}\right)$ , 0, 12  $\left(\frac{EI}{L^3}\right)$ , -6  $\left(\frac{EI}{L^2}\right)$ },
    {0, 6  $\left(\frac{EI}{L^2}\right)$ , 2  $\frac{EI}{L}$ , 0, -6  $\frac{EI}{L^2}$ , 4  $\frac{EI}{L}$ }};
```

```
MatrixForm[k[AE, EI, L]]
```

```
T[λx_, λy_] =
```

```
{{λx, λy, 0, 0, 0, 0}, {-λy, λx, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
  {0, 0, 0, λx, λy, 0}, {0, 0, 0, -λy, λx, 0}, {0, 0, 0, 0, 0, 1}};
```

```
ktruss[AE_, L_] :=  $\frac{AE}{L}$  {{1, -1}, {-1, 1}};
```

```
Ttruss[λx_, λy_] := {{λx, λy, 0, 0}, {0, 0, λx, λy}};
```

```
kbeam[EI_, L_] := {{ $\frac{12 EI}{L^3}$ ,  $\frac{6 EI}{L^2}$ ,  $-\frac{12 EI}{L^3}$ ,  $\frac{6 EI}{L^2}$ }, { $\frac{6 EI}{L^2}$ ,  $\frac{4 EI}{L}$ ,  $-\frac{6 EI}{L^2}$ ,  $\frac{2 EI}{L}$ },
  {- $\frac{12 EI}{L^3}$ ,  $-\frac{6 EI}{L^2}$ ,  $\frac{12 EI}{L^3}$ ,  $-\frac{6 EI}{L^2}$ }, { $\frac{6 EI}{L^2}$ ,  $\frac{2 EI}{L}$ ,  $-\frac{6 EI}{L^2}$ ,  $\frac{4 EI}{L}$ }};
```

```
Out[2]/MatrixForm=
```

$$\begin{pmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12 EI}{L^3} & \frac{6 EI}{L^2} & 0 & -\frac{12 EI}{L^3} & \frac{6 EI}{L^2} \\ 0 & \frac{6 EI}{L^2} & \frac{4 EI}{L} & 0 & -\frac{6 EI}{L^2} & \frac{2 EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12 EI}{L^3} & -\frac{6 EI}{L^2} & 0 & \frac{12 EI}{L^3} & -\frac{6 EI}{L^2} \\ 0 & \frac{6 EI}{L^2} & \frac{2 EI}{L} & 0 & -\frac{6 EI}{L^2} & \frac{4 EI}{L} \end{pmatrix}$$

```
In[195]:= (* connectivity *)
```

```
n1 = {1, 2, 3, 4};
```

```
n2 = {3, 4, 5, 6};
```

```
n3 = {7, 3};
```

```
(* material properties and geometry *)
```

```
val1 = {EI → 200 × 500, L → 10};
```

```
val2 = {AE →  $\frac{10\,000}{3}$ , L → 10};
```

```
(* member stiffness *)
```

```
k1 = kbeam[EI, L] /. val1;
```

```
k2 = kbeam[EI, L] /. val1;
```

```
k3 = ktruss[AE, L] /. val2;
```

```
(* assemble stiffness matrix *)
```

```
Kg = ConstantArray[0, {11, 11}];
```

```
Kg[[n1, n1]] = Kg[[n1, n1]] + k1;
```

```
Kg[[n2, n2]] = Kg[[n2, n2]] + k2;
```

```
Kg[[n3, n3]] = Kg[[n3, n3]] + k3;
```

Solving various parts of the problem

```

In[207]:= (* obtain unknown degrees of freedom *)
nu = {1, 2, 3, 4, 6};
k11 = Kg[[nu, nu]];
Qk = {-20, 80, 0, 0, 0};
Du = LinearSolve[k11, Qk] // N

Out[210]= {-0.488, 0.0417333, -0.144, 0.0237333, 0.00973333}

In[213]:= (* to find the support reactions *)
nk = {5, 7};
k21 = Kg[[nk, nu]].Du

Out[214]= {-28., 48.}

```